The Role of Gender, Birth Order, and Ability in Intra-household Educational Inequality: Evidence from Benin

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Introduction and Motivation

- Lack of education is a major barrier to development.
- Despite reforms, Sub-Saharan Africa struggles with high illiteracy, educational inequality, and many out-of-school children.
- Intra-household variation explains $\approx 40\%$ of child human capital variation in the developing world (Giannola 2023).

Distribution (Benin, 2013) Distribution (USA, 2015)

- Possible reasons
  - Parents: # of children, average human capital investment in children, education, wealth.
  - Children: Gender, birth order, and different initial talent.

How effective are different education policies in reducing intra-households inequality?

This Paper ...

Model parents' decision on human capital investments as it relates to:

1. Gender,
2. Birth order,
3. Innate ability.

Decompose ave absolute $\neq$ce in children's educ into gender, birth order and unobserved residuals effects.

and ...

Model parents' human capital dist decisions & analyze diverse counterfactual situations.

Preview of Findings (In Benin)

From the reduced form analysis

1. Inverted U-shaped relationship between mean and SD of education.
2. Non-educated parents: **gender + birth order effects $\approx$ 70 % $\times$ Average inequality**.
3. College educated parents: **gender + birth order effects $\approx$ 33 %$\times$ Average inequality**.

From the structural model

	Eliminate gender & birth order effects	Reduce average inequality
Education voucher (1)	✗	✗
Remove barriers to school entry (2)	✗	✓
Targeted educ cost reduction (3)	✓	✗

(2)+(3) is effective in both dimensions.

Literature Review and Contributions

This paper contributes to 4 strands of literature

1. Fertility choice model: (De Tray 1970, Becker and Lewis 1973, Becker and Tomes 1976, Vogl 2016, Weng et al. 2019).

2. Within household schooling decision: Gender and birth order disadvantages (Nerlove, Razin, and Sadka 1984, Biswas 2000, Ota and Moffatt 2007, Fergusson, Horwood, and Boden 2006,Lachaud et al. 2014, Moshoeshoe et al. 2016, Weng et al. 2019, Esposito, Kumar, and Villaseñor 2020).

3. Within household disparities in children's education and distribution of material resources (Rosenzweig and Schultz 1982, Akresh et al. 2012 Giannola 2023).

4. Educational Kuznets Theory: Ram 1990, Londoño 1990, Thomas, Wang, and Fan 2003.

First structural modeling and estimation of household's distribution of human capital resources as it relates to gender, birth order and children's innate ability.

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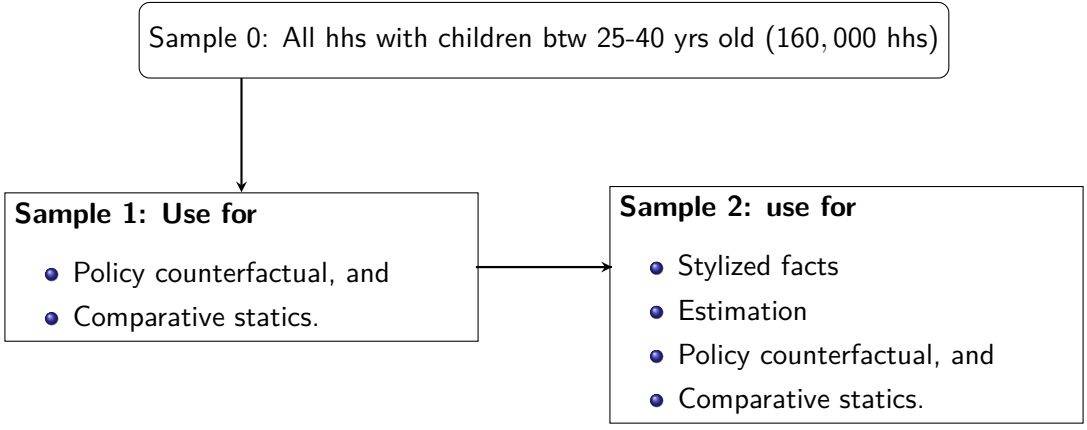
Data

- Population and habitation census data (Benin, 2013).
- Siblings and parents characteristics for about 90,000 individuals between 25 and 40 years old from about 33,000 hhs.
- Variables include gender, age, family size, education attainment, education attainment of parents and of a sub-sample of siblings.

Parents-Children Educ , Parents' educ-N_c

Data

1. **Sample 1:** hhs with at least 2 children btw 25 and 40 years old. ($\approx$ 51,600 hhs)
   Gender composition by hh type
2. **Sample 2:** hhs with at least one educated child btw 25 and 40 years old.($\approx$ 32,800 hhs)



Stylized Facts

Fact I: Inverted U-shaped structure btw Mean & SD of education

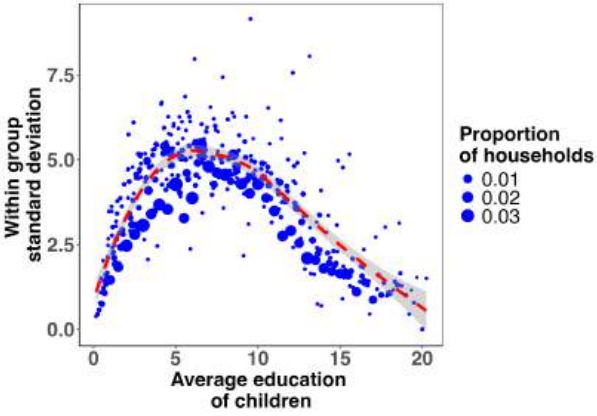


Figure 1: Within group mean and standard deviation of children's education

- Reinforced by the following regression:

$$SD_h = \beta_1 \bar{q}_h + \beta_2 \bar{q}_h^2 + \varepsilon_h, \qquad (1)$$

Estimation results , Map (Benin, 2013),
observed vs. min vs. max inequality (Benin, 2013)

This educational Kuznets curve theory was empirically shown using cross countries data (see Ram 1990, Londoño 1990, Thomas, Wang, and Fan 2003)

Fact II: Inequality Decomposition (Number of children equals to 2)

$$\Delta_{daughter-son} Educ_h = \beta_0 + \beta_1 Firstborn\_daughter_h + \varepsilon_h, \quad (2)$$

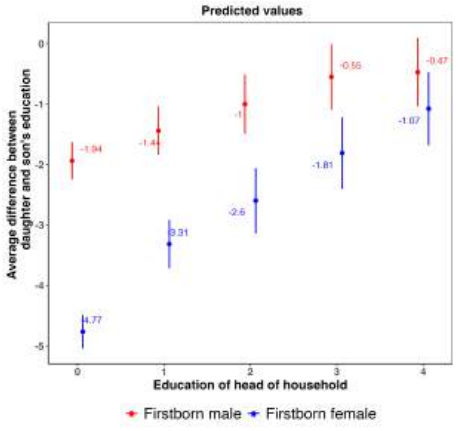


Figure 2: Effect of gender and birth order on within household inequality ($N_c = 2$)

- In blue: the average gender effect + the average birth order effect ($\beta_0 + \beta_1$),
- In red: the average effect of gender - the average effect of birth order (β_0),

Let $q_h = (q_{1,h}, q_{2,h})$, and

$$Range_h = max(q_h) - min(q_h) = |q_{1,h} - q_{2,h}|$$

$$E[\text{Range}|\text{household has a firstborn daughter}]$$

$$\geq |\beta_0 + \beta_1| \quad \text{Proof}$$

Fact II: High education of parents is associated with smaller shares of gender and birth order effects

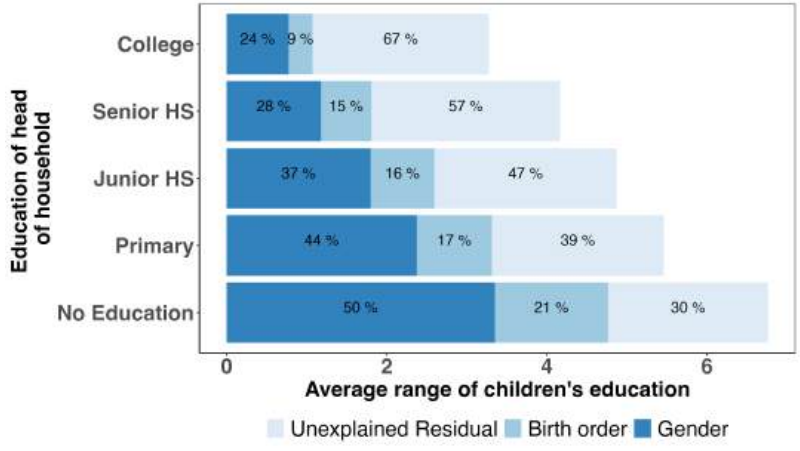
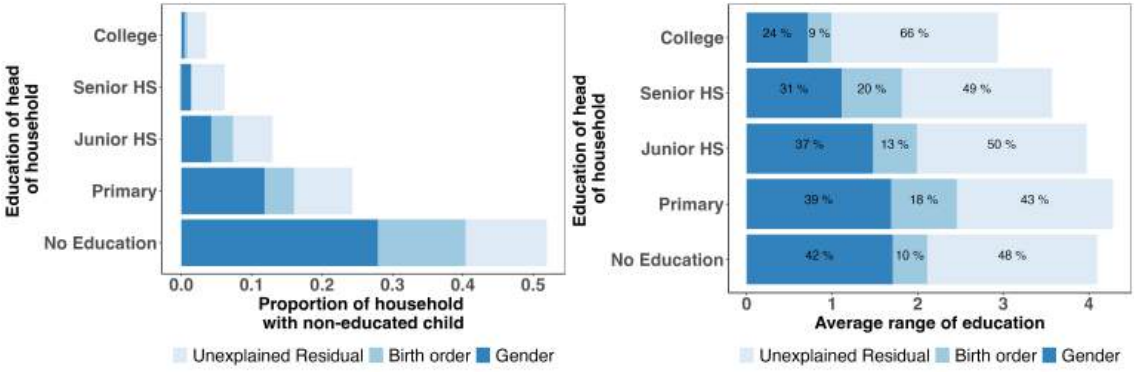


Figure 3: Inequality decomposition ($N_c = 2$)

- High education of parents is associated with a smaller average inequality.

Inequality Decomposition: Extensive v.s Intensive margin



(a) Extensive margin (b) Intensive margin

Figure 4: Inequality decomposition ($N_c = 2$)

- Compared to the extensive margin, the unexplained component has higher share in the average inequality for the intensive margin.
- The decrease in inequality by parents' education is mostly present in the extensive margin.

Facts to Model ...

Stylized Facts

- Increase in average education is associated with an increase in average inequality first, then with a decrease.
- Parents' education matters for average education of children, inequality, and its decomposition.
- Intra-household educational inequality is present both at extensive & intensive margin.

...

Model Features

- Gender and birth order are the 2 main observed sources of inequality.
- The key unobserved source is children's innate ability.

...

The Structural Model

The Structural Model: Set up

Each household is characterized by an unobserved type, which determined the observed number of educated children.

The parents' decision involves 2 stages:

1. In the 1st stage parents decides the number of children (N_c) and the aggregate education (q_T).
2. In the second stage, they distribute the education resources between children solving the following maximization problem.

$$\max_{q_i} U(q, \theta) \tag{3}$$

$$\sum q_i \leq q_T, \ q_i \geq 0, \ q_i \leq q_{\max}$$

$q_{\max}$ is the maximum years of education a child can get.

I model the second stage taking decisions in the first stage as given.

The Structural Model: Set up

I use a generalized utilitarian social welfare function with concave utility function for each child. For $N_c = 2$, Let

$$U(q_h) = \nu_h \Big[\sum_{i=1}^{2} \underbrace{a_{i,h}.(q_{i,h})^{\delta_{i,h}^{low}}}_{U_{B_{i,h}}^{low}} - \underbrace{\alpha_i^{low} q_{i,h}}_{U_{C_{i,h}}^{low}} \Big] + (1-\nu_h) \Big\{ \sum_{i=1}^{2} \Big[e_{i,h}.(\underbrace{a_{i,h}.(q_{i,h})^{\delta_{i,h}^{high}}}_{U_{B_{i,h}}^{high}} - \underbrace{\alpha_i^{high} q_{i,h}}_{U_{C_{i,h}}^{high}}) \Big] \Big\},$$

Utility function for any $N_c > 1$ Simulated, Parameters, Choice variables.

The Structural Model: Set up

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Utility function for any $N_c > 1$ Simulated, Parameters, Choice variables.

- ν_h, is parents' constraint for having a non-educated child. $[\nu_h = 1\{T_{\text{educ}_h} = 2\}]$
- $a_{i,h} = \frac{\omega_{i,h}}{\sum_{j=1}^{N_{c_h}} \omega_{j,h}}$, where $a_{i,h}$ is ability-based weight on child i's educ.
- $(a_{1,h}, a_{2,h}) \sim^{i.i.d}$ *Dirichlet*(β_1, β_2),

The Structural Model: Set up

I use a generalized utilitarian social welfare function with concave utility function for each child. For $N_c = 2$, Let

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Utility function for any $N_c > 1$ Simulated, Parameters, Choice variables.

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- $(a_{1,h}, a_{2,h}) \sim^{i.i.d}$ *Dirichlet*(β_1, β_2),
- $e_{i,h} = 1\{a_{i,h}.(q_T)^{\delta_{i,h}^{high}} - \alpha_i^{high} q_T > a_{j,h}.(q_T)^{\delta_{j,h}^{high}} - \alpha_j^{high} q_T\}$, $e_{j,h} = 1 - e_{i,h}$.
- $\delta_{i,h}^{type} = \gamma - \theta_1^{type} Female_i(1 - Female_j)$, $type \in \{low, high\}$. [γ is normalized to 0.5]
- α_i^{low}, and α_i^{high} are the marginal educ costs (financial and opportunity costs) of child i.

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Utility function for any $N_c > 1$ Simulated, Parameters, Choice variables.

- ν_h, is parents' constraint for having a non-educated child. $[\nu_h = 1\{T_{\text{educ}_h} = 2\}]$
- $a_{i,h} = \frac{\omega_{i,h}}{\sum_{j=1}^{N_{ch}} \omega_{j,h}}$, where $a_{i,h}$ is ability-based weight on child i's educ.
- $(a_{1,h}, a_{2,h}) \sim^{i.i.d} Dirichlet(\beta_1, \beta_2)$,
- $e_{i,h} = 1\{a_{i,h}.(q_T)^{\delta_{i,h}^{high}} - \alpha_i^{high} q_T > a_{j,h}.(q_T)^{\delta_{j,h}^{high}} - \alpha_j^{high} q_T\}$, $e_{j,h} = 1 - e_{i,h}$.
- $\delta_{i,h}^{type} = \gamma - \theta_1^{type} Female_i(1 - Female_j)$, $type \in \{low, high\}$. [γ is normalized to 0.5]
- α_i^{low}, and α_i^{high} are the marginal educ costs (financial and opportunity costs) of child i.

T_{educ_h}, $Female_{1,h}$, $Female_{2,h}$, and q_{T_h} are drawn from the empirical joint distribution of those variables

The Structural Model: Set up

- We allow θ_1^{high} to be function of daughter's birth order,
- $\theta_1^{high} = \theta_1^{ds,high} 1\{gender\_comp = b^{d,s}\} + \theta_1^{sd,high} 1\{gender\_comp = b^{s,d}\}$
- $b_h^{d,s} = Female_{1,h}.(1 - Female_{2,h}),\ b_h^{s,d} = (1 - Female_{1,h}).Female_{2,h}.$

$$\theta = \left(\theta_1^{low}, \theta_1^{ds,high}, \theta_1^{sd,high}, (\alpha_1^{high} - \alpha_2^{high}), (\alpha_1^{low} - \alpha_2^{low})\right)$$

- α_2^{high}, and α_2^{low} are normalized to 0.

Estimation

The Structural Model: Estimation

Parameters are estimated using simulated method of moments.

List of moment matched

- Ave diff in the educ of daughters and sons [both gender=1 and int margin=1] (θ_1^{low}).
- Ave diff in the educ of 1st born and second [both gender=0 and int margin=1] ($\alpha_{(1)}^{low}$).
- Prop of educated 1st born [both gender=0, and ext margin = 1] ($\alpha_{(1)}^{high}$) .
- Prop of educated firstborn daughters [both genders=1, and ext margin=1] ($\theta_1^{ds,high}$).
- Prop of educated second born daughters [both genders = 1, ext margin =1] ($\theta_1^{sd,high}$).

Def: Ext margin: Households with one non-educated child,

Int margin: households with no non-educated child,

Both gender: Households with one son, and one daughter.

The Structural Model: Estimation

The corresponding sample objective function is the following expression:

$$\hat{Q}(\theta) = (\bar{Y}^d_{d,z} - \bar{Y}^d_{b,z} - (\hat{\mu}^d_{d,z} - \hat{\mu}^d_{b,z}))^2 + (\hat{m}_2 - \hat{\pi}_1)^2 + (\hat{m}_3 - \hat{\pi}_{fb,d})^2 + (\hat{m}_4 - \hat{\pi}_{sb,d})^2 + \quad (4)$$

$$\sum_{l \in \{d,s\}} (\bar{Y}^l_{2,z} - \bar{Y}^l_{1,z} - (\hat{\mu}^l_{2,z} - \hat{\mu}^l_{1,z}))^2$$

$$\hat{\theta} = \text{argmin}_{\theta \in \Theta} \hat{Q}(\theta).$$

Model prediction , Data. Moments

Estimation of G(.)

$$a_{i,h} \sim^{i.i.d} Dirichlet(\beta_1, \ldots, \beta_{N_{c_h}})$$

$(\beta_1, \ldots, \beta_{N_{c_h}})$ are estimated using auxiliary data on GPA. [Using MLE]

Results

The Structural Model: Results (Extensive Margin)

Table 1: Estimates of $\hat{\theta}^{high}$, ($N_c = 2$)

	Non-educated parents		
	$\hat{\theta}_1^{ds,high}$	$\hat{\theta}_1^{sd,high}$	$\hat{\alpha}_1^{high}$
Estimates	0.119**	0.13**	0.02**
Standard errors	0.0004	0.0036	0.0013
Number of observations		5083	

** significant at 5% level of significance.

- The average cost difference at the extensive margin is 0.02.
- The perceived average utility of graduating high school is $\approx$ 41% higher for the 2nd born son compared to the 1st born daughter.
- It is $\approx$ 17% higher for 1st born son compared to 2nd born daughter.

The Structural Model: Results (Intensive Margin)

Table 2: Estimates of $\hat{\theta}^{low}$, ($N_c = 2$)

	Non-educated parents		College educated parents	
	$\hat{\theta}_1^{low}$	$\hat{\alpha}_1^{low}$	$\hat{\theta}_1^{low}$	$\hat{\alpha}_1^{low}$
Estimates	0.0239**	0.0013**	0.00759**	0.00045**
Standard errors	0.0014	0.0028	0.0006	0.0012
Number of observations	6187		942	

** significant at 5% level of significance.

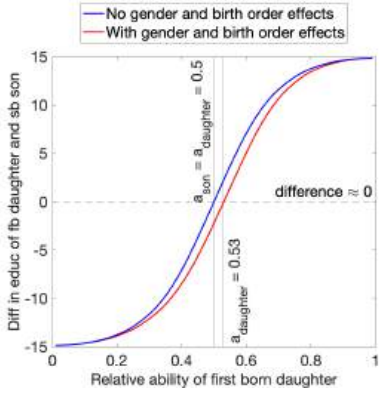
- parents without formal education perceive $\approx 6\%$ higher utility on graduating high school for sons compared to daughters,
- for college-educated parents, the difference is $\approx 2\%$.

Model Fit

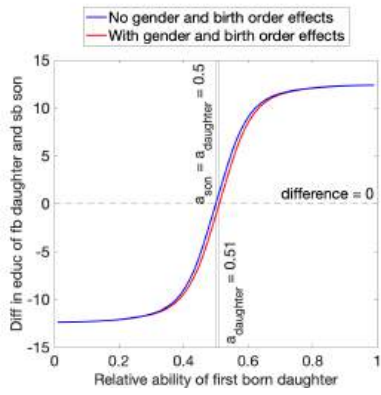
Estimates for $N_c = 3$

Counterfactual Analysis

Counterfactual Analysis: Heterogeneity in the effect of disadvantages by level of ability



(a) Non-educated head of hhs

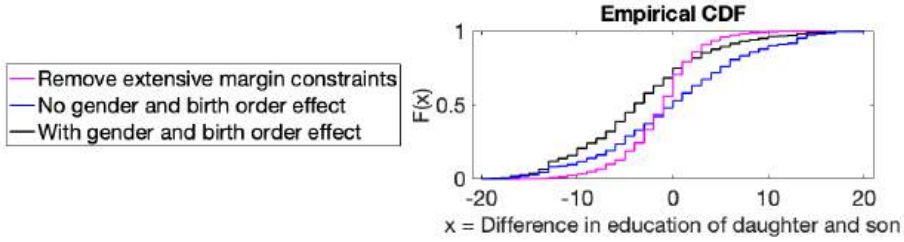


(b) College educated head of hhs

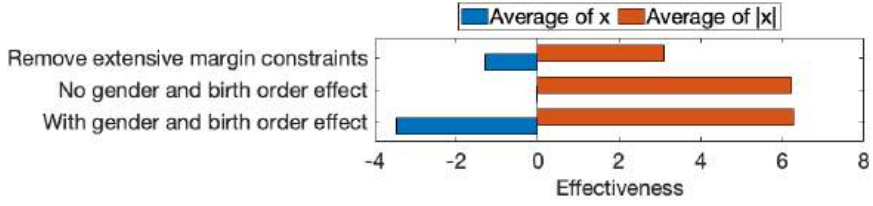
Figure 5: Effect of gender and birth order effects on inequality ($N_c = 2$)

- First-born daughters in non-educated hhs receive more education than younger brothers if their ability is 13% higher compared to their brothers.
- That number is 4% among college educated parents.

Counterfactual Analysis: Remove constraints for having a non-educated child

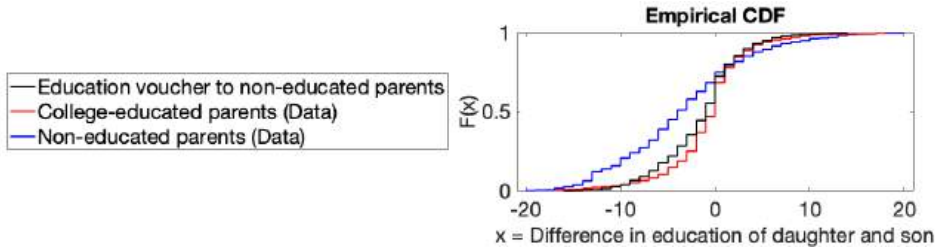


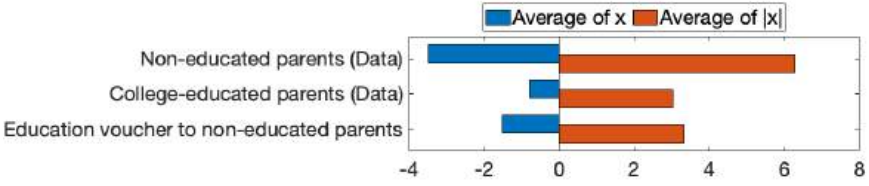
The dist in pink second order stochastically dominates both the one in blue and the on in **black**.



Comparative Statics: Increase Education Resources

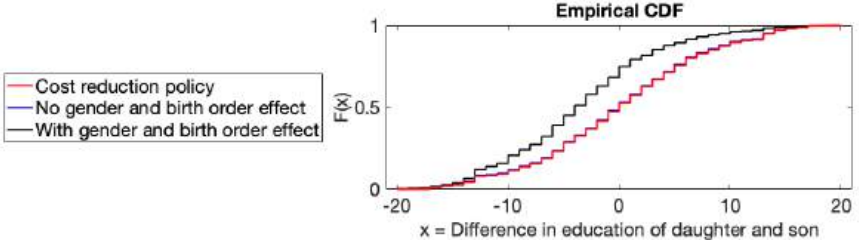
- Remove the high constraint for having an uneducated child among non-educated parents. and
- Increase q_T of non-educated parents s.t the ave is the same as college educated parents.



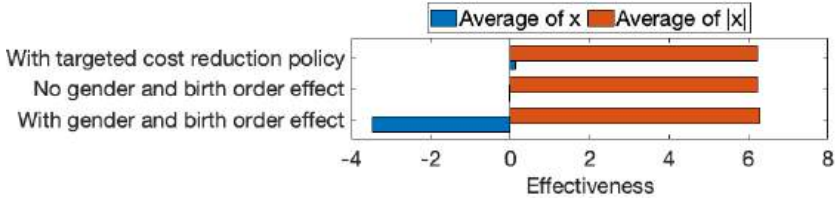


Counterfactual Analysis: Education cost reduction policy

1. At ext (resp. int) margin $\downarrow \approx 2\%$ (resp. $\approx 0.13\%$) in schooling cost for 1st born children.
2. At ext (resp. int) margin $\downarrow \approx 4\%$ (resp. $\approx 0.82\%$) in schooling cost for daughters.



It was calibrated s.t the dist with it matches the one without gender and birth order effect.



Ave utility gap of educ lower than college is smaller than the actual salary gap

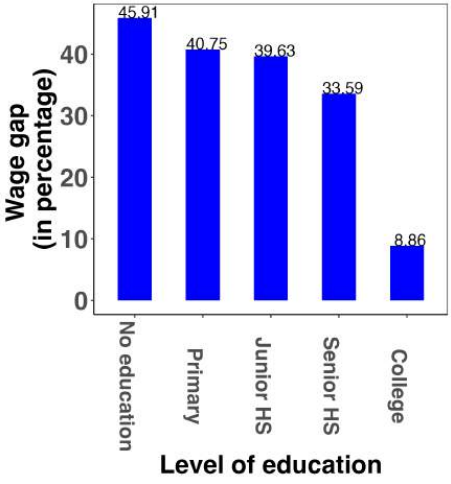


Figure 6: Salary gap (Benin, 2010)

Recall: $U_{B_{i,h}} = a_{i,h}.(q_{i,h})^{\delta_{i,h}}$, and define

$$\widehat{\text{Utility gap}} = 1 - \frac{\widehat{E}[U_B^{female}|q_{i,h},\widehat{\delta_{i,h}}]}{\widehat{E}[U_B^{male}|q_{i,h},\widehat{\delta_{i,h}}]}$$

Assumption: $E[U_B^{gender}] = L(E[Wage|gender])$, where $L(.)$ is a linear function.

- The average salary gap for senior secondary school level is 33.59%.

- For non-educated parents with unobserved constraints, the utility gap for senior secondary school level is 24.85% [26.75% for Bachelor degree].

Conclusion

Conclusion

- Sharp inverted U shape btw the within household ave and sd of children's education.
- Among non-educated parents, more than $\frac{2}{3}$ of within household inequality is due to gender and birth order effect.
- That number is $\frac{1}{3}$ among college educated parents.
- Addressing gender and birth order effect does not reduce average inequality in the sample.
- Addressing economic and social constraints that prevent parents from educating their children reduces average inequality but not the proportion due to gender effects.
- A targeted education cost reduction policy eliminate the effect of gender and birth order, but does not reduce average inequality in the sample.

Appendix

Appendix A1: Average level of education of children as function of parents' education

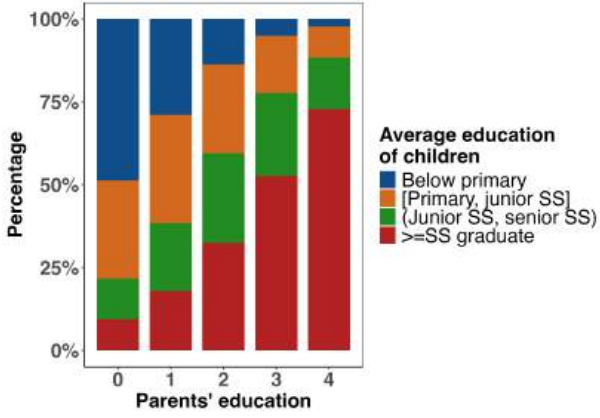


Figure 7: Average children's education as function of parents' education (Benin, 2013). return

Appendix A2: Number of children as function of parents' education

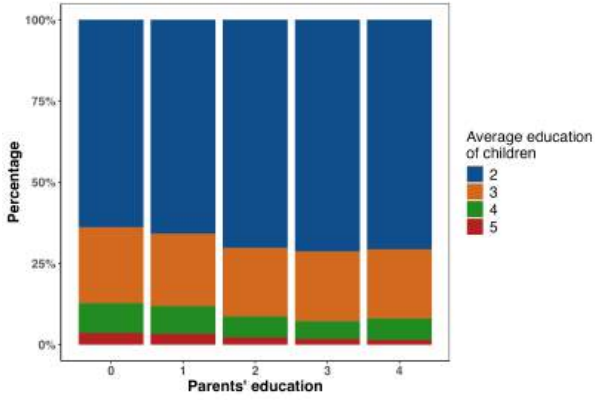


Figure 8: Distribution of number of children as function of parents' education (Benin, 2013). return

Appendix A3: Gender composition by type of households (households with 2 children)

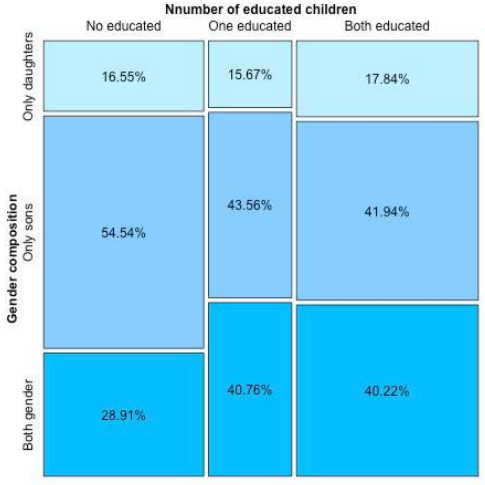
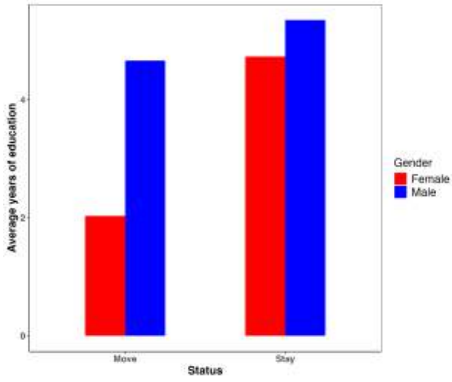
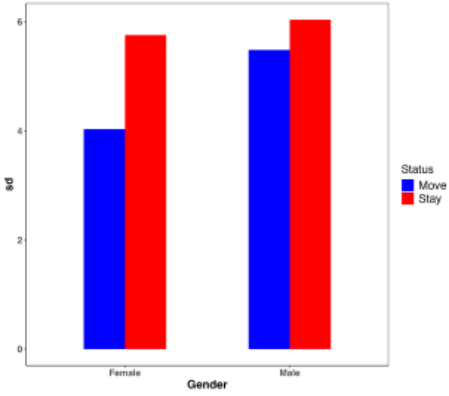


Figure 9: Gender composition by number of educated children (Benin, 2013). return

Appendix A: Move and Stay back adult children



(a) Mean



(b) Standard Deviation

Figure 10: Mean and standard deviation of adult children

Appendix A4: Average level of education and educational inequality

Table 3: Educational inequality on average education

	(1)	(2)	(3)	(4)	(5)	(6)
	Full sample		Non-educated hh		College educated hh	
Average education ($\bar{q}$)	1.25*	1.04*	1.44*	1.26*	0.87*	0.64*
	[1.24; 1.27]	[1.02; 1.07]	[1.42; 1.46]	[1.23; 1.29]	[0.81; 0.94]	[0.54; 0.75]
$\bar{q}^2$	−0.08*	−0.06*	−0.09*	−0.08*	−0.05*	−0.04*
	[−0.08; −0.08]	[−0.06; −0.06]	[−0.09; −0.09]	[−0.08; −0.08]	[−0.05; −0.04]	[−0.04; −0.03]
R^2	0.69	0.71	0.73	0.74	0.61	0.62
Covariates		✓		✓		✓
Num. obs.	32729	32729	19558	19558	1438	1438

* Null hypothesis value outside the confidence interval. Covariates include parents' education, area of residence, religion, number of children, and gender composition of the household.

return

Appendix A5: Average level of education and educational inequality

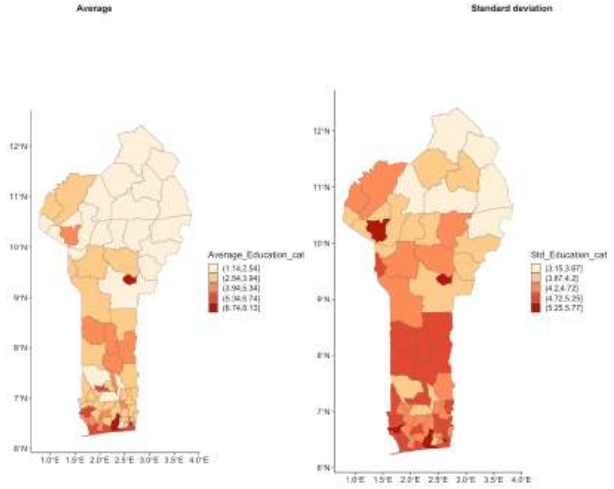


Figure 11: Across region mean and standard deviation of education for people btw 18-40yrs (Benin, 2013). return

Appendix A6: Average level of education and educational inequality

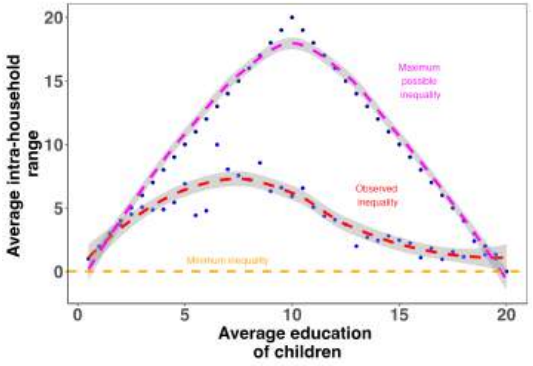


Figure 12: Average inequality (observed, min, and max) as function of average years of education for households with 2 children(Benin, 2013). return

Appendix A7: Proof

$$\Delta_{daughter-son} Educ_h = \beta_0 + \beta_1 Firstborn\_daughter_h + \varepsilon_h$$

$$E\Big[|\Delta_{daughter-son} Educ|\Big] = E\Big[|\beta_0 + \beta_1 Firstborn\_daughter + \varepsilon|\Big]$$

$$E\Big[|\beta_0 + \beta_1 Firstborn\_daughter + \varepsilon|\Big] \geq \Big|E\Big[\beta_0 + \beta_1 Firstborn\_daughter + \varepsilon\Big]\Big|$$

By Jensen's inequality.

$$E\Big[\Big|\beta_0 + \beta_1 Firstborn\_daughter + \varepsilon\Big|\Big] \geq \Big|\beta_0 + \beta_1 E[Firstborn\_daughter] + \underbrace{E[\varepsilon]}_{\text{Assumed to be 0}}\Big|$$

Therefore,

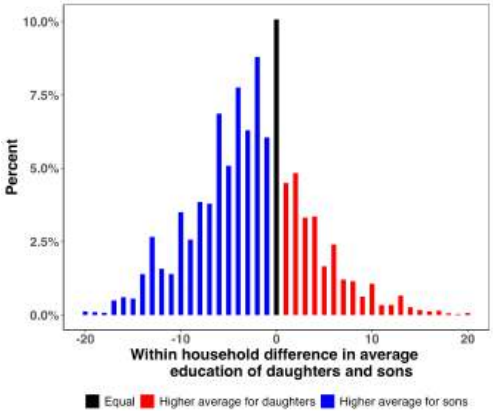
$$E[Range] \geq |\beta_0 + \beta_1 E[Firstborn\_daughter]|$$

Hence,

$$E[Range| \text{ household has firstborn daughter}] \geq |\beta_0 + \beta_1|$$

return

Appendix B: Histogram of within household difference in average education of daughters and sons (Benin, 2013)



Gender disadvantage in education attainment $\Longrightarrow$ Higher mass on the negative side.

Figure 13: Histogram of within household difference in average education of daughters and sons (Benin, 2013)

return

Appendix B: Histogram of within household difference in average education of daughters and sons (USA, 2015)

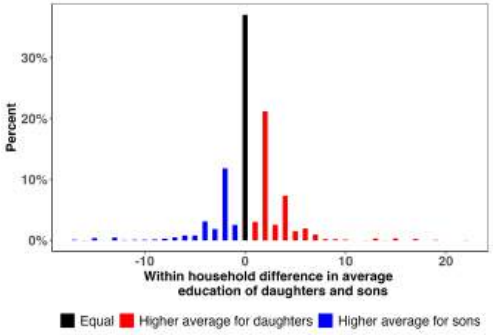


Figure 14: Histogram of within household difference in average education of daughters and sons (USA, 2015)

return

Appendix C1: Moments

Let $\theta = (\theta_1, \theta_2)$, where, $\theta_1 = (\theta_1^{low}, \theta_1^{ds,high}, \theta_1^{sd,high})$, and $\theta_2 = (\alpha_{(1)}^{low}, \alpha_{(1)}^{high})$.

Let T_{educ} be the total number of educated children.

Data Moments Matched:

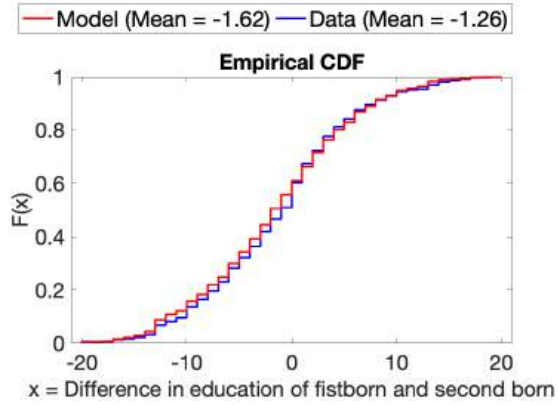
- $m_1 = E[Y^d | Gender\_Comp = b, T_{educ} = 2] - E[Y^s | Gender\_Comp = b, T_{educ} = 2]$,
- $m_2 = E[1\{Y^1 > 0\} | Gender\_Comp = s, T_{educ} = 1]$,
- $m_3 = E[1\{Y^d > 0\} | Gender\_Comp = b^{d,s}, T_{educ} = 1]$,
- $m_4 = E[1\{Y^d > 0\} | Gender\_Comp = b^{s,d}, T_{educ} = 1]$,
- $m_{t+4} = E[Y | birth\_order = t+1] - E[Y | birth\_order = t],\ t \in \{1, \ldots, N_c\}$.

I matched the following data and model moments to estimate θ.

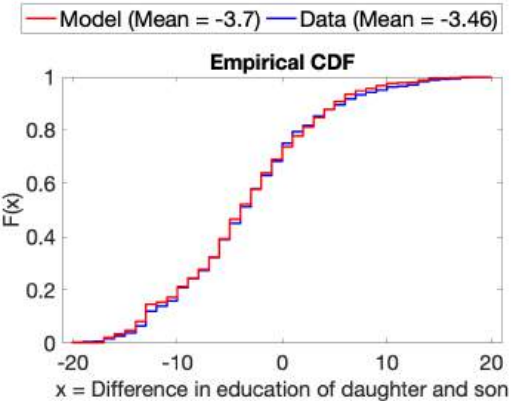
$m_1 = \hat{\mu}_d^d - \hat{\mu}_b^d,\ m_2 = \hat{\pi}_1,\ m_3 = \hat{\pi}_{fb,d},\ m_4 = \hat{\pi}_{sb,d}$ and $m_{t+4} = \hat{\mu}_{t+1} - \hat{\mu}_t;\ t \in \{1, \ldots N_c - 1\}$.

return

Appendix C2: Model vs. Data



(a) Firstborn vs. second born



(b) Daughter vs. son

Figure 15: Distribution of the difference in education (Data vs. Model)

return

Appendix C3: Model vs. Data

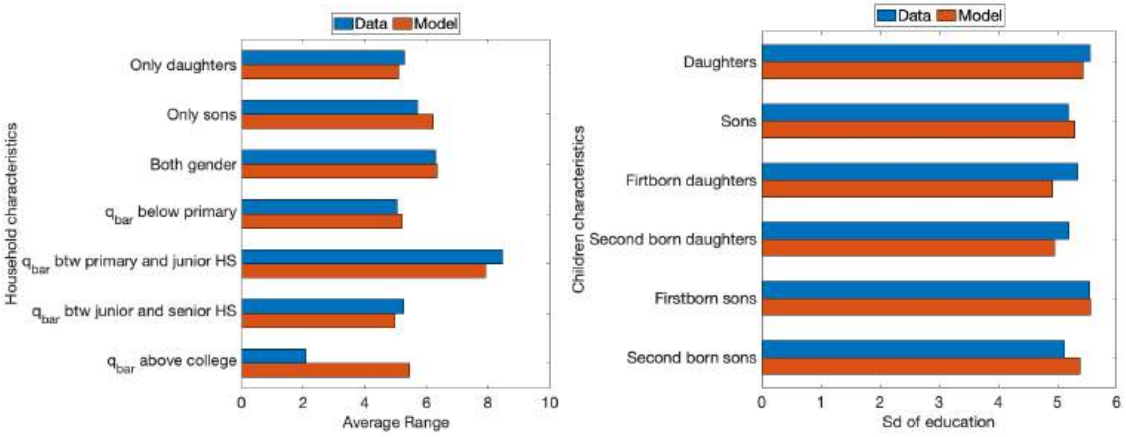


Figure 16: Moments (Data vs. Model)

return

Model for $N_c = 3$

Appendix D1: The Structural Model

For $N_c = 3$, Let

$$U(q_h, \theta) = \nu_h^L \Big[\sum_{i=1}^{N_c} a_i.(q_i)^{\delta_{i,h}^L} - \alpha_i^L q_i \Big] + \nu_h^M \Big\{ \sum_{i=1}^{N_c} \Big[e_i^M.(a_i.(q_i)^{\delta_{i,h}^M} - \alpha_i^M q_i) \Big] \Big\} \tag{5}$$

$$+\nu_h^H \Big\{ \sum_{i=1}^{N_c} \Big[e_i^H.\Big[(a_i.(q_i)^{\delta_{i,h}^H} - \alpha_i^H q_i) \Big] \Big] \Big\}$$

where,

- $\delta_{i,h}^{type} = \gamma - \theta_1^{type} Female_i \frac{1}{N_c - 1} \sum_{\{i,j \in h\}, j \neq i} (1 - Female_j)$, with

  $type \in \{low(L), medium(M), high(H)\}$

The Structural Model ($N_c = 3$)

$$e_i^M = 1\{\exists j : (a_i.(q_i)^{\delta_{i,h}^M} - \alpha_i^M q_i) > (a_j.(q_i)^{\delta_{j,h}^M} - \alpha_j^M q_j)\},$$

$$e_i^H = 1\{(a_i.(q_i)^{\delta_{i,h}^H} - \alpha_i^H q_i) > (a_j.(q_i)^{\delta_{j,h}^H} - \alpha_j^H q_j), \forall j \neq i\},$$

They are derived from the following constraints:

$$\sum_{i=1}^{3} e_i^M = 2 \text{ and } \sum_{i=1}^{3} e_i^H = 1$$

The vector of parameters of interest is

$$\theta = \left(\theta_1^L, \theta_1^M, \theta_1^H, \alpha_{(1)}^L - \alpha_{(2)}^L, \alpha_{(2)}^L - \alpha_{(3)}^L, \alpha_{(1)}^M - \alpha_{(2)}^M, \alpha_{(2)}^M - \alpha_{(3)}^M, \alpha_{(1)}^H - \alpha_{(2)}^H\right)$$

The dimension of θ is 1×9. return

Estimates ($N_c = 3$)

Table 4: Estimates of $\hat{\theta}$, ($N_c = 3$)

	$\hat{\theta}_1^L$	$\hat{\alpha}_1^L - \hat{\alpha}_2^L$	$\hat{\alpha}_2^L - \hat{\alpha}_3^L$	$\hat{\theta}_1^M$	$\hat{\alpha}_1^M - \hat{\alpha}_2^M$	$\hat{\alpha}_2^M - \hat{\alpha}_3^M$	$\hat{\theta}_1^H$	$\hat{\alpha}_1^H - \hat{\alpha}_2^H$	$\hat{\alpha}_2^H - \hat{\alpha}_3^H$
Estimates	0.033**	0.0026**	0.0010*	0.0761**	0.0094*	0.0001**	0.1596**	0.0120**	0.0105**
Standard errors	0.0052	0.0006	0.0006	0.0058	0.0016	0.0019	0.0073	0.0019	0.0015
Number of observations					3644				

$**$ significant at 5% level of significance, $*$ significant at 10% level of significance.

return

Model generalization for $N_c \geq 2$

The Structural Model ($N_c > 1$): Set up

Let $type \in \{0, 1, \ldots N_c - 1\}$, where

- 0 corresponds to the least constrained/no constraints to have a non-educated child,
- and $N_c - 1$ corresponds to constrained to have $N_c - 1$ non-educated children.

$$U(q_h, \theta) = \sum_{c=2}^{N_{c_h}} \left\{ 1\{N_{c_h} = c\}. \sum_{type} \left\{ \nu_h^{c,type}. \sum_{i=1}^{c} e_i^{c,type}. \left[a_i.(q_i)^{\delta_{i,h}^{type}} - \alpha_i^{type} q_i \right] \right\} \right\} \text{ where,} \quad (6)$$

- $\nu_h^{c,\cdot} \sim Multinomial(p_{c,\cdot})$, with $p_{c,\cdot} = (p_{c,0}, p_{c,1}, \ldots, p_{c,N_c-1})$, and $\sum_{type=0}^{N_c-1} p_{c,type} = 1$
- $\delta_{i,h}^{type} = \gamma - \theta_1^{type} Female_i \frac{1}{N_{c_h}-1} \sum_{\{i,j \in h\}, j \neq i} (1 - Female_j)$,
- $\sum_{i=1}^{c} e_i^{c,type} = N_{c_h} - type$,
- $q_h = (q_{1,h}, \ldots, q_{N_c,h})$

The Structural Model: Set up

Let, $M = C(N_{c_h}, N_{c_h} - type)$ be the total number of possible combination of $\left\{1, 2, \ldots, N_{c_h}\right\}$ s.t $e_k^{c,type} = 1$,

and, $S = \{S_1, S_2, S_M\}$ denotes the set of possible combinations.

Compute

$$q_h^{c,\star}(S_m) = argmax\Big\{ \sum_{k \in S_m} a_i.(q_k)^{\delta_{k,h}^{type}} - \alpha_k^{type} q_k \text{ subject to } \sum_k q_k = q_T \text{ and } 0 \leq q_k \leq q_{max} \Big\} \tag{7}$$

Compute $U(q_h^\star(S_m), \theta)$, for $m \in \{1, 2, \ldots M\}$,

The optimal educational distribution is

$$q_h^\star(S_m) \mid U(q_h^\star(S_m) > U(q_h^\star(S_{m'}),\ \forall m' \neq m, \text{ with } q_i^\star = 0 \text{ if } i \notin S_m$$

The vector of parameter is

$$\theta = (\theta_1^{type}, \alpha_1^{type}, \ldots, \alpha_{N_c - 1}^{type})$$

Illustrative Example

- Let $N_{c_h} = 3$, and $type = 1$,
- $C(N_{c_h}, N_{c_h} - type) = C(3, 2) = 3$, the possible combinations are $\{1, 2\}, \{1, 3\}$, and $\{2, 3\}$.
- We compute $q_h^\star(\{1, 2\})$, $q_h^\star(\{1, 3\})$, and $q_h^\star(\{2, 3\})$, and
- $U(q_h^\star(\{1, 2\}), \theta)$, $U(q_h^\star(\{1, 3\}), \theta)$, and $U(q_h^\star(\{2, 3\}), \theta)$
- Suppose that $U(q_h^\star(\{1, 2\}), \theta) > U(q_h^\star(\{1, 3\}), \theta) > U(q_h^\star(\{2, 3\}), \theta)$
- The optimal educational distribution is $(q_h^\star(\{1, 2\}), 0)$.

Appendix E: GMM Standard Errors

1. Variance covariance matrix for parameter estimates is given by:

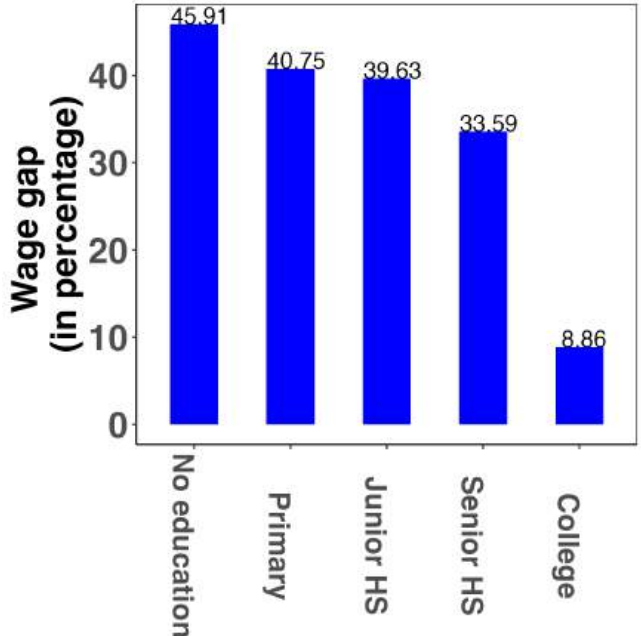
$$\Omega = \Big[\frac{\partial m(\theta)'}{\partial \theta}\Big|_{\hat{\theta}} V^{-1} \frac{\partial m(\theta)}{\partial \theta}\Big|_{\hat{\theta}}\Big]^{-1}$$

2. $\frac{\partial m(\theta)}{\partial \theta}$ is obtained numerically,

$$\frac{\partial m(\theta)}{\partial \theta}\Big|_{\hat{\theta}} = \frac{m(\hat{\theta} + h) - m(\hat{\theta})}{h}$$

3. V, the variance-covariance matrix of the data moments is calculated by bootstrapping the data - calculate the moments N times, then use these N obs of the moments and calculate the covariance between them.
4. The std errors will be the square roots of the diagonal elements of Ω.

Appendix F: Salary Gap Analysis



Appendix F: Salary Gap Analysis

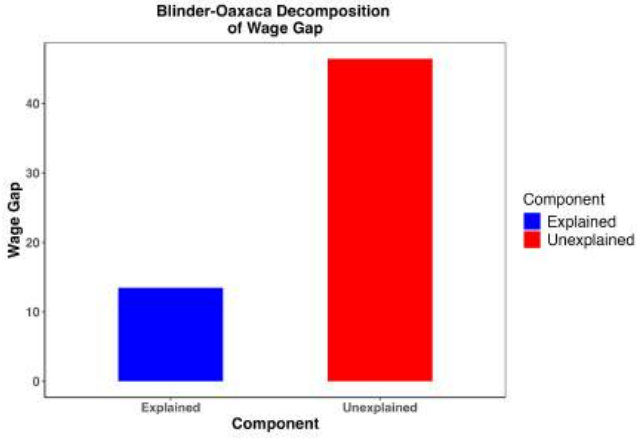


Figure 18: Salary gap decomposition (Benin, 2010)

References I

- Akresh, Richard et al. (2012). "Child ability and household human capital investment decisions in Burkina Faso". In: *Economic Development and Cultural Change* 61.1, pp. 157–186.
- Becker, Gary S and H Gregg Lewis (1973). "On the Interaction between the Quantity and Quality of Children". In: *Journal of political Economy* 81.2, Part 2, S279–S288.
- Becker, Gary S and Nigel Tomes (1976). "Child endowments and the quantity and quality of children". In: *Journal of political Economy* 84.4, Part 2, S143–S162.
- Biswas, Debashis (2000). "Gender Disparities in Education in the Developing World". In.
- De Tray, Dennis N (1970). *An Economic Analysis of Quantity-Quality Substitution in Household Fertility Decisions*. Tech. rep. RAND CORP SANTA MONICA CALIF.
- Esposito, Lucio, Sunil Mitra Kumar, and Adrián Villaseñor (2020). "The importance of being earliest: birth order and educational outcomes along the socioeconomic ladder in Mexico". In: *Journal of Population Economics* 33, pp. 1069–1099.
- Fergusson, David M, L John Horwood, and Joseph M Boden (2006). "Birth order and educational achievement in adolescence and young adulthood". In: *Australian Journal of Education* 50.2, pp. 122–139.

References II

- Giannola, Michele (2023). "Parental investments and intra-household inequality in child human capital: evidence from a survey experiment". In: *The Economic Journal*, uead086.
- Lachaud, James et al. (2014). "Family size and intra-family inequalities in education in Ouagadougou". In: *Demographic Research* 31, pp. 1455–1476.
- Londoño, Juan Luis (1990). "Kuznetsian tales with attention to human capital". In: *Third Inter-American Seminar in Economics, Rio de Janeiro, Brazil*.
- Moshoeshoe, Ramaele et al. (2016). "Birth order effects on educational attainment and child labour: Evidence from lesotho". In: *Economic Research Southern Africa (ERSA)*.
- Nerlove, Marc, Assaf Razin, and Efraim Sadka (1984). "Investment in human and nonhuman capital, transfers among siblings, and the role of government". In: *Econometrica: Journal of the Econometric Society*, pp. 1191–1198.
- Ota, Masako and Peter G Moffatt (2007). "The within-household schooling decision: a study of children in rural Andhra Pradesh". In: *Journal of Population Economics* 20.1, pp. 223–239.
- Ram, Rati (1990). "Educational expansion and schooling inequality: International evidence and some implications". In: *The Review of Economics and Statistics*, pp. 266–274.

References III

- Rosenzweig, Mark R and T Paul Schultz (1982). "Market opportunities, genetic endowments, and intrafamily resource distribution: Child survival in rural India". In: *The American Economic Review* 72.4, pp. 803–815.
- Thomas, Vinod, Yan Wang, and Xibo Fan (2003). "Measuring education inequality: Gini coefficients of education for 140 countries, 1960-2000". In: *Journal of Education Planning and Administration* 17.1, pp. 5–33.
- Vogl, Tom S (2016). "Differential fertility, human capital, and development". In: *The Review of Economic Studies* 83.1, pp. 365–401.
- Weng, Qian et al. (2019). "Family size, birth order and educational attainment: Evidence from China". In: *China Economic Review* 57, p. 101346.