When Green Investors Are Green Consumers

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Abstract

We introduce investors with preferences for green assets to a general equilibrium setting in which they also prefer consuming green goods. Their preference for green goods induces consumption premia on expected returns, which counterbalance the

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green premium stemming from their preferences for green assets. Because they provide a hedge when green goods become expensive, brown assets command lower consumption premia, while green investors allocate a larger share of their portfolios toward them. Empirically, the green-minus-brown consumption-premia differential reached 80 basis points annually, and contributes to explaining the limited impact of green investing on the cost of capital of polluting firms.

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JEL codes: G11, G12.

1 Introduction

The same proportion of U.S. individual investors surveyed by the Morgan Stanley Institute for Sustainable Investing (2019), namely, 33% of them, declare both that they "screen their investments according to their interests and values" and "purchase from a brand particularly because of the company's environmental or social impact." This survey suggests that the ethical motives underpinning green investors' capital allocation decisions (Riedl and Smeets, 2017; Hartzmark and Sussman, 2019; Krüger et al., 2020) are also—at least partly—reflected in their consumption practices.

Recent research has characterized a *green premium* on expected asset returns, which is induced by pro-environmental investment preferences (Pástor et al., 2021; Pedersen et al., 2021; Zerbib, 2022). This premium is higher on brown assets than green assets because green investors require a higher expected return to hold the assets they dislike in equilibrium. The existence of a green premium is of major importance, especially for investors willing to have an impact on corporate practices, because it can incentivize companies to mitigate their environmental footprints so as to decrease their cost of capital. However, the literature is silent on the effects of preferences for green consumption. How do pro-environmental preferences for consumption translate into investment decisions and expected returns? How do they interact with pro-environmental preferences for investment? Answering these questions is key to understanding whether the impact of green investors—once their preferences for green consumption is taken into account—on the cost of capital of companies can be an effective channel for prompting them to reduce their environmental footprints.

In this paper, we address these questions by building a general equilibrium model that features a green and a neutral investor, as well as a green and a brown equity asset that produce a green and a brown consumption good, respectively. The green investor has preferences toward both investing in the green asset and consuming the green good, while the neutral investor has no preferences for tilting his investment portfolio or consumption basket. We show that the green investor's preference for consuming the green good gives rise to *consumption premia* on expected returns.[1] Because the brown asset has higher payoffs when the green good becomes expensive, it offers a good hedge for the green investor, and

[1] An equivalent term, closer to the terminology in asset pricing, is *hedging premia*.

thus, commands lower consumption premia than the green asset in equilibrium, as detailed below. These consumption premia counterbalance the green premium that stems from the green investor's preference for the green asset. This effect on expected returns arises as the green investor allocates a larger share of her wealth to the brown asset compared to the case with no preferences for green consumption. Empirically as well as in the model, the impact can offset a large part of the green premium.

Consumption premia are primarily related to the relative supply of different consumption goods in the economy. Specifically, the main effect is driven by the willingness of the green investor to hedge against a decline in the relative supply of the green good or, equivalently, an increase in its relative price.[2] This risk may materialize as a result of the election of a new government (e.g., the withdrawal from the Paris climate agreement, the repeal of the Clean Power Act, and the suspension of federal subsidies to the renewable energy sector following the election of Donald Trump in the U.S. in 2017), a contraction of international trade (e.g., the 300% increase in the price of silicon—an essential component of solar panels, mostly produced by China—between August 2021 and October 2021 due to the Covid-19 crisis), the outbreak of an armed conflict (e.g., the increase in the share of coal in electricity production in Germany following the restrictions imposed on Russian gas imports since March 2022), or global energy shortages and the fear of an economic slump (e.g., the increase in coal production in China by 10% in the first two months of 2022 compared with the same period in 2021).

The intuition is as follows. When the green good becomes scarcer (equivalently, when its price increases), the satisfaction of the average investor in the economy decreases because she wants to consume more of that good but cannot.[3] In other words, her marginal value of wealth is large and she values a lot an asset that pays in these conditions. Coincidentally, in those situations, the relative payoffs of the brown asset also increase, while those of the green asset decrease. This effect occurs because the decrease in the relative supply of the green good is only partially compensated by the increase in its price, provided that the elasticity

[2] A second effect comes from the desire of investors to hedge movements in their relative wealth. This effect is, however, quantitatively small both in the model and empirically. Therefore, we mostly abstract from it in our main analysis.

[3] The consumption preferences for the average investor in the economy are tilted toward the green good because the green investor prefers the green good, while the neutral investor has equal preferences for green and brown goods.

of substitution between the goods is not too low (specifically, greater than one), as suggested by empirical evidence (Papageorgiou et al., 2017). As a result, the brown asset is a good hedge against the adverse event that the green good becomes scarcer (or its price increases), while the green asset is a poor hedge against these situations. Therefore, in equilibrium, the *green asset is riskier* from this perspective and commands larger consumption premia than the brown asset. This stems from the combination of a positive price of relative-supply risk and a larger loading of the green asset on this risk. For the same financial hedging reasons, the green investor also keeps more of her wealth invested in the brown asset than she would without green consumption motives because the brown asset comoves positively with the price of the green good that she favors.

Methodologically, we build a general equilibrium model with multiple heterogeneous agents, multiple equity assets, multiple consumption goods, and general preferences. Specifically, we augment the setting in Sauzet (2022a) to embed preferences for specific assets, in the spirit of Pástor et al. (2021), Pedersen et al. (2021), and Zerbib (2022). This setup allows us to (*i*) derive explicit expressions for risk premia, portfolios, and other variables; (*ii*) study these variables not only on average but also in their dynamic evolution with the state of the economy, which is a key aspect in our analysis; and (*iii*) highlight the significant impact of various parameters such as the elasticity of substitution across goods, preference for green consumption, and preference for green investing. Each of those advances is made possible by the unique combination of general preferences, the use of continuous time, and a global solution method.

We then provide empirical evidence supporting the existence of consumption premia in U.S. stock returns. Empirically as well as in the model, the difference in *consumption premia* between the greenest and brownest companies has reached from 40 basis points (bps) to up to 80 bps annually in recent years. This counterbalances and offsets a substantial part of the *green premium* that has been estimated in the recent literature (Pástor et al., 2022; Zerbib, 2022).

In more details, we test these predictions by estimating the beta-representation implied by the model for risk premia. We focus on the consumption premium driven by the relative supply of green goods, using U.S. common stocks from 2007 to 2019 at a monthly frequency.[4] We construct the factor associated with the relative supply of green goods at a granular 6-digit-NAICS industry level, based on the monthly Industrial Production and Capacity Utilization Indices from the Federal Reserve (publication G17). This factor is defined as the difference in supply changes between the greenest and brownest industries' terciles over the last two quarters, to account for the fact that macroeconomic data can take a while to affect asset markets. We measure the greenness of companies using two different measures: environmental ratings and carbon intensities provided by MSCI and S&P–Trucost, respectively. To capture and control for the preferences of investors for green *assets*, we include the green premium in the estimation by building a green factor, which is the portfolio taking a long position on the tercile of greenest companies and shorting the tercile of brownest ones (Pástor et al., 2022). We also control for the five Fama and French (2015) factors and the momentum factor (Carhart, 1997), as is standard.

Through four main results, we confirm the predictions of our asset pricing model regarding the risk of changes in the relative supply of green goods. First, across all specifications, the price of this risk is positive and highly significant, with t-statistics above four, five, or more in most specifications. Second, we confirm that the betas on relative-supply risks are lower for brown stocks than for green stocks across all months, emphasizing that brown assets provide a better hedge against situations with decreasing relative supply (or equivalently, increasing relative prices) of the green goods. Third, as a result, the consumption premium has gradually increased for the tercile of greenest assets with respect to the tercile of brownest assets to reach a spread of 80 bps per year, thereby substantially counterbalancing the green premium.[5] In short, *green assets are riskier than brown assets from the perspective of consumption premia*, as implied by the model. Fourth, irrespective of their environmental ratings, the difference in consumption premium between assets with betas in the top tercile and those with betas in the bottom tercile beta is substantial: assets that can hedge shocks to the relative supply of green goods carry lower returns of up to 1.56% annually. Finally, we perform robustness checks using changes in producer *price* indices from the Bureau of

[4] We end the analysis in December 2019 to avoid the major disruptions to production and other variables caused by the Covid pandemic.

[5] This baseline result focuses on NAICS codes 1 (Agriculture, Forestry, Fishing and Hunting), 2 (Mining, Quarrying, and Oil and Gas Extraction, Utilities, Construction) and 3 (Manufacturing), which encompass sectors that *produce* goods and are therefore most consistent with our model. However, the result also holds when we include NAICS 4 (Wholesale Trade, Retail Trade, Transportation and Warehousing).

Labor Statistics to construct the consumption factor as opposed to relative *supply*, and we find equivalently that the price of risk for this factor is negative and highly significant. We also verify that focusing on a more recent sample (2012–2019), consistent with Pástor et al. (2022), does not change our results and even reinforces them.

The results in this paper have implications not only for asset pricing, but also in terms of the real impact of sustainable investing. Through their preferences for green goods, green investors reduce their upward pressure on the cost of capital of polluting firms. Therefore, consumption premia help explain the low impact of green investing on mitigating the environmental footprints of companies through the cost of capital channel, as suggested by the literature (Berk and van Binsbergen, 2021; De Angelis et al., 2022). Instead, the preference of investors for green consumption makes the case for a stronger focus on shareholder engagement to impact companies' practices for two reasons: by allocating a larger share of their wealth to brown assets, green investors (i) reduce their impact on the cost of capital of brown firms, and simultaneously, (ii) increase their ability to actively engage with them. We briefly discuss the broader implications of these results as well as potential tools available to policymakers to counteract the effects of comsumption premia in Section 5.

Related literature. This paper contributes to several strands of the literature in asset pricing and sustainable finance. First, this is the first paper that studies the effects that investors' preferences toward sustainable consumption have on asset prices and investors' asset allocation. The construction of a general equilibrium model allows us to uncover these effects. From a theoretical viewpoint, Pástor et al. (2021), Pedersen et al. (2021), and Zerbib (2022) characterize the green premium driven by investors' preferences for green assets in equilibrium on financial markets. This premium is higher on brown assets than on green assets, and corresponds to the compensation required by sustainable investors for holding the assets they like least. Empirical evidence supports the existence of a green premium that is higher on the stock returns of the carbon-intensive companies (Bolton and Kacperczyk, 2021, 2022), polluting companies (Hsu et al., 2022), companies most exposed to climate change risk (Bansal et al., 2016; Barnett, 2022), and least held by green funds (Zerbib, 2022) than on the stock returns of green companies. A similar effect is documented on the cost of equity (ElGhoul et al., 2011; Chava, 2014), expected returns approximated from optionimplied information (Sautner et al., 2022), bond yields (Chava, 2014; Baker et al., 2018; Zerbib, 2019; Painter, 2020; Goldsmith-Pinkham et al., 2021; Huynh and Xia, 2021; Seltzer et al., 2022), venture capital funds (Barber et al., 2021), and real estate prices (Bernstein et al., 2019; Baldauf et al., 2020; Giglio et al., 2021). However, opposite effects can emerge in a dynamic setting: the cost of capital of green firms may increase because investors' preferences for green assets reduce asset price informativeness (Goldstein et al., 2021) or as a result of shocks on preferences for green assets (Avramov et al., 2021). In addition, performing empirical analyses on more recent time frames, Ardia et al. (2021) and Pástor et al. (2022) find higher green premia on the greenest stock returns driven by recent capital inflows, reflecting changes in investors' preferences in a transitory phase. Lontzek et al. (2023) study disagreement about climate risks and report a similar effect on the relative prices of green versus brown stocks when investors update their perception of climate risk upwards. Likewise, Zhang (2021) suggests that brown-minus-green returns turn negative in the U.S. and insignificant globally once accounting for the fact that carbon emissions embed forward-looking information about firm performance. Distinct from the work on the green premium, Albuquerque et al. (2019) show that green assets have lower systemic risk than brown assets and that this effect is stronger for firms with high product differentiation, and Campbell and Martin (2023) study how much consumption can be sustained in the long run in a risky world. Another body of the literature on sustainable asset pricing studies the impact of climate risks on asset prices (e.g., Hong et al., 2019; Alok et al., 2020; De Angelis et al., 2022). Notably, a couple of recent papers analyze climate-related financial risks in general equilibrium. Barnett (2022) shows that the price of climate risk is significantly negative, particularly driven by the risk of transition to a low-carbon economy. Hambel et al. (2022) highlight that investors' willingness to diversify their assets complements the attempt to mitigate economic damages from climate change in the short run, while in the longer run, a trade-off between diversification and climate action emerges. Baker et al. (2022) show that green investors may want to overinvest in brown assets because they provide a hedge against the pollution-related financial risks of their portfolios. This channel could complement our mechanism, which instead focuses on the effect of green consumption via the relative prices of goods, in a multi-good economy. Engle et al. (2020) and Alekseev et al. (2021) propose portfolio construction methods that allow to efficiently hedge these climate risks. In this paper, we depart from the asset pricing literature on the impact and hedging of environmental risks, and we focus on the preferences of green investors for green *consumption* by constructing a two-investor, two-tree, two-good general equilibrium model with heterogeneous preferences for investment and consumption. We provide the first theoretical and empirical evidence for the existence of significant consumption premia that counterbalance the effect of the green premium on asset returns. In a recent subsequent paper, Chen et al. (2023) propose a similar idea in a setting with a representative investor and habit in consumption. The authors concentrate more specifically on the role of the elasticity of substitution across goods—something that our framework allows but that is not our central focus. Notably, they show that the effect on risk premia is concentrated on sectors with high elasticity of substitution across goods, which is consistent with our baseline calibration. In contrast, we build a setup with heterogenous investors that allows us to elucidate and discuss risk premia as well as portfolio choices. We also embed preferences for green assets so as to compare our novel mechanism with the green premium that has been documented in the literature. In addition, we test the model by building the new factor implied by our theoretical results and estimating its pricing abilities. Overall, despite different approaches, their findings are consistent with the sign and magnitude of our consumption premia, which further emphasizes that green consumption is indeed an important channel to account for.

Second, this paper contributes more broadly to the literature on theoretical general equilibrium asset pricing with multiple heterogeneous agents, multiple equity assets, multiple consumption goods, and general preferences, such as recently developed in Sauzet (2022a,b). This framework, in turn, combines models with multiple agents—they have a long and distinguished history since the seminal contributions of Dumas (1989, 1992), Wang (1996), Basak and Cuoco (1998), Dumas et al. (2000), Dumas and Uppal (2001), Chan and Kogan (2002), and more recently Brunnermeier and Pedersen (2009), Weinbaum (2009), Bhamra and Uppal (2009, 2014), Brunnermeier and Sannikov (2014), Gârleanu and Pedersen (2011), Chabakauri (2013), Gârleanu and Panageas (2015), Drechsler et al. (2018), Borovička (2020)—with settings with multiple equity securities but one investor such as Cochrane et al. (2008), Martin (2013), and two consumption goods (Fang, 2019). In other words, the framework generalizes the contributions of Zapatero (1995), Pavlova and Rigobon (2007, 2008, 2010), Martin (2011), Stathopoulos (2017), to non-log preferences, and a general aggregation of goods. The unique combination of general preferences, the use of continuous time, and a global solution method, is key in allowing us to derive most of our results. To this literature, we also add the preferences for specific assets: in our case, the green investor prefers the green asset, a central element in the sustainable asset pricing literature. Our formulation of this asset preference is general and could be used in other contexts. The framework in this paper can also be extended along several directions. For instance, Sauzet (2024) studies what happens when investors can only invest through index funds or financial intermediaries, which can impede the transmission of their preferences to asset prices. We explore various related avenues in ongoing work, such as stochastic demand and production.[6]

This paper also contributes to the literature on environmental and ecological economics. Specifically, using two goods to capture green and brown consumption is in the spirit of Guesnerie (2004), Hoel and Sterner (2007), Sterner and Persson (2008), Gollier (2010), Traeger (2011), Barro and Misra (2016), and Gollier (2019), in which the two goods are taken to represent aggregate economic capital (physical capital, labor, scientific knowledge, *etc.*) on the one hand, and various ecosystem services generated by natural capital on the other. While most of these contributions are based on a representative agent or social planner, we bring this intuition to a general equilibrium economy with several investors. The investors are heterogenous in their (general) preferences for consumption and investment, and we solve for the decentralized equilibrium, which allows a meaningful discussion of portfolios, in addition to risk premia, and other variables. Broadly speaking, our study is also related to contributions in environmental macroeconomics such as Pindyck and Wang (2013), Golosov et al. (2014), Cai and Lontzek (2019), van den Bremer and van der Ploeg (2021) on the theoretical side, and Papageorgiou et al. (2017) on the empirical side.

Fourth, and importantly, this paper contributes to the literature on impact investing. Building on the seminal paper by Heinkel et al. (2001), De Angelis et al. (2022) find that the increase in the cost of capital driven by green investing has a limited impact on the practices of the most polluting companies. This conclusion is consistent with Berk and van Binsbergen (2021) who show that the effect of impact investing on the cost of capital is too small to meaningfully affect real investment decisions. However, Hakenes and Schliephake

[6]On the theoretical front, our study is also related to contributions introducing recursive preferences in continuous-time, for example, Duffie and Epstein (1992), and contributions focusing on the existence and uniqueness of equilibria in the presence of multiple agents, and possibly multiple goods and incomplete markets, for example, Polemarchakis (1988), Geanakoplos and Polemarchakis (1986), Geanakoplos and Mas-Colell (1989), Geanakoplos (1990), Duffie et al. (1994), Berrada et al. (2007), Anderson and Raimondo (2008), Hugonnier et al. (2012a,b), Ehling and Heyerdahl-Larsen (2015, 2017).

(2023) show that this positive impact could be reinforced when agents combine responsible consumption and investment. In addition, through two different approaches, Oehmke and Opp (2019) and Green and Roth (2020) emphasize the importance of investor coordination to finance the companies that need it most and increase their impact on the economy as a whole. In addition, Landier and Lovo (2020) highlight the effects of search frictions in capital markets, which increase the impact of investors on corporate practices. From an impact perspective, Broccardo et al. (2020) suggest that in most cases, shareholder engagement is more effective than the effect of sustainable investors' asset allocation on companies' cost of capital. This paper reinforces that suggestion for a different reason: green investors' preferences for green goods weaken the cost of capital channel via the consumption premia and increase the allocation of green investors toward the brownest companies, which are the preferred targets for shareholder engagement campaigns.

Outline. The paper is organized as follows. Section 2 describes the set-up of the economy and introduces the two state variables that drive economic mechanisms—the wealth share of the green investor and the relative supply of the green good. Section 3 revisits the impact of green investors on asset prices when they also have preferences for green goods. Section 4 provides empirical evidence supporting our findings. Section 5 discusses the results in light of impact investing challenges and Section 6 concludes. Proofs and additional material are provided in Appendix.

2 The Economy

This section presents the theoretical setup. We introduce a pure-exchange economy with a green and a neutral investor ($i \in \{G, N\}$), and a green and a brown tree ($j \in \{g, b\}$). The trees produce differentiated green and brown goods, respectively, and are traded as equity assets à la Lucas (1978). The green investor has preferences not only for investing in the green asset (Pástor et al., 2021; Pedersen et al., 2021; Zerbib, 2022), but also for consuming the green good (Sauzet, 2022a). We show that the equilibrium can be characterized as a function of two state variables: the relative wealth of the green investor, x_t, and the relative supply of the green good, y_t. The setup is summarized in Figure B.1 of the Appendix.

Appendix A gathers additional results that are omitted in the main text.

Time is continuous and the horizon is infinite, $t \in [0, \infty)$. Uncertainty is represented by a probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ supporting a two-dimensional Brownian motion $\vec{Z} \equiv (Z_g, Z_b)^T \in \mathbb{R}^2$. The filtration $\mathbb{F} = (\mathcal{F}_t)_{t \in [0,\infty)}$ is the usual augmentation of the filtration generated by the Brownian motions, and $\mathcal{F} \equiv \mathcal{F}_\infty$.

2.1 Endowments, prices, assets

The two trees produce differentiated, green and brown, goods. Their outputs follow geometric Brownian motions

$$\frac{dY_{j,t}}{Y_{j,t}} = \mu_{Y_j} dt + \sigma_{Y_j}^\top d\vec{Z}_t, \quad j \in \{g, b\}.$$

The prices of the green and brown goods are $p_{g,t}$ and $p_{b,t}$, respectively. We also define the terms of trade $q_t \equiv p_{g,t}/p_{b,t}$, which is the relative price of the green good, and the real exchange rate $\mathcal{E}_t \equiv P_t^G / P_t^N$, which is the relative price of the consumption basket of the green investor. All prices are defined with respect to a numéraire taken to be a CES-basket with weight $a = 1/2$ on both goods.[7]

The green and brown trees are traded as equity assets, with returns given by

$$dR_{j,t} = \frac{dQ_{j,t}}{Q_{j,t}} + \frac{p_{j,t} Y_{j,t}}{Q_{j,t}} dt = \frac{d\left(p_{j,t} Y_{j,t}/F_{j,t}\right)}{p_{j,t} Y_{j,t}/F_{j,t}} + F_{j,t} dt \equiv \mu_{j,t} dt + \sigma_{j,t}^T d\vec{Z}_t, \quad j \in \{g, b\}, \qquad (1)$$

where $Q_{j,t}$ are the equity prices, and $F_{j,t} \equiv p_{j,t} Y_{j,t}/Q_{j,t}$ are the dividend yields, for both assets. Drifts $\mu_{j,t}$, which measure conditional expected returns, and diffusion terms $\sigma_{j,t}$, which measure the loadings on the shocks and, therefore, the conditional volatilities, are obtained from Itô's Lemma and given in Appendix A.

The supply of each equity asset is normalized to unity, and there also exists a bond in net zero supply, which is locally riskless in units of numéraire. Its price is B_t, and the corresponding instantaneous interest rate is r_t, so that $dB_t/B_t = r_t dt$.

[7] Specifically, we normalize $\left[(1/2) p_{g,t}^{1-\theta} + (1/2) p_{b,t}^{1-\theta}\right]^{1/(1-\theta)}$ to unity.

2.2 Preferences

Investors have recursive preferences à la Duffie and Epstein (1992) that are defined over consumption, but also over the weights on each asset in their portfolios, $\boldsymbol{w}^i$. Specifically, for the green and neutral investors, $i \in \{G, N\}$,

$$V_t^i = \max_{\{C_{g,u}^i, C_{b,u}^i, w_{g,u}^i, w_{b,u}^i\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} f^i \left(C_u^i, V_u^i, \boldsymbol{w_u^i} \right) du \right] \tag{2}$$

$$f^i(C, V, \boldsymbol{w}) \equiv \left(\frac{1-\gamma}{1-1/\psi} \right) V \left[\left(\frac{C}{[(1-\gamma)V]^{1/(1-\gamma)}} \right)^{1-1/\psi} - \rho + \Phi^i(\boldsymbol{w}) \right]$$

where γ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution (EIS), and ρ is the discount rate.

Recursive preferences are relevant for three reasons. First, contrary to the case with *log* utility, investors are not myopic and hedging terms arise, which are important drivers of risk premia and portfolios. Second, the coefficient of relative risk aversion is not equal to the reciprocal of the EIS, $\psi \neq 1/\gamma$, which matters quantitatively to obtain risk premia that are closer to their empirical counterparts as well as for the quantitative impact of a potential tax on brown assets as discussed in Section 5. In addition, it leads the wealth share of the green investor to be a separate state variable, which is interesting from the perspective of comparing the model to the existing literature. Third, and relatedly, recursive preferences make it possible to obtain not only plausible risk premia, $\mu_{j,t} - r_t$, but also a reasonable riskfree rate, r_t.[8] This turns out to be important in this context so that the differential impact on risk premia for green and brown assets is not dwarfed by the magntiude of r_t, and can therefore meaningfully impact discount rates. This aspect has not been emphasized in the sustainable asset pricing literature so far, and is discussed in Section 3.2.

In what follows, parameters γ, ψ, ρ are taken to be identical for both investors. However, the resolution allows for any value so that exploring additional asymmetries stemming from these parameters could be an interesting avenue for future work.

[8]This relates to the well-known "risk-free rate puzzle" (Weil, 1989), which comes as a flip side to the "equity premium puzzle" (Mehra and Prescott, 1985), in a world with CRRA preferences. Historically, it constitutes one of the reasons why recursive preferences were introduced in the literature.

The green investor expresses her pro-environmental motives, in part, by displaying a preference toward the green asset. In this general equilibrium context, we introduce it as functions of the portfolio weights for both investors, $\Phi^i(\boldsymbol{w})$, where $\boldsymbol{w}_t^i \equiv \left(w_{g,t}^i, w_{b,t}^i\right)$, and $w_{g,t}^i$ ($w_{b,t}^i$) is the share of wealth on the green (brown) asset in the portfolio of investor $i \in \{G, N\}$. Specifically, we take[9]

$$\Phi^i(\boldsymbol{w}^i) \equiv (1 - 1/\psi)\left(w_g^i \phi_g^i + w_b^i \phi_b^i\right) \tag{3}$$

Parameter $\phi_g^G \equiv \phi > 0$ captures the additional value that the green investor derives from holding the green asset, in the spirit of Pástor et al. (2021) and Zerbib (2022). Without loss of generality, we assume that the neutral investor has no preference for the green asset ($\phi_g^N = 0$), and that neither investors have preferences for the brown asset ($\phi_b^G = \phi_b^N = 0$). In Section 3, we show that the preference of the green investor for the green asset gives rise to a green premium reducing the expected return on the green asset.

In terms of consumption, the basket of each investor is composed of the green and brown goods, which are combined according to an aggregator with constant elasticity of substitution θ, and bias in consumption α^i,

$$C_t^i = \left[\alpha^{i\frac{1}{\theta}} C_{g,t}^{i\frac{\theta-1}{\theta}} + (1-\alpha^i)^{\frac{1}{\theta}} C_{b,t}^{i\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{4}$$

While the neutral investor has no particular preference toward any of the goods ($\alpha^N = 1/2$), the green investor also expresses her pro-environmental preferences by tilting her consumption toward the green good ($\alpha^G > 1/2$). This preference for green consumption is the key novel element in this paper. In the theoretical characterization of Section 3, we show that it underpins large consumption premia on expected returns that can offset a substantial share of the green premium stemming from green asset preferences.

Allowing for a general elasticity of substitution across goods, θ, is also important because its value determines the relative magnitude of the movement in the relative price of the goods for a given shock to relative supply. In turn, this relative magnitude governs the movements in the relative dividends of the two assets, and ultimately the tilt in portfolios

[9] The $(1 - 1/\psi)$ factor serves purely as a normalization so that parameter ϕ drives the green premium like in the literature (cf. Section 3.2).

and consumption premia.[10]

From the share of wealth that investors allocate to the green and brown equity assets, $w^i_{g,t}, w^i_{b,t}$, they earn expected returns $\mu_{g,t}, \mu_{b,t}$. They allocate the remainder of their wealth $(1 - w^i_{g,t} - w^i_{b,t})$ to the riskless bond. They use the proceeds of their investments to purchase their desired baskets of consumption $c^i_t \equiv C^i_t / W^i_t$, at price P^i_t. In other words, investors $i \in \{G, N\}$ choose their consumption and portfolios to maximize (2) subject to the following budget constraint

$$\begin{aligned} \frac{dW^i_t}{W^i_t} = &\left(r_t + w^i_{g,t}\left(\mu_{g,t} - r_t\right) + w^i_{b,t}\left(\mu_{b,t} - r_t\right) - P^i_t c^i_t\right) dt \\ &+ \left(w^i_{g,t}\sigma_{g,t} + w^i_{b,t}\sigma_{b,t}\right)^T d\vec{Z}_t \end{aligned} \tag{5}$$

To complete the definition of their optimization problems, investors are subject to a standard transversality condition, and W^i_0 is given. Note also that $W^i_t \geqslant 0$.

The framework also allows for additional ingredients such as taxes on the dividends of each asset. This extension is discussed in Section 5.

2.3 Equilibrium and state variables

The definition of the equilibrium is standard: (1) investors solve their optimization problems by taking aggregate stochastic processes as given, and (2) goods and equity markets clear. The detailed definition of the equilibrium is given in Appendix A.4. The bond market clears by Walras's law, which gives rise to the following useful relationship: $W^G_t + W^N_t = Q_{g,t} + Q_{b,t}$. In words, total wealth has to be held in the form of the two equity assets in aggregate.

Stationary recursive Markovian equilibrium. Most importantly, the equilibrium can be recast as a stationary recursive Markovian equilibrium in which all variables of interest

[10]For instance, the common Cobb-Douglas case ($\theta = 1$) leads the relative price of the goods to move exactly enough to compensate relative supply so that relative dividends are unaffected. The relative payoffs of the two assets can then be perfectly correlated, and the portfolio choice between them indeterminate, at least without additional preferences for specific assets. Empirically, $\theta > 1$, which will drive the direction of the hedging terms as discussed in Section 3.

are expressed as a function of a pair of state variables $X_t \equiv (x_t, y_t)'$, whose dynamics are also solely a function of X_t. x_t is the wealth share of the green investor, and y_t is the relative supply of the green good.[11] Both are defined below.

The characterization of the solution as a system of coupled algebraic and second-order partial differential equations is the focus of Section 3. For now, let us discuss the intuition behind both state variables. Note that an additional variable, which is not a state variable *per se* but is useful throughout, is $w^M_{g,t}$, the ratio of the green equity price to total wealth. It captures the weight of the green asset in the market portfolio, and it can be shown that[12]

$$w^M_{g,t} \equiv \frac{Q_{g,t}}{Q_{g,t} + Q_{b,t}} = \left(1 + \left(\frac{F_{g,t}}{F_{b,t}}\right) q_t^{-1} \left(\frac{1 - y_t}{y_t}\right)\right)^{-1}. \tag{6}$$

Wealth share. The wealth share of the green investor is defined as

$$x_t \equiv \frac{W^G_t}{W^G_t + W^N_t}. \tag{7}$$

In this setting, the wealth share is neither constant nor solely a monotonic function of the current relative supply of the green good, y_t. It is therefore required as an additional state variable even when risk sharing is perfect (e.g., even when there are no taxes on dividends). This occurs because preferences are recursive, and due to the fundamental heterogeneity stemming from the green investor's bias toward consuming and investing green.

Relative supply. The relative supply of the green good captures the effect of current fundamentals and is defined as

$$y_t \equiv \frac{Y_{g,t}}{Y_{g,t} + Y_{b,t}}. \tag{8}$$

The relative supply is a key driver of the marginal values of wealth of both investors due

[11]Formally, this is shown using a guess and verify approach like, for example, in Gârleanu and Panageas (2015). The variables of interest are: $\{c^G_{g,t}, c^G_{b,t}, c^N_{g,t}, c^N_{b,t}, w^G_{g,t}, w^G_{b,t}, w^N_{g,t}, w^G_{b,t}, \mu_{R_g,t}, \mu_{R_b,t}, r_t, F_{g,t}, F_{b,t}, p_{g,t}, p_{b,t}, P^G_t, P^N_t, q_t, \mathcal{E}_t\}$.

[12]Because the bond is in zero net supply, the weight of the brown asset in the market portfolio is $w^M_{b,t} = 1 - w^M_{g,t}$ in equilibrium.

to their desire to consume both goods, which stems from their CES consumption baskets.[13] This is particularly true for the green investor who has a strong willingness to consume more green goods, as discussed in Section 3. For the same reason, we show in Section 3 that the relative supply is also the main driver of the relative price of the green good, q_t, and therefore, of the relative price of the consumption basket of the green investor, $\mathcal{E}_t$.

As discussed in introduction, a decline in the relative supply of green goods or, equivalently, an increase in their relative prices, may result from a variety of political and economic risk factors such as energy shortages, a contraction of international trade, the election of a new government, or the outbreak of an armed conflict.

Note that because $W_t^i \geqslant 0$ and $Y_{j,t} \geqslant 0$, x_t and y_t are both evolving in the bounded interval $[0, 1]$. This has the advantage that solving for unknown functions on a bounded domain is numerically more stable. Conceptually, as x_t gets closer to either of the boundaries, the economy converges (continuously) to a natural one-investor environment. As y_t gets closer to either of the boundaries, the economy converges to a one-good one-equity asset economy, but this has consequences in terms of marginal values of wealth as the investors still want to consume both goods.

Throughout, we focus on the solution to the decentralized, that is, Radner equilibrium instead of relying on the social planner's problem. The existence and uniqueness of the equilibrium should be guaranteed, for instance, following the work of Duffie and Epstein (1992), who use partial differential equation techniques to prove them in an infinite-horizon Markov diffusion setting with stochastic differential utility, or Chabakauri (2013) and Bhamra and Uppal (2014), who do so constructively for economies with heterogeneous agents and incomplete and complete markets, respectively. Both are also shown in situations with potentially dynamically complete markets[14] using a planner solution in Anderson and Raimondo (2008), and under complete markets with a full set of Arrow-Debreu securities in Hugonnier et al.

[13] Note that the ratio involves *quantities* of the two different goods. This poses no particular theoretical issue and is used because it simplifies the characterization of the equilibrium. This definition is a monotonic transformation of $Y_{b,t}/Y_{g,t}$: $y_t \equiv (1 + Y_{b,t}/Y_{g,t})^{-1}$, which ensures that the state variable evolves in the bounded interval $[0, 1]$. $Y_{b,t}/Y_{g,t}$ has the clear interpretation of the output of brown good produced per unit of green good. An economic intuition is that one compares the economy to the symmetric point in which relative prices are $q_t = \mathcal{E}_t = 1$.

[14] A securities market is potentially dynamically complete if the number of securities with non-colinear payoffs is equal to one plus the number of risk factors (Brownian motions) to be spanned.

(2012a). As has been known since the seminal example of Hart (1975), however, the introduction of multiple goods could complicate the matter, for instance, because markets can become dynamically incomplete even if the number of assets should technically be sufficient to span risks. Those multiple-good contexts are discussed, for example, in Berrada et al. (2007) and Ehling and Heyerdahl-Larsen (2015), mostly through the lens of the Pareto efficient allocation obtained from a social planner. Overall, equilibrium existence and uniqueness in the context of this paper with multiple goods, a bias in consumption and investment, potential imperfect risk sharing (when there exists a tax on dividends), and a decentralized Radner solution, could therefore be analyzed further from a theoretical perspective. This represents an interesting avenue for further research.

2.4 Computation of the equilibrium

Section 3 characterizes all variables of interest as a function of the state variables, $X_t = (x_t, y_t)'$, and a set of unknown functions $\mathcal{G} \equiv \{J_t^G,\ J_t^N,\ F_{g,t},\ F_{b,t},\ q_t,\ w_{g,t}^G,\ w_{b,t}^G\}$.[15] Due to the stationary recursive Markovian structure of the equilibrium, those unknown functions are themselves solely functions of X_t, and are determined by a set of coupled algebraic and second-order partial differential equations.

The resolution is based on projection methods and orthogonal collocation. Specifically, each of the unknown function $g : [0,1]^2 \to \mathcal{D}^g \subseteq \mathbb{R}$ in $\mathcal{G}$ is approximated using Chebyshev polynomials and the equilibrium is solved on an grid based on the zeros of the Chebyshev polynomials. Details are provided in Sauzet (2022a).

The main appeal of this approach is that this is a global solution method, which allows us to trace out the evolution of our variables of interest as a function of the state of the economy. Combined with continuous-time, it makes it possible to cleanly express and solve for the exact subcomponents of the main variables—risk premia, portfolios, goods prices—as well as our mechanisms of interest, in particular hedging components induced by consumption preferences. Our methodology will prove crucial, for example, when discussing the dynamic aspects of those mechanisms, and how they can be state-dependent.

[15] J_t^G, J_t^N are introduced in Section 3 and capture (an increasing monotonic transformation of) the marginal values of wealth of each investor. In addition, as a point of notation, for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to the infinite horizon).

Projection methods are also well-suited to contexts with multiple state variables. For settings with additional state variables that could become computationally too costly, such as those that might arise when generalizing the framework, extensions of those methods to higher-dimensional settings could prove necessary. One such method consists in naturally extending the concept of projection approaches, but to replace the Chebyshev polynomials in the approximation by neural networks, which are designed specifically to handle high-dimensional (and non-linear) contexts. Those "projection methods via neural networks" for continuous-time models are proposed in Sauzet (2022c).

3 Characterization of the Equilibrium

We now characterize the equilibrium theoretically. In Section 3.1, we start by discussing the marginal values of wealth of both investors, consumption, and good prices, which are important underpinnings for other variables in the economy. Section 3.2 discusses asset prices, and we show that a preference for green consumption gives rise to consumption premia that counterbalance the green premium on green assets stemming from the preference for green investing. Section 3.3 focuses on portfolios, and describes how a preference for green consumption leads investors to allocate a larger share of their wealth to brown assets compared to when they have solely preferences for green investing. Appendix A discusses additional theoretical results such as the evolution of the state variables (Appendix A.5).

Calibration. Unless otherwise specified, parameters are set according to the calibration of Assumption 1. What matters for the preference for green consumption of the green investor is that $\alpha^G > 1/2$. Similarly, what matters for her preference for green investing is that $\phi > 0$. Their exact values mostly have a quantitative impact that is discussed below. In practice, we pick $\alpha^G = 0.85$, and $\phi_g^G = \phi = 1\%$ to broadly match the green premium that has been estimated in the recent literature (Pástor et al., 2022; Zerbib, 2022) and the consumption premia that we obtain empirically in Section 4.[16] The elasticity of substitution across goods, θ, is also of particular interest for the direction of portfolio biases and risk

[16]The value of α^G and ϕ_g^G can easily be updated to more closely match the larger green premium and consumption premia estimated in the last few years.

premia in equilibrium (see Sections 3.2 and 3.3). We follow estimations in the environmental economics literature and set it to $\theta = 2 > 1$. In an influential contribution, Papageorgiou et al. (2017) provide evidence that this parameter significantly exceeds unity, a condition that is favorable for promoting green growth.[17]

The value of other parameters mostly have a quantitative impact, as long as (i) risk aversion γ is above 1 so that there are hedging terms, and (ii) risk aversion is not equal to the reciprocal of the EIS, $\gamma \neq 1/\psi$, so that preferences are recursive. We pick a relatively large risk aversion of $\gamma \in \{15, 25, 50\}$, to obtain average risk premia that are in line with the data. Indeed, as is well-known, consumption-based asset pricing models tend to generate somewhat modest risk premia. The effect is purely quantitative, however, and it impacts mostly the "market" component of risk premia, which is not our focus. Our novel consumption premia arise regardless of the exact value of γ, and remain quantitatively large.[18] Similarly, we pick $\psi = 1.25$ to keep a relatively low average riskfree rate, r_t. Consistent with the literature (e.g., Bansal and Yaron, 2004), $\psi > 1$, and investors have preference for early resolution of uncertainty ($\gamma > 1/\psi$). In what follows, parameters γ, ψ, ρ are also taken to be identical for both investors. However, the resolution allows for any value.

Assumption 1 (Baseline calibration)**.** *Unless otherwise specified, the results in this section are obtained under the following calibration, $i \in \{G, N\}, j \in \{g, b\}$:*

- *Preference for green consumption: $\alpha^G = \alpha = 0.85$, $\alpha^N = 1/2$,*
- *Preference for green investing: $\phi_g^G = \phi = 1\%$.*
- *Elasticity of substitution across goods: $\theta^i = \theta = 2$,*

[17] Some particular subsets of goods could be closer substitutes, for example within very narrow industries. We stay conservative, however, and stick to a moderate $\theta = 2$, because some goods and services in the basket are also likely to be more different and therefore less substitutable. Such a value is also consistent with the sign of consumption premia that we encover empirically in Section 4. The impact of the elasticity of substitution is studied in more details in the recent work of Chen et al. (2023), who show that the effect on risk premia is concentrated on sectors with high θ, consistent with our baseline calibration. The elasticity could also differ in the short- and long-run, an aspect that could be interesting to explore empirically. Finally, such a calibration with $\theta > 1$ is also consistent with the elasticity of substitution across goods in other settings. For instance, this is the case in an international context, as discussed in Imbs and Méjean (2015), among others.

[18] An alternative could be to introduce additional elements such as consumption habits. We refrain from doing so in this paper because the main benefit of such additions in this context would be to increase average risk aversion, as we do, while they could obscure the main mechanisms that we uncover. However, this represents an interesting avenue for future work.

- *Numéraire basket:* $a = 1/2$,
- *Risk aversion:* $\gamma^i = \gamma \in \{15, 25, 50\}$,
- *Elasticity of intertemporal substitution:* $\psi^i = \psi = 1.25$,
- *Discount rate:* $\rho^i = \rho = 1\%$,
- *Output:* $\mu_{Y_j} = \mu_Y = 2\%, \sigma_{Y_1} = (4.1\%, 0)^T, \sigma_{Y_2} = (0, 4.1\%)^T$ *(no fundamental correlation).*

3.1 Marginal values of wealth, consumption, goods prices

The marginal value of wealth of the investors underly many decisions in the economy. To characterize them, note that due to the homotheticity of preferences, the value functions of the investors $i \in \{G, N\}$ can be expressed as

$$V^i(W_t^i, x_t, y_t) = \left(\frac{W_t^{i1-\gamma}}{1-\gamma}\right) J^i(x_t, y_t)^{\frac{1-\gamma}{1-\psi}} \tag{9}$$

Because W_t^G, W_t^N mostly have an impact in levels, the marginal values are driven primarily by functions J_t^G, J_t^N. Therefore, in the remainder of the text, we refer to them as (monotonic transformations of) the marginal values of wealth. Those quantities underpin the dynamics of the stochastic discount factors of both investors in the economy, which in turn determine portfolios, asset prices, and other economic decisions.[19]

The evolutions of $J_t^i, i \in \{G, N\}$, are governed by two Hamilton-Jacobi-Bellman equations, summarized in Proposition A.4 in Appendix. Figure 1 shows the result for both investors in the baseline calibration as a function of the relative supply of the green good (y_t), shown on the horizontal axis, and the wealth share of the green investor (x_t), shown as different curves.

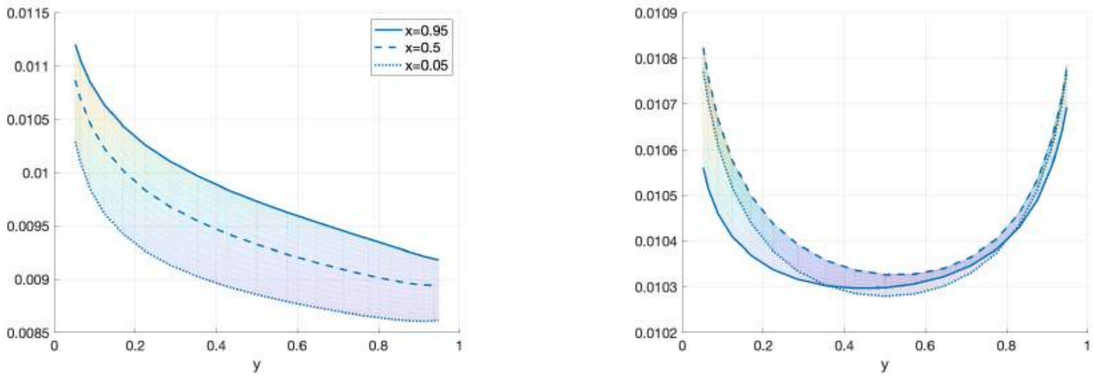
[19]The stochastic discount factors for investors $i \in \{G, N\}$ are given by

$$\xi_t^i \equiv \xi_0^i \exp\left\{\int_0^t \left(\Theta_1 P_u^{i1-\psi} J_u^i + \Theta_2\right) du\right\} W_t^{i-\gamma} J_t^{i\frac{1-\gamma}{1-\psi}}$$

with $\Theta_1 \equiv -(\gamma - 1/\psi)/(1 - 1/\psi)$ and $\Theta_2 \equiv \rho(\gamma - 1)/(1 - 1/\psi)$.

Figure 1: Marginal values of wealth

(a) Green investor (J_t^G) (b) Neutral investor (J_t^N)



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

The intuition is as follows, and will be at the core of the consumption premia and portfolio biases. As the green good becomes relatively scarce, that is, as y_t decreases, both investors have to switch some of their consumption to the brown good. The green investor is particularly negatively affected: she prefers consuming more of the green good ($\alpha^G > 1/2$), but cannot due to its low relative supply, or equivalently its high relative price. Therefore, her marginal value of consumption, which is the same as her marginal value of wealth, J_t^G, following a standard envelope argument, strongly increases. The neutral investor does not have a specific preference toward the green good ($\alpha^N = 1/2$), but still likes consuming both, due to his CES consumption basket. He is, therefore, also negatively impacted, and his marginal value of wealth, J_t^N, increases as any of the goods becomes relatively scarce ($y_t \to 0$ or $y_t \to 1$) because he would prefer a more balanced basket, that is, a more comparable relative supply or relative price of both goods. This effect for the neutral investor is, however, much more muted.

Similarly, as her share of wealth, x_t, increases, the preference of the green investor for green consumption puts upward pressure on the price of her preferred green good. This induces her to reluctantly tilt her consumption slightly toward the brown good, and her marginal value of wealth, J_t^G, increases. On the other hand, because the neutral investor has no particular bias in consumption, his marginal value of wealth, J_t^N, is little affected by x_t. In practice, the changes in the economy-wide marginal value of wealth $\widetilde{J}_t \equiv x_t J_t^G + (1-x_t) J_t^N$ are, therefore, dominated by those of J_t^G.

From the Hamilton-Jacobi-Bellman equations in A.4, a first set of first-order conditions yield expressions for consumptions, summarized in Proposition A.5, which emphasize once again the underlying role of J_t^i: $c_t^i \equiv C_t^i / W_t^i = P_t^{i-\psi} J_t^i$. Details are shown in Appendix A.7. Combining with market-clearing conditions, one obtains Equation (10) for the relative price of the green good, q_t, shown in Proposition 1.

Proposition 1. *The relative price of the green good, $q_t = q(X_t) \equiv p_{g,t}/p_{b,t}$, solves the following non-linear equation*

$$q_t = S_t^{1/\theta} \left(\frac{1-y_t}{y_t} \right)^{1/\theta}, \tag{10}$$

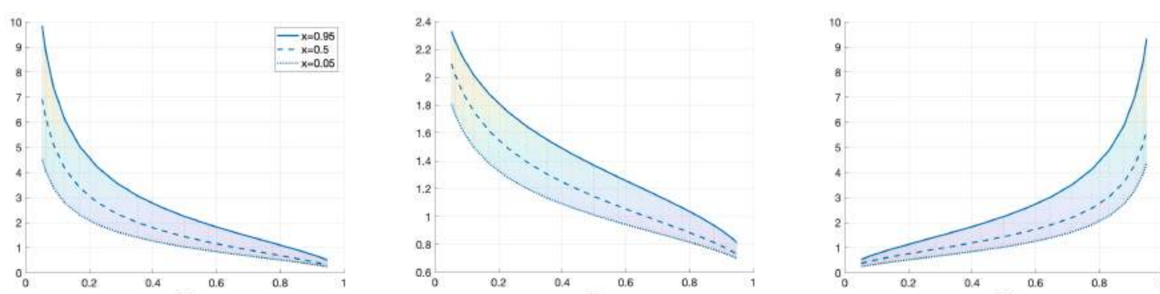
where

$$S_t = \frac{\alpha^G J_t^G x_t P_t^{G\theta-\psi} + \alpha^N P_t^{N\theta-\psi} J_t^N (1-x_t)}{(1-\alpha^G) P_t^{G\theta-\psi} J_t^G x_t + (1-\alpha^N) P_t^{N\theta-\psi} J_t^N (1-x_t)}.$$

Prices $p_{g,t}, p_{b,t}, P_t^G, P_t^N, \mathcal{E}_t$ follow from the definition of the numéraire and Proposition A.5, and are shown in Proposition A.6.

Figure 2: Relative prices and dividends

(a) Green good ($q_t \equiv p_{g,t}/p_{b,t}$) | (b) Green consumption basket ($\mathcal{E}_t \equiv P_t^G / P_t^N$) | (c) Relative dividends ($p_{g,t} Y_{g,t} / (p_{b,t} Y_{b,t})$)



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure 2 shows the resulting relative price in the baseline calibration of Assumption 1. As expected, the relative price of the green good, q_t, strongly decreases as the green good becomes more abundant, that is, as y_t increases (Panel [a]). The pattern is similar for the relative price of the consumption basket of the green investor, $\mathcal{E}_t \equiv P_t^G/P_t^N$ (Panel [b]), whose evolutions are driven by q_t as shown in Proposition A.6.

Beyond the relative prices, which drive relative consumption decisions, the relative dividends of the green asset are also of particular interest. They are shown in Panel (c) of Figure 2 and are obtained as

$$\frac{p_{g,t}Y_{g,t}}{p_{b,t}Y_{b,t}} = q_t\left(\frac{y_t}{1-y_t}\right) = S_t^{\frac{1}{\theta}}\left(\frac{1-y_t}{y_t}\right)^{\frac{1-\theta}{\theta}}. \tag{11}$$

Let us consider a situation in which the green good becomes scarce (y_t decreases). In that case, the relative quantity of output of the green tree, $Y_{g_t}/Y_{b,t} = y_t/(1-y_t)$, decreases. As discussed above, the relative price of the green good, $p_{g,t}/p_{b,t}$, therefore, increases. However, because the green and brown goods remain substitutable enough ($\theta > 1$), the effect on the relative price remains muted and the effect of the relative quantity of output dominates; thus, the relative dividends of the green tree decrease overall. In other words, relative dividends and relative supply move in the same direction, an observation that will prove important for the direction of portfolio biases and risk premia. Indeed, as we discuss in Section 3.2, relative dividends are the main drivers of the relative returns on the two assets, while changes in dividend yields (i.e., equity prices relative to fundamentals) play a limited role.

The case in which green and brown goods are very poor substitutes (broadly $\theta < 1$)[20] would have the counterintuitive implication that the payoff of an asset would be *low* when the quantity of goods that it produces is *high*. Most importantly, it is also inconsistent with empirical estimates in the environmental economics literature that put θ strongly above unity, a condition that is also favorable for promoting green growth (see, for instance, Papageorgiou et al., 2017), as well as with the sign of consumption premia that we encover empirically. Therefore, although our setup accomodates different calibrations for θ (as in the recent work of Chen et al., 2023), we focus on the case $\theta > 1$ that seems most relevant. Note that Chen

[20]Coeurdacier (2009) shows in a CRRA context based on zero-order approximations that the exact value at which the switch occurs is a non-linear function of all parameters, although it is close to 1.

et al. (2023) also show that the effect on risk premia is concentrated on sectors with high θ, consistent with our baseline calibration.

Finally, the relative dividends of the green asset also increase as the wealth share of the green investor increases, consistent with her preference for green consumption, which puts an upward pressure on the relative price of the green good. However this effect is much more muted in the baseline calibration.

3.2 Asset prices

Second moments. Let us start with second moments, which underpin part of the intuition for risk premia, and portfolios.

Recall that the diffusion terms for both asset returns, $j \in \{g, b\}$, are $\sigma_{j,t} \equiv \sigma_{p_j,t} + \sigma_{Y_j} - \sigma_{F_j,t}$.[21] Those capture how returns load on the different shocks in the economy. In practice, although dividend yields, $F_{j,t}$, are time-varying in our setting with recursive preferences and heterogeneity, the effect of their changes via $\sigma_{F_j,t}$ remains comparatively muted. The patterns in returns diffusions are, therefore, mostly driven by $\sigma_{p_j,t}, \sigma_{Y_j}$, that is, by movements in the relative dividends of both assets. In turn, the relative dividends of the green asset, $p_{g,t}Y_{g,t}/(p_{b,t}Y_{b,t})$, evolve in the same direction as relative supply, $Y_{g,t}/Y_{b,t}$. As described in Section 3.1 above, the intuition is that goods are good enough substitutes ($\theta > 1$), so that the effect on the relative price of the goods, $p_{g,t}/p_{b,t}$, is moderate enough to not overturn the impact of the relative supply, $Y_{g,t}/Y_{b,t}$. *In fine*, this implies that for most of the state space, the returns on the green asset tend to load more on the output shocks to the green tree ($\sigma_{gZ_g,t} > \sigma_{gZ_b,t}$) because those shocks increase the relative dividends of the green tree, while the returns on the brown asset load more on the output shocks to the brown tree ($\sigma_{bZ_b,t} > \sigma_{bZ_g,t}$) because those shocks increase the relative dividends of the brown tree.[22]

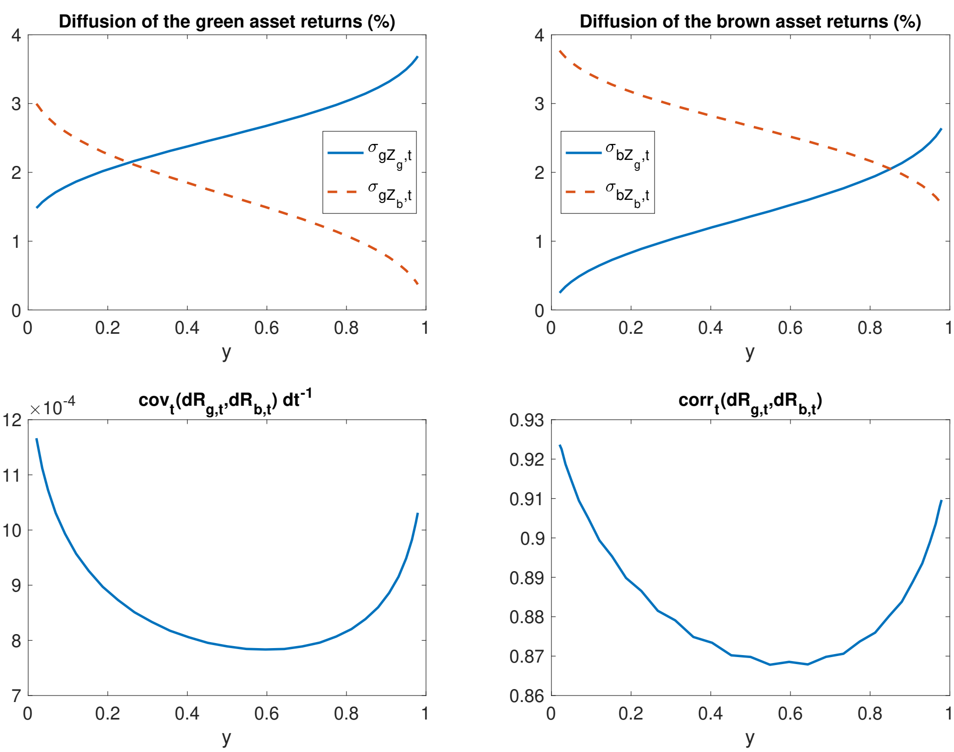
In short: the green asset tends to earn higher returns when the green good is relatively

[21] This expression comes from an application of Itô's Lemma to the definition of returns in Equation (1), $dR_{j,t} = dQ_{j,t}/Q_{j,t} + F_{j,t}dt$, with $Q_{j,t} = p_{j,t}Y_{j,t}/F_{j,t}$. See Section 2 and Appendix A.

[22] Note that if θ were to be below unity, movements in the relative prices of the goods would be so extreme that relative dividends would move invertedly with relative supply, so that the returns on the *green* asset would ultimately load more on the output shock to the *brown* tree. This implication is both counterintuitive and inconsistent with empirical estimates in the environmental economics literature, which puts θ strongly above unity (see, for instance, Papageorgiou et al., 2017), as well as with the sign of the consumption premia that we encover. See discussion in Section 3.1.

more abundant, that is, when positive shocks to the supply of the green tree occur, which increase its relative supply, y_t. Figure 3 shows the diffusion terms for the returns on both assets in the baseline calibration, and confirms that this is indeed the case: on average, the loading of the green asset returns on the green output shock ($\sigma_{gZg,t}$, blue curve) is larger than their loading on the brown output shock ($\sigma_{gZb,t}$, orange curve), and vice versa for the brown asset returns.

Figure 3: Second moments of returns



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows a cut in which $x_t = 1/3$, consistent with empirical estimates.

Beyond the main patterns discussed above, the returns on both assets also load on both shocks, albeit in a more limited way. This leads the assets to be strongly correlated: on average, $\text{corr}_t(dR_{g,t}, dR_{b,t}) \approx 0.9$, even though the outputs of the trees themselves have no fundamental correlation ($\sigma_{Y_g Z_b} = \sigma_{Y_b Z_g} = 0$). In other words, the large correlation between asset returns emerges endogenously. This phenomenon is driven by movements in the relative prices of the goods and in the allocation of wealth, as well as by the patterns of the marginal values of wealth of both investors J_t^i (and hence of their stochastic discount factors). Economically, this emphasizes the financial contagion taking place through asset markets in this economy: a shock to the output of a given tree has a sizable impact on the returns of the other tree, and can, therefore, impact both investors beyond its impact on goods markets. The bottom panels of Figure 3, together with Figure B.8 in the Appendix, also emphasize that second moments are inherently time-varying and depend on the current state of the economy.

Risk premia. Proposition 2 presents the expected excess returns on the green and brown assets. Proposition A.1 in the Appendix generalizes those expressions to the case in which risk aversion and EIS differ across investors, and in which both investors have preferences toward both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t.

Proposition 2. *The expected risk premia on the green and brown equity assets are*

$$\begin{aligned} \mu_{g,t} - r_t &= \gamma \sigma_{g,t}^T \sigma_{\widetilde{W},t} - x_t \phi - \gamma \sigma_{g,t}^T \sigma_{\tilde{J},t} \\ \mu_{b,t} - r_t &= \gamma \sigma_{b,t}^T \sigma_{\widetilde{W},t} \qquad\quad - \gamma \sigma_{b,t}^T \sigma_{\tilde{J},t} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \sigma_{\widetilde{W},t} &\equiv w_{g,t}^M \sigma_{g,t} + (1 - w_{g,t}^M) \sigma_{b,t} \\ \sigma_{\tilde{J},t} &\equiv \left(\frac{1}{\gamma}\right) \left(\frac{1-\gamma}{1-\psi}\right) \left(x_t \sigma_{J^G,t} + (1 - x_t) \sigma_{J^N,t}\right) \end{aligned}$$

and $\widetilde{W}_t$ is the total wealth, $\widetilde{J}_t$ is the economy-wide marginal value of wealth, and $\sigma_{J^G,t}, \sigma_{J^N,t}$ are the geometric diffusion terms of J_t^G, J_t^N obtained as in Remark A.1.

The expressions for risk premia are composed of three terms.

The first term is a *total wealth component* driven by the covariance of each risky asset return with the total wealth in the economy, $\widetilde{W}_t$. It can be thought of as a "market" component. Intuitively, an asset that comoves a lot with total wealth provides little diversification benefits, is therefore risky, and commands a high risk premium in equilibrium. This is the usual financial diversification component that exists even when investors are myopic, and makes them want to hold some of both assets to maximize the Sharpe ratio of their portfolios.

The second term is the green premium characterized by Pástor et al. (2021) and Zerbib (2022), among others. Because the green investor has a preference toward investing in the green asset ($\phi > 0$), she accepts a lower expected return to hold it and the expected returns on that asset decrease. In addition, this effect scales with the wealth share of the green investor, x_t. Because we set the preferences of both investors toward the brown asset to zero ($\phi_b^G = \phi_b^N = 0$), the brown asset does not display any such premium. This is without loss of generality, however, and the green premium term should be understood in a relative sense between green and brown assets.

The third term is a hedging component that constitutes our novel consumption premia, and deserves more emphasis.

From a broad perspective, this term is driven by the comovement of asset returns with the economy-wide wealth-weighted marginal value of wealth, $\widetilde{J}_t \equiv x_t J_t^G + (1 - x_t) J_t^N$. Intuitively, an asset whose returns are large when $\widetilde{J}_t$ is large is a good hedge because it pays when it is most valuable for the average investor, that is, for the economy as a whole. Thus, such an asset is less risky, and commands a lower risk premium in equilibrium.[23]

Importantly, note that such hedging components—and, therefore, our novel consumption premia—would be completely absent with log, mean-variance, or CARA preferences that have been popular in the literature, because investors would be myopic under those specifications.

To make the intuition more precise, note that in our Markovian setting, we can break

[23]In the terminology of the asset pricing literature, those consumption premia embed the desire of investors to hedge against changes in their investment opportunities, captured here by the state variables $X_t = (x_t, y_t)'$.

down the hedging term as follows[24]

$$
\begin{aligned}
-\gamma \sigma_{j,t}^T \sigma_{\widetilde{J},t} = & - \sigma_{j,t}^T \sigma_{x,t} x_t \left(\frac{1-\gamma}{1-\psi} \right) \left\{ x_t \frac{J_{x,t}^G}{J_t^G} + (1-x_t) \frac{J_{x,t}^N}{J_t^N} \right\} \\
& - \sigma_{j,t}^T \sigma_{y,t} y_t \left(\frac{1-\gamma}{1-\psi} \right) \left\{ x_t \frac{J_{y,t}^G}{J_t^G} + (1-x_t) \frac{J_{y,t}^N}{J_t^N} \right\}.
\end{aligned} \tag{13}
$$

In words, the novel consumption premia are composed of a wealth-hedging premium (hedging of movements in the wealth share of the green investor, x_t), and a relative-supply-hedging premium (hedging of movements in the relative supply of the green good, y_t, or, equivalently, of its relative price).

Establishing the sign of those consumption premia for green versus brown assets requires eliciting the patterns of quantities of risk, as well as the prices of those risks.

The quantities of risk are driven by the (instantaneous) covariances of the asset returns with the state variables, x_t, y_t, that fully characterize the state of the economy: $\text{cov}_t\left(dR_{j,t}, dx_t\right) dt^{-1} = \sigma_{j,t}^T \sigma_{x,t} x_t$ and $\text{cov}_t\left(dR_{j,t}, dy_t\right) dt^{-1} = \sigma_{j,t}^T \sigma_{y,t} y_t$. On average, we expect the latter, $\text{cov}_t\left(dR_{j,t}, dy_t\right) dt^{-1}$, to be positive for the green asset, and negative for the brown asset. That is, we expect the returns on the green (brown) asset to increase (decrease) with the relative supply of the green good, y_t. This is because, as explained above, the returns on the green asset tend to load more on shocks to the green output, $dZ_{g,t}$, which also increase $Y_{g,t}$ and, therefore, increase the relative supply of the green good, $y_t \equiv Y_{g,t}/(Y_{g,t} + Y_{b,t})$. Conversely, the returns on the brown asset tend to load more on $dZ_{b,t}$, which also increase $Y_{b,t}$ and, therefore, decrease y_t. The sign of $\text{cov}_t\left(dR_{j,t}, dx_t\right) dt^{-1}$ depends on the covariance between x_t and y_t, which is endogenous and depends on investors' portfolios, which in turn depend on ϕ. It is discussed below.

The remaining pieces are the prices of those risks, which summarize how investors *value* an asset with certain quantities of risk. Those prices of risk are driven by preference parameters γ, ψ, but most importantly by how the economy-wide wealth-weighted marginal value of wealth evolves with those state variables: this is captured by $J_{x,t}^i$, $J_{y,t}^i$, the derivatives of

[24]Again, the framework allows for potentially different γ^i, ψ^i, ρ^i for both investors. In that case, the economy-wide risk aversion is state-dependent, $\gamma_t \equiv \left(x_t/\gamma^G + (1-x_t)/\gamma^N\right)^{-1}$, and the weighting in the economy-wide marginal value of wealth $\widetilde{J}_t$ also reflects differences in those parameters.

the marginal values of wealth of both investors $i \in \{G, N\}$ with respect to each state variable. The economy is composed of an investor with a preference for green consumption and an investor who is neutral, so that it has, on average, a tilt toward preferring the green good. In other words, because J_t^G strongly decreases with the relative supply of the green good ($J_{y,t}^G \ll 0$), the economy-wide wealth-weighted marginal value of wealth $\tilde{J}_t$ is, on average, a decreasing function of y_t. That is, situations in which y_t is low—and, thus, the relative price of the green good is high—are adverse states of the world, and the price of y_t-risk is positive (recall the minus sign in Equation (13)). Therefore, an asset that comoves with the relative supply of the green good y_t is risky, because it is a poor hedge against those adverse states and hence, commands a higher risk premium in equilibrium. Conversely, the price of x_t-risk is expected to be negative on average. Indeed, as shown in Section 3.1, the evolution of $\widetilde{J}_t$ with x_t is again dominated by the marginal value of wealth of the green investor J_t^G, which tends to increase with her wealth share x_t ($J_{x,t}^G > 0$) due to the upward pressure she puts on the price of her preferred green good.

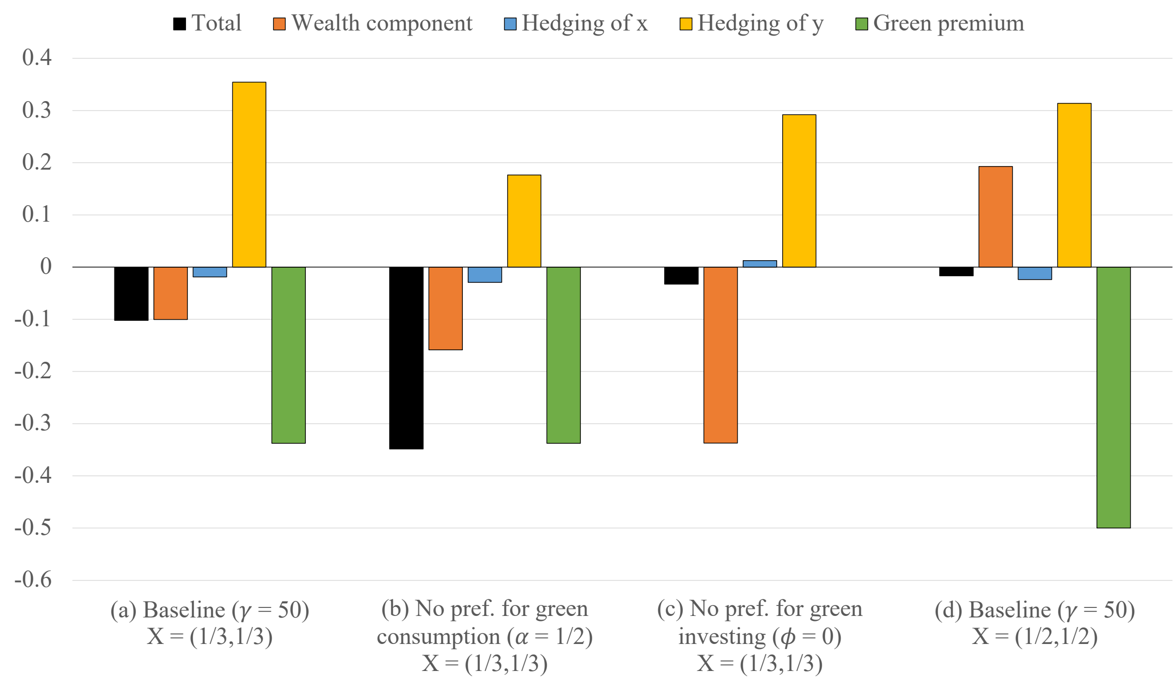
Therefore, taken together, we expect the green asset, whose returns comove positively with y_t, to be riskier in terms of relative supply risk than the brown asset whose returns comove negatively with y_t. Therefore, the green asset is expected to command a higher relative-supply-hedging premium on average. The sign of the hedging of the wealth share risk is more ambiguous and is discussed below. It turns out to be negative but small in our benchmark.

Panel (a) of Figure 4 shows the *difference* in expected returns between the green and the brown asset, $(\mu_{g,t} - r_t) - (\mu_{b,t} - r_t)$, as well as its components, in the baseline calibration of Assumption 1. To get a sense of the average premia differentials, they are shown at the point at which the green investor holds one third of the wealth ($x_t = 1/3$), and the relative supply of the green good is one third ($y_t = 1/3$), broadly consistent with empirical estimates in Morgan Stanley Institute for Sustainable Investing (2019). The wealth component, $\gamma \sigma_{j,t}^T \sigma_{\widetilde{W},t}$, which can be understood as a market component, is important to get risk premia that are quantitatively broadly in line with the data: on average, $\mu_{j,t} - r_t \approx 4.2\%$, only slightly lower than their empirical counterparts.[25] In practice, this component mostly depends on how

[25] Getting such values for the average risk premia is the main reason for which we pick a high calibration of risk aversion, γ. Indeed, as is well-known, consumption-based asset pricing models tend to generate somewhat modest risk premia (see footnote 18).

dominant a given asset is in total wealth, that is, on the weights of the assets in the market portfolio $(w^M_{g,t}, w^M_{b,t})$. The wealth component, therefore, drives the overall shape of the risk premia on both assets with the state of the economy, especially with respect to the relative supply.[26] Because this term is more common, however, it is not our focus in this paper.

Figure 4: Average difference in risk premia between green and brown asset $(\mu_{g,t} - r_t) - (\mu_{b,t} - r_t)$ (%)



Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows the difference between risk premia on the green and brown asset, and their components, at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d).

[26]The market weights $(w^M_{g,t}, w^M_{b,t})$ are inherently related to y_t, the relative supply of both goods, as seen in Panels (a) and (b) of Figure B.12 in Appendix. They are equal for both assets broadly around the point at which their relative supply is equal, $y_t = 1/2$, although the preference for green investing leads the green asset to be slightly overvalued so that its weight in the market portfolio $(w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}))$ is slightly larger on average. In other words, the wealth component is slightly larger for the green asset on average, even though the difference is dominated by variations with the state of the economy.

Of more interest are the green premium and the novel consumption premia.

When the green investor holds about one third of total wealth, like in the data, the green premium is $-x_t\phi \approx -0.333\%$ in this baseline calibration. Recall that this green premium should be interpretated in a relative way, so that on average the expected excess returns on the green asset is 33.3 basis points (bps) smaller than those on the brown asset, when we focus purely on the effect of the preference of investors for green investing. This is also visible from Panel (c), which sets $\phi = 0$. There is considerable uncertainty about the magnitude (and even sign) of the green premium in practice, and 33.3 bps could be consistent with some empirical estimates (Zerbib, 2022). We do not take a strong stance on this magnitude, but note that the ϕ parameter can be easily adjusted to yield a green premium consistent with larger estimates (e.g., Pástor et al., 2022), without any change to the main intuition.

Most importantly—and this is our main result—consumption premia can broadly compensate the green premium: on average (i.e., at $X_t = (1/3, 1/3)$), the expected excess returns on the green asset is 36.6 basis points larger than those on the brown asset, when we focus purely on the effect of the preference of investors for green consumption. In other words, the only reason why $\mu_{g,t} - r_t$ is larger overall at $X_t = (1/3, 1/3)$ is the mostly mechanical total wealth component. This is visible in our baseline calibration of Panel (a). This can be compared to the case in which the green investor has no preference for green consumption ($\alpha = 0.5$) so that the green premium dominates,[27] and in which she has no preference for green investing ($\phi = 0$) so that the consumption premia dominate. Such a magnitude for the consumption premia is consistent with the empirical counterpart that we estimate at the beginning of our sample (see Section 4) but the effect can in fact be larger, for example, for larger risk aversion γ, an EIS closer to $\psi = 1$, larger bias toward consumption α, a slighty lower θ, and in some parts of the state space, as discussed below. Such calibrations could readily deliver a larger magnitude, in line with its empirical value in more recent years.

Consistent with our intuition above, the bulk of the consumption premia is driven by a positive relative-supply-hedging premium. To say a brief word about the hedging of relative wealth risk: it is negative for both assets, and in particular, slightly more negative

[27]Note that there still exist consumption premia even without preference for green consumption ($\alpha = 1/2$). This is because investors still want to consume both goods, and, therefore, remain sensitive to movements in relative prices (i.e., they still want to hedge against changes in their investment opportunities). The effect is quantitatively more muted, but continues to show the appeal of bringing green investors to this general equilibrium context in which they also consume, even in that case.

for the green asset. This stems from the larger covariance of green asset returns with the wealth share, x_t, that is, from a larger quantity of "x_t"-risk (Figure B.6), combined with the negative price for that risk described above (Figure B.7).[28] In practice, this conclusion depends on the magnitude of the bias in portfolio holdings, and therefore, on the calibration. For instance, this premium is positive if preferences for green consumption, α, are strong enough, while preferences for green investing, ϕ, are moderate enough, like in Panel (c) of Figure 4. However, in most cases, and regardless of its sign, the magnitude of this effect remains quantitatively small (e.g., -1.9 bp in the baseline). It is, therefore, not our focus here, and we leave it aside in the empirical part of Section 4.[29]

Dynamics of risk premia. Interestingly, those patterns of the risk premia and their subcomponents also vary strongly with the state of the economy, an aspect that our global solution allows to explore.

Panels (a) of Figure 5 shows that the expected risk premium on the green asset $\mu_{g,t} - r_t$ increases, in particular, as the relative supply of the underlying green tree, y_t, increases. This pattern is driven by the wealth component shown in Panel (c): as y_t becomes large, the green good starts to dominate the economy, so that the green asset also starts to dominate total wealth ($w^M_{g,t} \equiv Q_{g,t}/[Q_{g,t} + Q_{b,t}]$, the weight of the green asset in the market portfolio, increases toward 1). In such situations, the risk on the green asset is difficult to diversify away so that the green asset is risky and commands a higher risk premium.[30] Conversely,

[28]In more details, the intuition for this negative relative premium is as follows. In the baseline, the wealth share loads more on shocks to the output of the green tree ($dZ_{g,t}$): Figures A.3 and A.4 in Appendix indeed show that $\sigma_{xZ_g,t}x_t > \sigma_{xZ_b,t}x_t$ in magnitudes, and $\sigma_{xZ_g,t}x_t > 0$ for any X_t, while the loading of x_t on shocks to the brown output $\sigma_{xZ_b,t}x_t$ flips sign, for example, as y_t increases. As shown in Proposition A.3 in Appendix, the loadings of the wealth share are themselves endogenous and follow those patterns provided that the portfolio of the green investor is biased enough toward the green asset ($w^G_{g,t} - w^M_{g,t} > w^G_{b,t} - w^M_{b,t}$). In short, in the baseline, a positive shock to the output of the green tree tends to increase the wealth share x_t. (Relatedly, because the relative supply y_t also tends to increase with positive shocks to the green output, the wealth share x_t and relative supply y_t are positively correlated in the baseline.) Because such a shock also tends to increase the returns on the green asset more, as explained previously, this leads the covariance of the green asset returns with x_t to be larger than that of the brown asset returns. In other words, the quantity of x_t "risk" is larger for the green asset (Figure B.6). Combined with the negative price of this risk described above (Figure B.7), this leads to the negative relative wealth-share-hedging premium for the green asset in the baseline calibration. In most cases, however, the magnitude of this effect remains small.

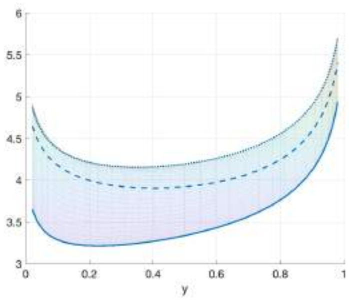
[29]The introduction of a tax on dividends, as discussed in Section 5, can reinforce the impact of x_t-hedging, because it can lead to imperfect risk sharing across investors.

[30]In other words, because the green asset dominates total wealth as y_t becomes large, the covariance of

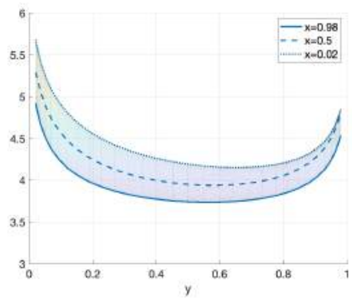
the expected returns on the brown asset increases as y_t decreases, as shown in Panel (b).[31]

Figure 5: Returns

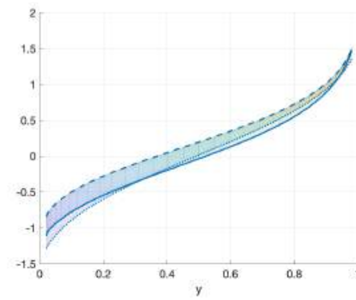
(a) Total risk premium on green asset $(\mu_{g,t} - r_t)$



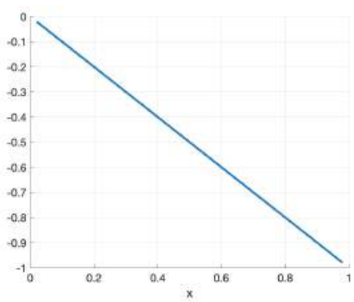
(b) Total risk premium on brown asset $(\mu_{b,t} - r_t)$



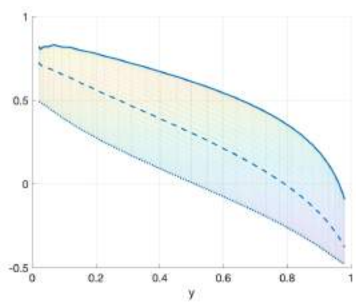
(c) Relative "market" component $(g - b)$



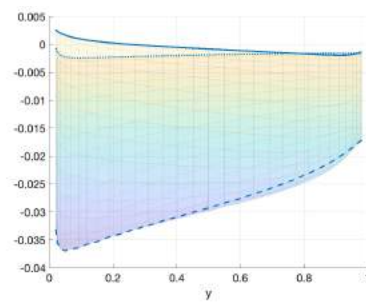
(d) Relative green premium $(-x_t\phi, g - b)$



(e) Relative y_t-hedging premium $(g - b)$



(f) Relative x_t-hedging premium $(g - b)$



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

this asset with total wealth is large because it is broadly equal to the covariance of the asset with itself. This leads the wealth component of the risk premia, which is driven by the covariance with total wealth, to be large for the green asset.

[31]Figure B.3 in Appendix also shows that states of the world in which one of the good becomes scarce (low or high y_t) are associated with a lower riskfree interest rate r_t, consistent with higher precautionary saving motives. Note that in some calibrations, for example, with $\gamma = 50$, r_t is negative, which is in line with real interest rates being negative empirically in the recent period even for longer maturities (e.g., Figure B.2 in Appendix shows that this is case for the 10-year market yield on inflation-indexed U.S. Treasury Securities since 2019). This has no particular impact on the equilibrium. For instance, Figure B.3 in Appendix shows that $r_t > 0$ for $\gamma = 15$, and risk premia and portfolios in that case are similar to those with a larger γ except in terms of magnitude.

The expected excess returns on both assets also decrease with the share of wealth held by the green investor, x_t (Panels (a) and (b)). For the green asset, this is mostly driven by the increasing impact of the preference for green investing, ϕ, as the green investor becomes larger in the economy, that is, by an increasing green premium. This can be seen in Panel (d), which plots the green premium on the green asset relative to the brown asset as a function of x_t. For the brown asset, however, this pattern is driven by the state-dependence in the hedging of relative supply risk, which becomes more strongly negative for the brown asset as the green investor—who is more worried about this risk—holds increasingly more wealth. The riskfree interest rate also increases with x_t, a fact that is consistent with the pattern of borrowing and saving discussed in Section 3.3.

Panels (e) and (f) confirm that consumption premia are themselves very time-varying. The hedging of relative-supply risk for the green asset relative to the brown asset is positive and large for most of the state-space, as shown in Panel (e). It increases as the green investor—who is particularly worried about this risk—becomes larger in the economy, that is, as x_t increases. This positive relative premium on the green asset also strongly increases as the relative supply of the green good, y_t, decreases: for example, it reaches close to 1% for large x_t and small y_t. This is consistent with the green investor being *especially* worried about relative-supply risk when her preferred good becomes very scarce, and suggests that hedging terms can grow and continue to compensate the green premium, even as the latter becomes larger when the green investor becomes dominant. Finally, as discussed above and shown in Panel (f), the relative premium on the green asset stemming from wealth share hedging is negative on average in the baseline, and largest in magnitude around $x_t = 1/2$, the point at which the identity of the investor dominating the economy flips. However, again, because it is quantitatively much more muted, it is not our focus here, and we leave it aside in the empirical part of Section 4.[32]

Risk premia vs. discount rates. Lastly, obtaining quantitatively plausible risk premia, $\mu_{j,t} - r_t$, as well as a reasonable riskfree rate, r_t, as is made possible by recursive preferences (Weil, 1989), turns out to be particularly important in this context. In an economy with CRRA preferences, $\gamma = 1/\psi$, the large riskfree rate can quantitatively dwarf any differential

[32]The patterns of the x_t- and y_t-premia are in turn driven by changes in both the quantities and prices of risk with the state of the economy $X_t = (x_t, y_t)'$, as shown in Figures B.6 and B.7 in Appendix.

impact on risk premia between the green and brown assets emerging from green preferences. Indeed, Figure B.4 in Appendix shows that, in this case, the riskfree rate, r_t, is so large that its increase, as the green investor gets larger, can dominate any change in risk premia, $\mu_{j,t} - r_t$. In other words, the *discount rates* on both assets, which are the sum of riskfree rate and risk premia, $\mu_{j,t} = r_t + (\mu_{j,t} - r_t)$, can *in*crease with the wealth share of the green investor, x_t. Even though the increase is smaller for the green asset, because the risk premia differential remains negative (despite the counterbalancing effect of the consumption premia), the difference can be small as compared to the magnitude of the increase in discount rate. Instead, recursive preferences ensure that the riskfree rate remains moderate in magnitude, consistent with empirical estimates, so that green preferences can have a meaningful differential impact on discount rates, as show in Figure B.5. This aspect has not been emphasized in the sustainable asset pricing literature so far, but is not innucuous: ultimately, the difference in relative cost of capital between the green and brown assets must be large enough quantitatively, compared to other changes, to have a sizable impact and foster the green transition.

3.3 Portfolios

We conclude this characterization by a discussion of the optimal portfolios of both investors. Proposition 3 shows that those are Merton (1973)-type portfolios that are composed of two pieces.[33]

The first term is similar for both investors and corresponds to the myopic portfolio that would be chosen by a one-period mean-variance investor. It is the usual financial diversification component driven by the risk premia on both assets, normalized by volatilities, partly related to the market portfolio ($w^M_{g,t}$, $w^M_{b,t}$).

In this context, however, this first term also embeds the preference of the green investor for green assets, ϕ. Equation (14) shows that it is isomorphic to the expected returns on the green asset being perceived as (relatively) larger by the green investor. As expected, this term, therefore, makes her tilt her portfolio allocation toward the green asset in equilibrium. In other words, it is the manifestation of the green premium for portfolios.

[33] Again, Proposition A.2 in Appendix generalizes those expressions to the case in which risk aversion and EIS differ across investors, and in which both investors have preferences toward both assets, $\phi^i_j \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t.

Proposition 3. *The optimal portfolios of the green and neutral investors $j \in \{G, N\}$ are given by*

$$\begin{pmatrix} w^G_{g,t} \\ w^G_{b,t} \end{pmatrix} = \frac{1}{\gamma}\left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi \\ \mu_{b,t} - r_t \end{pmatrix} + \left(\frac{1-\gamma}{1-\psi}\right) \Sigma_t^T \left(\frac{J^G_{x,t}}{J^G_t} x_t \sigma_{x,t} + \frac{J^G_{y,t}}{J^G_t} y_t \sigma_{y,t} \right) \right\}$$
$$b^G_t = 1 - w^G_{g,t} - w^G_{b,t} \tag{14}$$

$$\begin{pmatrix} w^N_{g,t} \\ w^N_{b,t} \end{pmatrix} = \frac{1}{\gamma}\left(\Sigma_t^T \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t \\ \mu_{b,t} - r_t \end{pmatrix} + \left(\frac{1-\gamma}{1-\psi}\right) \Sigma_t^T \left(\frac{J^N_{x,t}}{J^N_t} x_t \sigma_{x,t} + \frac{J^N_{y,t}}{J^N_t} y_t \sigma_{y,t} \right) \right\}$$
$$b^N_t = 1 - w^N_{g,t} - w^N_{b,t} \tag{15}$$

where $w^i_{g,t}, w^i_{b,t}, b^i_t$ are the portfolio weights (as a share of wealth) allocated to the green equity asset, the brown equity asset, and the riskless bond, and $\Sigma_t \equiv \begin{bmatrix} \sigma_{g,t} & \sigma_{b,t} \end{bmatrix}$.

The second component are hedging terms. They are driven by preferences for green consumption and are absent with log or myopic preferences (such as CARA). They are the counterpart for portfolios of the consumption risk premia, and capture the way investors tilt their allocation to insure against changes in the state of the economy, summarized by $X_t = (x_t, y_t)'$. Investors do so by overweighting assets whose payoffs are large when they find it most valuable, that is, when their individual marginal values of wealth are high, so that hedging terms are governed by the covariance between risky returns and individual marginal values of wealth, J^G_t, J^N_t.

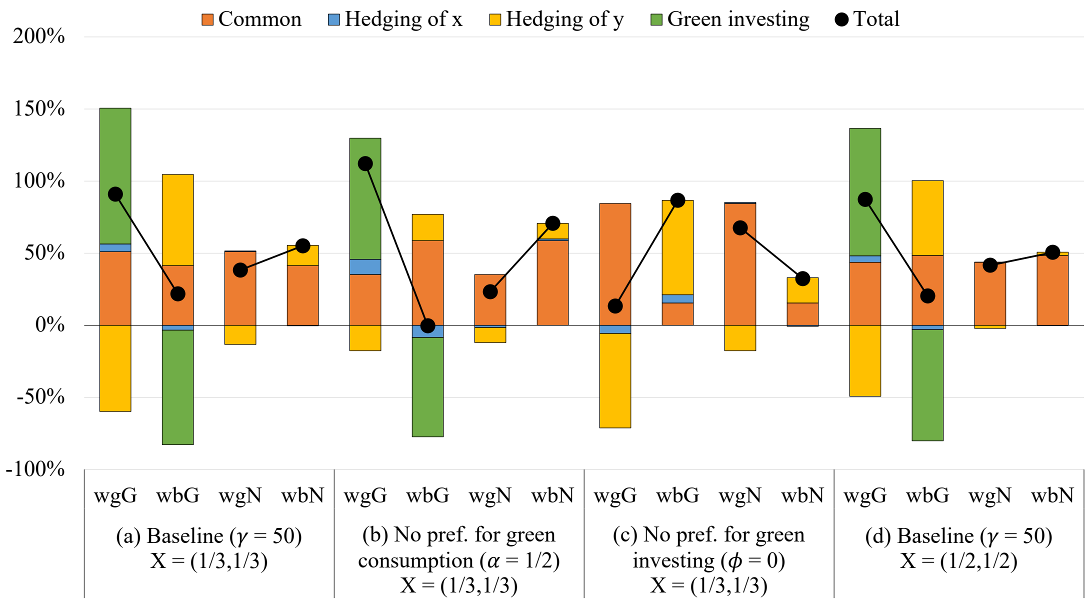
Overall, the common term drives the broad pattern of the portfolios of both investors throughout the state space, corrected for the preference of the green investor for green investing, ϕ, while the hedging term captures how investors differentially deviate from this broad pattern. Hedging terms are, therefore, a prime quantity of interest in our economy with heterogeneous investors.

Figure 6 shows the corresponding portfolio weights on the risky assets for each investor as a percentage of their wealth ($w^i_{g,t}$, $w^i_{b,t}$ for $i \in \{G, N\}$), as well as their components, for various calibrations.[34] Like for risk premia, to get a sense of average portfolios, all those variables are shown at the point at which the green investor holds one third of the wealth ($x_t = 1/3$),

[34]We plot the portfolio weight that each investor allocates to the riskless bond to borrow or save (b^i_t)

and the relative supply of the green good is one third ($y_t = 1/3$), broadly consistent with empirical estimates in Morgan Stanley Institute for Sustainable Investing (2019) (except for Panel (d) for which $X_t = (1/2, 1/2)$).

Figure 6: Portfolios at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows portfolios and their components at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d). $w^i_{g,t}, w^i_{b,t}$ are the weights (as % of wealth) on the green and brown asset in the portfolio of investor $i \in \{G, N\}$.

Panel (a) shows that in the baseline calibration, the green investor significantly tilts her allocation toward the green asset: on average (i.e., at $X_t = (1/3, 1/3)$), she invests $w^G_{g,t} \approx 91\%$ of her wealth in it, as opposed to $w^G_{b,t} \approx 22\%$ in the brown asset. This is significantly more

in Figure B.10 in Appendix. Figure B.9 also shows the weights in the *market* portfolio for comparison: $w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$, $w^M_{b,t}$. Recall that the bond is in zero net supply so that for the market overall, $b^M_t = 0$, and $w^M_{b,t} = 1 - w^M_{g,t}$.

biased toward the green asset than the market portfolio, $w_{g,t}^M \approx 56\%$, $w_{b,t}^M \approx 44\%$ (Figure B.9 in Appendix). The neutral investor, because he is less sensitive to changes in relative supply and wealth share, is willing to take the other side of this trade: on average, he invests more of his wealth in the brown asset, $w_{b,t}^N = 55\%$, than in the green asset, $w_{g,t}^N = 38\%$.

As expected, the overweighting of the green asset in the portfolio of the green investor is driven by her preference toward green investing ($\phi > 0$), shown in green in Figure 6. In the baseline, this component taken separately would lead her to overweight the green asset by an *additional* 94% of her wealth, substantially beyond the 51% dictated by the common component (shown in orange) that is identical for both investors. Conversely, it would lead her to underweight the brown asset by 79% of her wealth, compared to the 41% dictated by the common component.

Most importantly—and this is our main novel result in terms of portfolios—the impact of green investing is again strongly counterbalanced once it is brought to our general equilibrium context. Indeed, the hedging term related to the relative supply (shown in yellow in Figure 6), which is mostly stemming from the preferences of the green investor toward green consumption ($\alpha > 1/2$), leads her to *under*weight the green asset by about 60% of her wealth. This arises because the returns on the green asset are comparatively much smaller when the relative supply of the green good, y_t, is low (i.e., when the relative price of green good is high; see Section 3.2), that is, when the green investor values it most because her marginal value of wealth, J_t^G, is high in those states of the world (Section 3.1).

Overall, her preference for green consumption leads the green investor to cut the overweighting stemming from green investing substantially. This is also visible in Panel (c), which shows that when she has no preference for green investing ($\phi = 0$), she would end up investing much more of her wealth in the brown asset overall ($w_{g,t}^G \approx 13\%$, $w_{b,t}^G \approx 87\%$). Therefore, the green investor would pick a portfolio that is biased toward the *brown* asset in equilibrium, compared to the market portfolio. Conversely, the counterbalancing impact of hedging terms is also visible in Panel (b): without preference for green consumption ($\alpha = 1/2$), the green investor would invest an even larger share of her wealth in the green asset ($w_{g,t}^G \approx 112\%$, $w_{b,t}^G \approx 0\%$).

A few remaining comments on portfolios are in order.

First, like for risk premia, the impact of the hedging of the wealth share depends on the calibration, but remains in most cases more muted. In the baseline, it leads the green investor to increase back the weight in her portfolio on the green asset slightly as seen in Figure 6 (blue component).

Second, because the green investor is more sensitive to the risks associated with consumption preferences, especially the one related to the relative supply, she is more eager to strongly tilt her portfolio according to her preferences. In practice, she is in fact willing to borrow in the riskless bond to *lever* her risky portfolio weights slightly: at $X_t = (1/3, 1/3)$, she borrows $|b_t^G| = |1 - w_{g,t}^G - w_{b,t}^G| = 13\%$ of her wealth (Figure B.10 in Appendix). The remaining investor, because he is neutral, is willing to accommodate the green investor by lending $b_t^N = 6\%$ of his wealth. Those patterns of borrowing and lending are also reflected in the riskfree rate: r_t increases as x_t increases, that is, as the green investor, who is a borrower, gets a larger share of total wealth. Introducing portfolio constraints, for example, such as borrowing or shorting limits, could be an interesting avenue for further research that could enrich those phenomena.

Third, because of the preference for green investing and green consumption of the "average investor" in the baseline, the equity price of the green asset $Q_{g,t}$ is slightly overvalued compared to an economy with $\phi = 0$ and $\alpha = 1/2$. In other words, the weight on the green asset in the market portfolio, which is nothing but its equity price divided by total wealth $w_{g,t}^M \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$, is slightly larger than the weight on the brown asset in the market portfolio, $w_{b,t}^M \equiv Q_{b,t}/(Q_{g,t} + Q_{b,t})$.[35] This is visible in Figure B.9 in the Appendix, which plots the market portfolios in various cases, especially in Panel (d), which focuses on the symmetric point $X_t = (1/2, 1/2)$. At that point, the weight in the market portfolio would be $w_{g,t}^M = w_{b,t}^M = 50\%$ for $\alpha = 0.5$ (or more generally symmetric αs) and $\phi = 0$, as opposed to $w_{g,t}^M = 65\%, w_{b,t}^M = 35\%$ in the baseline calibration. Similarly, absent preference for green consumption and green investing, the market weights at $X_t = (1/3, 1/3)$ would be strongly tilted toward the *brown* asset unlike Panels (a), (b), (c) of Figure B.9 for which either $\phi > 0$, $\alpha^G > 1 - \alpha^N = 1/2$, or both.

Finally, portfolio weights, as well as how biased they are when compared to the market portfolio ($w_{g,t}^M \equiv Q_{g,t}/[Q_{g,t} + Q_{b,t}]$, $w_{b,t}^M = 1 - w_{g,t}^M$), are also strongly state-dependent. This is shown in Figures B.11 and B.12 in the Appendix, which plot both as a function of the state of the economy $X_t = (x_t, y_t)'$ in the baseline calibration. For instance, both investors increase the share of their wealth invested in the brown asset as the relative supply of the

[35] In equilibrium, the latter is $w_{b,t}^M = 1 - w_{g,t}^M$ because the bond is in zero net supply.

green good, y_t, decreases (consistent with the market portfolio).[36]

All in all, and even though those time variations in portfolios are not the focus of our empirical analysis in Section 4, they could provide interesting avenues for further tests and research.

4 Empirical Evidence

In this section, we provide empirical evidence for the effect of consumption premia on asset returns. We focus on the effect of the relative supply of green goods, as suggested by our findings in previous sections. Strongly supporting our theoretical results, we find that this factor is significantly priced in the cross-section of U.S. equity assets, both statistically and in terms of economic magnitude. In recent years, the annual consumption premium on a basket of green assets has steadily increased to being 80 bps larger than that on a basket of brown assets. More generally, assets that can hedge shocks to the relative supply of green goods, based on their betas regardless of their environmental ratings, carry lower returns of up to 1.5% annually.

4.1 Data and factor construction

Based on our theoretical findings, our baseline empirical analysis focuses on the consumption premium related to relative supply for two reasons: *(i)* quantitaitvely, the impact of preferences for *green consumption* is mainly driven by this effect, while the effect related to the hedging of wealth share changes is muted; *(ii)* the impact of the wealth share is partly captured by the green premium, which is driven by preferences for *green assets* and, therefore, very related to the share of wealth held by green investors (Pástor et al., 2022; Zerbib, 2022).

[36]They do so because of heightened relative-supply hedging motives, and despite the fact that the common component should make them want to *decrease* their portfolio weight in that asset. The green investor also increases her weight on the green asset as y_t decreases because the impact of her preference for green assets is heightened by a strongly increasing correlation across assets. Because the weight of the green asset in the market portfolio *decreases* at the same time, however, the green investor has to rely on an increasing amount of borrowing in the riskfree bond $|b_t^G|$ to tilt her risky portfolio as she desires in what she perceives as bad times (i.e., when y_t decreases).

We test the existence of the consumption premium by estimating the following beta-representation, implied by the equilibrium equations for expected returns in Proposition 2, for all assets j:

$$\mu_{j,t} - r_t = \lambda_{M,t}\beta_{j,M,t} + \lambda_{GMB,t}\beta_{j,GMB,t} + \lambda_{ConsHedge,t}\beta_{j,ConsHedge,t} + \varepsilon_{j,t}. \tag{16}$$

We perform the estimation in a standard two-stage procedure à la Fama and MacBeth (1973a). First, quantities of risk, $\beta_{j,M}, \beta_{j,GMB}, \beta_{j,ConsHedge}$ are obtained from time-series regressions of asset excess returns on the market factor, the green-minus-brown factor reflecting investors' preferences for green assets (Pástor et al., 2022), and the consumption factor related to the relative supply of green goods. More details on each are provided below. Second, prices of risk λ_M, λ_{GMB}, and $\lambda_{ConsHedge}$ are estimated from a cross-sectional regression of the average excess returns on each asset on the betas.[37]

Given our theoretical results, we expect the price of risk associated with the relative *supply* of green goods, $\lambda_{ConsHedge}$, to be strongly and significantly positive. Indeed, as discussed in Section 3, the average investor in the economy values assets whose returns are negatively correlated with the relative supply of green goods, because these assets offer a hedge against those adverse states of the world.[38]

We start our analysis from all common stocks (share type codes 10 and 11) listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and National Association of Securities Dealers Automated Quotations exchange (NASDAQ; exchange codes 1, 2, and 3) in the CRSP database. We map them to the 6-digit North American Industry Classification System (NAICS) using CRPS/Compustat data. Given the fact that green investing rose only recently (Zerbib, 2022) and due to the availability of environmental ratings (Pástor et al., 2021), our analysis starts in January 2007 and ends in December 2019.[39]

[37]Theoretical expressions for the quantities of risk, $\beta_{j,M,t}, \beta_{j,GMB,t}, \beta_{j,ConsHedge,t}$, and prices of risk $\lambda_{M,t}$, $\lambda_{GMB,t}$, and $\lambda_{ConsHedge,t}$, can be derived from Equation (12) in Proposition 2.

[38]Equivalently, the average investor values assets whose returns are large when the relative *prices* of green goods are large, so that the price of risk for such a measure is expected to be strongly and significantly negative. This is consistent with the strong negative relationship between the relative supply of green goods and their relative prices (see Figures 2 (a) and (b).)

[39]We end the analysis in December 2019 to avoid the major disruptions to production and other variables caused by the Covid pandemic.

We construct four versions of the consumption hedging factor by combining different measures for (i) the environmental footprints of companies, and (ii) the relative supply/prices of green goods. Let us start with the main version. We rank companies according to their environmental ratings provided by MSCI, which is the world's largest provider of ESG ratings (Eccles and Stroehle, 2020) and covers more firms than the other ESG raters (Berg et al., 2022).[40] To capture supply changes, we use the Industrial Production and Capacity Utilization Indices (also refered to as Production Indices, publication G17) constructed by the Federal Reserve, which are available at the granular 6-digit NAICS level, and we compute the MSCI rating of each industry as the value-weighted MSCI rating of all firms in that industry. We use the log change in production indices over the last two quarters, as is common with macroeconomic data (e.g., Herskovic et al., 2019), to account for the fact that real variables can take a while to affect asset markets. Finally, we lag the measure to avoid any look-ahead bias due to the lagged release schedule of those macroeconomic data.[41] Taken together, we construct the consumption factor for relative supply as the difference between the value-weighted six-month changes in production of the industries in the greenest tercile and those in the brownest industry tercile:

$$ConsHedge_t \equiv \left(\sum_{j \in G} w_{j,t}^{M,G} dPI_{j,t}\right) - \left(\sum_{j \in B} w_{j,t}^{M,B} dPI_{j,t}\right), \tag{17}$$

where dPI is the change in Production Index, and $w^{M,G}$ and $w^{M,B}$ are the market weights of the industries in the greenest and brownest terciles, respectively, both with the appropriate lags.

The second version of the consumption factor uses carbon intensities provided by S&P-Trucost to measure companies' environmental footprints, instead of MSCI scores. The carbon intensity (CI) of a company is defined as the annual amount of greenhouse gases emitted by the company across its value chain, normalized by its annual revenues.[42] The goal of this

[40]Specifically, we compute the green score at the firm-level using the procedure in Pástor et al. (2022).

[41]Specifically, around the 15th or 20th of a given month, the Fed only releases preliminary G17 data for the previous month and revised/final data for the month before that. We therefore lag the values by two months to avoid any issue. See https://www.federalreserve.gov/releases/g17/default.htm for details and examples.

[42]We use Trucost's default emission scope, which includes direct and first-tier indirect emissions, that is, for a given firm, the emissions related to the its activity (scope 1), induced by the generation of its purchased energy (scope 2), and those of its suppliers (upstream scope 3).

version is to ensure that our results are not driven by a specific way of measuring greenness.

The third and fourth versions of our consumption factor follow the first (main, supply + MSCI score) and second (supply + CI) versions, but use a measure of the relative *prices* of green goods, as opposed to relative *supply*. This approach is common in the international finance literature (see, e.g., Coeurdacier, 2009; Coeurdacier and Rey, 2013) and is helpful in cases in which there are no high-frequency measures of relative supply. They constitute interesting robustness checks, although we consider our versions based on production indices as cleaner measures that are more direclty related to the relative supply shocks on which we focus in this paper. Specifically, for these versions, we replace the 6-month changes in the Production Indices with those of the Producer *Price* Indices constructed by the U.S. Bureau of Labor Statistics. We lag the measure to avoid any look-ahead bias due to the lagged release schedule of those data. The third version uses MSCI scores (prices + MSCI score), and the fourth one uses carbon intensities (prices + CI) to measure the environmental footprints of companies. Given the negative relationship between the relative supply of green goods and their relative prices (Figures 2 (a) and (b)), we expect to obtain a significantly *negative* price of risk for these versions. We carry out several additional robustness tests, which we detail in the next section.

We construct the green factor (also referred to as green-minus-brown factor, GMB_t, in the literature) using the environmental ratings provided by MSCI. By closely following Pástor et al. (2022), we construct this factor as a green minus brown value-weighted portfolio that is long on the tercile of the greenest firms and short on the tercile of the brownest firms, excluding firms without ratings. We make sure to use the ratings in the previous month to construct the factor in the current month.

Finally, we proxy for the market component by using excess returns on the market, that is, the standard market factor (market return minus riskfree rate) from Fama and French (2015). In the estimations, as is usual, we also control for the small-minus-big (SMB), high-minus-low (HML), conservative-minus-aggressive (CMA), robust-minus-weak (RMW) factors (Fama and French, 2015), and the momentum (MOM) factor (Carhart, 1997). We obtain all those factors from Kenneth French's website.[43]

[43]Link: `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

Table 1: Summary statistics (%, monthly)

Variable	$\mu_j - r$	*ConsHedge*	*GMB*	*Mkt-RF*	*SMB*	*HML*	*RMW*	*CMA*	*MOM*
Mean	0.312	-1.793	0.187	0.741	0.016	-0.263	0.270	-0.010	0.059
Standard deviation	12.892	7.093	2.229	4.340	2.399	2.686	1.574	1.470	4.713
Min	-38.629	-17.311	-5.757	-17.230	-4.930	-11.290	-3.880	-3.250	-34.300
25% percentile	-5.566	-7.471	-0.918	-1.585	-1.837	-1.860	-0.643	-1.005	-1.775
Median	0.289	-0.651	0.233	1.240	0.180	-0.455	0.370	-0.040	0.260
75% percentile	5.698	3.669	1.369	3.348	1.435	1.073	1.198	0.887	2.680
Max	49.057	11.518	8.736	11.350	7.130	8.210	4.960	3.700	12.750

Notes: Variables are described in Section 4.1. *ConsHedge* is the baseline consumption factor, based on producer prices and MSCI scores. Summary statistics are similar for other versions of the factor.

All in all, we work with a scope of 3388 stocks and estimate the specification of Equation (16) using a two-pass (Fama and MacBeth, 1973b) regression from January 2007 to December 2019. In the second pass, we run cross-sectional regressions of the time-series average of each asset returns on the betas, wherein returns and betas are winsorized at the 1% level. Table 1 provides summary statistics.

4.2 Estimation

Consistent with the characterization of the model, the results of the estimation strongly support the existence of the relative-supply consumption premium in the cross-section of stock returns. They are summarized in Table 2.

First, the price of risk for our baseline factors based on Production Indices, $\widehat{\lambda}_{ConsHedge}$, is positive and highly significant across all estimated specifications (Panel A, columns [1] to [4]): it ranges from 0.62% to 0.95% per month with t-stats ranging from 4.39 to 6.34. Moreover, the price of risk for the versions of the factors constructed using Producer *Price* Indices as robustness check is also highly significant across all estimated specifications (Panel A, columns [5] to [8]) and negative: it ranges from -0.62% to -0.93% bps per year with t-stats between -3.86 and -5.44. Both are exactly in line with our theoretical predictions.

Table 2: Empirical estimation of consumption premia

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
ConsHedge	0.623	0.671	0.819	0.947	-0.675	-0.620	-0.935	-0.858
	(5.218)	(4.393)	(5.570)	(6.339)	(-4.164)	(-3.856)	(-4.830)	(-5.441)
GMB	0.006	0.082	0.109	0.142	0.026	0.144	0.139	0.232
	(5.918)	(0.131)	(1.939)	(2.324)	(3.633)	(0.585)	(3.590)	(3.068)
Panel B: Premia difference (annual, %)								
ConsHedge	1.245	1.239	1.479	1.561	-0.928	-0.842	-1.187	-1.076
GMB	0.042	0.624	0.719	0.994	0.175	1.076	0.918	1.665
Measure	Supply	Supply	Supply	Supply	Prices	Prices	Prices	Prices
Greenness	CI	CI	MSCI	MSCI	CI	CI	MSCI	MSCI
Controls	FF3	FF5MOM	FF3	FF5MOM	FF3	FF5MOM	FF3	FF5MOM

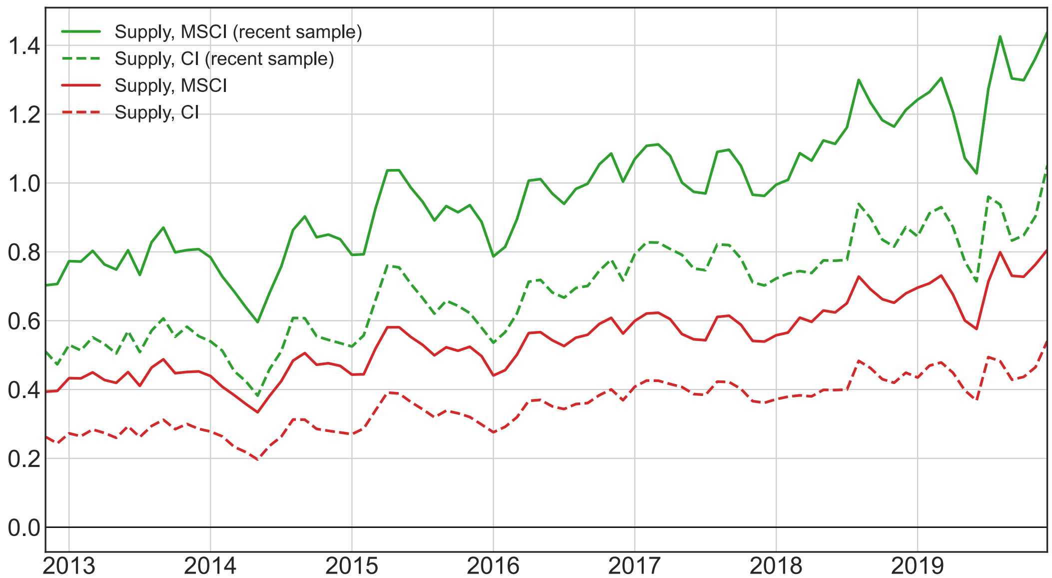
Notes: $ConsHedge$ and GMB refer to the consumption hedging factor and the green-minus-brown factor, respectively. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

Second, as they provide a hedge against a decrease in the relative supply of green goods, assets whose returns covary most with the factor constructed from the Production Indices have a higher risk premium than assets that do not. Panel B, specifications (1) to (4), reports the difference in the risk premium between the assets with loadings in the top tercile (high $\widehat{\beta}_{ConsHedge\_Supply}$) and those with loadings in the bottom tercile (low $\widehat{\beta}_{ConsHedge\_Supply}$).[44] The difference in risk premium is substantial: it ranges from 1.24% to 1.56% per year depending on the specification considered, and highlights that investors undervalue assets whose payoffs are large when green goods are abundant. In other words, assets whose payoffs are large when

[44] More precisely, for each tercile, we calculate the "capped value-weighted" average of the betas, wherein market capitalizations are winsorized at the 20% and 80% levels. This follows Jensen et al. (2023) among others, and prevent outliers from having outsized influence on the results.

green goods are relatively scarce are indeed good hedges, consistent with our predictions. Similarly, when we consider the alternative consumption factor constructed from Producer *Price* Indices, the difference in premium between the assets with loadings in the top tercile (high $\widehat{\beta}_{ConsHedge\_Prices}$) and those in the bottom tercile (low $\widehat{\beta}_{ConsHedge\_Prices}$) ranges from -0.84% to -1.19% (Panel B, specifications [5] to [8]). This highlights that assets whose returns are large when green goods are expensive are valued as good hedges by investors empirically, again in line with our predictions.

Figure 7: Relative-consumption premium difference between green and brown assets



Notes: This figure shows the difference in consumption premium between the tercile of greenest assets (high MSCI rating or low carbon intensity) and that of brownest assets (low MSCI rating or high carbon intensity), using our baseline factor constructed from Production Indices. NAICS: 1 (Agriculture), 2 (Mining, Utilities, Construction), 3 (Manufacturing). Red lines are based on estimates obtained from the full sample (Jan 2007-Dec 2019), whereas green lines are based on estimates from the recent sample (Nov 2012-Dec 2019, consistent with Pástor et al., 2022).

Third, we test whether green assets are indeed riskier from the perspective of our consumption premium. Focusing on the environmental footprint of U.S. equity assets, that is, their MSCI environmental scores and their carbon intensities, the tercile of greenest assets had a lower average beta than that of brownest assets across all months between January 2007 and December 2019, consistent with our findings.[45] Our baseline results focus on firms with NAICS codes 1 (Agriculture, Forestry, Fishing and Hunting), 2 (Mining, Quarrying, and Oil and Gas Extraction, Utilities, Construction) and 3 (Manufacturing), which encompass sectors that *produce* goods and are therefore most consistent with our model.[46] In addition, the beta gap has steadily widened over time. As a result, the difference in annual consumption premium between the tercile of greenest assets and that of brownest assets, $(\widehat{\beta}_{G,ConsHedge,t} - \widehat{\beta}_{B,ConsHedge,t})\widehat{\lambda}_{ConsHedge}$, has been positive throughout the sample, and gradually increased to reach 80 bps using MSCI scores and 50 bps using carbon intensities (Figure 7). When estimating prices of risks over a more recent sample, from November 2012 to December 2019 (consistent with the recent rise in environmental concerns and sustainable investing documented, for example, in Pástor et al., 2022), we find that the difference in annual consumption premium has had similar dynamics and reached higher magnitudes, around 1% and 1.4% using MSCI environmental grades and carbon intensities, respectively.

These findings again strongly support the predictions of the model: brown assets provide a better financial hedge against changes in the relative supply of green goods as compared to green assets, which are riskier from the perspective of consumption premia. As such, the steady growth of this premium and its current level are consistent with an asset allocation that reflects a growing preference for green consumption.

In Appendix C, we show that our findings are stable across different robustness checks. First, the several versions of our consumption factor remain strongly priced across various specifications based on different factors used as controls (Appendix C.1). This is true in terms of economic magnitude, statistical significance, and implied average risk premia. Second, results are broadly unchanged whether we estimate the second stage without a constant (our baseline in the main text and in C.1) or with a constant (Appendices C.2 and C.3). Third, the difference in annual consumption premium across green and brown assets also remain significant and sizable when we include firms with NAICS code 4 (Wholesale Trade, Retail Trade, Transport, Warehousing), although we see those sectors as less consistent with our model, which is primarily about firms producing consumption goods. In that case, the premia

[45] As a reminder, we compute MSCI green scores at the firm-level using the procedure in Pástor et al. (2022).

[46] In a robustness check below, we show that results do not change much when we widen the scope.

differential reaches around 60 bps and 1.1% when estimated based on MSCI environmental scores over the whole sample (Jan 2007–Dec 2019) and the recent sample (Nov 2012–Dec 2019), respectively.

Finally, the price of risk associated with the GMB factor is significant for almost all specifications, but contrary to what the theory predicts in equilibrium, the premium for the 33% greenest firms is higher than that for the 33% brownest firms by 17 bps to 1.66% per year. As documented by Pástor et al. (2021) and Bolton and Kacperczyk (2022), this effect could be due to the unexpected increase in investors' preferences for green assets over the last few years, which has pushed up their realized returns and does not permit to capture the green premium on *expected* returns.[47] Several papers have developed methods to control for this effect (e.g., Ardia et al., 2021; Pástor et al., 2022; Sautner et al., 2022; Zerbib, 2022). For example, Pástor et al. (2022) find that the green premium on U.S. equities—corresponding to the difference between the premium on the expected returns of green stocks and the premium on the expected returns of brown stocks—ranges from −0.50% to −2% between 2013 and 2020. Zerbib (2022) estimates the premium at an average of −1.50% between 2013 and 2019. As a result, the 0.5% to 1.4% consumption premium that we estimate over the most recent years accounts for a substantial share of the green premium estimated by Pástor et al. (2022), thereby almost entirely offsetting its effect.

Overall, these estimations strongly support the model predictions: our consumption factor is strongly priced, and the consumption premia can offset a substantial part of the green premium on green assets. As such, the consumption premia, related to pro-environmental preferences for green *goods*, help explain the limited effect of green investing on the cost of capital of brown firms as discussed in Section 5.

5 Implications for Impact Investing

Impact investing covers several investment strategies that aim at encouraging companies to change their practices. By inducing a green premium that increases the cost of capital of polluting companies, investors' preferences for green assets are supposed to incentivize com-

[47]Note that our consumption factor is unlikely to be plagued by similar issues, given that it is based on macroeconomic data (production indices and producer price indices).

panies to mitigate their environmental footprints. Yet, empirical evidence suggests that the real impact is low. De Angelis et al. (2022) find that by internalizing the climate externalities of the companies in which they invest, green investors drive companies to reduce their carbon footprint at a low rate, in the range of 1% to 3% per year. In addition, Oehmke and Opp (2019), Landier and Lovo (2020), and Green and Roth (2020) emphasize that green investors do not maximize their global impact by internalizing only the environmental footprints of the companies in which they invest.[48]

Our findings have dual implications from the perspective of impact investing. First, by showing that the green premium is counterbalanced by green investors' preferences for green consumption, we contribute to explaining why the impact of green investors on the cost of capital and practices of polluting firms can be limited. Second, the overweighting of polluting companies in the portfolios of green investors is an opportunity to leverage their shareholder position so as to increase their engagement with these companies (e.g., private or public communications, votes in general assemblies) and push them to become greener. This conclusion reinforces the findings of Broccardo et al. (2020), among others, who suggest that shareholder engagement is often more effective than green investment (without accounting for consumption preferences).

Be it to accelerate the ecological transition in general, or specifically to mitigate the effect of the consumption premia on firms' cost of capital, policymakers could have different options, such as capping (or providing a guidance on caps moving forward) green good prices or introducing a dividend tax. For example, one can show (see Sauzet, 2022a, for details) that when investors pay a tax τ on dividends from brown firms, the expected returns are as follows:

$$\begin{aligned} \mu_{g,t} - r_t &= \gamma \sigma_{j,t}^{\top} \sigma_{\widetilde{W},t} - \gamma \sigma_{g,t}^{\top} \sigma_{\tilde{J},t} - x_t \phi^G, \\ \mu_{b,t} - r_t &= \gamma \sigma_{j,t}^{\top} \sigma_{\widetilde{W},t} - \gamma \sigma_{b,t}^{\top} \sigma_{\tilde{J},t} + \tau F_{b,t}. \end{aligned} \tag{18}$$

Taxes on dividends can counterbalance the consumption premia on the assets of brown

[48] The impact is larger when they internalize the environmental footprints of all firms in the economy, irrespective of whether they invest in them (Oehmke and Opp, 2019; Green and Roth, 2020), and by prioritizing firms where the inefficiencies induced by the externalities are particularly acute and the capital search frictions are strong (Landier and Lovo, 2020).

firms through $\tau F_{b,t}$ and hence, increase their cost of capital. From a quantitative viewpoint, dividend taxation has a substantial impact on expected returns if the dividend yields are sufficiently high, or equivalently, if asset prices at a given dividend level are sufficiently low, that is, when firms' cost of capital is high.[49] Therefore, introducing a dividend tax could be all the more effective because the brownest companies are subject to transition risks (environmental regulations, carbon prices increases, changes in consumer preferences, technological and reputational risks, etc.), which increase their cost of capital relative to green companies. We leave a deeper exploration of those policy options for future work.

6 Conclusion

In this paper, we show how the preferences of investors for green consumption substantially moderate the effect of the green premium associated with their preferences for green assets on expected asset returns. Indeed, green assets are riskier from the perspective of consumption premia, while brown assets provide a financial hedge against situations in which green goods are in low supply and expensive. We find strong empirical support for our theoretical predictions in the cross-section of U.S. equity assets. In addition to being relevant for asset pricing and capital allocation, the main effect documented in this paper has important implications for investors willing to contribute to the ecological transition: an increase in the cost of capital of brown firms is dampened as soon as green goods are subject to shocks that may impact their relative supply or relative prices. The allocation of a larger share of green investors' capital to brown firms could, therefore, provide a welcome opportunity to reinforce their engagement with the most polluting firms.

The general equilibrium framework that we propose in this paper represents a basis that can open up multiple avenues for applications and extensions. For instance, Sauzet (2024) studies what happens when investors can only invest through index funds or financial intermediaries, which can impede the transmission of their preferences to asset prices. We also explore various related avenues in ongoing work, such as stochastic demand and production. It would also be valuable to analyze alternative forms of investments and account for shareholder engagement with a view to maximizing investors' impact in a general equilibrium

[49] Technically, this occurs for instance when the elasticity of intertemporal substitution is not too large.

model. Another promising avenue is to include environment-related financial risks (van den Bremer and van der Ploeg, 2021; Hambel et al., 2022; Barnett, 2022) into this setting. Finally, constructing portfolios that are hedged against several types of risks, notably climate risks but also the risk of a rise in green good prices, by building on Engle et al. (2020) and Alekseev et al. (2021), constitutes an interesting direction for future work.

References

Albuquerque, Rui, Yrjö Koskinen, and Chendi Zhang, 2019, Corporate Social Responsibility and Firm Risk: Theory and Empirical Evidence, *Management Science* 65, 4451–4469.

Alekseev, Georgij, Stefano Giglio, Quinn Maingi, Julia Selgrad, and Johannes Stroebel, 2021, A quantity-based approach to constructing climate risk hedge portfolios, Working Paper.

Alok, Shashwat, Nitin Kumar, and Russell Wermers, 2020, Do fund managers misestimate climatic disaster risk, *Review of Financial Studies* 33, 1146–1183.

Anderson, Robert M., and Roberto C. Raimondo, 2008, Equilibrium in continuous-time financial markets: Endogenously dynamically complete markets, *Econometrica* 76, 841–907.

Ardia, David, Keven Bluteau, Kris Boudt, and Koen Inghelbrecht, 2021, Climate change concerns and the performance of green versus brown stocks, *Working Paper,* SSRN.

Avramov, Doron, Abraham Lioui, Yang Liu, and Andrea Tarelli, 2021, Dynamic ESG Equilibrium, *Working Paper,* SSRN.

Baker, M., D. Bergstresser, G. Serafeim, and J. Wurgler, 2018, Financing the Response to Climate Change: The Pricing and Ownership of U.S. Green Bonds NBER WP.

Baker, Steven, Burton Hollifield, and Emilio Osambela, 2022, Asset Prices and Portfolios with Externalities*, *Review of Finance* 26, 1433–1468.

Baldauf, Markus, Lorenzo Garlappi, Constantine Yannelis, and José Scheinkman, 2020, Does climate change affect real estate prices? only if you believe in it, *Review of Financial Studies* 33, 1256–1295.

Bansal, Ravi, Dana Kiku, and Juan Ochoa, 2016, Price of long-run temperature shifts in capital markets, *Working Paper,* NBER.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance* 59, 1481–1509.

Barber, Brad M., Adair Morse, and Ayako Yasuda, 2021, Impact investing, *Journal of Financial Economics* 139, 162–185.

Barnett, Michael, 2022, Climate Change and Uncertainty: An Asset Pricing Perspective, *Working Paper,* Stanford Libraries.

Barro, Robert J., and Sanjay Misra, 2016, Gold Returns, *The Economic Journal* 126, 1293–1317.

Basak, Suleyman, and Domenico Cuoco, 1998, An Equilibrium Model with Restricted Stock Market Participation, *The Review of Financial Studies* 11, 309–341.

Berg, Florian, Julian F Kölbel, and Roberto Rigobon, 2022, Aggregate Confusion: The Divergence of ESG Ratings, *Review of Finance* rfac033.

Berk, Jonathan B., and Jules H. van Binsbergen, 2021, The Impact of Impact Investing, *Working Paper,* SSRN.

Bernstein, Asaf, Matthew T. Gustafson, and Ryan Lewis, 2019, Disaster on the horizon: The price effect of sea level rise, *Journal of Financial Economics* 134, 253–272.

Berrada, Tony, Julien Hugonnier, and Marcel Rindisbacher, 2007, Heterogeneous preferences and equilibrium trading volume, *Journal of Financial Economics* 83, 719 – 750.

Bhamra, Harjoat S., and Raman Uppal, 2009, The Effect of Introducing a Non-Redundant Derivative on the Volatility of Stock-Market Returns When Agents Differ in Risk Aversion, *The Review of Financial Studies* 22, 2303–2330.

Bhamra, Harjoat S., and Raman Uppal, 2014, Asset Prices with Heterogeneity in Preferences and Beliefs, *The Review of Financial Studies* 27, 519–580.

Bolton, Patrick, and Marcin Kacperczyk, 2021, Do investors care about carbon risk?, *Journal of Financial Economics* 142, 517–549.

Bolton, Patrick, and Marcin Kacperczyk, 2022, Global pricing of carbon-transition risk, *Working Paper,* SSRN.

Borovička, Jaroslav, 2020, Survival and long-run dynamics with heterogeneous beliefs under recursive preferences, *Journal of Political Economy* 128, 206–251.

Broccardo, Eleonora, Oliver Hart, and Luigi Zingales, 2020, Exit vs. Voice NBER WP.

Brunnermeier, Markus, and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421.

Brunnermeier, Markus K., and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *The Review of Financial Studies* 22, 2201–2238.

Cai, Yongyang, and Thomas S. Lontzek, 2019, The Social Cost of Carbon with Economic and Climate Risks, *Journal of Political Economy* 127, 2684–2734.

Campbell, John Y, and Ian Martin, 2023, Sustainability in a risky world, Working Paper 28899, National Bureau of Economic Research.

Carhart, M. M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57–82.

Chabakauri, Georgy, 2013, Dynamic Equilibrium with Two Stocks, Heterogeneous Investors, and Portfolio Constraints, *The Review of Financial Studies* 26, 3104–3141.

Chan, Yeung Lewis, and Leonid Kogan, 2002, Catching up with the joneses: Heterogeneous preferences and the dynamics of asset prices, *Journal of Political Economy* 110, 1255–1285.

Chava, S., 2014, Environmental externalities and cost of capital, *Management Science* 60, 2223–2247.

Chen, Xuhui, Lorenzo Garlappi, and Ali Lazrak, 2023, Responsible Consumption, Demand Elasticity, and the Green Premium, *Working Paper* .

Cochrane, John H., Francis A. Longstaff, and Pedro Santa-Clara, 2008, Two Trees, *The Review of Financial Studies* 21, 347–385.

Coeurdacier, Nicolas, 2009, Do trade costs in goods market lead to home bias in equities?, *Journal of International Economics* 77, 86 – 100.

Coeurdacier, Nicolas, and Hélène Rey, 2013, Home bias in open economy financial macroeconomics, *Journal of Economic Literature* 51, 63–115.

De Angelis, Tiziano, Peter Tankov, and Olivier David Zerbib, 2022, Climate Impact Investing, *Management Science,* Forthcoming.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2018, A model of monetary policy and risk premia, *The Journal of Finance* 73, 317–373.

Duffie, D., J. Geanakoplos, A. Mas-Colell, and A. McLennan, 1994, Stationary markov equilibria, *Econometrica* 62, 745–781.

Duffie, Darrell, and Larry Epstein, 1992, Stochastic differential utility, *Econometrica* 60, 353–94.

Dumas, Bernard, 1989, Two-Person Dynamic Equilibrium in the Capital Market, *The Review of Financial Studies* 2, 157–188.

Dumas, Bernard, 1992, Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World, *The Review of Financial Studies* 5, 153–180.

Dumas, Bernard, and Raman Uppal, 2001, Global Diversification, Growth, and Welfare with Imperfectly Integrated Markets for Goods, *The Review of Financial Studies* 14, 277–305.

Dumas, Bernard, Raman Uppal, and Tan Wang, 2000, Efficient intertemporal allocations with recursive utility, *Journal of Economic Theory* 93, 240–259.

Eccles, Robert G., and Judith C. Stroehle, 2020, Exploring social origins in the construction of esg measures, *Working Paper,* SSRN.

Ehling, Paul, and Christian Heyerdahl-Larsen, 2015, Complete and incomplete financial markets in multi-good economies, *Journal of Economic Theory* 160, 438 – 462.

Ehling, Paul, and Christian Heyerdahl-Larsen, 2017, Correlations, *Management Science* 63, 1919–1937.

ElGhoul, S., O. Guedhami, C. C.Y. Kowk, and D. R. Mishra, 2011, Does corporate social responsibility affect the cost of capital?, *Journal of Banking and Finance* 35, 2388–2406.

Engle, Robert F, Stefano Giglio, Bryan Kelly, Heebum Lee, and Johannes Stroebel, 2020, Hedging Climate Change News, *Review of Financial Studies* 33, 1184–1216.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and James D. MacBeth, 1973a, Risk, return, and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Fama, Eugene F., and James D. MacBeth, 1973b, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Fang, Xiang, 2019, Intermediary leverage and currency risk premium, *Working Paper,* SSRN.

Gârleanu, Nicolae, and Stavros Panageas, 2015, Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing, *Journal of Political Economy* 123, 670 – 685.

Gârleanu, Nicolae, and Lasse Heje Pedersen, 2011, Margin-based asset pricing and deviations from the law of one price, *The Review of Financial Studies* 24, 1980–2022.

Geanakoplos, John, 1990, An introduction to general equilibrium with incomplete asset markets, *Journal of Mathematical Economics* 19, 1 – 38.

Geanakoplos, John, and Andreu Mas-Colell, 1989, Real indeterminacy with financial assets, *Journal of Economic Theory* 47, 22 – 38.

Geanakoplos, John D, and Heraklis M Polemarchakis, 1986, Existence, regularity, and constrained suboptimality of competitive allocations, *Essays in Honor of Kenneth J. Arrow: Volume 3, Uncertainty, Information, and Communication* 3, 65.

Giglio, Stefano, Matteo Maggiori, Krishna Rao, Johannes Stroebel, Andreas Weber, and Stijn Van Nieuwerburgh, 2021, Climate Change and Long-Run Discount Rates: Evidence from Real Estate [Abrupt climate change], *Review of Financial Studies* 34, 3527–3571.

Goldsmith-Pinkham, Paul, Matthew T. Gustafson, Ryan C. Lewis, and Michael Schwert, 2021, Sea Level Rise Exposure and Municipal Bond Yields, *Working Paper,* SSRN.

Goldstein, Itay, Alexandr Kopytov, Lin Shen, and Haotian Xiang, 2021, On esg investing: Heterogeneous preferences, information, and asset prices, *Working Paper,* SSRN.

Gollier, Christian, 2010, Ecological discounting, *Journal of Economic Theory* 145, 812–829, Judgment Aggregation.

Gollier, Christian, 2019, Valuation of natural capital under uncertain substitutability, *Journal of Environmental Economics and Management* 94, 54–66.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski, 2014, Optimal Taxes on Fossil Fuel in General Equilibrium, *Econometrica* 82, 41–88.

Green, Daniel, and Benjamin Roth, 2020, The allocation of socially responsible capital, *Working Paper,* SSRN.

Guesnerie, Roger, 2004, Calcul économique et développement durable, *Revue économique* 55, 363–382.

Hakenes, Hendrik, and Eva Schliephake, 2023, Responsible investment and responsible consumption, *Working Paper,* SSRN.

Hambel, Christoph, Holger Kraft, and Frederick van der Ploeg, 2022, Asset Diversification versus Climate Action, *Working Paper,* SSRN.

Hart, Oliver D, 1975, On the optimality of equilibrium when the market structure is incomplete, *Journal of Economic Theory* 11, 418 – 443.

Hartzmark, Samuel, and Abigail Sussman, 2019, Do Investors Value Sustainability? A Natural Experiment Examining Ranking and Fund Flows, *Journal of Finance* 74, 2789–2837.

Heinkel, R., A. Kraus, and J. Zechner, 2001, The effect of green investment on corporate behaviour, *Journal of Financial and Quantitative Analysis* 36, 377–389.

Herskovic, Bernard, Alan Moreira, and Tyler Muir, 2019, Hedging risk factors SSRN.

Hoel, Michael, and Thomas Sterner, 2007, Discounting and relative prices, *Climatic Change* 84, 265–280.

Hong, Harrison, Frank Weikai Li, and Jiangmin Xu, 2019, Climate risks and market efficiency, *Journal of Econometrics* 208, 265–281.

Hsu, Po-Hsuan, Kai Li, and Chi-Yang Tsou, 2022, The Pollution Premium, *Journal of Finance,* Forthcoming.

Hugonnier, J., S. Malamud, and E. Trubowitz, 2012a, Endogenous completeness of diffusion driven equilibrium markets, *Econometrica* 80, 1249–1270.

Hugonnier, Julien, Florian Pelgrin, and Pascal St-Amour, 2012b, Health and (Other) Asset Holdings, *The Review of Economic Studies* 80, 663–710.

Huynh, Thanh D., and Ying Xia, 2021, Climate Change News Risk and Corporate Bond Returns, *Journal of Financial and Quantitative Analysis* 56, 1985–2009.

Imbs, Jean, and Isabelle Méjean, 2015, Elasticity optimism, *American Economic Journal: Macroeconomics* 7, 43–83.

Jensen, Theis Ingerslev, Bryan Kelly, and Lasse Heje Pedersen, 2023, Is there a replication crisis in finance?, *Journal of Finance* 78, 2465–2518.

Krüger, Philipp, Zacharias Sautner, and Laura Starks, 2020, The Importance of Climate Risk for Institutional Investors, *Review of Financial Studies,* 33, 1067–1111.

Landier, Augustin, and Stefano Lovo, 2020, ESG Investing: How to Optimize Impact?, *Working Paper,* SSRN.

Lontzek, Thomas, Walter Pohl, Karl Schmedders, Marco Thalhammer, and Ole Wilms, 2023, Asset pricing with disagreement about climate risks, *Working Paper,* SSRN.

Lucas, Robert, 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–45.

Martin, Ian, 2011, The forward premium puzzle in a two-country world, Working Paper 17564, National Bureau of Economic Research.

Martin, Ian, 2013, The lucas orchard, *Econometrica* 81, 55–111.

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145 – 161.

Merton, Robert, 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–87.

Morgan Stanley Institute for Sustainable Investing, 2019, Sustainable Signals: Individual Investor Interest Driven by Impact, Conviction and Choice, Technical report.

Oehmke, Martin, and Marcus Opp, 2019, A Theory of Socially Responsible Investment, *Working Paper,* SSRN.

Painter, Marcus, 2020, An inconvenient cost: The effects of climate change on municipal bonds, *Journal of Financial Economics* 135, 468–482.

Papageorgiou, Chris, Marianne Saam, and Patrick Schulte, 2017, Substitution between Clean and Dirty Energy Inputs: A Macroeconomic Perspective, *The Review of Economics and Statistics* 99, 281–290.

Pástor, Lubos, Robert Stalbaugh, and Lucian Taylor, 2022, Dissecting Green Returns, *Journal of Financial Economics,* Forthcoming.

Pástor, Lubos, Robert F. Stambaugh, and Lucian A. Taylor, 2021, Sustainable Investing in Equilibrium, *Journal of Financial Economics,* Forthcoming.

Pavlova, Anna, and Roberto Rigobon, 2007, Asset Prices and Exchange Rates, *The Review of Financial Studies* 20, 1139–1180.

Pavlova, Anna, and Roberto Rigobon, 2008, The Role of Portfolio Constraints in the International Propagation of Shocks, *The Review of Economic Studies* 75, 1215–1256.

Pavlova, Anna, and Roberto Rigobon, 2010, An asset-pricing view of external adjustment, *Journal of International Economics* 80, 144 – 156, Special Issue: JIE Special Issue on International Macro-Finance.

Pedersen, Lasse Heje, Shaun Fitzgibbons, and Lukasz Pomorski, 2021, Responsible Investing: The ESG-Efficient Frontier, *Journal of Financial Economics,* Forthcoming.

Pindyck, Robert S., and Neng Wang, 2013, The economic and policy consequences of catastrophes, *American Economic Journal: Economic Policy* 5, 306–39.

Polemarchakis, Heraklis M, 1988, Portfolio choice, exchange rates, and indeterminacy, *Journal of Economic Theory* 46, 414–421.

Riedl, Arno, and Paul Smeets, 2017, Why Do Investors Hold Socially Responsible Mutual Funds?, *Journal of Finance* 72, 2505–2550.

Sautner, Zacharias, Laurence van Lent, Grigory Vilkov, and Ruishen Zhang, 2022, Pricing Climate Change Exposure, *Management Science,* Forthcoming.

Sauzet, Maxime, 2022a, Two investors, two trees, two goods, Working paper.

Sauzet, Maxime, 2022b, Asset prices, global portfolios, and the international financial system, Working paper.

Sauzet, Maxime, 2022c, Projection methods via neural networks for continuous-time models, Working paper.

Sauzet, Maxime, 2024, Green intermediary asset pricing, Working paper.

Seltzer, Lee, Laura T. Starks, and Qifei Zhu, 2022, Climate Regulatory Risks and Corporate Bonds, *Working Paper,* SSRN.

Stathopoulos, Andreas, 2017, Asset Prices and Risk Sharing in Open Economies, *The Review of Financial Studies* 30, 363–415.

Sterner, Thomas, and U. Martin Persson, 2008, An even sterner review: Introducing relative prices into the discounting debate, *Review of Environmental Economics and Policy* 2, 61–76.

Traeger, Christian P., 2011, Sustainability, limited substitutability, and non-constant social discount rates, *Journal of Environmental Economics and Management* 62, 215–228.

van den Bremer, Ton S., and Frederick van der Ploeg, 2021, The Risk-Adjusted Carbon Price, *American Economic Review* 111, 2782–2810.

Wang, Jiang, 1996, The term structure of interest rates in a pure exchange economy with heterogeneous investors, *Journal of Financial Economics* 41, 75 – 110.

Weil, Philippe, 1989, The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* 24, 401–421.

Weinbaum, David, 2009, Investor heterogeneity, asset pricing and volatility dynamics, *Journal of Economic Dynamics and Control* 33, 1379 – 1397.

Zapatero, Fernando, 1995, Equilibrium asset prices and exchange rates, *Journal of Economic Dynamics and Control* 19, 787–811.

Zerbib, Olivier David, 2019, The effect of pro-environmental preferences on bond prices: Evidence from green bonds, *Journal of Banking and Finance* 98, 39–60.

Zerbib, Olivier David, 2022, A Sustainable Capital Asset Pricing Model (S-CAPM): Evidence from green investing and sin stock exclusion, *Review of Finance,* Forthcoming.

Zhang, Shaojun, 2021, Carbon Returns Across the Globe, *Journal of Finance,* Conditionally Accepted.

Internet Appendix

A Additional theoretical results

A.1 Drift and diffusion terms for any variable

Remark A.1. *By Itô's Lemma, the geometric drift and diffusion term for any function* $g_t = g(X_t)$ *are given by:*

$$\frac{dg_t}{g_t} = \frac{dg(X_t)}{g(X_t)} \equiv \mu_{g,t}dt + \sigma_{g,t}^{\top} d\vec{Z}_t \tag{A.1}$$

where:

$$\mu_{g,t} = \frac{g_{x,t}}{g_t} x_t \mu_{x,t} + \frac{g_{y,t}}{g_t} y_t \mu_{y,t} + \frac{1}{2}\frac{g_{xx,t}}{g_t} x_t^2 \sigma_{x,t}^{\top}\sigma_{x,t} + \frac{1}{2}\frac{g_{yy,t}}{g_t} y_t^2 \sigma_{y,t}^{\top}\sigma_{y,t} + \frac{g_{xy,t}}{g_t} x_t y_t \sigma_{x,t}^{\top}\sigma_{y,t} \tag{A.2}$$

$$\sigma_{g,t} = \frac{g_{x,t}}{g_t} x_t \sigma_{x,t} + \frac{g_{y,t}}{g_t} y_t \sigma_{y,t} \tag{A.3}$$

This result is used repeatedly throughout the paper.

As a point of notation, recall that for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to infinite horizon). $g_{x,t}, g_{y,t}, g_{xx,t}, g_{yy,t}, g_{xy,t}$ denote the partial derivatives of $g(X_t)$.

A.2 Returns, and risk premia

The (geometric) drifts and diffusion terms for asset returns are obtained from Itô's Lemma and are as follows, for $j \in \{g, b\}$

$$\begin{aligned} dR_{j,t} &= \mu_{j,t}dt + \sigma_{j,t}^{\top} d\vec{Z}_t \\ &\equiv \left(F_{j,t} + \mu_{p_j,t} + \mu_{Y_j} + \sigma_{p_j,t}^{\top}\sigma_{Y_j} - \mu_{F_j,t} + \sigma_{F_j,t}^{\top}\sigma_{F_j,t} - \left(\sigma_{p_j,t} + \sigma_{Y_j}\right)^{\top}\sigma_{F_j,t}\right) dt \\ &+ \left(\sigma_{p_j,t} + \sigma_{Y_j} - \sigma_{F_j,t}\right)^{\top} d\vec{Z}_t \end{aligned} \tag{A.4}$$

where $\mu_{p_j,t}$, $\mu_{F_j,t}$, $\sigma_{p_j,t}$, $\sigma_{F_j,t}$ are obtained using Remark A.1 above.

Proposition A.1 generalizes Proposition 2 to the case in which investors have different risk aversions, $\gamma^G \neq \gamma^N$, different elasticity of intertemporal substitutions, $\psi^G \neq \psi^N$, and in which both investors have preferences toward both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t. This poses no particular problem for the resolution, as our method allows for any value of the parameters. Exploring additional asymmetries stemming from those could be an interesting avenue for future work.

Proposition A.1. *The expected risk premia on the green and brown equity assets are*

$$\mu_{g,t} - r_t = \gamma_t \sigma_{g,t}^\top \sigma_{\widetilde{W},t} - \gamma_t \sigma_{g,t}^\top \sigma_{\tilde{J},t} - \gamma_t \left(x_t \frac{\phi_g^G}{\gamma^G} + (1 - x_t) \frac{\phi_g^N}{\gamma^N} \right) \tag{A.5}$$

$$\mu_{b,t} - r_t = \gamma_t \sigma_{b,t}^\top \sigma_{\widetilde{W},t} - \gamma_t \sigma_{b,t}^\top \sigma_{\tilde{J},t} - \gamma_t \left(x_t \frac{\phi_b^G}{\gamma^G} + (1 - x_t) \frac{\phi_b^N}{\gamma^N} \right)$$

where $\widetilde{W}_t$ *is the total wealth,* $\tilde{J}_t$ *is the economy-wide marginal value of wealth,* γ_t *is the wealth-weighted risk aversion,* $\sigma_{J^G,t}, \sigma_{J^N,t}$ *are the geometric diffusion terms of* J_t^G, J_t^N *obtained as in Remark A.1 above, and*

$$\begin{aligned}
\sigma_{\widetilde{W},t} &\equiv w_{g,t}^M \sigma_{g,t} + (1 - w_{g,t}^M) \sigma_{b,t} \\
\sigma_{\tilde{J},t} &\equiv x_t \left(\frac{1}{\gamma^G} \right) \left(\frac{1 - \gamma^G}{1 - \psi^G} \right) \sigma_{J^G,t} + (1 - x_t) \left(\frac{1}{\gamma^N} \right) \left(\frac{1 - \gamma^N}{1 - \psi^N} \right) \sigma_{J^N,t} \\
\gamma_t &\equiv \left(\frac{x_t}{\gamma^G} + \frac{1 - x_t}{\gamma^N} \right)^{-1}
\end{aligned}$$

A.3 Portfolios

Proposition A.2 generalizes Proposition 3 to the case in which investors have different risk aversions, $\gamma^G \neq \gamma^N$, different elasticity of intertemporal substitutions, $\psi^G \neq \psi^N$, and in which both investors have preferences toward both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t. This poses no particular problem for the resolution, as our method allows for any value of the parameters. Exploring additional asymmetries stemming from those could be an interesting avenue for future work.

Proposition A.2. *The optimal portfolios of the green and neutral investors $j \in \{G, N\}$ are given by*

$$\begin{pmatrix} w_{g,t}^G \\ w_{b,t}^G \end{pmatrix} = \frac{1}{\gamma^G} \left(\Sigma_t^\top \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi_g^G \\ \mu_{b,t} - r_t + \phi_b^G \end{pmatrix} + \left(\frac{1-\gamma^G}{1-\psi^G}\right) \Sigma_t^\top \left(\frac{J_{x,t}^G}{J_t^G} x_t \sigma_{x,t} + \frac{J_{y,t}^G}{J_t^G} y_t \sigma_{y,t} \right) \right\}$$

$$b_t^G = 1 - w_{g,t}^G - w_{b,t}^G \tag{A.6}$$

$$\begin{pmatrix} w_{g,t}^N \\ w_{b,t}^N \end{pmatrix} = \frac{1}{\gamma^N} \left(\Sigma_t^\top \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi_g^N \\ \mu_{b,t} - r_t + \phi_b^N \end{pmatrix} + \left(\frac{1-\gamma^N}{1-\psi^N}\right) \Sigma_t^\top \left(\frac{J_{x,t}^N}{J_t^N} x_t \sigma_{x,t} + \frac{J_{y,t}^N}{J_t^N} y_t \sigma_{y,t} \right) \right\}$$

$$b_t^N = 1 - w_{g,t}^N - w_{b,t}^N \tag{A.7}$$

where $w_{g,t}^i, w_{b,t}^i, b_t^i$ are the portfolio weights (as a share of wealth) allocated to the green equity asset, the brown equity asset, and the riskless bond, and $\Sigma_t \equiv \begin{bmatrix} \sigma_{g,t} & \sigma_{b,t} \end{bmatrix}$.

A.4 Equilibrium

The definition of the equilibrium is standard.

Definition 1. *A competitive equilibrium is a set of aggregate stochastic processes adapted to the filtration generated by $\vec{Z}$: the price of the equity asset $(Q_{g,t}, Q_{b,t})$, and the interest rate (r_t), together with a set of individual stochastic processes for each investor: consumption of each good $(C^G_{g,t}, C^G_{b,t}, C^N_{g,t}, C^N_{b,t})$, wealth (W^G_t, W^N_t), and portfolio shares $(w^G_{g,t}, w^N_{b,t}, w^G_{g,t}, w^N_{b,t})$, such that, given the output of the two endowment trees $(Y_{g,t}, Y_{b,t})$:*

1. *Given the aggregate stochastic processes, individual choices solve the investor optimization problem given in Section 2.*

2. *Markets clear.*

   (a) *Good markets:*

$$\begin{aligned} C^G_{g,t} + C^N_{g,t} &= Y_{g,t} \\ C^G_{b,t} + C^N_{b,t} &= Y_{b,t} \end{aligned} \tag{A.8}$$

   (b) *Equity markets:*

$$\begin{aligned} w^G_{g,t} W^G_t + w^N_{g,t} W^N_t &= Q_{g,t} \\ w^G_{b,t} W^G_t + w^N_{b,t} W^N_t &= Q_{b,t} \end{aligned} \tag{A.9}$$

Most importantly, as shown in Section 2.3 of the main text, the equilibrium can be recast as a stationary recursive Markovian equilibrium in which all variables of interest are expressed as a function of a pair of state variables $X_t \equiv (x_t, y_t)'$, whose dynamics are also solely a function of X_t. x_t is the wealth share of the green investor, and y_t is the relative supply of the green good.

A.5 Evolutions of the state variables

Due to the Markovian nature of the equilibrium, the laws of motion of the state variables underlie the dynamics of the economy. They are summarized in Proposition A.3.

Proposition A.3. *The laws of motion for the wealth share of the green investor x_t, and the relative supply of the green good y_t are*

$$\begin{aligned}\frac{dx_t}{x_t} &\equiv \mu_{x,t}dt + \sigma_{x,t}^{\top} d\vec{Z}_t \\ \frac{dy_t}{y_t} &\equiv \mu_{y,t}dt + \sigma_{y,t}^{\top} d\vec{Z}_t\end{aligned} \tag{A.10}$$

where

$$\begin{aligned}\mu_{x,t} &= \left(w_{g,t}^G - w_{g,t}^M\right)(\mu_{g,t} - r_t) + \left(w_{b,t}^G - w_{b,t}^M\right)(\mu_{b,t} - r_t) \\ &\quad + \left(F_{g,t} w_{g,t}^M + w_{b,t}^M F_{b,t}\right) - P_t^G c_t^G \\ &\quad - \left((w_{g,t}^G - w_{g,t}^M)\sigma_{g,t} + (w_{b,t}^G - w_{b,t}^M)\sigma_{b,t}\right)^{\top}\left(w_{g,t}^M \sigma_{g,t} + w_{b,t}^M \sigma_{b,t}\right) \\ \sigma_{x,t} &= \left(\left(w_{g,t}^G - w_{g,t}^M\right)\sigma_{g,t} + \left(w_{b,t}^M - w_{b,t}^M\right)\sigma_{b,t}\right) \\ \mu_{y,t} &= (1 - y_t)\left(\mu_{Y_g} - \mu_{Y_b}\right) - (1 - y_t)\left(\sigma_{Y_g} - \sigma_{Y_b}\right)^{\top}\left(y_t \sigma_{Y_g} + (1 - y_t)\sigma_{Y_b}\right) \\ \sigma_{y,t} &= (1 - y_t)\left(\sigma_{Y_g} - \sigma_{Y_b}\right)\end{aligned}$$

and $w_{g,t}^M \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$, $w_{b,t}^M \equiv Q_{b,t}/(Q_{g,t} + Q_{b,t})$ are the weights of the green and brown equity assets in the market portfolio, with $w_{g,t}^M$ defined in Equation (6) and $w_{b,t}^M = 1 - w_{g,t}^M$ in equilibrium because the bond is zero net supply.

Figure A.1 show the drift and diffusion terms for y_t, the relative supply of the green good. They do not depend on the wealth share of the green investor x_t or on parameters beyond $\mu_{Y_g}, \mu_{Y_b}, \sigma_{Y_g}, \sigma_{Y_b}$, because y_t is purely determined by the outputs of the green and brown trees.

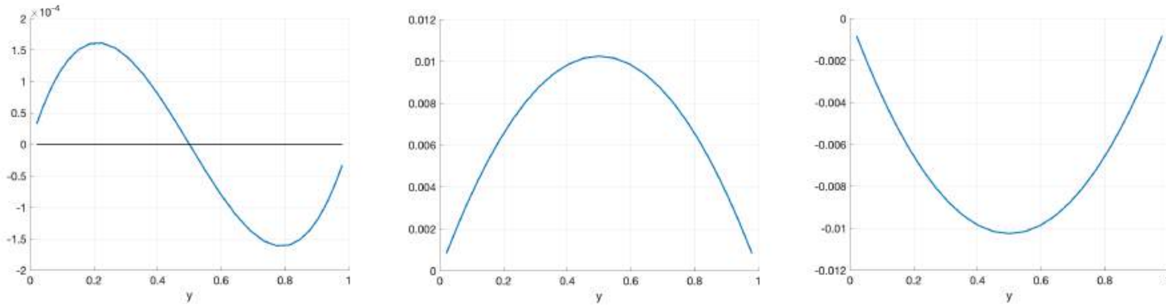
Figures A.2, A.3, A.4 show the drift and diffusion terms for x_t, the wealth share of the green investor, for various calibrations. As mentioned in the main text, the diffusion terms for x_t, and therefore the covariance between state variables, are inherently dependent on the portfolio bias of the green investor, which in turn depends strongly on her preference for green consumption (α) and green investing (ϕ).

Figure A.1: Drift and diffusion terms for the relative supply of the green good y_t

(a) Drift of y_t: $\mu_{y,t} y_t$

(b) Loading of y_t on shock to green output: $\sigma_{yZ_g,t} y_t$

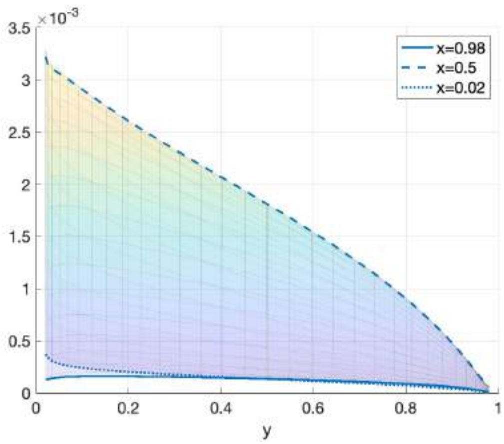
(c) Loading of y_t on shock to brown output: $\sigma_{yZ_b,t} y_t$



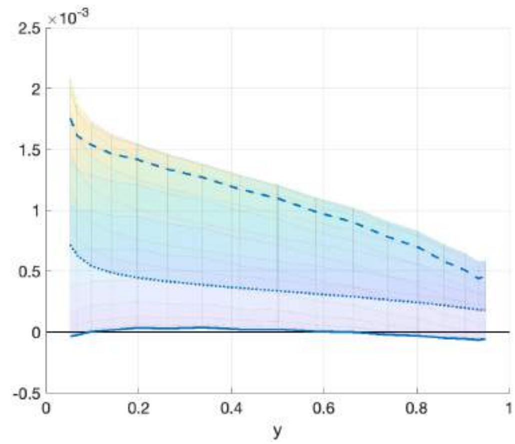
Notes: Based on the calibration of Assumption 1. y_t is the relative supply of the green good, which is exogenous so that its drift and diffusion terms do not depend on the wealth share of the green investor x_t.

Figure A.2: Drift for the wealth share of the green investor x_t: $\mu_{x,t} x_t$

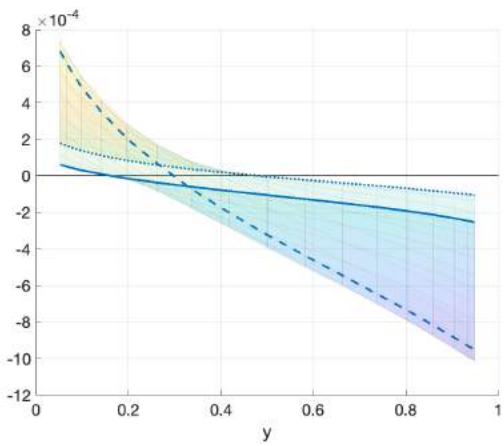
(a) Baseline calibration
($\alpha = 0.85, \phi = 1\%$)



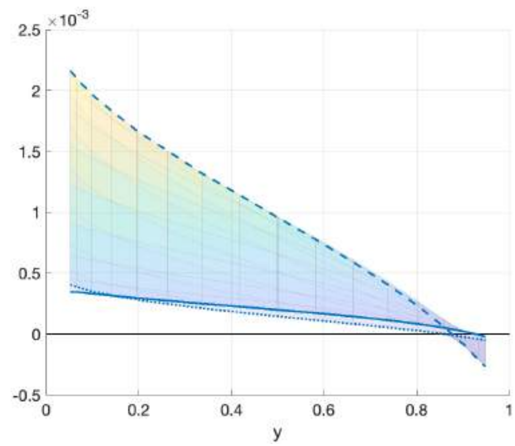
(b) No preference for green consumption
($\alpha = 1/2, \phi = 1\%$)



(c) No preference for green investing
($\alpha = 0.85, \phi = 0$)



(d) Limited preference for green investing
($\alpha = 0.85, \phi = 0.5\%$)

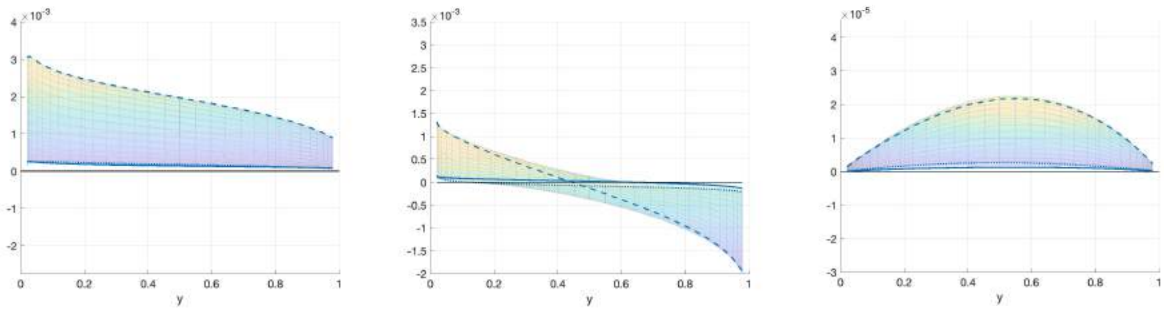


Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure A.3: Diffusion terms for the wealth share of the green investor x_t

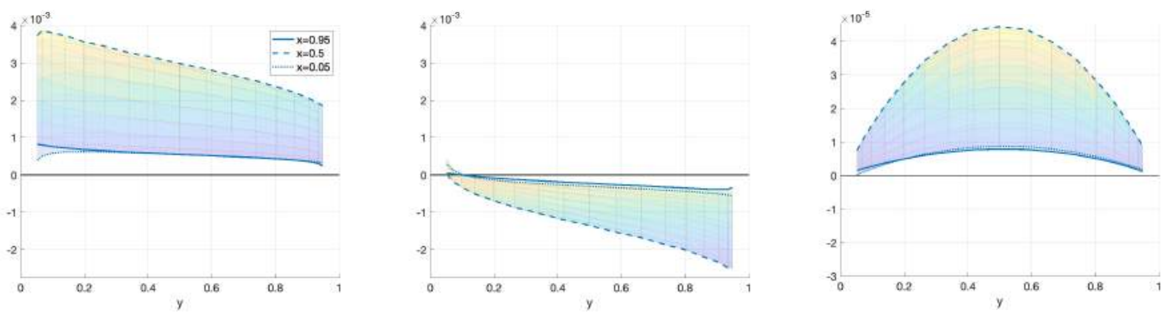
(a) Baseline calibration ($\alpha = 0.85, \phi = 1\%$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$



(b) No preference for green consumption ($\alpha = 1/2, \phi = 1\%$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$ | Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$

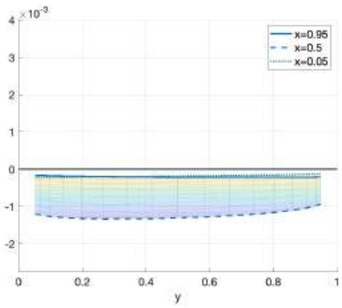


Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

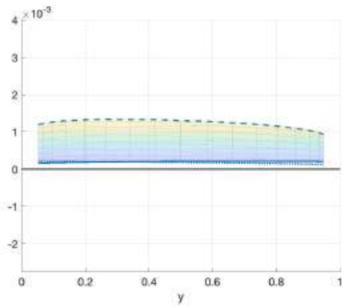
Figure A.4: Diffusion terms for the wealth share of the green investor x_t

(c) No preference for green investing ($\alpha = 0.85, \phi = 0$)

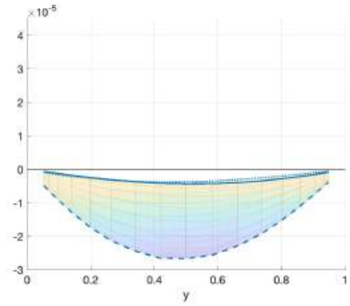
Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$



Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$

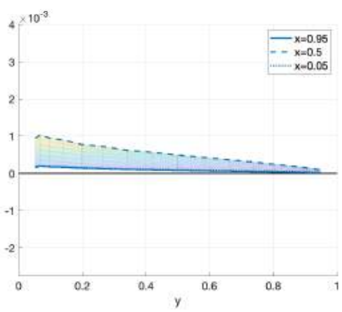


Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$

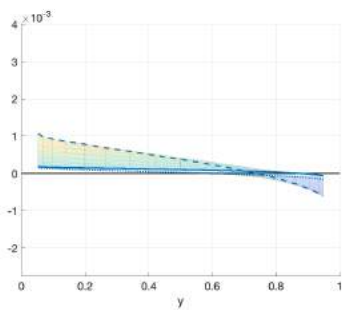


(d) Limited preference for green investing ($\alpha = 0.85, \phi = 0.5\%$)

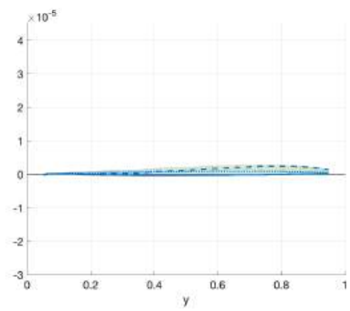
Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$



Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$



Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

A.6 Marginal values of wealth and Hamilton-Jacobi-Bellman equations

Proposition A.4. *J_t^G, J_t^N satisfy the Hamilton-Jacobi-Bellman equations for $i \in \{G, N\}$*

$$0 = \left(\frac{1}{\psi - 1}\right) P_t^{i1-\psi} J_t^i - \left(\frac{1}{1 - 1/\psi}\right) \rho + r_t + \frac{\gamma}{2} \left(w_{g,t}^i \sigma_{g,t} + w_{b,t}^i \sigma_{b,t}\right)^\top \left(w_{g,t}^i \sigma_{g,t} + w_{b,t}^i \sigma_{b,t}\right)$$
$$+ \left(\frac{1}{1-\psi}\right) \mu_{J^i,t} + \frac{1}{2} \left(\frac{1}{1-\psi}\right) \left(\frac{\psi - \gamma}{1-\psi}\right) \sigma_{J^i,t}^\top \sigma_{J^i,t} \tag{A.11}$$

where $\mu_{J^i,t}, \sigma_{J^i,t}$ are the geometric drift and diffusion terms of J_t^i obtained as in Remark A.1:

$$\frac{dJ_t^i}{J_t^i} \equiv \mu_{J^i,t} dt + \sigma_{J^i,t}^\top d\vec{Z}_t \tag{A.12}$$

A.7 Consumptions, goods prices

Proposition A.5. *The consumption of each investor $i \in \{G, N\}$ is given by*

$$c_t^i \equiv \frac{C_t^i}{W_t^i} = P_t^{i-\psi} J_t^i \tag{A.13}$$

$$c_{g,t}^i = \alpha^i \left(\frac{p_{g,t}}{P_t^i}\right)^{-\theta} c_t^i \tag{A.14}$$

$$c_{b,t}^i = (1 - \alpha^i) \left(\frac{p_{b,t}}{P_t^i}\right)^{-\theta} c_t^i \tag{A.15}$$

$$P_t^i = \left[\alpha^i p_{g,t}^{1-\theta} + (1 - \alpha^i) p_{b,t}^{1-\theta}\right]^{1/(1-\theta)} \tag{A.16}$$

Proposition A.6. *The relative price of the green good, $q_t = q(X_t) \equiv p_{g,t}/p_{b,t}$, solves the following non-linear equation*

$$q_t = S_t^{1/\theta} \left(\frac{1 - y_t}{y_t} \right)^{1/\theta} \tag{A.17}$$

where

$$S_t = \frac{\alpha^G J_t^G x_t P_t^{G\theta-\psi} + \alpha^N P_t^{N\theta-\psi} J_t^N (1 - x_t)}{(1 - \alpha^G) P_t^{G\theta-\psi} J_t^G x_t + (1 - \alpha^N) P_t^{N\theta-\psi} J_t^N (1 - x_t)}$$

Using the defintion of the numéraire, with $a = 1/2$, prices follow

$$p_{g,t} = \left(a + (1 - a) q_t^{\theta-1}\right)^{1/(\theta-1)} \tag{A.18}$$

$$p_{b,t} = p_{g,t} q_t^{-1} = \left(a q_t^{1-\theta} + (1 - a)\right)^{1/(\theta-1)} \tag{A.19}$$

$$P_t^i = \left[\alpha^i p_{g,t}^{1-\theta} + (1 - \alpha^i) p_{b,t}^{1-\theta}\right]^{1/(1-\theta)} \tag{A.20}$$

$$\mathcal{E}_t = P_t^G / P_t^N \tag{A.21}$$

B Additional figures

B.1 Economic set-up

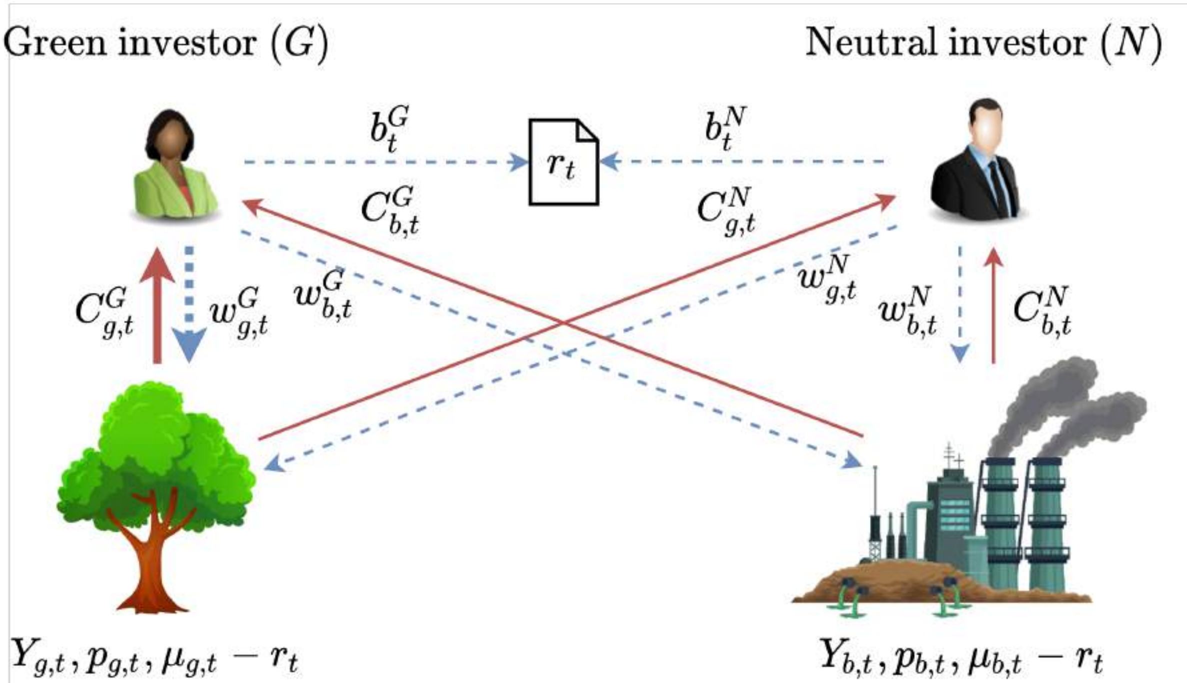


Figure B.1: The Economy

Source: Vecteezy.com. Back to main text: Section 2.

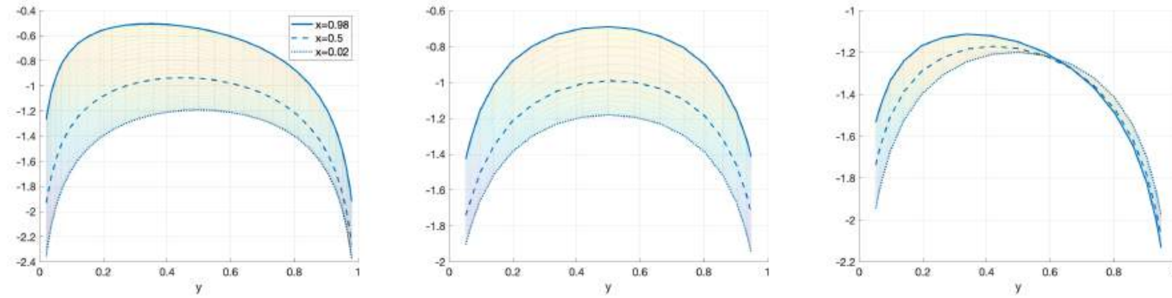
B.2 Riskfree interest rate, risk premia, discount rates

Figure B.2: Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Inflation-Indexed

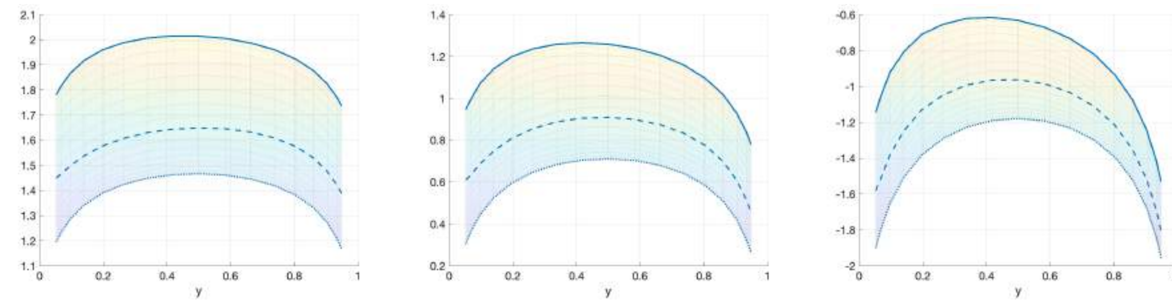
Source: Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis.

Figure B.3: Riskfree interest rate (r_t) for various calibrations

(a) Baseline ($\gamma = 50, \alpha = 0.85, \phi = 1\%$) | (b) No pref. green cons. ($\gamma = 50, \alpha = 1/2, \phi = 1\%$) | (c) No pref. green inv. ($\gamma = 50, \alpha = 0.85, \phi = 0\%$)



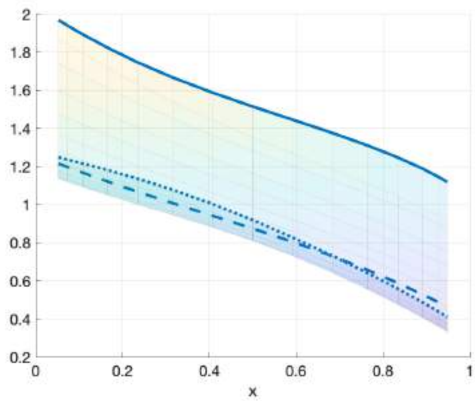
(d) $\gamma = 15, \alpha = 0.75, \phi = 1\%$ | (e) $\gamma = 25, \alpha = 0.75, \phi = 1\%$ | (f) $\gamma = 50, \alpha = 0.75, \phi = 1\%$



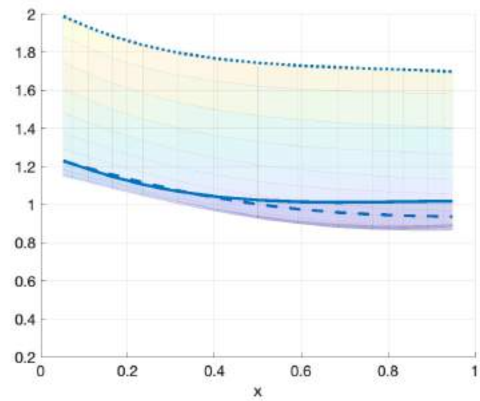
Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure B.4: Risk premia under CRRA preferences (%)

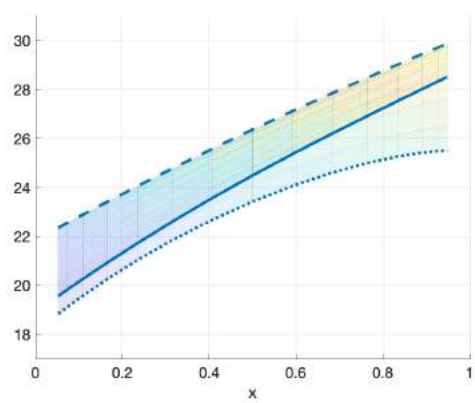
(a) Green risk premium ($\mu_{g,t} - r_t$)



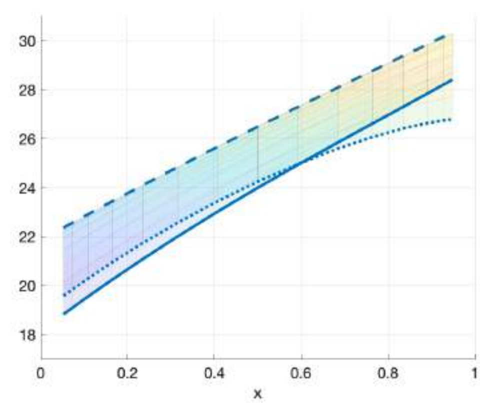
(b) Brown risk premium ($\mu_{b,t} - r_t$)



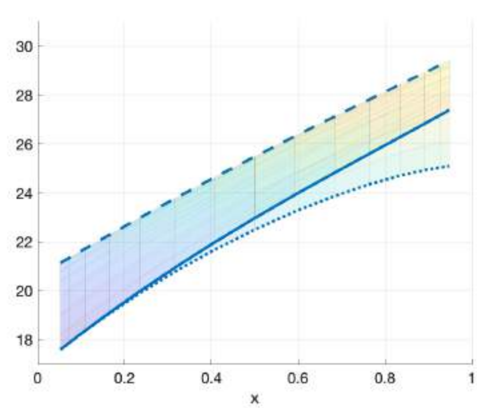
(c) Green discount rate ($\mu_{g,t}$)



(d) Brown discount rate ($\mu_{b,t}$)



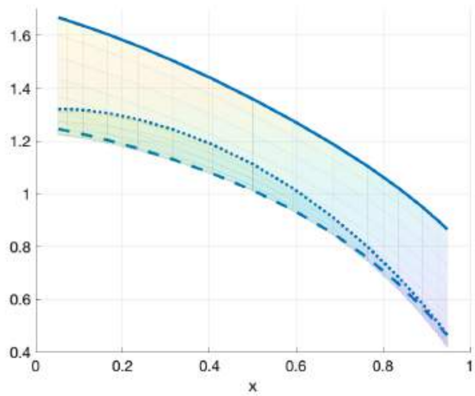
(e) Riskfree rate (r_t)



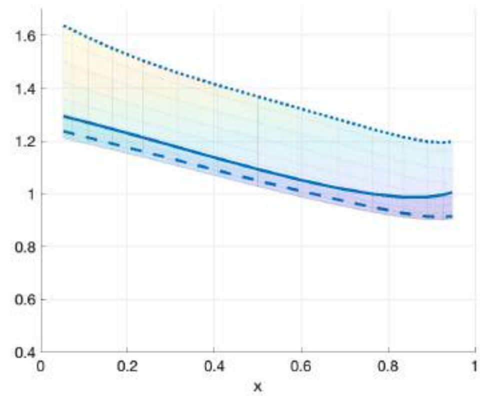
Notes: Based on the calibration of Assumption 1, with $\gamma = 25$ and $\psi = 1/\gamma$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure B.5: Risk premia under baseline recursive preferences (%)

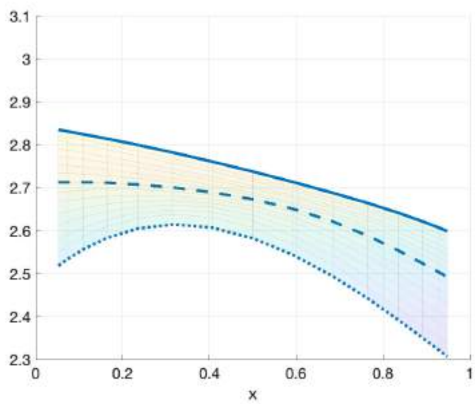
(a) Green risk premium ($\mu_{g,t} - r_t$)



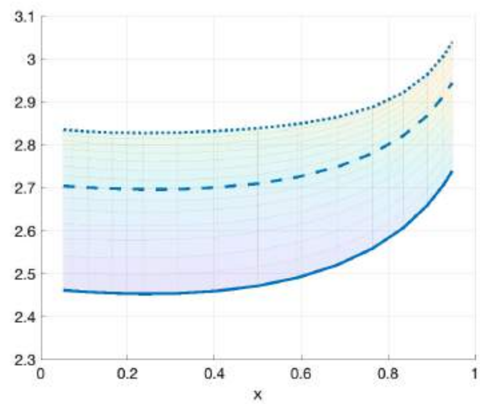
(b) Brown risk premium ($\mu_{b,t} - r_t$)



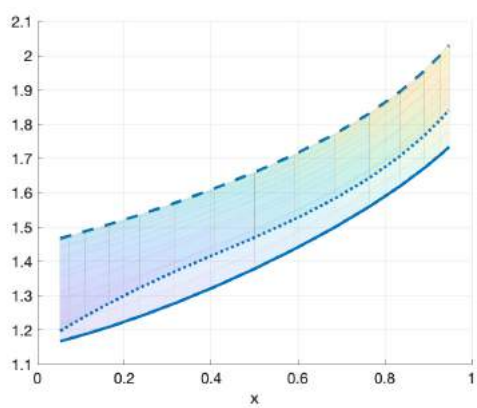
(c) Green discount rate ($\mu_{g,t}$)



(d) Brown discount rate ($\mu_{b,t}$)



(e) Riskfree rate (r_t)

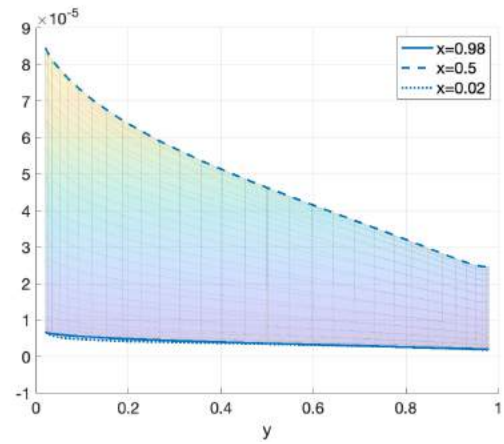


Notes: Based on the calibration of Assumption 1, with $\gamma = 25$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

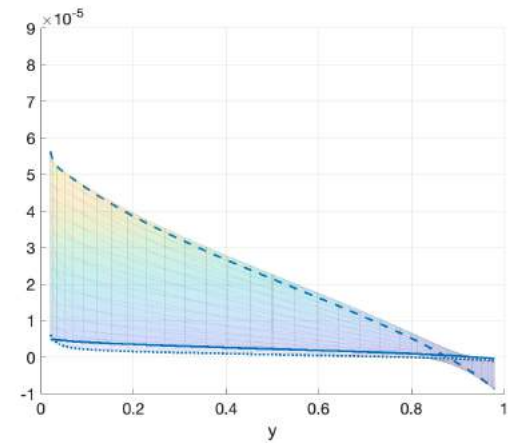
B.3 Quantities and prices of risk

Figure B.6: Quantities of risk

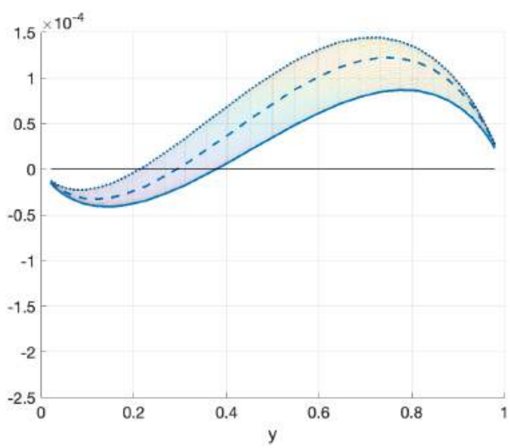
(a) Green asset returns on x_t risk
$\text{cov}_t(dR_{g,t}, dx_t)dt^{-1} = \sigma_{g,t}^{\top}\sigma_{x,t}x_t$



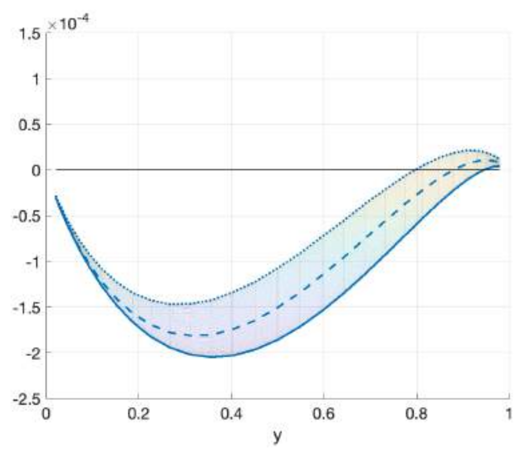
(b) Brown asset returns on x_t risk
$\text{cov}_t(dR_{b,t}, dx_t)dt^{-1} = \sigma_{b,t}^{\top}\sigma_{x,t}x_t$



(c) Green asset returns on y_t risk
$\text{cov}_t(dR_{g,t}, dy_t)dt^{-1} = \sigma_{g,t}^{\top}\sigma_{y,t}y_t$



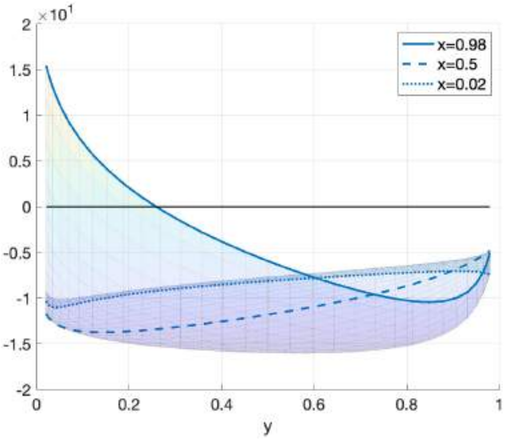
(d) Brown asset returns on y_t risk
$\text{cov}_t(dR_{b,t}, dy_t)dt^{-1} = \sigma_{b,t}^{\top}\sigma_{y,t}y_t$



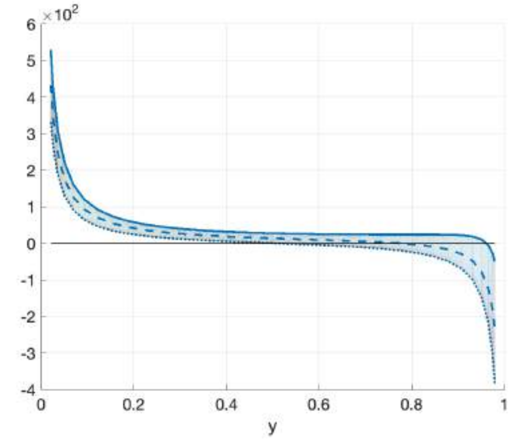
Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure B.7: Quantities of risk

(a) Price of x_t risk
$-\left(\frac{1-\gamma}{1-\psi}\right)\left\{x_t\frac{J^G_{x,t}}{J^G_t}+(1-x_t)\frac{J^N_{x,t}}{J^N_t}\right\}$



(b) Price of y_t risk
$-\left(\frac{1-\gamma}{1-\psi}\right)\left\{x_t\frac{J^G_{y,t}}{J^G_t}+(1-x_t)\frac{J^N_{y,t}}{J^N_t}\right\}$



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

B.4 Second moment of returns

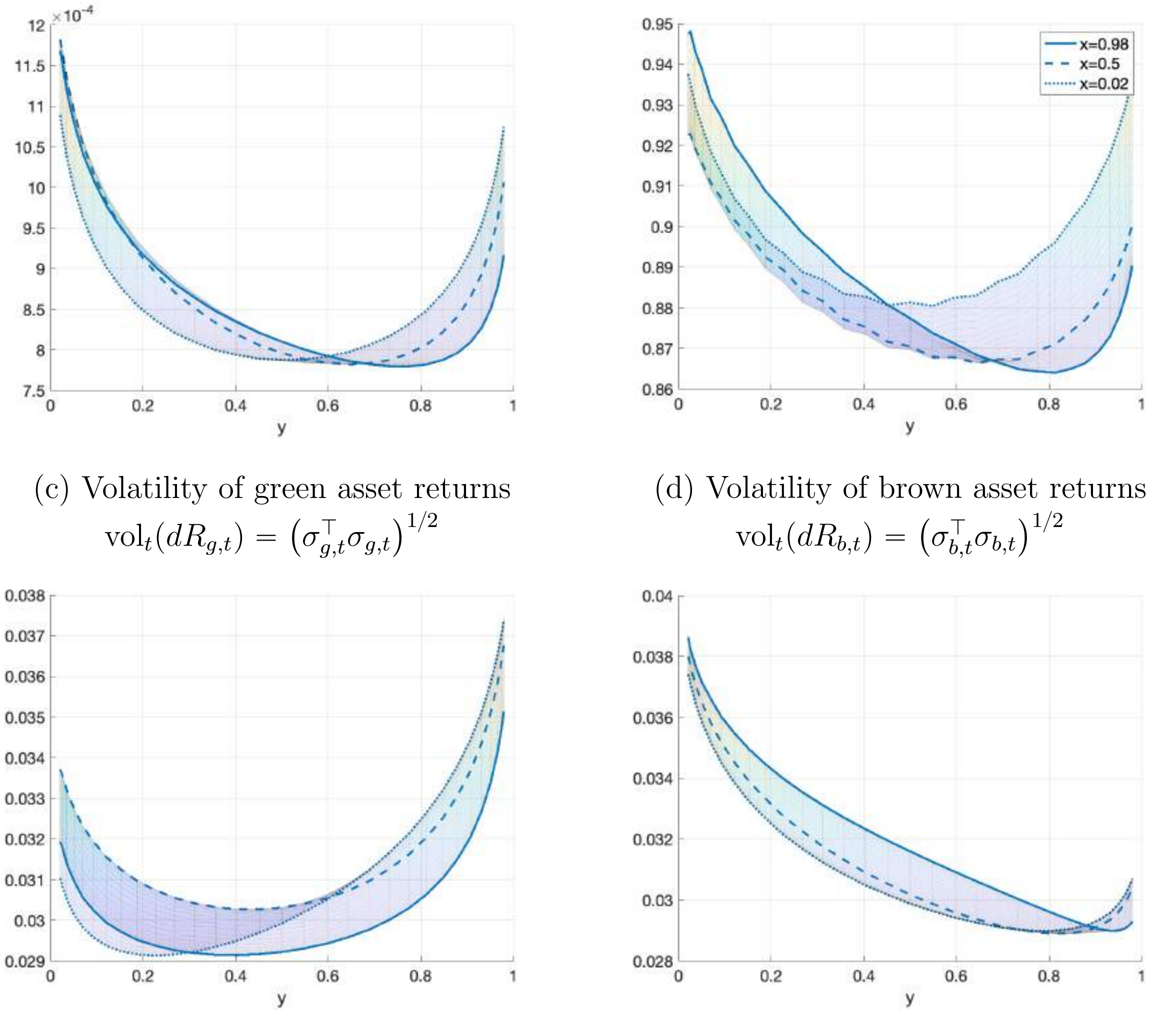
Figure B.8: (Instantaneous) Second moment of returns

(a) Covariance of returns
$\text{cov}_t(dR_{g,t}, dR_{b,t})dt^{-1} = \sigma_{g,t}^\top \sigma_{b,t}$

(b) Correlation of returns
$\text{corr}_t(dR_{b,t}, dR_{b,t})dt^{-1}$

(c) Volatility of green asset returns
$\text{vol}_t(dR_{g,t}) = \left(\sigma_{g,t}^\top \sigma_{g,t}\right)^{1/2}$

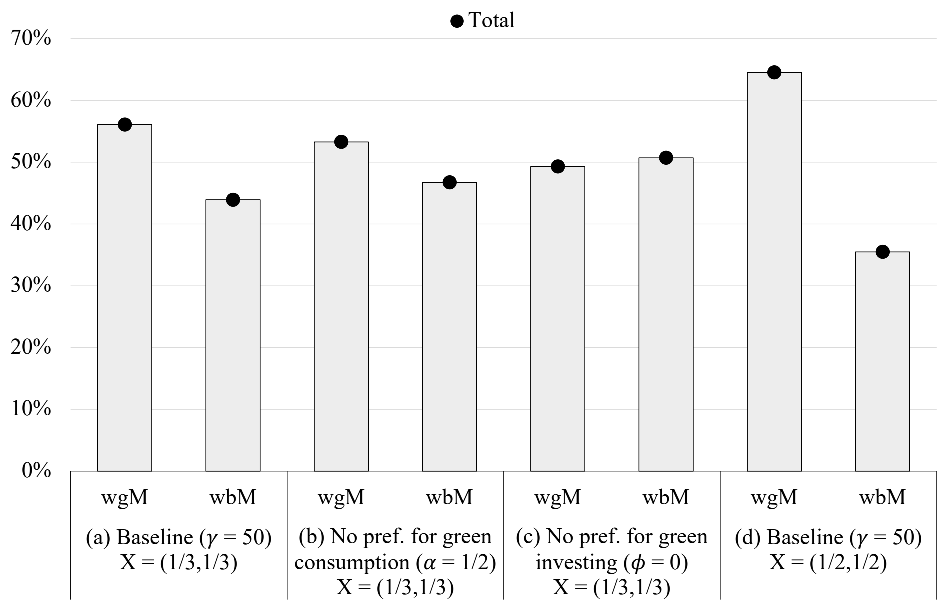
(d) Volatility of brown asset returns
$\text{vol}_t(dR_{b,t}) = \left(\sigma_{b,t}^\top \sigma_{b,t}\right)^{1/2}$



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

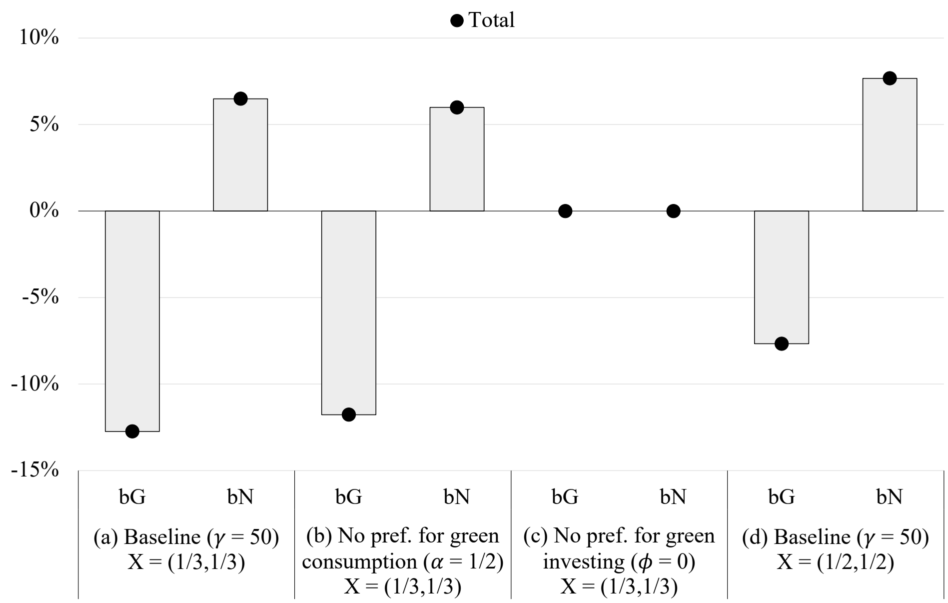
B.5 Portfolios

Figure B.9: Market portfolio at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



Notes: Based on the calibration of Assumption 1, except for the specified parameters. The figure shows the market portfolio at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d). $w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}), w^M_{b,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$ are the weights (as % of wealth) on the green and brown asset in the market portfolio. In equilibrium, $w^M_{b,t} = 1 - w^M_{g,t}$ because the bond is in zero net supply.

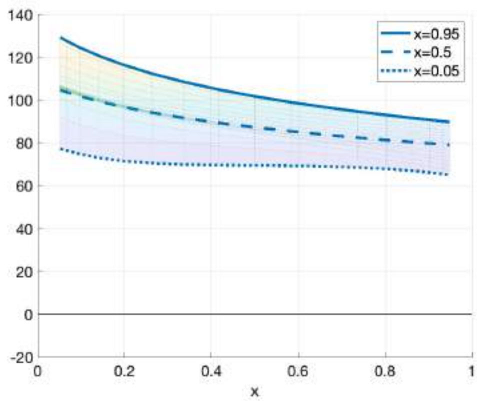
Figure B.10: Borrowing and saving in the riskless bond at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



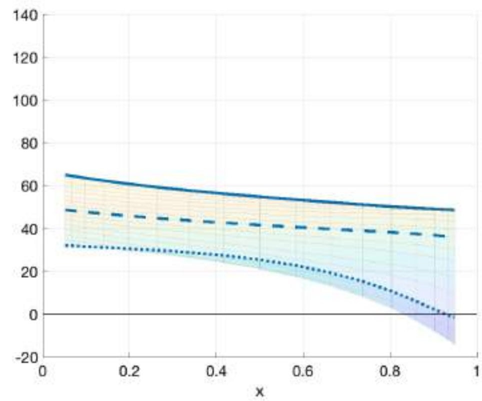
Notes: Based on the calibration of Assumption 1, except for the specified parameters. The figure shows $b_t^i = 1 - w_{g,t}^i - w_{b,t}^i$, the weight (as % of wealth) allocated to the riskfree bond by each investor, $i \in \{G, N\}$. $b_t^i > 0$ corresponds to saving in the bond, $b_t^i < 0$ corresponds to borrowing.

Figure B.11: Portfolios of both investors, $i \in \{G, N\}$ (% of wealth)

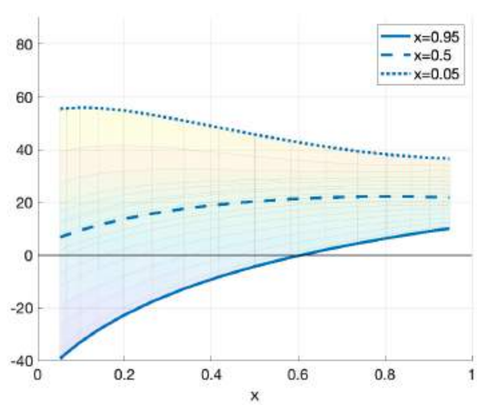
(a) Green asset in green portfolio ($w^G_{g,t}$)



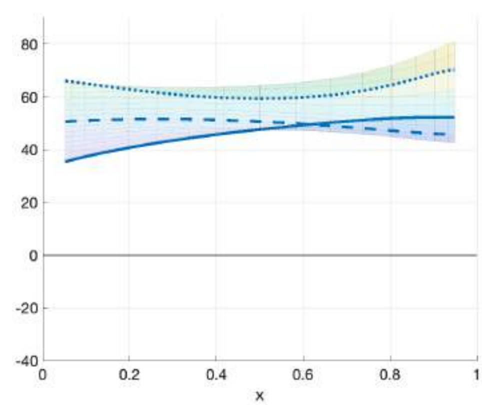
(b) Green asset in neutral portfolio ($w^N_{g,t}$)



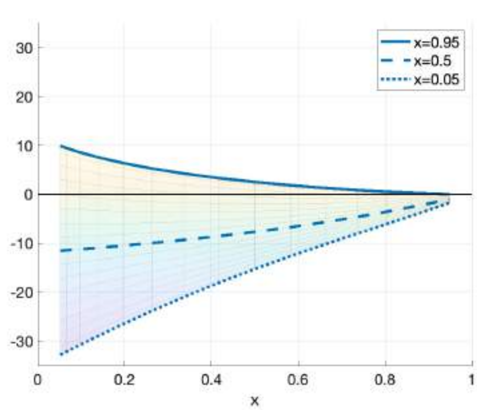
(c) Brown asset in green portfolio ($w^G_{b,t}$)



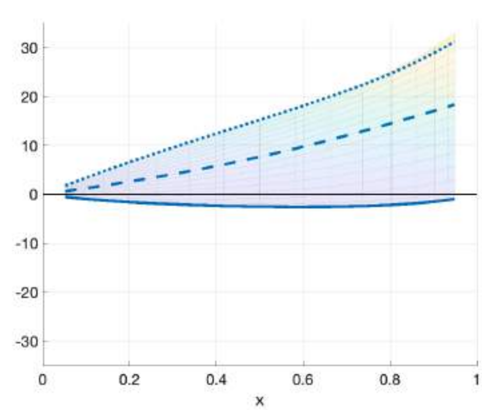
(d) Brown asset in neutral portfolio ($w^N_{b,t}$)



(e) Riskfree bond in green portfolio ($b^G_t = 1 - w^G_{g,t} - w^G_{b,t}$)



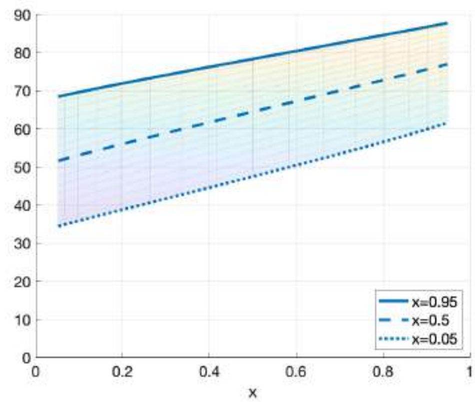
(f) Riskfree bond in neutral portfolio ($b^N_t = 1 - w^N_{g,t} - w^N_{b,t}$)



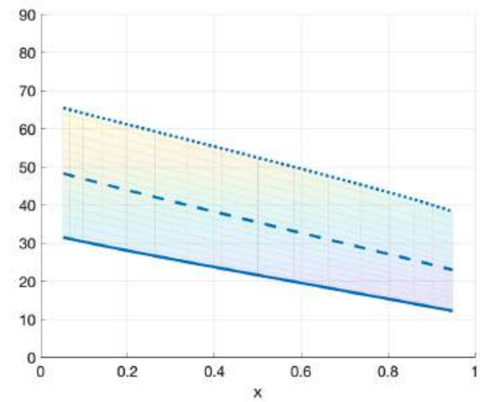
Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure B.12: Portfolios of both investors, $i \in \{G, N\}$, vs. market portfolio (%)

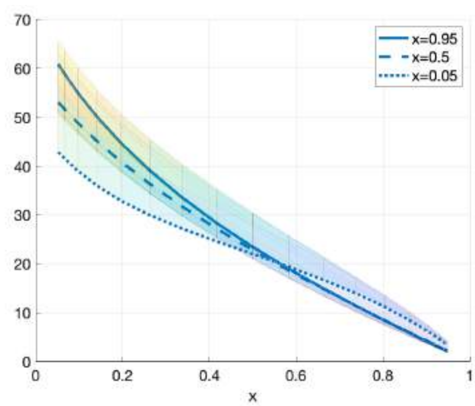
(a) Green asset in market portfolio ($w_{g,t}^{M}$)



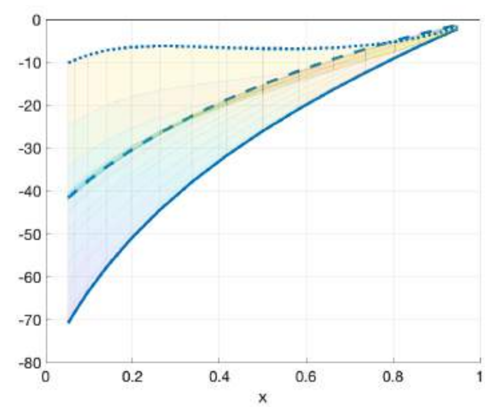
(b) Brown asset in market portfolio ($w_{g,t}^{M}$)



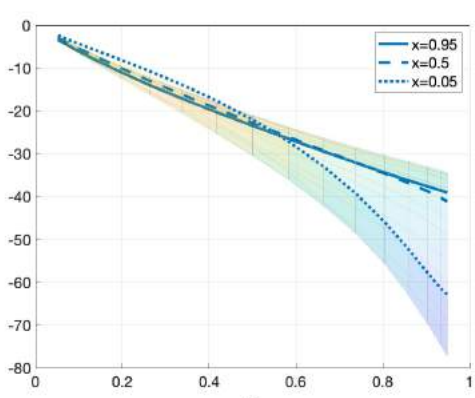
(c) Bias on green asset in green vs. market portfolio ($w_{g,t}^{G} - w_{g,t}^{M}$)



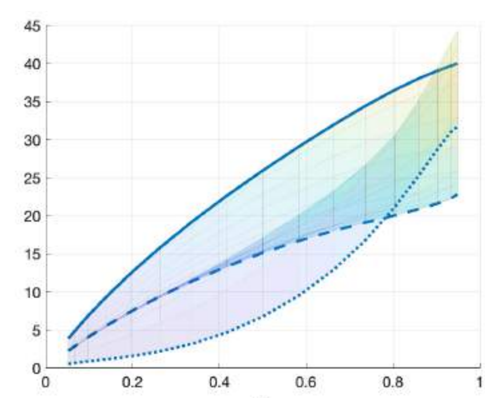
(d) Bias on brown asset in green vs. market portfolio ($w_{b,t}^{G} - w_{b,t}^{M}$)



(e) Bias on green asset in neutral vs. market portfolio ($w_{g,t}^{N} - w_{g,t}^{M}$)



(f) Bias on brown asset in neutral vs. market portfolio ($w_{b,t}^{N} - w_{b,t}^{M}$)



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

C Empirics – Additional results

C.1 Estimation without constant (baseline), detailed tables

Table C.1: Empirical estimation of consumption premia (without constant, consumption factor based on production indices, and MSCI environmental scores)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	0.702		0.995	0.491	0.819	0.747	0.877	0.947
	(5.146)		(7.673)	(3.381)	(5.570)	(5.135)	(5.911)	(6.339)
GMB		0.124	0.052	-0.025	0.109	0.145	0.111	0.142
		(2.085)	(0.918)	(-0.444)	(2.324)	(3.379)	(2.586)	(3.633)
Panel B: Premia difference (annual, %)								
Cons. hedge	1.633		2.305	0.884	1.479	1.228	1.562	1.561
GMB		0.838	0.352	-0.161	0.719	0.939	0.762	0.994
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: $ConsHedge$ and GMB refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

Table C.2: Empirical estimation of consumption premia (without constant, consumption factor based on production indices, and carbon intensities)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	0.610		0.822	0.542	0.623	0.785	0.391	0.671
	(4.153)		(5.776)	(3.887)	(5.218)	(5.359)	(3.426)	(4.393)
GMB		0.124	0.139	0.095	0.006	0.026	0.033	0.082
		(2.085)	(2.409)	(1.899)	(0.131)	(0.559)	(0.865)	(1.939)
Panel B: Premia difference (annual, %)								
Cons. hedge	1.436		2.004	1.065	1.245	1.474	0.754	1.239
GMB		0.838	0.975	0.639	0.042	0.177	0.250	0.624
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: $ConsHedge$ and GMB refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

Table C.3: Empirical estimation of consumption premia (without constant, consumption factor based on producer *price* indices, and MSCI environmental scores)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	-0.029		-0.294	-0.697	-0.935	-0.852	-0.844	-0.858
	(-0.149)		(-1.568)	(-3.821)	(-4.830)	(-5.016)	(-4.820)	(-5.441)
GMB		0.124	0.199	0.067	0.139	0.174	0.203	0.232
		(2.085)	(3.826)	(1.365)	(3.068)	(4.126)	(5.112)	(5.918)
Panel B: Premia difference (annual, %)								
Cons. hedge	-0.039		-0.405	-0.927	-1.187	-1.041	-1.067	-1.076
GMB		0.838	1.370	0.429	0.918	1.163	1.465	1.665
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: $ConsHedge$ and GMB refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

Table C.4: Empirical estimation of consumption premia (without constant, consumption factor based on producer *price* indices, and carbon intensities)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	-0.256		-0.499	-0.621	-0.675	-0.469	-0.790	-0.620
	(-1.470)		(-3.488)	(-3.695)	(-4.164)	(-3.119)	(-5.088)	(-3.856)
GMB		0.124	0.125	0.004	0.026	0.089	0.061	0.144
		(2.085)	(2.500)	(0.073)	(0.585)	(2.156)	(1.709)	(3.590)
Panel B: Premia difference (annual, %)								
Cons. hedge	-0.384		-0.752	-0.895	-0.928	-0.622	-1.102	-0.842
GMB		0.838	0.884	0.024	0.175	0.606	0.454	1.076
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: ConsHedge and *GMB* refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

C.2 Estimation with constant, main table

Table C.5: Empirical estimation of consumption premia (with constant)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	0.577	0.638	0.717	0.856	-0.690	-0.564	-0.949	-0.819
	(4.739)	(4.249)	(4.952)	(5.850)	(-4.269)	(-3.611)	(-4.966)	(-5.341)
GMB	-0.004	0.074	0.099	0.135	0.007	0.129	0.122	0.223
	(-0.092)	(1.759)	(2.102)	(3.479)	(0.150)	(3.225)	(2.693)	(5.703)
Panel B: Premia difference (annual, %)								
Cons. hedge	1.149	1.176	1.289	1.403	-0.949	-0.768	-1.204	-1.030
GMB	-0.030	0.560	0.652	0.944	0.046	0.960	0.808	1.598
Measure	Supply	Supply	Supply	Supply	Prices	Prices	Prices	Prices
Greenness	CI	CI	MSCI	MSCI	CI	CI	MSCI	MSCI
Controls	FF3	FF5MOM	FF3	FF5MOM	FF3	FF5MOM	FF3	FF5MOM

Notes: ConsHedge and *GMB* refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

C.3 Estimation with constant, detailed tables

Table C.6: Empirical estimation of consumption premia (with constant, consumption factor based on production indices, and MSCI environmental scores)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	0.668		0.995	0.402	0.717	0.626	0.782	0.856
	(4.495)		(7.013)	(2.763)	(4.952)	(4.390)	(5.407)	(5.850)
GMB		0.118	0.052	-0.042	0.099	0.150	0.099	0.135
		(1.990)	(0.920)	(-0.720)	(2.102)	(3.556)	(2.309)	(3.479)
Panel B: Premia difference (annual, %)								
Cons. hedge	1.553		2.305	0.718	1.289	1.022	1.385	1.403
GMB		0.799	0.352	-0.264	0.652	0.971	0.679	0.944
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: $ConsHedge$ and GMB refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

Table C.7: Empirical estimation of consumption premia (with constant, consumption factor based on production indices, and carbon intensities)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	0.584		0.822	0.489	0.577	0.736	0.363	0.638
	(3.665)		(5.369)	(3.484)	(4.739)	(5.135)	(3.190)	(4.249)
GMB		0.118	0.139	0.101	-0.004	0.036	0.035	0.074
		(1.990)	(2.411)	(2.037)	(-0.092)	(0.783)	(0.927)	(1.759)
Panel B: Premia difference (annual, %)								
Cons. hedge	1.374		2.002	0.957	1.149	1.378	0.699	1.176
GMB		0.799	0.975	0.679	-0.030	0.245	0.267	0.560
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: $ConsHedge$ and GMB refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

Table C.8: Empirical estimation of consumption premia (with constant, consumption factor based on producer *price* indices, and MSCI environmental scores)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	0.008		-0.275	-0.703	-0.949	-0.859	-0.836	-0.819
	(0.041)		(-1.456)	(-3.894)	(-4.966)	(-5.132)	(-4.890)	(-5.341)
GMB		0.118	0.185	0.045	0.122	0.174	0.182	0.223
		(1.990)	(3.530)	(0.897)	(2.693)	(4.152)	(4.629)	(5.703)
Panel B: Premia difference (annual, %)								
Cons. hedge	0.011		-0.379	-0.936	-1.204	-1.049	-1.057	-1.030
GMB		0.799	1.270	0.288	0.808	1.160	1.314	1.598
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: ConsHedge and *GMB* refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

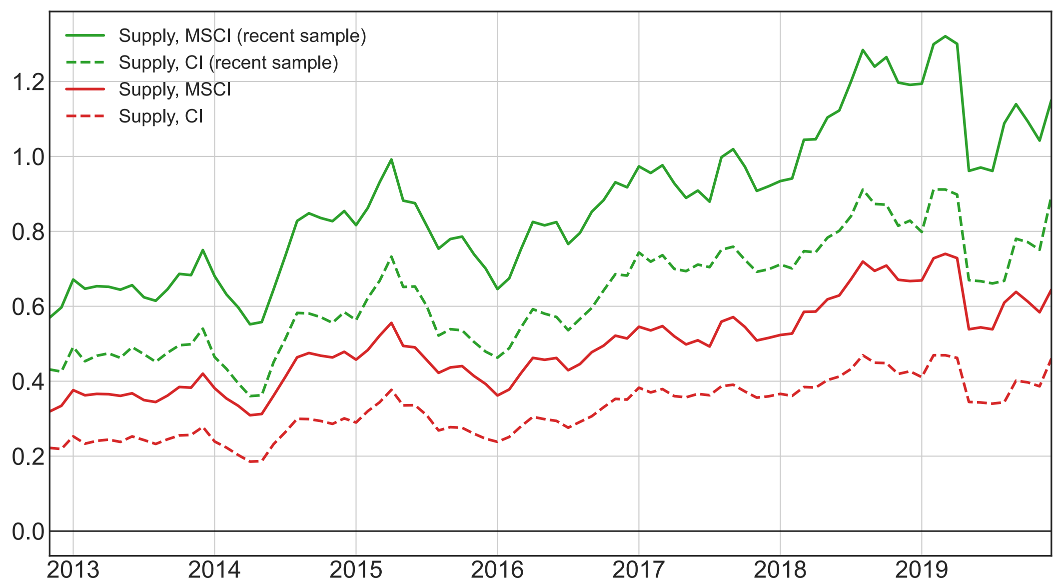
Table C.9: Empirical estimation of consumption premia (with constant, consumption factor based on producer *price* indices, and carbon intensities)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Risk prices (monthly, %)								
Cons. hedge	-0.219		-0.480	-0.612	-0.690	-0.408	-0.761	-0.564
	(-1.236)		(-3.306)	(-3.676)	(-4.269)	(-2.819)	(-4.999)	(-3.611)
GMB		0.118	0.119	-0.001	0.007	0.094	0.052	0.129
		(1.990)	(2.361)	(-0.015)	(0.150)	(2.306)	(1.459)	(3.225)
Panel B: Premia difference (annual, %)								
Cons. hedge	-0.330		-0.724	-0.883	-0.949	-0.544	-1.063	-0.768
GMB		0.799	0.838	-0.005	0.046	0.642	0.387	0.960
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: $ConsHedge$ and GMB refer to the consumption hedging factor and the green-minus-brown factor. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Jan. 2007-Dec. 2019. Returns and betas are winsorized at the 1% level. The premium difference for each variable is computed as the capped value-weighted annual premium on assets with loading in the top tercile (high $\hat{\beta}$), minus that on assets with loading in the bottom tercile (low $\hat{\beta}$).

C.4 Consumption premia, additional figures

Figure C.1: Relative-consumption premium difference between green and brown assets, including NAICS 4



Notes: This figure depicts the difference in relative-supply consumption premium, constructed using Production Indices, between the tercile of greenest assets (high MSCI rating or low carbon intensity) and that of brownest assets (low MSCI rating or high carbon intensity), in consumer-related sectors, including NAICS 4: NAICS: 1 (Agriculture), 2 (Mining, Utilities, Construction), 3 (Manufacturing), 4 (Wholesale Trade, Retail Trade, Transport, Warehousing).