Asymmetric Information, Asset Markets, and the Medium of Exchange[*]

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Abstract

Recent work in the New Monetarist literature has studied how *asymmetric information* can hinder the role of assets as means of payment in decentralized markets for goods/services. The present paper revisits this important question, incorporating a crucial ingredient that has so far been overlooked: agents who wish to use assets for liquidity motives also have the option to sell these assets for money in dedicated *secondary markets*, where, arguably, the asymmetry of information is less severe, and subsequently use money as a medium of exchange. In this environment, agents use secondary asset market trades both for rebalancing their portfolios and as a *signal* of asset quality in the goods market. The model delivers a number of new economic insights. For instance, a decrease in the severity of asymmetric information in the secondary asset market can hurt welfare. We also find that inflation is crucial for the determination of key equilibrium variables, such as the volume of trade in the secondary asset market, and the decision of agents to invest in information that reduces the degree of information asymmetry.

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1 Introduction

Consider an agent who owns some assets, such as corporate or municipal bonds, and wishes to purchase goods or services from a producer in an environment where credit is not feasible. This agent could attempt to use her assets as a medium of exchange; however, the producer, whose primary interest is in producing goods or services, may not be equipped with the necessary knowledge to determine the quality of these assets. Such asymmetric information can hinder the assets' use as a direct medium of exchange. Alternatively, the agent may choose to liquidate her assets for money in a secondary market and use that money to purchase the goods/services. While the secondary market may also suffer from asymmetric information, the severity of this asymmetry may be lower because asset buyers enter that market to purchase this specific type of assets and are more likely to learn about (and agree with sellers on) the quality of the assets. The New Monetarist literature has studied how the role of assets as media of exchange can be hindered by asymmetric information, but it has overlooked how trading assets in dedicated secondary markets can reduce information frictions and affect liquidity and welfare in general equilibrium.

The goal of this paper is to fill this gap. We develop a model where agents trade periodically in a decentralized *goods market*; due to standard frictions (e.g., anonymity and limited commitment), a medium of exchange is necessary. Agents are free to use either fiat money or a real asset as a medium of exchange. However, the producers of goods are uninformed about the quality of the real asset, which hinders its role as a (direct) medium of exchange. A novel ingredient of our model is that agents have the option to visit an over-the-counter (OTC) secondary *asset market* where the real assets can be liquidated for money. The addition of the secondary asset market is not only empirically relevant, but also an important theoretical innovation; beyond giving agents the ability to liquidate assets for money (a superior medium of exchange in equilibria where assets are only accepted at a discount), trade in this market can serve as a *signal to goods market participants* that the asset is of good quality (and hence should be accepted at not too much of a discount) – as long as the informational imperfection in the secondary market is not absolute.[1]

To capture the idea that buyers in a dedicated asset market may not be completely uninformed about the quality of the assets they are buying, we assume that a fraction of OTC meetings operate under full information. We refer to these meetings as "transparent", in contrast to "opaque" meetings where only the holder/seller of an asset knows its quality. To begin with, we assume that the fraction of transparent meetings, $\tau \in [0, 1]$, is exogenous (but we will relax this later). All meetings in the goods market are opaque (an assumption we will later relax, too), but the asset portfolio a consumer brings into the meeting carries

[1] This is true independent of how imperfect information is in the goods market; see Appendix C.1.

information about how they traded in the asset market, and therefore, indirectly, the quality of their assets. As a result, equilibrium in all three markets – transparent asset market, opaque asset market, and goods market – must be solved jointly. In a typical equilibrium, we find that trade in the transparent asset market is separating (trivially, since there is no information asymmetry), trade in the opaque asset market is pooling, and trade in the goods market is a mix of both.

Specifically, sellers of high-quality assets in transparent meetings will sell their assets for money; this helps them obtain money, of course, but also lets them reveal their type to producers in the subsequent goods market. Sellers of low-quality assets in those same transparent meetings are not under threat of being imitated by anyone, hence if they do sell any assets they do so at a price that reflects the low quality. Agents who are matched in opaque meetings trade at the pooling price; interestingly, they always choose to sell a positive amount of assets to make sure they are not imitated in the goods market by owners of low-quality assets who found themselves in transparent meetings. In other words, even in the subset of asset market trades where information is just as asymmetric as in the goods market, our framework predicts that agents are better off selling assets for money. Therefore, even though we do not impose a cash-in-advance constraint in the goods market, our framework offers an explanation of how money *emerges* as the dominant medium of exchange via asymmetric information and signaling.

One may expect that aggregate welfare should be increasing in the fraction τ of transparent asset market meetings, since those meetings reduce the degree of asymmetric information. However, in our model, aggregate welfare is non-monotone in τ. A larger fraction of transparent meetings makes it more likely that agents can avoid asymmetric information by liquidating assets in the secondary market and ultimately using money as medium of exchange in the goods market. As a consequence, a higher τ induces agents to reduce their money holdings, which lowers their consumption if their assets turn out to be of low quality, and if this latter effect dominates then welfare will decrease in τ.

Our model also predicts that the effect of asymmetric information on the liquidity role of assets depends on the level of inflation. When inflation is high, holding money is costly, which promotes the indirect liquidity role of assets, i.e., agents prefer to sell assets for money in the secondary market. However, when inflation is low, no agents except for those with high-quality assets in transparent meetings will sell assets in the secondary market. An interesting consequence is that while aggregate welfare is decreasing in inflation, as expected, the decrease is sometimes discontinuous. At higher levels of inflation, agents must rely more heavily on the liquidity role of assets, which exposes them to the *penalty* of asymmetric information. As a result, aggregate welfare can jump discontinuously as a function of inflation. This result is topical as it implies that the recent increases in inflation that economies all around the world have been experiencing may have an even larger impact on welfare than expected.

The next step is to endogenize the fraction of meetings that operate under complete information (i.e., the value of τ). Specifically, we give asset sellers the option to pay a cost κ that allows them to trade in transparent meetings. One can think of κ as the cost of producing a certificate of the asset's quality, or a fee to access a specialist intermediary who can guarantee the quality of the asset. We find that agents only pay κ when inflation is neither too high nor too low. When inflation is low, agents can use money as (cheap) insurance against the quality shock, i.e., they use money if their assets are of low quality. In that case, agents do not need to rely heavily on asset liquidity, so they prefer not to pay κ. But why would agents not pay this cost when inflation is high and money holdings are scarce? This is because opaque meetings allow agents to obtain liquidity from assets even if they receive the bad quality shock. In that sense, opaque meetings also serve as insurance against the quality shock, which is especially valuable to agents when their money holdings are low. Thus, allowing for an endogenous determination of τ highlights new, important insights regarding the liquidity role provided by assets, as money is no longer the unique form of insurance against the quality shock. Consequently, agents choose not to pay κ and trade in opaque meetings, precisely when the cost of the alternative form of insurance (money) is especially high, i.e., when inflation is high.

Finally, we examine how the size of the information cost κ affects aggregate welfare. Surprisingly, we find that as long as inflation is relatively low, the aggregate welfare can be increasing in κ, and this is independent of when κ is paid. Intuitively, a large κ discourages agents from investing in information to participate in transparent meetings, which diminishes the liquidity role of assets. When inflation is low, agents do not pay κ (i.e., equilibrium $\tau = 0$), and assets do not provide liquidity services because agents carry large amounts of real balances. This ultimately increases agents' consumption if they turn out to be low-types.

The model in this paper is based on Geromichalos and Herrenbrueck (2016), who extend the Lagos and Wright (2005) framework by introducing an over-the-counter asset market to allow agents to rebalance their portfolios when money is needed. Assets therefore have "indirect liquidity" since they can be exchanged for money, even though they are not "directly liquid" in the sense of being used as payment. In that paper, as in others that make use of the indirect liquidity concept, the exclusive ability of money to serve as a payment instrument is imposed as an assumption.[2] While often empirically relevant, this assumption is also subject to criticism: in his famous dictum, Wallace (1998) argued that the adoption of a medium of

[2] Papers that advance the indirect liquidity concept include Berentsen, Huber, and Marchesiani (2014),Mattesini and Nosal (2016), Herrenbrueck and Geromichalos (2017), Herrenbrueck (2019), Geromichalos and Herrenbrueck (2022), Altermatt, Iwasaki, and Wright (2022), and Geromichalos, Herrenbrueck, and Lee (2023). Other papers that explore the idea of rebalancing asset portfolios include Kocherlakota (2003), Boel and Camera (2006), Berentsen, Camera, and Waller (2007), and Berentsen and Waller (2011). None of these papers study private information.

exchange is endogenous and subject to changes if *conditions* favor a different asset. Here, the real asset competes with money as a medium of exchange, but due to asymmetric information and adverse selection – and, crucially, the existence of a secondary asset market – we find that money arises as the dominant medium of exchange endogenously. As predicted by Wallace (1998), the degree of its dominance depends on economic conditions such as inflation, access to asset markets, and information frictions in these markets.

The closest work to ours is Rocheteau (2011). In that paper, money and assets can serve as media of exchange in a decentralized goods market, where producers cannot observe the quality of the assets, which impairs their liquidity role. Our model adopts a similar framework; however, agents have now access to a secondary asset market where assets can be sold for money as needed, and where the degree of asymmetric information is typically less severe (but not necessarily; see Appendix C.1). Trading in this market serves not only to reallocate liquidity, but also to signal asset quality in subsequent trades (i.e., in the goods market). Like Rocheteau (2011), we find that money can be a dominant medium of exchange, but the notion of dominance is different. In that paper, media of exchange are used along a pecking order with money at the top; here, agents may take advantage of the secondary market to liquidate assets and obtain more money to be used in exchange. Another difference is that we endogenize the degree of asymmetric information, and thus derive results on how this degree itself depends on economic conditions (such as inflation).

In other related work, Madison (2019) and Wang (2020) study how asymmetric information affects the indirect liquidity of assets. Geromichalos, Jung, Lee, and Carlos (2021) also study the coexistence of direct and indirect asset liquidity, but they do not directly deal with asymmetric information, because they assume that producers in decentralized goods market never accept assets they do not recognize. Lu (2022) studies asset liquidity under the assumption that buyers of assets, instead of sellers, possess more information. Lester, Postlewaite, and Wright (2012) develop a framework where the degree of asset recognizability affects the degree of asset acceptability. Finally, our paper belongs to the vast literature on asymmetric information spurred by Akerlof (1970) and Leland and Pyle (1977). Some recent work that also studies asymmetric information in asset markets includes Eisfeldt (2004), Kurlat (2013), Guerrieri and Shimer (2014), Chiu and Koeppl (2016), and Choi (2018).[3]

Finally, our paper is also related to the literature that uses the *undefeated equilibrium* refinement proposed by Mailath, Okuno-Fujiwara, and Postlewaite (1993) as the equilibrium notion to study asymmetric information in asset markets (see for example Li and Rocheteau

[3] Other work that studies private information in asset markets includes Williamson and Wright (1994), Trejos (1997), Trejos (1999), Velde, Weber, and Wright (1999), Li, Rocheteau, and Weill (2012), Lester et al. (2012), Gorton and Ordonez (2014), Golosov, Lorenzoni, and Tsyvinski (2014), Camargo and Lester (2014), Chari, Shourideh, and Zetlin-Jones (2014), Carapella and Williamson (2015), Lauermann and Wolinsky (2016), Ozdenoren, Yuan, and Zhang (2019), and Cai and Dong (2020).

(2008); Bajaj (2018); Wang (2019, 2020)). A key difference is that in our setup, asymmetric information exists in two markets: the asset and the goods market. Hence, a contribution of our paper is to demonstrate how the undefeated equilibrium refinement can be adapted to environments where asymmetric information exists in multiple (and consecutive) markets. We show that trade in the asset market serves not only as a way to acquire money but also as a signal of the asset quality to sellers in the goods market.

The rest of the paper is organized as follows. Section 2 introduces the model. In Section 3, we describe the equilibrium with an exogenous τ, while we endogenize τ in Section 5. Section 6 concludes the paper, and it is followed by an Appendix which contains proofs and additional results, including the case of partial information in the goods market (C.2).

2 Environment

Time is discrete and continues forever. Each period is divided into three subperiods: the asset market (AM), the decentralized goods market (DM), and the centralized market (CM). There is a unit measure of "consumers" and a unit measure of "producers", named for their roles in the consumption and production of DM goods (q). In the CM, there is a single good (x) that can be produced and consumed by all agents, and which also serves as the numéraire. A consumer's instantaneous utility is given by:

$$\eta_t u(q_t) + x_t,$$

where q_t and x_t are the consumption of the DM good and the CM good, respectively. We assume $q_t \geq 0$, but x_t can be negative, in which case it is interpreted as production in the CM: one unit of labor in the CM can be turned into one unit of CM good. We assume η_t is stochastic. Specifically,

$$\eta_t = \begin{cases} 0, & \text{with probability } 1-\lambda; \\ 1, & \text{with probability } \lambda. \end{cases}$$

If $\eta_t = 0$, a consumer does not derive utility from the DM good and is referred to as an "N-type" consumer. If $\eta_t = 1$, a consumer derives utility from the DM good and is referred to as a "C-type" consumer. Consumers learn the value of η_t at the beginning of the AM. The realization of η_t is *i.i.d.* across consumers and time, and whether a consumer is a C-type (i.e., $\eta_t = 1$) or an N-type (i.e., $\eta_t = 0$) is common knowledge. We assume that $u(0) = 0$, $u'(.) > 0$, $u''(.) < 0$, $u'(0) = \infty$. We also define the first-best quantity of DM good as q^*, where $u'(q^*) = 1$. Next, the instantaneous utility of a producer is given by

$$-h_t + X_t,$$

where h_t is the amount of labor supplied in the DM, and X_t is the consumption of the CM good. Similar to x_t, X_t can be negative, in which case it is interpreted as production in the CM. We assume one unit of labor in the DM can be turned into one unit of the DM good. Neither goods can be carried across periods. All agents discount future utility using $\beta \in (0, 1)$.

There are two types of assets in the economy: (fiat) money and perfectly divisible real assets.[4] Money is issued by a government, and each consumer is endowed with a units of the real assets in each CM. Following Rocheteau (2011), we interpret the real asset as private equity, corporate bonds, or asset-backed securities. In the CM of the next period, each unit of assets produces a dividend of δ units of CM good before depreciating fully. Again in line with Rocheteau (2011), we assume that δ is stochastic, and that by holding the real assets, consumers learn their quality.[5] Specifically, at the beginning of the AM, with probability ρ, a consumer learns that her real assets have high quality, and each unit will produce $\delta = \delta_h > 0$ units of the CM good. With probability $1 - \rho$, a consumer learns that her real assets have low quality, and each unit of the assets will produce $\delta = \delta_l < \delta_h$ units of the CM good. We study $\delta_l = 0$ as a separate case, and we interpret it as asset holders having received private information that the asset is either fake or will default in the following CM. Finally, we assume that the realization of δ is independent across consumers and is independent of η_t.

We assume that agents are anonymous, and therefore, a medium of exchange is necessary for trade in the DM. We assume both money and real assets can be used as payment instruments in principle, though agents will form subjective beliefs about the quality of an asset before accepting it. Additionally, consumers can trade assets for money in the asset market, which conveniently opens before the DM but after all shocks are revealed: specifically, consumers have learned η_t (whether they will be active in that period's DM) and the quality of their assets. In this market, an asset seller is randomly matched with an asset buyer. We assume that a fraction, τ, of the meetings are "transparent" in the sense that the buyers can observe the quality of the real assets. The remaining $1 - \tau$ of the meetings are "opaque" in the sense that the quality of the assets, δ, is asset sellers' private information. Once matched, the asset seller makes a take-it-or-leave-it offer to the asset buyer. The offer consists of a unit price, ψ, and the quantity for sale, s. For simplicity, we assume asset sellers are matched to buyers with probability one, which requires that matching is efficient and that $\lambda \leq 1/2$.

[4] None of our results depend on the assets being real or nominal.

[5] Plantin (2009) shows that this assumption is of particular relevance for assets like collateralized debt obligations and privately placed debt, which are securities sold to selected investors and are bundled with future access to privileged information about the assets. Similar assumptions can be found in Rocheteau (2011), Madison (2024, 2019), and Wang (2020).

After the AM, each C-type consumer is matched with a producer with probability one in the DM, and C-type consumers make take-it-or-leave-it offers to producers. We assume that both assets and money can be used as means of payment. However, the quality of the assets is the consumers' private information in all meetings between C-type consumers and producers.

Since this is a signaling game, it is important to carefully describe the information sets of various players. We assume that in the AM, asset buyers can observe sellers' asset portfolios. In the DM, producers can observe C-type consumers' asset portfolios both at the beginning of each period and at the beginning of the DM. This is consistent with the "open-pocket" bargaining assumption adopted widely in the New Monetarist literature. Furthermore, as we will demonstrate in Sections 3 and 4, some consumers use asset trade in the AM to differentiate themselves from other consumers with low-quality assets. This means that such consumers have the incentive to disclose their asset portfolios to producers as proof of their trade in the AM. For this reason, consumers who do not volunteer such information will be (correctly) identified as low-quality asset holders. Hence, this feature arises naturally in our framework.[6]

Finally, in the CM, agents produce, trade, and consume the CM good. Agents also choose how much money to bring to the next period. Since our focus is on trade in the AM and DM, for simplicity we assume that the real assets are not traded in the CM. Lastly, let M_t denote the supply of money. It satisfies:

$$M_{t+1} = (1 + \mu)M_t,$$

where μ is the money growth rate. We assume $\mu > \beta - 1$. Money is injected into (or withdrawn from) the economy by a monetary authority via a lump-sum transfer (or tax) in each CM.

3 Definition of Equilibrium

In this section, we describe the agents' problems and define the equilibrium. It is convenient to start with the DM. Consider a C-type consumer who has z units of money (in real terms) and a units of assets at the beginning of the AM, and $\tilde{z}$ units of money and $\tilde{a}$ units of assets at the beginning of the DM. If the consumer is matched with a producer, she makes a take-it-or-leave-it offer $(q, \hat{z}, \hat{a})$ to the producer, where q is the quantity of goods she wants to purchase, and $\hat{z}$ and $\hat{a}$ are money and assets the producer will receive, respectively.

[6] More generally, our model aims to focus on asymmetric information about the *quality* of the assets. Not assuming open pockets would add asymmetric info about the *quantity* of the assets. We know from the literature (e.g., Nosal and Rocheteau (2011), Chapter 7) that solving a model with asymmetric info about quantity alone is very difficult, and adding this dimension of asymmetric information will make the model intractable.

Since the asset quality is the consumer's private information, the producer must form beliefs regarding asset quality. Let $\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)$ denote the producer's belief about the probability of the asset quality being high (i.e., $\delta = \delta_h$), conditional on the consumer's offer $(q, \hat{z}, \hat{a})$ and asset portfolios. Note that by the open-pocket bargaining assumption, producers can observe both (z, a) and $(\tilde{z}, \tilde{a})$.

As is standard in the Lagos and Wright (2005) literature, both the consumer's and producer's value functions in the DM are linear in money and assets, and the price of the DM good in terms of the CM good is one. The consumer solves the following problem.

$$\begin{aligned} &\max_{q,\hat{z},\hat{a}} \{u(q) - \hat{z} - \delta\hat{a}\} \qquad (3.1) \\ \text{s.t. } &[\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)\delta_h + (1 - \gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)\delta_l]\hat{a} + \hat{z} \geq q, \\ &\hat{z} \leq \tilde{z}, \;\; \hat{a} \leq \tilde{a}. \end{aligned}$$

In words, the consumer maximizes her utility $u(q)$ from consuming the DM good versus giving up $\hat{z}$ units of money and $\hat{a}$ units of assets, subject to the producer finding her offer acceptable. For this to happen, the producer's subjective expected value of the assets, $[\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)\delta_h + (1 - \gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}), z, a)\delta_l]\hat{a}$, plus the value of money, must at least compensate for the cost of producing the DM good, q. We define $q(\tilde{z}, \tilde{a}, \delta, \gamma^g)$ to be the solution to problem (3.1) conditional on a C-type consumer's asset portfolios, the quality of the consumer's assets, and the producers' beliefs.

Next, we turn to the agents' problems in the AM. Recall that only consumers participate in the AM. Denote by z and a the amounts of money and assets held by a C-type consumer, respectively, and denote by z_b the amount of money held by an N-type consumer. We restrict our attention to the case where $z < q^*$, which ensures that a consumer will not be able to consume the efficient amount in the DM using only the money they have carried from the previous CM, thereby giving C-type consumers an incentive to either use assets directly in the DM or trade them for money in the AM. (We will show later that this always holds as long as $\mu > \beta - 1$.)

Recall that whether a consumer is a C-type (i.e., $\eta_t = 1$) or an N-type (i.e., $\eta_t = 0$) is common knowledge. This means that asset sellers can only be C-type consumers and asset buyers can only be N-type consumers, since there is no surplus to be gained from asset trade between two C-type consumers or two N-type consumers. Once a seller is matched with a buyer, the seller makes a take-it-or-leave-it offer (ψ, s) to the buyer, where ψ is the price of the asset, and s is the quantity of the asset for sale.

Now, consider an asset seller in a transparent meeting with a buyer who has z_b units of money. If the seller's offer is accepted by the buyer, she enters the DM with $\tilde{z} = z + \psi s$ units of money and $\tilde{a} = a - s$ units of assets. To determine (ψ, s), she solves the following problem:

$$\max_{\psi,s}\{u(q(z+\psi s, a-s, \delta, \gamma^g)) - u(q(z, a, \delta, \gamma^g)) - \delta s\} \tag{3.2}$$
$$\text{s.t. } \psi \le \delta,\ \psi s \le z_b,\ s \le a.$$

In words, if the offer is accepted, the seller obtains a surplus equal to $u(q(z+\psi s, a-s, \delta_h, \gamma^g)) - u(q(z, a, \delta_h, \gamma^g))$ but has to give up s units of assets, which will generate δs of CM goods in the CM. In addition, for the offer to be accepted by the asset buyer, the unit price of the asset cannot be higher than the dividend per unit of the asset (which is common knowledge in a transparent meeting).

Finally, consider asset sellers in opaque meetings. Since the asset quality is the seller's private information, buyers must form beliefs regarding asset quality conditional on the offers. Let $\gamma^a(\psi, s; z, a)$ denote a buyer's belief about the probability of the asset quality being high (i.e., $\delta = \delta_h$) conditional on the seller's offer. The seller solves the following problem:

$$\max_{\psi,s}\{u(q(z+\psi s, a-s, \delta_h, \gamma^g)) - u(q(z, a, \delta_h, \gamma^g)) - \delta s\} \tag{3.3}$$
$$\text{s.t. } \psi \le (1-\gamma^a(\psi, s; z, a))\delta_l + \gamma^a(\psi, s; z, a)\delta_h,\ \psi s \le z_b,\ s \le a.$$

Compared to problem (3.2), the only difference is that the asset price cannot exceed the buyer's expected amount of dividend per unit of assets, $(1-\gamma^a(\psi, s; z, a))\delta_l + \gamma^a(\psi, s; z, a)\delta_h$. In both problems (3.2) and (3.3), the presence of γ^g indicates that the outcome of this trade may affect, and is affected by, the future belief of a producer that this particular consumer has assets of high quality.

There are four kinds of C-type consumers in the AM and the DM depending on the quality of their assets and the types of their AM meetings. We denote these C-type consumers as type-jk consumers, where $j \in \{l, h\}$ stands for the quality of their assets (l for low quality and h for high quality), and $k \in \{O, T\}$ stands for the AM meeting type (T for transparent and O for opaque). Now, we define the Perfect Bayesian Equilibrium in the AM and the DM.

Definition 1 *A Perfect Bayesian Equilibrium in the AM and the DM consists of offers from consumers* $\{(\psi^{jk}, s^{jk}; q^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$ *where* $j \in \{l, h\}$ *and* $k \in \{O, T\}$*, a decision rule by asset buyers* $\mathbb{1}^a(\psi, s; z, a)$*, a belief function by asset buyers* $\gamma^a(\psi, s; z, a)$*, a decision rule by producers* $\mathbb{1}^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)$*, a belief function by producers* $\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)$ *such that*
(1) (ψ^{jT}, s^{jT}) *solves* (3.2) *for* $j \in \{l, h\}$*;* (ψ^{jO}, s^{jO}) *solves* (3.3) *for* $j \in \{l, h\}$*;* $\{(q^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$ *solves* (3.1) *for* $j \in \{l, h\}$ *and* $k \in \{O, T\}$*.*
(2) $\mathbb{1}^a(\psi, s; z, a) = 1$ *if and only if* $\psi \le \gamma^a(\psi, s; z, a)\delta_h + (1-\gamma^a(\psi, s; z, a))\delta_l$*.*
(3) $\mathbb{1}^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a) = 1$ *if and only if* $[\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)\delta_h + (1-\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a))\delta_l]a + z \ge q$*.*
(4) $\gamma^a(\psi, s; z, a)$ *and* $\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)$ *are derived from Bayes' rule whenever possible.*

In words, conditions (1), (2), and (3) guarantee that agents' strategies are optimal given asset buyers' and producers' beliefs. Condition (4) guarantees that asset buyers' and producers' beliefs are consistent with agents' strategies.

Before we solve for the equilibrium in the AM and DM, it is worth pointing out the unique structure of the signaling game in our environment. The game appears to have two stages, i.e., the interactions between asset sellers and buyers in the AM, and the interactions between C-type consumers and producers in the DM. However, the two stages cannot be solved separately. This is because, firstly, in the AM, asset sellers' strategies and (consequently) asset buyers' beliefs depend on what they think producers' beliefs will be like in the following DM, since producers' beliefs affect asset's continuation value and by extension asset sellers' payoff in the *AM*. Secondly, producers' beliefs in the DM also depend on what they think asset buyers' beliefs were like in the preceding AM, since asset buyers' beliefs affect asset sellers' strategies and asset portfolios that the producers observe at the beginning of the DM. For these reasons, agents' problems in the AM and DM must be solved jointly. In the following proposition, we describe the full set of Perfect Bayesian Equilibria in AM and DM.

Proposition 1 *The full set of Perfect Bayesian Equilibria in the AM and the DM consists of offers* $\{(\psi^{jk}, s^{jk}; q^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$ *where* $j \in \{l, h\}$ *and* $k \in \{O, T\}$ *and belief functions* $\gamma^a(\psi, s; z, a)$ *and* $\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)$ *that satisfy the following:*
(1) *For all* j *and* k*,*

$$\psi^{jk} \leq \gamma^a(\psi^{jk}, s^{jk}; z, a)\delta_h + (1 - \gamma^a(\psi^{jk}, s^{jk}; z, a))\delta_l, \text{ and}$$
$$[\gamma^g(q^{jk}, \hat{z}^{jk}, \hat{a}^{jk}; \tilde{z}, \tilde{a}, z, a)\delta_h + (1 - \gamma^g(q^{jk}, \hat{z}^{jk}, \hat{a}^{jk}; \tilde{z}, \tilde{a}, z, a))\delta_l]\hat{a}^{jk} + \hat{z}^{jk} \geq q^{jk},$$

where $\tilde{z} = z + \psi^{jk}s^{jk}$ *and* $\tilde{a} = a - s^{jk}$*.*
(2) *Define* $\tilde{v}^{jO}(\tilde{z}, \tilde{a})$ *to be C-type consumers' DM surplus with DM portfolio* $(\tilde{z}, \tilde{a})$

$$\tilde{v}^{jO}(\tilde{z}, \tilde{a}) \equiv \max_{q, \hat{z} \leq \tilde{z}, \hat{a} \leq \tilde{a}} \{u(q) - \hat{z} - \delta_j \hat{a}\}$$
$$\text{s.t. } [\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)\delta_h + (1 - \gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)\delta_l]\hat{a} + \hat{z} \geq q.$$

Define $\underline{v}^{jO}$ *to be C-type consumers' lowest possible surplus in the AM*

$$\underline{v}^{jO} \equiv \max_{\psi, s} \{\tilde{v}^{jO}(z + \psi s, a - s) - \tilde{v}^{jO}(z, a) - \delta_j s\} \text{ s.t. } \psi \leq \delta_l, \ \psi s \leq z_b, \ s \leq a.$$

Then, $\gamma^a(\psi^{jO}, s^{jO}; z, a)$ *is given by*

$$\gamma^a(\psi^{jO}, s^{jO}; z, a) = \begin{cases} 1, \text{ if } \tilde{v}^{lO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) \leq \underline{v}^{lO} \ \& \ \tilde{v}^{hO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) \geq \underline{v}^{hO}; \\ \rho, \text{ if } \tilde{v}^{lO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) > \underline{v}^{lO} \ \& \ \tilde{v}^{hO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) \geq \underline{v}^{hO}; \\ 0, \text{ if } \tilde{v}^{lO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) \geq \underline{v}^{lO} \ \& \ \tilde{v}^{hO}(z^\dagger + \psi^{jO}s^{jO}, a^\dagger - s^{jO}) < \underline{v}^{hO}, \end{cases}$$

while $\gamma^a(\psi^{jT}, s^{jT}; z, a) = 1$ *if and only if* $j = h$.

(3) *Define* $\tilde{\psi} = \frac{\tilde{z}-z}{a-\tilde{a}}$ *to be the price of assets sold in the AM. Let* $\tilde{\psi} = 0$ *if* $(\tilde{z}, \tilde{a}) = (z, a)$. *Define* $\underline{\tilde{v}}^j(\tilde{z}, \tilde{a})$ *to be C-type consumers' lowest possible surplus in the DM*

$$\underline{\tilde{v}}^j(\tilde{z}, \tilde{a}) \equiv \max_{q,\hat{z},\hat{a}} \{u(q) - \hat{z} - \delta_j \hat{a}\} \text{ s.t. } \delta_l \hat{a} + \hat{z} \geq q,\ \hat{z} \leq \tilde{z},\ \hat{a} \leq \tilde{a}.$$

Then, $\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)$ *is given by*

$$\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a) = \begin{cases} 1, & \text{if } \ u(q) - \hat{z} - \delta_l \hat{a} \leq \underline{\tilde{v}}^l(\tilde{z}, \tilde{a}) \ \text{ and } \ u(q) - \hat{z} - \delta_h \hat{a} \geq \underline{\tilde{v}}^h(\tilde{z}, \tilde{a}), \\ & \text{or if } \ \tilde{\psi} \in (\rho\delta_h + (1-\rho)\delta_l, \delta_h]; \\ \rho, & \text{if } \tilde{\psi} \in (\delta_l, \rho\delta_h + (1-\rho)\delta_l]; \\ 0, & \text{if otherwise.} \end{cases}$$

(4) $\gamma^a(\psi, s; z, a) = 0$ *and* $\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a) = 0$ *for all* $(\psi, s; q, \hat{z}, \hat{a}) \notin \{(\psi^{jk}, s^{jk}; q^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$.

Proof: See Appendix B.

In words, condition (1) ensures that it is optimal for asset buyers in the AM and producers in the DM to accept C-type consumers' offers conditional on asset buyers' and producers' beliefs. Condition (2) guarantees that asset buyers' beliefs on the equilibrium path, $\gamma^a(\psi^{jk}, s^{jk}; z, a)$, are consistent with C-type consumers' strategies in the AM. Specifically, asset buyers believe that an offer comes from a C-type with quality-j assets, $j \in \{l, h\}$, if and only if the C-types with quality-j assets have the incentive to make such an offer. In the AM, the offers can either be pooling (i.e., $(\psi^{lO}, s^{lO}) = (\psi^{hO}, s^{hO})$) or separating (i.e., $(\psi^{lO}, s^{lO}) \neq (\psi^{hO}, s^{hO})$). Condition (3) requires $\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)$ to be consistent with C-type consumers' strategies:

a. Similar to condition (2), producers believe that an offer comes from a C-type with high-quality assets if and only if C-types with high-quality assets are the sole type that have the incentive to make such an offer.

b. Producers can also infer from consumers' portfolios the quality of their assets:

(i) Because asset buyers' beliefs must satisfy Bayes' rule, in the AM only C-types with high-quality assets (either in transparent meetings or opaque meetings) may sell at a price that is strictly higher than $\rho\delta_h + (1-\rho)\delta_l$. To see why, note that C-types with low-quality assets in transparent meetings can at most sell at $\psi = \delta_l$. Next, if C-types in opaque meetings make pooling offers, they can at most sell at $\psi = \rho\delta_h + (1-\rho)\delta_l$, which is the average asset quality in opaque meetings. If C-types in opaque meetings make separating offers, those with low-quality assets can at most sell at $\psi = \delta_l$.

(ii) The belief in (i) ensures that C-types with high-quality assets in transparent meetings will not sell at price less that $\rho\delta_h + (1-\rho)\delta_l$. This in turn ensures that only C-types in opaque meetings may sell at a price that is strictly higher than δ_l but weakly less than

$\rho\delta_h + (1-\rho)\delta_l$, which only happens when they make pooling offers. It should be noted that $\tilde{\psi} > 0$ implies that the C-type consumer managed to sell assets in the AM, which rules out the possibility that the consumer is of type-lT.

(iii) The belief in (ii) ensures that C-types in opaque meetings, when making pooling offers, will choose $\psi \leq \delta_l$. Hence, if $\tilde{\psi} \leq \delta_l$, the consumer must have low-quality assets.

Finally, condition (4) is a sufficient but not necessary condition on off-equilibrium path beliefs that ensures no consumers have the incentive to deviate and make an offer not in the set of equilibrium offers $\{(\psi^{jk}, s^{jk}; q^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$.

Proposition 1 shows that a plethora of equilibria exist under Definition 1, because no restrictions are put on off-equilibrium path beliefs. To conduct a meaningful analysis, some equilibrium refinement is necessary. In Definition 2, we adapt the *undefeated equilibrium refinement* proposed by Mailath et al. (1993) to our environment.[7]

Definition 2 *A Perfect Bayesian Equilibrium in the AM and the DM (PBE-1) is defeated by another Perfect Bayesian Equilibrium in the AM and the DM (PBE-2) if there exists* $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ *that satisfies*

(1) *There exist* $K \subseteq \{lO, hO, lT, hT\}$ *such that* $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ *is played by type-*K *consumers in PBE-2 but not in PBE-1.*

(2) *There exist* $J \subseteq K$ *such that type-*J *consumers play* $(\hat{\psi}, \hat{s}; q, \hat{z}, \hat{a})$ *and obtain strictly higher surplus in PBE-2.*

(3) *In PBE-1, at least one of the following conditions is satisfied:*

$$\gamma^a(\hat{\psi}', \hat{s}'; z, a) \neq \frac{\rho \mathbb{1}(hO \in J)}{\rho \mathbb{1}(hO \in K) + (1-\rho)\mathbb{1}(lO \in K)}, \tag{3.4}$$

$$\gamma^g(q', \hat{z}', \hat{a}'; z + \hat{\psi}'\hat{s}', a - \hat{s}', z, a) \neq \frac{\rho(1-\tau)\mathbb{1}(hO \in J) + \rho\tau\mathbb{1}(hT \in J)}{\rho(1-\tau)\mathbb{1}(hO \in K) + (1-\rho)(1-\tau)\mathbb{1}(lO \in K) + \rho\tau\mathbb{1}(hT \in K) + (1-\rho)\tau\mathbb{1}(lT \in K)}. \tag{3.5}$$

A Perfect Bayesian Equilibrium in the AM and the DM is undefeated if and only if there does not exist $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ *that satisfies conditions* (1)-(3).

In words, Definition 2 requires a Perfect Bayesian Equilibrium to be undefeated in the sense that there does not exist a strategy that satisfies: (1) it is part of another Perfect Bayesian Equilibrium; (2) it benefits some C-types; and (3) the off-equilibrium path beliefs of asset buyers and/or producers in the original Perfect Bayesian Equilibrium fail to take into account such a strategy. In particular, the right-hand side of Condition (3.4) is the fraction of

[7] Undefeated equilibrium refinement has also been used in other papers that study asymmetric information in asset markets. See for example Rocheteau (2008, 2011), Bajaj (2018), Madison (2024), and Wang (2020).

asset sellers with high-quality assets that play $(\hat{\psi}', \hat{s}')$ in the AM, while the right-hand side of Condition (3.5) the fraction of asset sellers with high-quality assets that play $(\hat{\psi}', \hat{s}')$ in the AM and play $q', \hat{z}', \hat{a}'$ in the DM. Requirement (3) says that the beliefs of asset buyers and/or producers must be the reason why the strategy $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ is not played in PBE-1. However, because this alternative equilibrium is profitable for some consumers, asset buyers and/or producers should have expected it. In other words, the beliefs of asset buyers and/or producers in PBE-1 are arguably "unreasonable". So, PBE-1 is "defeated" by PBE-2.

The main benefit of using the undefeated equilibrium refinement in our environment, especially when compared to the popular Intuitive Criterion refinement (Cho and Kreps, 1987), is that it allows pooling equilibria (where C-types with low-quality and high-quality assets make the same offer in the AM and/or the DM) to exist as long as they are Pareto-optimal. However, we cannot use the definition in Mailath et al. (1993) "off the shelf", because in that paper, trading happens between two parties; in our paper, three parties are involved, as both asset buyers *and* producers must form beliefs regarding the quality of the asset offered by an asset-selling consumer. In Appendix A, we explain in detail (a) the logic behind the Undefeated Equilibrium refinement and why we use it instead of the Intuitive Criterion, and (b) how we adapt the Undefeated Equilibrium refinement of Mailath et al. (1993) to our environment and why the version in this paper captures the logic of the original version.

Definitions 1 and 2 define undefeated equilibria in the AM and the DM, taking as given consumers' asset portfolios. Recall that in each CM, each consumer is endowed with a unit of the real asset, but their money holdings for the following period are determined endogenously. In the following section, we characterize undefeated equilibria in the AM and the DM as well as the consumers' optimal choice of z.

4 Characterization of Equilibrium

4.1 The case where $\delta_l = 0$

We start with the case where $\delta_l = 0$. One can think of this case as a scenario where the asset holders receive private information that the asset is counterfeit or will default in the following CM. In what follows, we first solve for a partial equilibrium in the AM and DM given consumers' portfolios (z, z_b, a). Then, we solve for the general equilibrium where consumers choose their money holdings in the CM.

The following proposition describes undefeated equilibria in the AM and the DM, taking as given consumers' portfolios (z, z_b, a).

Proposition 2 *Let $\tilde{q}$ be such that $u'(\tilde{q}) = 1/\rho$. Conditional on (z, z_b, a), undefeated equilibria in the AM and the DM satisfy the following conditions:*

(1) $(\psi^{hO}, s^{hO}; q^{hO}, \hat{z}^{hO}, \hat{a}^{hO}) = (\psi^{lO}, s^{lO}; q^{lO}, \hat{z}^{lO}, \hat{a}^{lO}) = (\psi_p, s_p, q_p; \hat{z}_p, \hat{a}_p)$, *where*

(a) $\psi_p = \rho\delta_h$; (b) $s_p \in (0, \min\{z_b/(\rho\delta_h), (q_p - z)/(\rho\delta_h), a\}]$;

(c) $q_p = \max\{z, \min\{z + \rho\delta_h a, \tilde{q}\}\}$; (d) $\hat{z}_p = z + \psi_p s_p$; (e) $\hat{a}_p = \frac{q_p - \hat{z}_p}{\rho\delta_h}$.

(2) $(\psi^{hT}, s^{hT}; q^{hT}, \hat{z}^{hT}, \hat{a}^{hT})$ *is given by*

(a) $\psi^{hT} = \delta_h$; (b) $q^{hT} = \min\{q^*, z + \delta_h a\}$;

(c) $s^{hT} \in (0, \min\{z_b/\delta_h, a\}]$; (d) $\hat{z}^{hT} = z + \psi^{hT} s^{hT}$; (e) $\hat{a}^{hT} = \frac{q^{hT} - \hat{z}^{hT}}{\delta_h}$.

(3) $(\psi^{lT}, s^{lT}; q^{lT}, \hat{z}^{lT}, \hat{a}^{lT}) = (0, 0; z, z, 0)$.

Proof: See Appendix B.

The proposition says that there are three different offers being made in the AM:
(1) Type-hT consumers sell at price $\psi^{hT} = \delta_h$.
(2) Type-lO and type-hO consumers sell at pooling price $\psi_p = \rho\delta_h$ provided that $z < \tilde{q}$.
(3) Type-lT consumers cannot sell assets for money in the AM.

By selling assets in the AM, type-hT, type-lO, and type-hO consumers differentiate themselves from type-lT consumers, who are the only types that cannot obtain money from the AM. The existence of the AM also allows type-hT consumers to differentiate themselves from type-lO and type-hO consumers, since only they can sell at price $\psi^{hT} = \delta_h$, which in turn lets type-hT consumers obtain the same allocation as they would under complete information.

More interestingly, type-lO and type-hO consumers also benefit from trading in the AM, because otherwise they will have to pool with type-lT consumers in the DM and suffer a large discount if they use the assets as payment in the DM. Nevertheless, because type-hO consumers make pooling offers in the AM, the price for their assets is still discounted due to the information asymmetry. It is straightforward to show that the marginal cost of selling assets at a pooling price is $1/\rho$ for type-hO consumers. If $z \geq \tilde{q}$, then for type-hO consumers, the marginal benefit of using assets to either obtain money in the AM or purchase goods directly in the DM exceeds the marginal cost. Consequently, in such case, type-hO consumers only use money in the DM and do not trade assets in the AM. Type-lO consumers always mimic the type-hO consumers' strategy, because otherwise they would be identified by producers.

Next, consider the consumers' choice of real balances in the CM. We focus on symmetric solutions where $z = z_b$. Taking into account the AM and DM outcomes described in Proposition 2, consumers solve the following problem in the CM:

$$\begin{aligned}\max_z \ & \lambda\{(1-\rho)\tau[u(q^{lT}) + z - q^{lT}] + \rho\tau[u(q^{hT}) + z + \delta_h a - q^{hT}] \\ & + (1-\tau)[u(q_p) + z + \rho\delta_h a - q_p]\} + (1-\lambda)(z + \rho\delta_h a) - \frac{(1+\mu)z}{\beta},\end{aligned} \tag{4.1}$$

where q^{lT}, q^{hT} and q_p are given by Proposition 2.

In words, the problem says that with probability λ, a consumer becomes a C-type consumer. With probability $(1-\rho)\tau$, the C-type consumer has low quality assets and is in a transparent meeting in the AM. With probability $\rho\tau$, the C-type consumer has high quality assets and is in a transparent meeting in the AM. Finally, with probability $1-\tau$, the C-type consumer is in an opaque meeting in the AM. Recall that in this case, consumers with high-quality assets and consumers with low-quality assets make the same offers in both the AM and the DM. With probability $1-\lambda$, a consumer becomes an N-type consumer. Note that the expected AM surplus of an N-type consumer is zero. The cost of accumulating z units of real balances in the last CM is $(1+\mu)z/\beta$.

The first-order condition is:

$$\frac{1+\mu}{\beta} - 1 = \lambda\{(1-\rho)\tau[u'(q^{lT}) - 1] + \rho\tau[u'(q^{hT}) - 1] \\ + (1-\tau)[u'(q_p) \cdot \mathbf{1}(z > \tilde{q} \text{ or } z + \rho\delta_h a < \tilde{q}) + \mathbf{1}(\tilde{q} \leq z + \rho\delta_h a \leq \tilde{q} + \rho\delta_h a) - 1]\}, \quad (4.2)$$

where $\mathbf{1}(.)$ is an indicator function that equals one if the condition in the bracket is satisfied, and zero otherwise. To see how the equation is derived, first note that $q^{lT} = z$ and $q^{hT} = \min\{q^*, z + \delta_h a\}$ from Proposition 2. The first line on the right-hand side of (4.2) follows directly from these results.

Next, consider the second line, which represents the scenario where the consumer is an opaque AM meeting. From Proposition 2, we know that in this case, the assets of type-hO consumers can only be sold at a discount due to information asymmetry, and the marginal cost of selling assets at the discounted price is $1/\rho$ for type-hO consumers. Let us first suppose that $z + \rho\delta_h a < \tilde{q}$, where $\tilde{q}$ solves $u'(\tilde{q}) = 1/\rho$. From Proposition 2, we know that in the DM, type-hO consumers consume $q = z + \rho\delta_h a$. Hence, the derivative with respect to z is $u'(q_p)$. Next, suppose $z > \tilde{q}$. Proposition 2 states that in this case, type-hO consumers consume $q = z$. Recall that this is because marginal utility at $q = z$ is already lower than the cost of selling assets for type-hO consumers, $1/\rho$. Hence, type-lO and type-hO consumers will not sell assets in the AM, and they use only money to purchase the DM good. It follows that the derivative with respect to z is $u'(q_p)$.

Now, assume that $\tilde{q} \leq z + \rho\delta_h a \leq \tilde{q} + \rho\delta_h a$. Proposition 2 shows that $q_p = \tilde{q}$ regardless of the value of z in this case, because consumers are not able to consume $q > \tilde{q}$ using real balances alone. However, based on our earlier arguments, as long as type-hO consumers opt to sell assets in the AM, they will not consume more than $\tilde{q}$. Consequently, in this case, having more real balances does not change the consumption of consumers in opaque AM meetings – it only leads to fewer assets being sold by type-hO consumers. Hence, accumulating one more unit of real balances in the CM means $1/(\rho\delta_h)$ units fewer assets being sold in the next period (recall that type-hO consumers sell assets at price $\rho\delta_h$), which has an expected value of $\rho\delta_h$ (because the quality shock is realized in the next period). In other words, the marginal value of accumulating one more unit of real balances in the CM is one in this case.

To simplify the equilibrium equations, we define $i \equiv (1+\mu)/\beta - 1$ as a variable which incorporates money growth μ and impatience $1/\beta$, and thereby summarizes the opportunity cost of holding money over time. Solving for equilibrium, we first consider the case where $z + \rho\delta_h a < \tilde{q}$. In such case, $q^{lT} = z$, $q^{hT} = \max\{z + \delta_h a, q^*\}$, and $q_p = z + \rho\delta_h a$. Equation (4.2) then becomes:

$$i = \lambda\{(1-\rho)\tau[u'(z) - 1] + \rho\tau[u'(\min\{z + \delta_h a, q^*\}) - 1] + (1-\tau)[u'(z + \rho\delta_h a) - 1]\},\tag{4.3}$$

It is clear that there exists a unique z that solves (4.3). Now let us consider other cases. If $\tilde{q} \leq z + \rho\delta_h a \leq \tilde{q} + \rho\delta_h a$, the first order conditions become:

$$i = \lambda\{(1-\rho)\tau[u'(z) - 1] + \rho\tau[u'(\min\{z + \delta_h a, q^*\}) - 1]\},\tag{4.4}$$

If $z > \tilde{q}$, the first order condition becomes:

$$i = \lambda\{(1-\rho)\tau[u'(z) - 1] + \rho\tau[u'(\min\{z + \delta_h a, q^*\}) - 1] + (1-\tau)[u'(z) - 1]\},\tag{4.5}$$

The following proposition summarizes the solution to the portfolio problem in the CM.

Proposition 3 *Suppose $\rho\delta_h a < \tilde{q}$. For any given i, there exists a unique solution to problem* (4.1)*. Specifically, given a, there exist $i_1 > i_2 \geq i_3 > 0$ such that*
(1) *for all $i \geq i_1$, z solves* (4.3)*;* (2) *for all $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$;*
(3) *for all $i_3 < i < i_2$, z solves* (4.4)*;* (4) *for all $i \leq i_3$, z solves* (4.5)*.*
If $\rho\delta_h a \geq \tilde{q}$, there exists $i^\dagger$ such that
(1) *for all $i > i^\dagger$, z solves* (4.4)*;* (2) *for all $i \leq i^\dagger$, z solves* (4.5)*.*
Proof: See Appendix B.

When $\rho\delta_h a < \tilde{q}$, there exist either two or three cutoff values of i:

(1) When $i \geq i_1$, the cost of holding real balances is high, so z is sufficiently small that type-lO and type-hO consumers are constrained in the DM (i.e., $z + \rho\delta_h a \leq \tilde{q}$ in equilibrium). Note that $z + \rho\delta_h a = \tilde{q}$ when $i = i_1$.

(2) Recall our earlier discussion about the scenario where $z + \rho\delta_h a > \tilde{q}$. As long as $z < \tilde{q}$, due to the price discount in opaque meetings, consumption of consumers in opaque meetings is equal to $\tilde{q}$ regardless of the value of z. This implies a drop in the marginal benefit of accumulating more real balances. We show that as long as $i \leq i_2$, the marginal cost of accumulating more real balances exceeds the marginal benefit. As a result, consumers choose $z = \tilde{q} - \rho\delta_h a$.

(3) When $i_3 < i < i_2$, the cost of real balances is relatively low so that despite consumers

not benefiting from having more money if they end up in opaque meetings, they will still choose to hold more money just in case they end up in transparent meetings.

(4) When $i \leq i_3$, the cost of holding real balances is sufficiently low that type-lO and type-hO consumers carry enough real balances to avoid selling assets completely.

Finally, if $\rho\delta_h a \geq \tilde{q}$, only cases (3) and (4) are possible, so there is only a single cutoff value.

Next, we turn to the comparative statics of equilibrium outcomes with respect to the interest rate (i) and the proportion of transparent meetings in the AM (τ). We look at consumers' holdings of real balances (z), their consumption in the DM (q_p, which is by consumers in opaque meetings, and q^{hT} which is by consumers with high-quality assets in transparent meetings), total assets sold in the DM and used as payment in the DM ($s_p+\hat{a}_p$ and $s^{hT}+\hat{a}^{hT}$), and aggregate welfare, which is given by:

$$W = \lambda\{(1-\rho)\tau[u(q^{lT}) - q^{lT}] + \rho\tau[u(q^{hT}) - q^{hT}] + (1-\tau)[u(q_p) - q_p]\}. \tag{4.6}$$

Production and consumption of the CM good do not appear in (4.6) because they sum to zero. Note that the consumption by consumers with low-quality assets in transparent meetings is simply equal to z. In addition, s_p or $\hat{a}_p$ cannot be pinned down individually, since selling any positive amount of assets in the AM is a sufficient signal to producers in the DM that a consumer is a type-lO or type-hO consumer. Similarly, s^{hT} or $\hat{a}^{hT}$ cannot be pinned down individually. However, the sum of the assets sold in the AM and the assets used as payment in the DM can be determined.

Proposition 4 *Suppose $\rho\delta_h a < \tilde{q}$. Then, for any a, there exist $i_1 > i_2 \geq i_3 > 0$ such that the comparative statics of z, q_p, q^{hT}, $s_p + \hat{a}_p$, $s^{hT} + \hat{a}^{hT}$, and W with respect to increases in i and τ are described by the following table.*

Cases	z	q_p	q^{hT}	$s_p + \hat{a}_p$	$s^{hT} + \hat{a}^{hT}$	W
$i \geq i_1$	↓	↓	–	–	↑†	↓
$i_2 \leq i < i_1$	–	–	–	–	–	–
$i_3 \leq i < i_2$	↓	–	↓*	↑	↑†	↓
$i \leq i_3$	↓	↓	↓*	–	↑†	↓

(a) Comparative statics: an increase in i

Cases	z	q_p	q^{hT}	$s_p + \hat{a}_p$	$s^{hT} + \hat{a}^{hT}$	W
$i \geq i_1$	↑	↑	↑*	–	↓†	↑
$i_2 \leq i < i_1$	–	–	–	–	–	↓
$i_3 \leq i < i_2$	↑	–	↑*	↓	↓†	↑
$i \leq i_3$	↓	↓	↓*	–	↑†	↓

(b) Comparative statics: an increase in τ

Note: ↑ means"increase"; ↓ means "decrease"; – means "no change"
*: no change if $z + \delta_h a \geq q^*$; †: no change if $s^{hT} + \hat{a}^{hT} = a$

Table 1: Comparative Statics: $\rho\delta_h a < \tilde{q}$

Next, suppose $\rho\delta_h a \geq \tilde{q}$. Then, for any a, there exists $i^\dagger$ such that such that the comparative statics are described by the following table.

Cases	z	q_p	q^{hT}	$s_p + \hat{a}_p$	$s^{hT} + \hat{a}^{hT}$	W
$i > i^\dagger$	$\downarrow$	$-$	$\downarrow^*$	$\uparrow$	$\uparrow^\dagger$	$\downarrow$
$i \leq i^\dagger$	$\downarrow$	$\downarrow$	$\downarrow^*$	$-$	$\uparrow^\dagger$	$\downarrow$

(a) Comparative statics: an increase in i

Cases	z	q_p	q^{hT}	$s_p + \hat{a}_p$	$s^{hT} + \hat{a}^{hT}$	W
$i > i^\dagger$	$\uparrow$	$-$	$\uparrow^*$	$\downarrow$	$\downarrow^\dagger$	$\uparrow$
$i \leq i^\dagger$	$\downarrow$	$\downarrow$	$\downarrow^*$	$-$	$\uparrow^\dagger$	$\downarrow$

(b) Comparative statics: an increase in τ

Note: $\uparrow$ means"increase"; $\downarrow$ means "decrease"; $-$ means "no change"

*: no change if $z + \delta_h a \geq q^*$; †: no change if $s^{hT} + \hat{a}^{hT} = a$

Table 2: Comparative Statics: $\rho\delta_h a \geq \tilde{q}$

Proof: See Appendix B.

In what follows, we use numerical examples to help explain the comparative statics with respect to i and τ. First, we fix τ and consider the effect of i.[8] Figure 1 shows how the consumption by type-lO and type-hO consumers, q_p, and the consumption by type-hT consumers, q^{hT}, depend on i. Note that the consumption by type-lT consumers is equal to z, which is shown in the left panel of Figure 3. When $i \geq i_1$, both q_p and q^{hT} are constrained by z, which is decreasing in i. When $i_2 \leq i < i_1$, both q_p and q^{hT} are constant because the choice of z does not depend on i. Intuitively, while carrying more real balances would allow consumers to reduce their reliance on selling assets in opaque meetings, the marginal benefit from this is too low compared to the cost of real balances. When $i_3 < i < i_2$, a larger z increases q^{hT} but $q_p = \tilde{q}$. In this case, type-lO and type-hO consumers simply reduce the amount of assets that they sell while increasing the payment made in money in the DM. This allows type-hO to reduce their loss from selling assets at a discount. Type-lO consumers must copy their strategies to avoid being identified as having low-quality assets in the DM.

Finally, if $i \leq i_3$, q_p is increasing in i, because type-lO and type-hO consumers do not sell assets in the AM, and they use only money in the DM to purchase the DM good. In this particular example, type-q^{hT} consumers' portfolio of money and assets allow them to consume the efficient amount when $i \leq i_3$. Notice that there is a discontinuous change in q's at the cutoff value i_3. This because when $i > i_3$, the cost of real balance is high enough that consumers are unwilling to carry more than $z = \tilde{q}$, which means that consumers in opaque meetings will need to use their assets to provide additional liquidity. Total consumption, however, remains at $\tilde{q}$. In such a scenario, the marginal benefit of having more real balances is low, because for consumers in opaque meetings, more real balances only lead to fewer assets being sold but not more consumption in the DM. When $i < i_3$, the cost of real balances is sufficiently low so that consumers are willing to carry enough to avoid asset sales in the AM. More importantly, because a marginal increase in z will always lead to an increase in consumption q^p, the marginal benefit of real balances jumps. As a result, z increases discontinuously (see Figure 3), and consumers in opaque meetings consume discontinuously more.

[8] Parameter values chosen are: $u(q) = \frac{q^{1-\sigma}}{1-\sigma}$, $\sigma = 0.5$, $\lambda = 0.5$, $\rho = 0.7$, $\tau = 0.5$, $a = 0.5$, and $\delta_h = 1$. Note that these imply $\rho\delta_h a < \tilde{q}$.

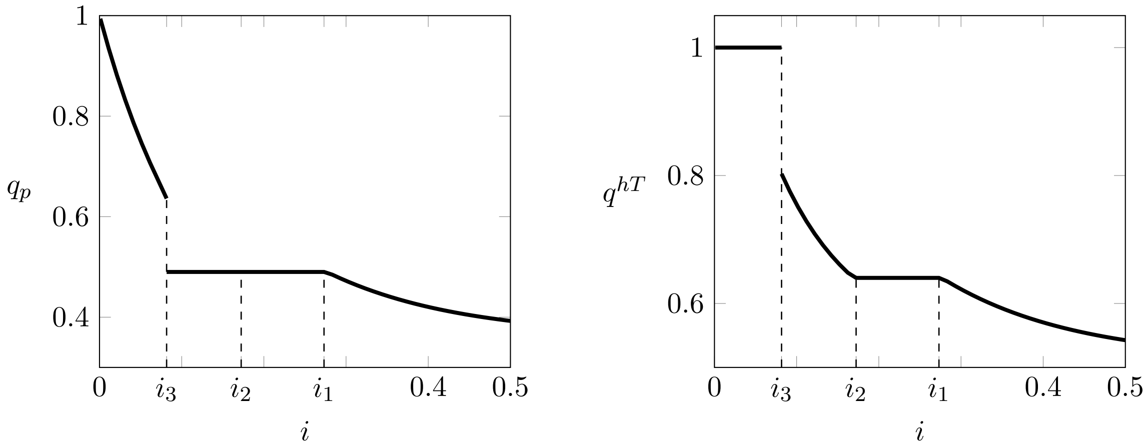


Figure 1: Equilibrium consumption and i

Figure 2 shows how the sum of the assets sold in the AM (s_p and s^{hT}) and the assets used as payment in the DM ($\hat{a}_p$ and $\hat{a}^{hT}$) depend on i. Note that when $i \geq i_1$, consumers of type-lO, type-hO, and type-hT are all constrained by their assets. When $i_3 \leq i < i_1$, on the other hand, type-hT consumers are still constrained by their assets but type-lO and type-hO consumers are not. Specifically, the latter two types of consumers will not end up consuming more even if they have more assets. This happens because the marginal benefit for type-hO consumers of additional consumption ($u'(q_p)$) is lower than the marginal cost ($u'(\tilde{q})$) of selling assets at a discounted price. (While the same is not true for type-lO consumers, attempting to sell more would reveal their low type, hence they do not.)

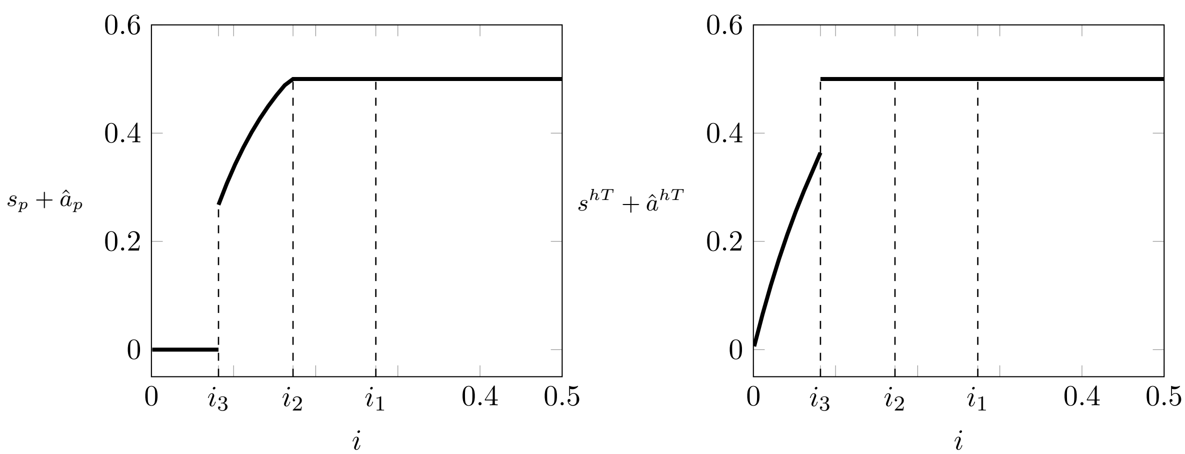


Figure 2: Assets sold in the AM (s_p and s^{hT}) and used as payment in the DM ($\hat{a}_p$ and $\hat{a}^{hT}$)

The right panel of Figure 3 shows how i affects aggregate welfare. Aggregate welfare exhibits a jump at $i = i_3$ because agents' holdings of real balances increase discontinuously at $i = i_3$. Intuitively, the information frictions in the AM and DM mean that type-hO consumers are unwilling to consume more than $\tilde{q}$, unless they have enough real balances to avoid using assets at all. This means that type-hO consumers' consumption is also low when i is high since they always mimic the strategies of type-hO consumers. When $i < i_3$, the cost of real balances is sufficiently low so that consumers are willing to carry enough to avoid selling assets in the AM. This leads to a discontinuous increase in welfare.

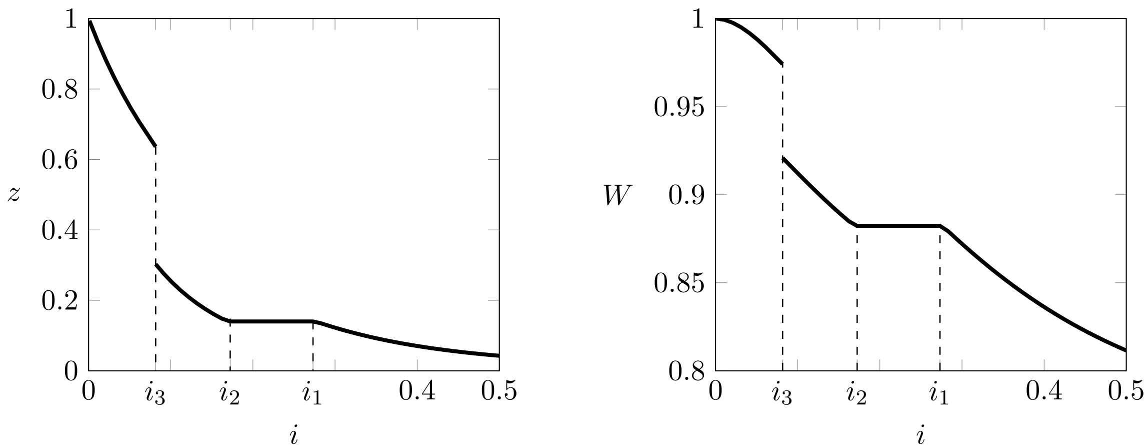


Figure 3: Equilibrium choice of real balances and equilibrium welfare

Next, we show how τ, the share of transparent meetings in the AM, affects equilibrium outcomes. We refer to the four scenarios in Proposition 3 when $\rho\delta_h a < \tilde{q}$ as Cases 1 to 4. As shown by Figure 4 and Figure 5, a higher τ leads to the equilibrium to switch from Case 2 to Case 4.[9] In Case 2, $q_p = \tilde{q}$ and $q_h = \tilde{q} + (1-\rho)\delta_h a$. Hence, both consumption levels are unaffected by the value of τ. Once the equilibrium switches to Case 4, however, an increase in τ lowers agents' incentive to carry money. This is because type-hT consumers can sell assets in the AM or use assets directly in the DM, and a higher τ makes it more likely that a consumer will become type-hT. Hence, we see a drop in the real balances agents hold as τ increases (see the left panel Figure 5). While type-hT consumers have enough real balances and assets to consume the efficient amount ($q^* = 1$), all the other consumers consume less in the DM due to the decrease in money holdings. Hence, aggregate welfare is decreasing in τ.

Notice that aggregate welfare decreases in τ when the equilibrium is in Case 2. This is because a higher τ means a larger share of consumers will become type-lT and will not be able to use assets at all in the AM or DM. This lowers the consumption in the DM. In other words, the existence of opaque meetings in this case is beneficial for aggregate welfare, because it allows some consumers with low-quality assets, i.e. type-lO consumers, to utilize their assets either to obtain money in the AM or to purchase goods in the DM. This finding is similar to that in Andolfatto, Berentsen, and Waller (2014), where disclosing the information about asset quality can lead to assets losing their function as payment instruments. This in turn prevents trade that relies on such payment instruments from happening, thereby hurting

[9] When i is high, it is also possible that the equilibrium switches from Case 2 to Case 3, or from Case 1 to Case 2, and then to Case 3. Examples of these scenarios can be found in Appendix C.3.

welfare. However, it should be emphasized that although welfare is also decreasing in τ in Case 4, it is for a different reason from Case 2; in Case 4, it happens because consumers carry fewer real balances, therefore consumers who do not end up becoming type-hT consume less in equilibrium.

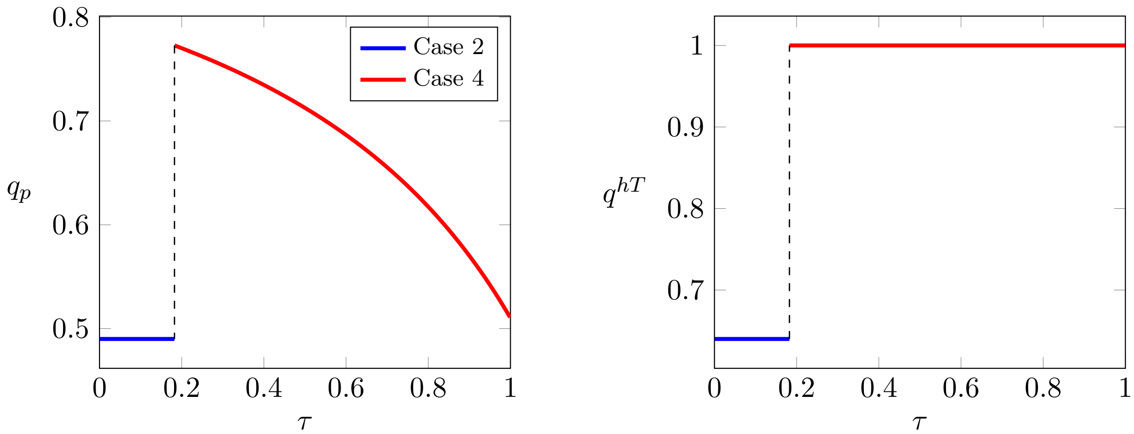


Figure 4: Equilibrium consumption and τ ($i = 0.06$)

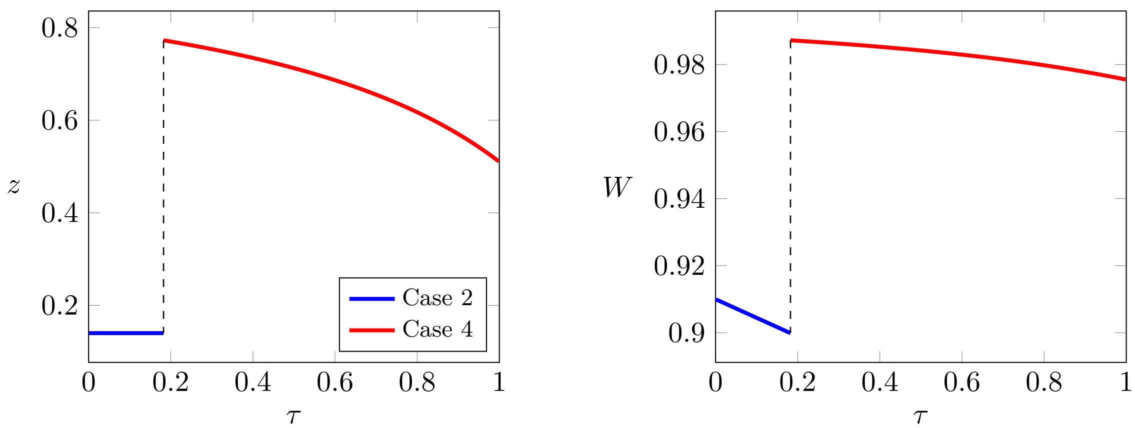


Figure 5: Equilibrium choice of real balances, and equilibrium welfare ($i = 0.06$)

4.2 The case where $\delta_l > 0$

In this section, we consider the case where $\delta_l > 0$. Compared to the case with $\delta_l = 0$, consumers with low-quality assets can now obtain money from the AM even if they are in transparent meetings. Furthermore, in both the AM and the DM, it is possible to have separating equilibria where consumers with low-quality and high-quality assets make different offers. Nevertheless, we show that the spirit of the main result remains unchanged. That is, consumers benefit from selling assets for money in the AM, even though assets have direct liquidity. This result holds even in opaque AM meetings, where the asymmetry of information is equally severe as in the DM.

First, we present a useful observation that simplifies the analysis: although C-type consumers can in principle make separating offers in the AM, only pooling offers can be part of an equilibrium.

Lemma 1 *In equilibrium, C-type consumers only pool in opaque meetings.*

If C-types make separating offers in the AM, they will enter the DM with different asset portfolios depending on the quality of their assets. This means that producers can identify the C-types with high-quality assets through their asset portfolios, which in turn suggests that in the AM, C-types with low-quality assets will always mimic the strategies of those with high-quality assets. Hence, C-types do not make separating offers in the AM.

We summarize undefeated equilibria in the AM and the DM given consumers' portfolios (z, z_b, a) in the following proposition.

Proposition 5 *Assume that $(1-\rho)\delta_l + (2\rho-1)\delta_h > 0$. Conditional on (z, z_b, a), undefeated equilibria in the AM and the DM satisfy the following conditions.*

(1) $(\psi^{hT}, s^{hT}; q^{hT}, \hat{z}^{hT}, \hat{a}^{hT})$ *is given by*

(a) $\psi^{hT} = \delta_h$; (b) $s^{hT} \in (0, \min\{z_b/\delta_h, a\}]$;

(c) $q^{hT} = \min\{q^*, z + \delta_h a\}$; (d) $\hat{z}^{hT} = z + \psi^{hT} s^{hT}$; (e) $\hat{a}^{hT} = \frac{q^{hT} - \hat{z}^{hT}}{\delta_h}$.

(2) $(\psi^{lT}, s^{lT}; q^{lT}, \hat{z}^{lT}, \hat{a}^{lT})$ *is given by*

(a) $\psi^{lT} = \delta_l$; (b) $s^{lT} \in [0, \min\{z_b/\delta_l, a\}]$;

(c) $q^{lT} = \min\{q^*, z + \delta_l a\}$; (d) $\hat{z}^{lT} = z + \psi^{lT} s^{lT}$; (e) $\hat{a}^{lT} = \frac{q^{lT} - \hat{z}^{lT}}{\delta_l}$.

(3) *Define $q_l(s) = \min\{z + \bar{\delta}s + \delta_l(a-s), q^*\}$ and $q_h(s) = z + \bar{\delta}s + \delta_h \hat{a}(s)$, where $\hat{a}(s)$ solves*

$$u(q_l(s)) - q_l(s) = u(q_h(s)) - z - \bar{\delta}s - \delta_l \hat{a},$$

and $\bar{\delta} = (1-\rho)\delta_l + \rho\delta_h$. Let $s^\dagger$ solve

$$\frac{[(1-\rho)\delta_l + (2\rho-1)\delta_h]u'(q_h(s^\dagger))}{\rho\delta_h} + [u'(q_l(s^\dagger)) - 1][u'(q_h(s^\dagger)) - 1] = 1.$$

There exist $0 \le z' < z''$ such that if $z \ge z''$,

(a) $s^{lO} = s^{hO} = s_p = 0$; (b) $q^{lO} = q_l(s_p)$ *and* $q^{hO} = q_h(s_p)$;

(c) $\hat{z}^{lO} = \hat{z}^{hO} = z$; (d) $\hat{a}^{lO} = \frac{q^{lO} - \hat{z}^{lO}}{\delta_l}$ *and* $\hat{a}^{hO} = \hat{a}(0)$.

If $z' \le z < z''$,

(a) $\psi^{lO} = \psi^{hO} = \psi_p = (1-\rho)\delta_l + \rho\delta_h$; (b) $s^{lO} = s^{hO} = s_p = \min\{s^\dagger, z_b\}$;

(c) $q^{lO} = q_l(s_p)$ *and* $q^{hO} = q_h(s_p)$; (d) $\hat{z}^{lO} = \hat{z}^{hO} = z + \psi_p s_p$;

(e) $\hat{a}^{lO} = \frac{q^{lO} - \hat{z}^{lO}}{\delta_l}$ *and* $\hat{a}^{hO} = \hat{a}(s_p)$.

If $z < z'$,

(a) $\psi^{lO} = \psi^{hO} = \psi_p = (1-\rho)\delta_l + \rho\delta_h$;

(b) $s^{lO} = s^{hO} = s_p \in (0, \min\{z_b/[(1-\rho)\delta_l + \rho\delta_h], a\}]$;

(c) $q^{lO} = q^{hO} = z + \psi_p a$; (d) $\hat{z}^{lO} = \hat{z}^{hO} = z + \psi_p s_p$; (e) $\hat{a}^{lO} = \hat{a}^{hO} = a - s_p$.

Proof: See Appendix B.

Undefeated equilibria in transparent meetings are similar to the case where $\delta_l = 0$, with the only difference being that type-lO consumers may also sell their assets. However, because type-hT, type-lO, and type-hO consumers can differentiate themselves from type-lT consumers through selling assets in the AM, type-lT consumers may only trade their assets at a price equal to δ_l in both the AM and the DM.

Compared to the case where $\delta_l = 0$, the equilibria in opaque meetings are now different, and they depend on C-type consumers' money holdings z. When z is large (i.e., $z \geq z''$), C-type consumers do not sell in the AM, and they make separating offers in the DM. However, when z is small (i.e., $z < z''$), separating in the DM is costly for C-types with high-quality assets, because they need to ration the use of assets as payment in order to signal asset quality to producers. This gives such consumers an incentive to sell some of their assets in the AM, provided that the price discount in the AM is not too severe.[10] Such a strategy allows them to bring more real balances to the DM and rely less on the real asset. As a result, C-types with high-quality assets achieve both higher consumption (when compared to only separating in the DM) and a higher average price for their assets (when compared to pooling in both the AM and the DM). This equilibrium in the AM and the DM, which combines the features of a separating equilibrium and a pooling equilibrium, is unique to our environment, because the real assets can be used to provide both indirect and direct liquidity. Finally, if $z < z'$, C-types will pool in the DM after selling assets in the AM, which is similar to the equilibria where $\delta_l = 0$. We show in the proof of Proposition 5 that the cutoff value, z', is strictly positive if and only if a is small. This is because when a is small, the amount of real balances C-types can obtain from the AM is low. In such a case, separating in the DM is costly for the C-types with high-quality assets if their money holding, z, is also small.

Despite the differences, the main insights from the cases where $\delta_l > 0$ and $\delta_l = 0$ are the same: consumers benefit from selling assets for money in the AM even though assets can provide direct liquidity, because such a strategy allows consumers to reduce/avoid the information asymmetry in the DM. Importantly, this result holds even for consumers that are in opaque AM meetings, where the information asymmetry is as severe as in the DM.

Finally, we solve for the consumers' optimal choice of real balances in the CM. Similar to the case where $\delta_l = 0$, we focus on symmetric solutions where $z = z_b$. The following proposition shows that how inflation affects C-type consumers' choices in the AM and the

[10] This is the reason for the assumption in Proposition 5 that $(1-\rho)\delta_l + (2\rho - 1)\delta_h > 0$, which holds when ρ is sufficiently large. If the condition is not satisfied, consumer may not sell in opaque AM meetings when $z < z''$. In such a case, consumers with high-quality assets prefer to make separating offers in the DM and avoid the price discount in the AM.

DM, and it corresponds directly to the results in Proposition 5. The solution to the optimal choice of real balances can be found in the proof of the proposition.

Proposition 6 *Assume that $z' \geq \bar{\delta} a$, where z' is given by* Proposition 5*, and $\bar{\delta} = (1-\rho)\delta_l + \rho\delta_h$. There exist $i' > 0$ and $i'' > i'$ such that*
(1) *If $i > i''$, consumers in opaque meetings sell assets in the AM and pool in DM.*
(2) *If $i' < i \leq i''$, consumers in opaque meetings sell assets in the AM and separate in DM.*
(3) *If $i \leq i'$, consumers in opaque meetings separate in the DM and do not trade in AM.*
Proof: See Appendix B.

5 Endogenizing the Probability of Transparent Meetings (τ)

In this section, we endogenize τ, the share of transparent meetings in the AM. Specifically, a consumer may pay a utility cost κ and enter a transparent meeting in the AM, while asset quality remains private information in the DM. One can think of κ as the cost of producing a certificate of the asset's quality, or a fee to access a specialist intermediary who can guarantee the quality of the asset. We consider two scenarios. In the first scenario, the consumer must make the choice of whether to pay κ after she learns her η_t (i.e., whether she is a C-type or N-type consumer) but before she learns the quality of her assets, δ. In the second scenario, the consumer must make the choice after she learns η_t and δ. The results are similar for the cases where $\delta_l > 0$ and $\delta_l = 0$; to keep the analysis concise, here we focus on the case where $\delta_l = 0$ and relegate the other case to Appendix C.2.

5.1 κ paid before the quality shock

Given z, define the benefit of paying κ and entering a transparent meeting as $\mathcal{B}_1$.

$$\mathcal{B}_1(z) = (1-\rho)[u(q^{lT}) + z - q^{lT}] + \rho[u(q^{hT}) + z + \delta_h a - q^{hT}] - [u(q_p) + z + \rho\delta_h a - q_p],$$

where $q^{lT} = \min\{z, q^*\}$, $q^{hT} = \min\{z + \delta_h a, q^*\}$, and $q_p = \max\{\min\{z, q^*\}, \min\{z + \rho\delta_h a, \tilde{q}\}\}$, where $u'(\tilde{q}) = 1/\rho$ as before. To understand the expression, note that if a consumer pays κ, she will be in a transparent meeting regardless of the quality of the assets. Her surplus then follows from Proposition 2. If she does not pay the cost, she will be in an opaque meeting, and her expected value is $u(q_p) + z + \rho\delta_h a - q_p$, where q_p is again given by Proposition 2. The following lemma shows how $\mathcal{B}_1(z)$ depends on z.

Lemma 2 *Assume $u'''(.) > 0$ and $\delta_h a < q^*$. Then, $\mathcal{B}_1'(z) > 0$ for all $z \leq \tilde{q}$ and $\mathcal{B}_1'(z) < 0$ for all $\tilde{q} < z < q^*$. In addition, there exists $0 < \tilde{z} < \tilde{q}$ such that $\mathcal{B}_1(\tilde{z}) = 0$. Finally, $\mathcal{B}_1(q^*) = 0$.*
Proof: See Appendix B.

The proposition shows that, firstly, being in a transparent meeting is not always beneficial *ex ante*. This is because opaque meetings allow agents with low-quality assets to also use their assets in the AM and DM, thereby providing insurance against the quality shock. However, as agents' money holdings increase, they will be able to consume more even if they end up having low-quality assets. As a result, the insurance benefit decreases with z (i.e., $\mathcal{B}_1'(z) > 0$ for all $z \leq \tilde{q}$). If $z > \tilde{q}$, agents in opaque meetings do not sell assets in the AM or use assets directly in the DM. Hence, they consume less compared to agents with high-quality assets. However, as z increases, the difference in consumption becomes smaller. Thus, $\mathcal{B}_1'(z) < 0$ for all $z > \tilde{q}$, and $\mathcal{B}_1(q^*) = 0$.

From the proposition, we can conclude that $\mathcal{B}_1(z)$ reaches its maximum at $z = \tilde{q}$. Therefore, if $\kappa \geq \mathcal{B}_1(\tilde{q})$, no consumers will pay to be in transparent meetings. We assume agents do not pay the cost if they are indifferent.

The following proposition summarizes the equilibrium when $\kappa < \mathcal{B}_1(\tilde{q})$.

Proposition 7 *Assume that* $0 < \kappa < \mathcal{B}_1(\tilde{q})$, $u'''(.) > 0$, *and* $a < \frac{\tilde{q}}{\rho\delta_h}$. *Then, there exist* $i_1 > i_2 \geq i_3 > 0$ *such that*
(1) *for all* $i \geq i_1$, z *solves* $i = \lambda[u'(z + \rho\delta_h a) - 1]$, *and agents do not pay* κ;
(2) *for all* $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$, *and agents do not pay* κ;
(3) *for all* $i_3 < i < i_2$, z *solves* $i = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1]\}$, *and agents pay* κ;
(4) *for all* $i \leq i_3$, z *solves* $i = \lambda[u'(z) - 1]$, *and agents do not pay* κ.
Proof: See Appendix B.

Our next proposition summarizes the comparative statics of equilibrium outcomes with respect to the cost of holding money (i). We look at consumers' holdings of real balances (z), consumption in the DM by consumers with high-quality assets (q_h), total assets sold in the AM and used as payment in the DM ($s + \hat{a}$), and aggregate welfare. Note that consumption by consumers with low-quality assets is either equal to q^h if consumers do not pay κ, or z if consumers do pay κ. In addition, if consumers do not pay κ, s or $\hat{a}$ cannot be pinned down individually since consumers will be indifferent between selling real assets in the AM and using them directly as payment in the DM. However, the sum of the assets sold in the AM and the assets used as payment in the DM can be determined. Finally, since κ is a fixed cost, within each of the four equilibrium cases (see Proposition 7), a change in κ does not affect z, q_h, or $s + \hat{a}$. Therefore, we focus on the comparative statics with respect to i.

Proposition 8 *There exist* $i_1 > i_2 \geq i_3 > 0$ *such that the comparative statics of* z, q_h, $s + \hat{a}$, *and* W *with respect to an increase in* i *are described by the following table.*

Cases	z	q_h	$s+\hat{a}$	W
$i \geq i_1$	$\downarrow$	$-$	$-$	$\downarrow$
$i_2 \leq i < i_1$	$-$	$-$	$-$	$-$
$i_3 \leq i < i_2$	$\downarrow$	$\downarrow^*$	$\uparrow^\dagger$	$\downarrow$
$i \leq i_3$	$\downarrow$	$\downarrow$	$-$	$\downarrow$

Note: $\uparrow$ means"increase"; $\downarrow$ means "decrease"; $-$ means "no change"

*: no change if $z + \delta_h a \geq q^*$; †: no change if $s + \hat{a} = a$

Table 3: Comparative Statics: κ Paid Before the Quality Shock

Proof: See Appendix B.

We use the following numerical example to illustrate the results.[11] Similar to the scenario where τ is exogenous (see Proposition 3), there are either two or three cutoff values of i. When i is either too high or too low, consumers choose not to pay κ, and therefore they are in opaque meetings in the AM. In these cases, consumers with high-quality assets consume the same amount (q_h) as consumers with low-quality assets.

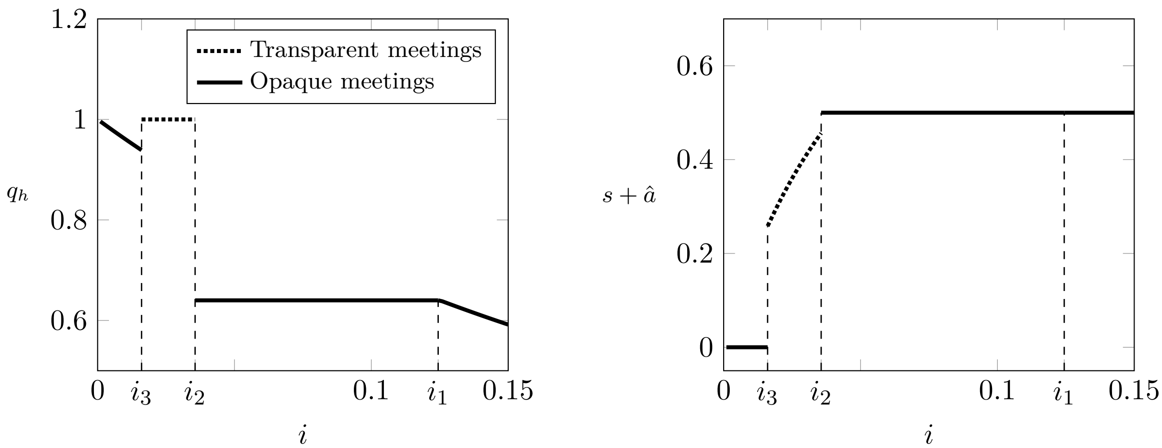


Figure 6: Consumption (q_h) and assets sold or used as payment ($s + \hat{a}$)

When $i \leq i_3$, consumers do not sell assets in the AM or use assets as a medium of exchange in the DM. This is because the information asymmetry means that for consumers with high-quality assets, the marginal cost of selling or using assets (i.e., sacrificing the dividend) exceeds the marginal benefit of selling or using assets (i.e., more consumption). When $i_2 \leq i < i_1$, however, the marginal cost of selling or using assets is equal to the marginal benefit of selling or using assets for consumers with high-quality assets. In this scenario, having more real balances simply means that consumers substitute assets for real balances. Since holding real balances is costly for any $i > 0$, agents only hold enough real balances so that their asset portfolios allow them to consume $\tilde{q}$. Recall that s or $\hat{a}$ cannot be pinned down individually. This is because when consumers have transparent meetings, selling any positive amount of assets in the AM is a sufficient signal to producers in the DM that a consumer has high-quality

[11] In this example, $u(q) = \frac{q^{1-\sigma}}{1-\sigma}$, $\sigma = 0.5$, $\lambda = 0.5$, $\rho = 0.7$, $\kappa = 0.004$, $a = 0.5$, and $\delta_h = 1$.

assets. If instead consumers are in opaque meetings, they are indifferent between selling assets in the AM or using them directly in the DM, since the information asymmetry is the same in the AM and the DM. However, in either case, the *sum* of the assets sold in the AM and the assets used as payment in the DM is determinate.

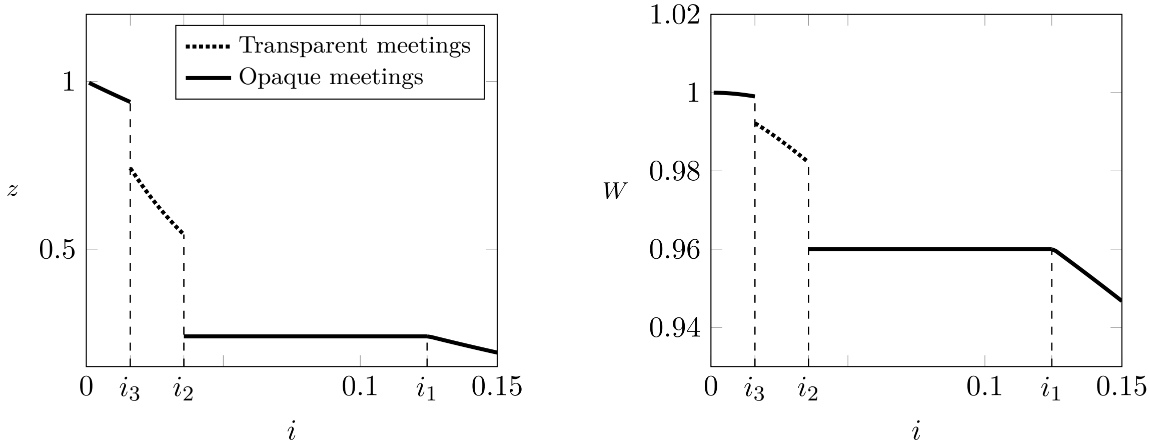


Figure 7: Real balances (z) and aggregate welfare (W)

We also check how the equilibrium depends on κ for a given i. Unsurprisingly, agents prefer opaque meetings in the AM when κ is sufficiently large. Since κ is a fixed cost, it does not otherwise affect equilibrium outcomes. Interestingly, aggregate welfare may increase discontinuously when consumers switch to not paying κ. This is because in this particular example i is low, so if consumers do not pay κ (i.e., the black lines in Figure 8), they also do not sell or use assets (see case (4) in Proposition 7). Therefore, consumers hold more real balances relative to when κ is low. This increases the consumption of consumers with low-quality assets relative to when consumers have transparent meetings in the AM. Consequently, aggregate welfare is higher. However, the opposite happens when inflation is high: consumers benefit more from pooling in the AM, which allows them to use the real assets either directly as MoE or to obtain money.

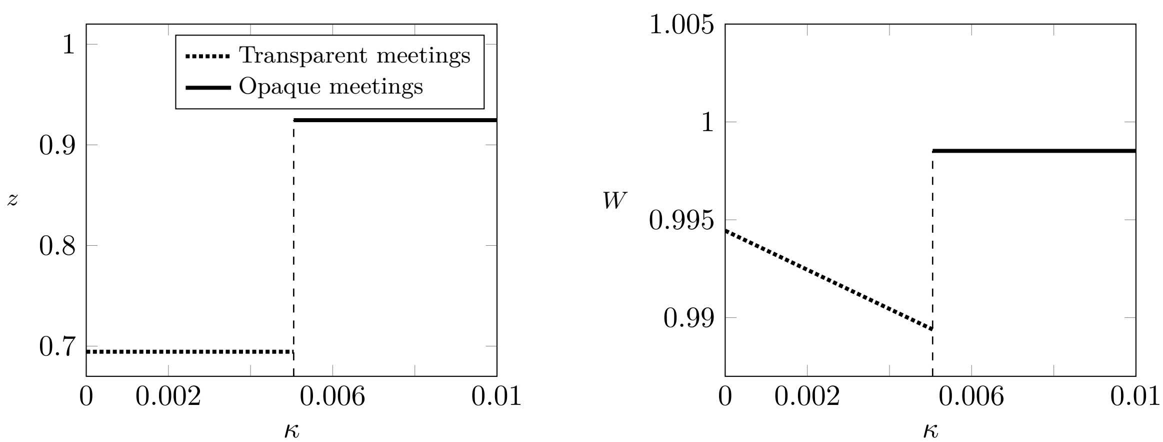


Figure 8: Real balances (z) and aggregate welfare (W) ($i = 0.02$)

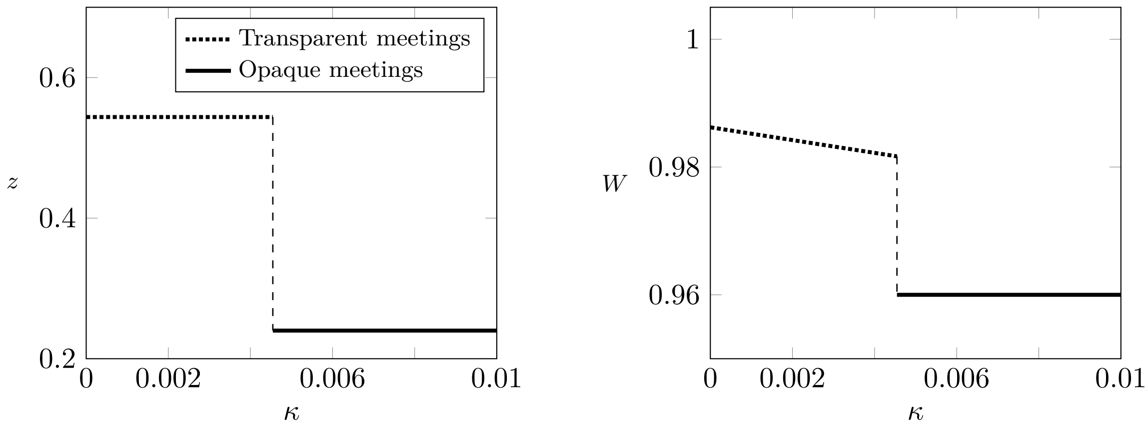


Figure 9: Real balances (z) and aggregate welfare (W) ($i = 0.04$)

5.2 κ paid after the quality shock

Next, we consider the second scenario, where consumers choose whether to pay κ after learning the quality of their assets. Given (z, a), define the benefit of paying κ and entering a transparent meeting for a consumer with high-quality assets as $\mathcal{B}_2$. Note that consumers with low-quality assets have no incentive to pay κ, since they do not benefit from disclosing the quality of their assets. Let $\tilde{\tau}$ denote the fraction of consumers who choose to pay κ.

$$\mathcal{B}_2(z, \tilde{\tau}) = u(q^{hT}) + z + \delta_h a - q^{hT} - [u(q(\tilde{\tau})) + z + \rho\delta_h a - q(\tilde{\tau})],$$

where $q^{hT} = \min\{z + \delta_h a, q^*\}$, and $q(\tilde{\tau}) = \max\{z, \min\{z + \xi(\tilde{\tau})\delta_h a, q^\dagger(\tilde{\tau})\}\}$ where $u'(q^\dagger(\tilde{\tau})) = 1/\xi(\tilde{\tau})$, and $\xi(\tilde{\tau}) = \frac{\rho(1-\tilde{\tau})}{\rho(1-\tilde{\tau})+1-\rho}$ is the average asset quality among consumers who do not pay κ. Notice that now the benefit of having transparent meetings also depends on $\tilde{\tau}$.

Lemma 3 *$\mathcal{B}_2(z, \tilde{\tau})$ is decreasing in z for any $z < q^\dagger(\tilde{\tau}) - \xi(\tilde{\tau})\delta_h a$ and $z > q^\dagger(\tilde{\tau})$ and increasing in z for any $z \in [q^\dagger(\tilde{\tau}) - \xi(\tilde{\tau})\delta_h a, q^\dagger(\tilde{\tau})]$, while $\mathcal{B}_2(z, \tilde{\tau})$ is increasing in $\tilde{\tau}$.*

Proof: See Appendix B.

Now, define $\bar{\mathcal{B}}_2(z)$ to be the upper bound of the benefit of paying κ (i.e., when $\tilde{\tau} = 1$).

$$\bar{\mathcal{B}}_2(z) = u(q^{hT}) + z + \delta_h a - q^{hT} - [u(z) + z + \rho\delta_h a - z].$$

Next, define $\underline{\mathcal{B}}_2(z)$ to be the lower bound of the benefit of paying κ (i.e., when $\tilde{\tau} = 0$).

$$\underline{\mathcal{B}}_2(z) = u(q^{hT}) + z + \delta_h a - q^{hT} - [u(q(0)) + z + \rho\delta_h a - q(0)].$$

If $\kappa \geq \bar{\mathcal{B}}_2(z)$, no consumers will pay κ. If $\kappa \leq \underline{\mathcal{B}}_2(z)$, all consumers will pay κ. If $\kappa \in (\underline{\mathcal{B}}_2(z), \bar{\mathcal{B}}_2(z))$, there exists $\tilde{\tau}'(z)$ such that for all $\tilde{\tau}' \geq \tilde{\tau}(z)$, agents will pay κ, where $\tilde{\tau}'$ solves

$$u(q^{hT}) + z + \delta_h a - q^{hT} - [u(q(\tilde{\tau}'(z))) + z + \rho\delta_h a - q(\tilde{\tau}'(z))] = \kappa.$$

Now, we solve for the optimal choice of z. The following lemma simplifies the analysis.

Lemma 4 *If z is chosen optimally in the CM, then either $\tilde{\tau} = 1$ or $\tilde{\tau} = 0$.*
Proof: See Appendix B.

The lemma shows that as long as z is chosen optimally, consumers do not play mixed strategies in the AM when deciding whether to pay κ. Intuitively, this is because the marginal value of real balances depends on whether κ is paid. Therefore, even if consumers are indifferent ex post regarding paying κ, they will strictly prefer different holdings of real balances ex ante. Now, let $\iota(\delta, z)$ denote the choice of whether to pay κ or not conditional on the realization of δ and z. The following proposition shows that except at one cutoff value (i'_2), there is a unique combination of z and $\iota(z, \delta)$ that maximizes consumers' expected utility.

Proposition 9 *Assume that $0 < \kappa < \bar{\mathcal{B}}_2(0)$, $u'''(.) > 0$, and $a < \frac{\tilde{q}}{\rho\delta_h}$. Then, there exist $i'_1 \geq i'_2 \geq i'_3 \geq i'_4 > 0$ such that*
(1) *for all $i \geq i'_1$, z solves $i = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$, and agents pay κ in the AM;*
(2) *for all $i'_2 \leq i < i'_1$, $z = z_1$ where z_1 solves $i'_1 = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$, and agents pay κ in the AM;*
(3) *for all $i'_3 \leq i < i'_2$, z solves $i = \lambda[u'(z + \rho\delta_h a) - 1]$, and agents do not pay κ in the AM.*
(4) *for all $i'_4 \leq i < i'_3$, z solves $i'_3 = \lambda[u'(z + \rho\delta_h a) - 1]$, and agents do not pay κ in the AM.*
(5) *for all $i < i'_4$, z solves $i = \lambda[u'(z) - 1]$, and agents do not pay κ in the AM.*
Proof: See Appendix B.

Recall that when κ had to be paid before consumers learn the quality of their assets, consumers did not pay κ when i was either too low or too high (see Proposition 7). However, if κ is instead paid after consumers learn asset quality, then they pay κ as long as i is high but does not pay when i is low. To understand the difference, note that in opaque meetings, consumers receive insurance against the quality shock, since those with low-quality assets are able to sell their assets. Such insurance is especially beneficial when i is high, and consumers' money holdings are small. This is why consumers do not pay κ if it has to be paid before they learn asset quality. However, if consumers pay κ after they learn asset quality, such insurance is no longer relevant. When i is high, and consumers' money holdings are small, paying κ and avoiding information asymmetry becomes beneficial for consumers with high-quality assets.

The following proposition summarizes the comparative statics of equilibrium outcomes.

Proposition 10 *There exist $i'_1 \geq i'_2 \geq i'_3 \geq i'_4 > 0$ such that such that the comparative statics of z, q_h, $s + \hat{a}$, and W with respect to an increase in i is given by the following table.*

Cases	z	q_h	$s+\hat{a}$	W
$i \geq i'_1$	$\downarrow$	$\downarrow^*$	$\uparrow^\dagger$	$\downarrow$
$i'_2 \leq i < i'_1$	$-$	$-$	$-$	$-$
$i'_3 \leq i < i'_2$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
$i'_4 \leq i < i'_3$	$-$	$-$	$-$	$-$
$i < i'_4$	$\downarrow$	$\downarrow$	$-$	$\downarrow$

Note: $\uparrow$ means“increase”; $\downarrow$ means “decrease”; $-$ means “no change”

*: no change if $z + \delta_h a \geq q^*$; †: no change if $s + \hat{a} = a$

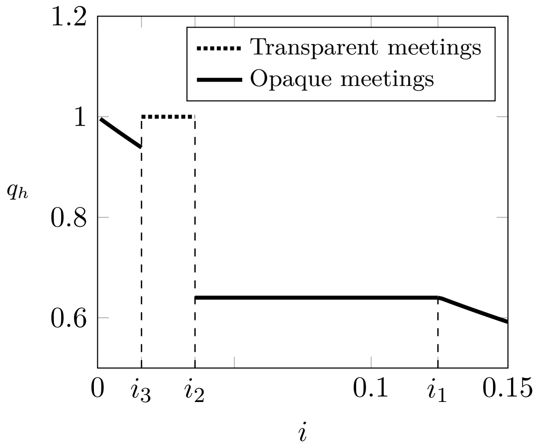
Table 4: Comparative Statics: κ After the Quality Shock

Proof: See Appendix B.

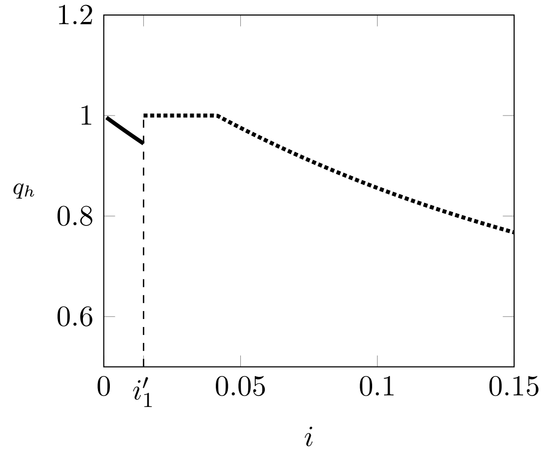
We again use a numerical example to illustrate the results, with the same parameter values as in the example shown in Figures 6 and 7. In Figure 10, we compare how equilibrium outcomes depend on *when* κ is paid by consumers. We use solid and dotted black lines, respectively, to represent equilibrium outcomes when consumers choose to pay or do not pay κ in the AM after they learn asset quality. (In this particular example, it turns out that $i'_1 = i'_2 = i'_3 = i'_4$, i.e., there is only one cutoff value in the case where κ is paid after the quality shock.) As shown in Proposition 9, the timing of the payment of κ makes a difference mainly when i is relatively large. When κ can be paid after the quality shock, consumers with high-quality assets pay it as long as $i > i'_1$; the value of their assets is then higher in the AM and DM, and they consume more in the DM, than consumers with low-quality assets.

Note that because $q_l = z$ when $i > i'_1$ and κ is paid after the quality shock, ex ante, consumers carry more money to insure against the quality shock. This explains why aggregate welfare, W, drops less significantly compared to to when κ has to be paid before the quality shock. However, if inflation is very high, the opposite happens: because compared to carrying real balances, the opaque meetings in the AM offers a better insurance against the quality shock. Consequently, welfare is higher if κ is paid before the quality shock.

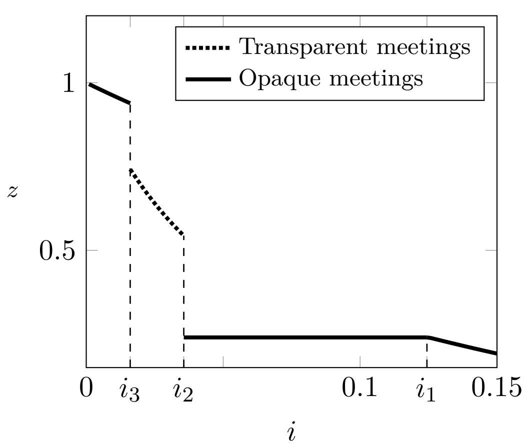
We also compare how equilibrium outcomes depend on κ in both scenarios. When inflation is low, the main difference is that the cutoff value at which consumers switch from paying κ to not paying κ increases when κ can be paid after the quality shock. This is because in that case, only consumers with high-quality assets pay the cost, which also explains why aggregate welfare is higher in Figure 11. When inflation is high, however, welfare is monotonically decreasing in κ when it is paid before the quality shock. This is because when inflation is high, consumers benefit more from pooling in the AM, which allows them to use the real assets to either obtain money or directly as a means of payment. However, when κ is high and inflation is low, consumers hold more real balances. This increases the consumption of consumers with low-quality assets. Consequently, aggregate welfare increases discontinuously when consumers switch to opaque meetings.



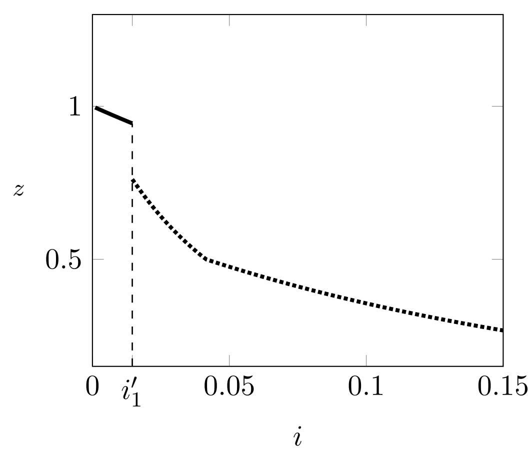
(a) κ paid before the quality shock



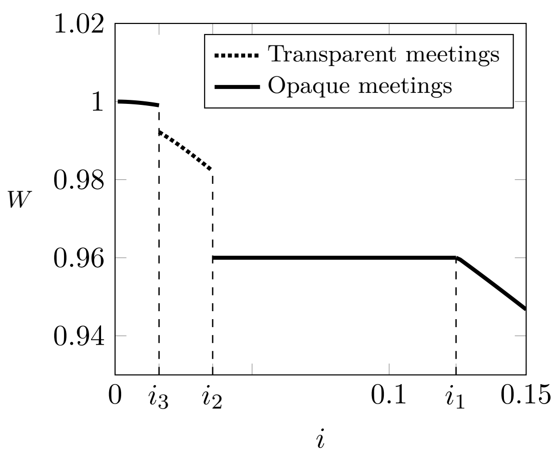
(b) κ paid after the quality shock



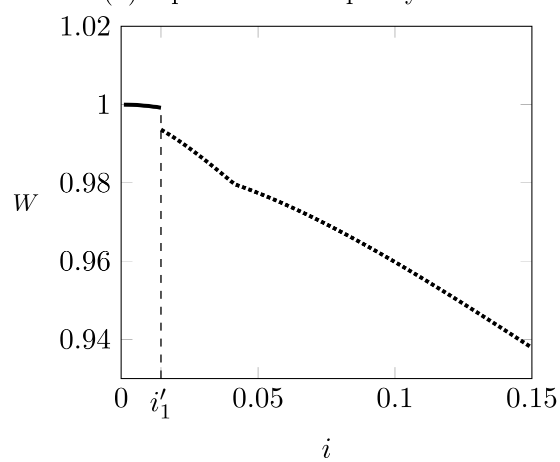
(c) κ paid before the quality shock



(d) κ paid after the quality shock

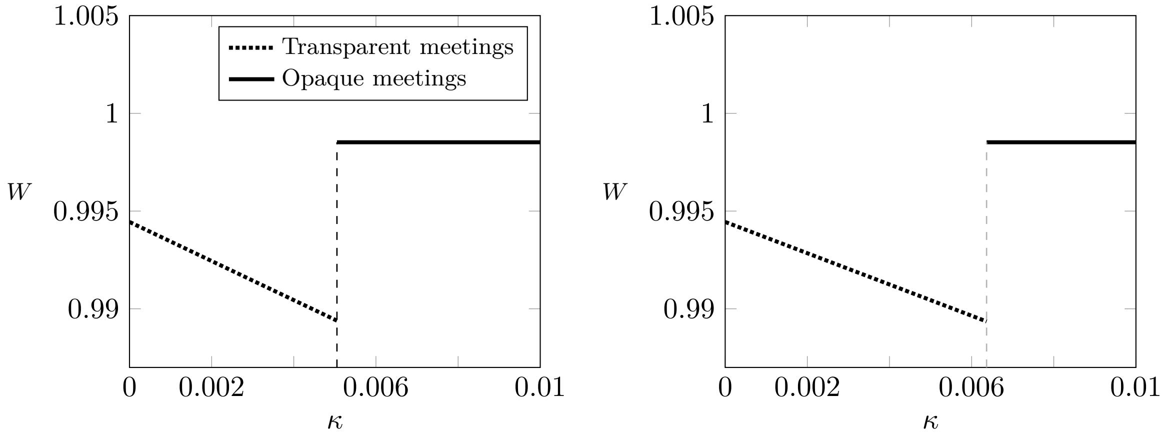


(e) κ paid before the quality shock



(f) κ paid after the quality shock

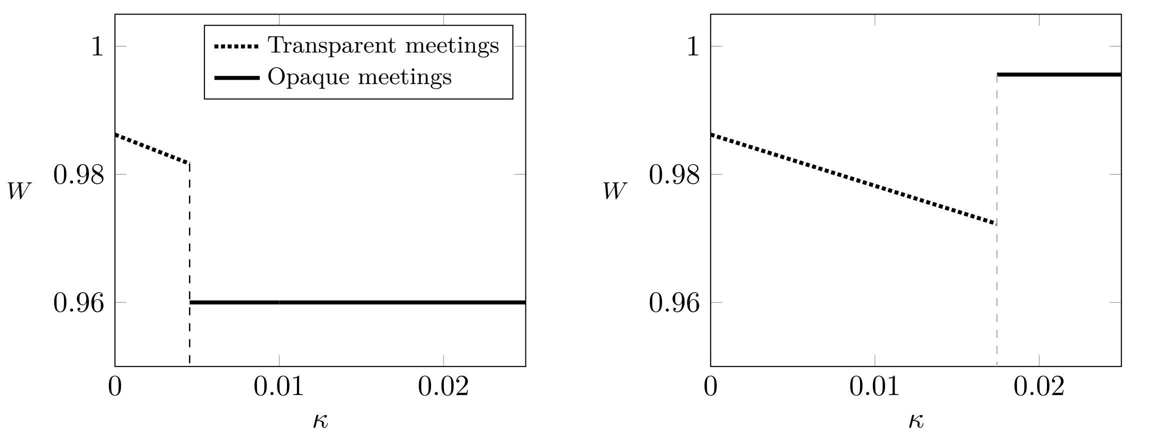
Figure 10: Consumption (q_h), real balances (z), and aggregate welfare (W)



(a) κ paid before the quality shock

(b) κ paid after the quality shock

Figure 11: Aggregate welfare (W) for low inflation ($i = 0.02$)



(a) κ paid before the quality shock

(b) κ paid after the quality shock

Figure 12: Aggregate welfare (W) for higher inflation ($i = 0.04$)

6 Conclusion

In this paper, we develop a model where a real asset is subject to asymmetric information and serves a double liquidity role: it can compete directly with money as a medium of exchange or it can be liquidated for money in an over-the-counter secondary market. Thus, our model allows us to study how the degree of asymmetric information in the secondary market affects asset liquidity and aggregate welfare.

We start with a version of the model where the degree of asymmetric information in the asset market is exogenous. We find that rather than using the asset directly as a medium of exchange, agents prefer to liquidate it for money in the asset market. Furthermore, we show that a decrease in severity of asymmetric information in the asset market can hurt welfare, and that high inflation can lead to a discontinuous decrease in aggregate welfare.

Our model delivers two novel insights. First, we show AM trade can be used both for rebalancing asset portfolios *and* for reducing information friction in the goods market. Specifically, by trading in the AM, consumers can differentiate themselves from certain agents with low-quality assets, which enables these consumers to obtain better terms of trade in the DM. Thus, adding a secondary asset market is not only empirically relevant but also allows us to demonstrate that trading in the asset market can serve as a signal of asset quality in the goods market. Second, we also endogenize the degree of information asymmetry in the asset market by allowing agents to invest in information. We find that there are two sources of insurance against the liquidity shock: money holdings and opaqueness of the asset market. The cost of holding money affects which kind of insurance is preferred in equilibrium and therefore the decision of agents to invest in information acquisition. This result highlights a novel channel through which inflation affects asset prices in the secondary market.

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Appendix A Undefeated Equilibrium Refinement

In the first part of this section, we use a simplified version of the model to illustrate the logic behind the (original) Undefeated Equilibrium refinement as defined by Mailath et al. (1993) and why we use it instead of another popular refinement method, the Intuitive Criterion (Cho and Kreps, 1987). In the second part, we show that the version of Undefeated Equilibrium refinement used in this paper (see Definition 2) retains the logic behind the original version.

Part I. Undefeated Equilibrium Refinement

First, consider a simplified version of the model without the AM between the CM and DM. Suppose also that $\delta_l > 0$ so that separating equilibria are possible. For simplicity, assume $\lambda = 1$ so that all consumers want to consume in the DM. The rest of the model environment remains unchanged. Now, let $j \in \{l, h\}$ denote the quality of consumer's asset. Define $\underline{v}^j$, $j \in \{l, h\}$, to be the lowest possible surplus j-type consumer can obtain

$$\underline{v}^j \equiv \max_{q,z,a} \{u(q) - z - \delta_j a\} \text{ s.t. } \delta_l a + z \geq q.$$

It is straightforward to show that the full set of perfect Bayesian equilibria (PBE) is given by the following.

(1) $u(q^j) - z^j - \delta_j a^j \geq \underline{v}^j$ for $j \in \{l, h\}$ (individual participation constraint).

(2) $[\gamma(q^j, z^j, a^j)\delta_h + (1 - \gamma(q^j, z^j, a^j))\delta_l]a^j + z^j \geq q^j$ for $j \in \{l, h\}$.

(3) $\gamma(q^j, z^j, a^j)$ is given by

$$\gamma(q^j, z^j, a^j) = \begin{cases} 1, \text{ if } u(q^j) - z^j - \delta_l a^j \leq \underline{v}^l \text{ and } u(q^j) - z^j - \delta_h a^j \geq \underline{v}^h; \\ \rho, \text{ if } u(q^j) - z^j - \delta_l a^j > \underline{v}^l \text{ and } u(q^j) - z^j - \delta_h a^j \geq \underline{v}^h; \\ 0, \text{ if } u(q^j) - z^j - \delta_l a^j \geq \underline{v}^l \text{ and } u(q^j) - z^j - \delta_h a^j < \underline{v}^h. \end{cases}$$

(4) Off-equilibrium path beliefs: $\gamma(q, z, a) = 0$ for all $(q, z, a) \notin \{(q^l, z^l, a^l), (q^h, z^h, a^h)\}$.

Similar to the model in this paper, there exist a continuum of equilibria, and an equilibrium refinement is necessary. Mailath et al. (1993) introduce a refinement method that can be used for two-player signaling games such as this one. It is defined as follows: a PBE (PBE-1) is defeated by another PBE (PBE-2) if there exists $(q, \hat{z}, \hat{a})$ that satisfies

Requirement 1 : There exists $K \subseteq \{l, h\}$ such that $(q, \hat{z}, \hat{a})$ is played by type-K consumers in PBE-2 but not in PBE-1.

Requirement 2: There exists $J \subseteq K$ such that J-type consumers obtain strictly higher surplus in PBE-2.

Requirement 3: In PBE-1,

$$\gamma^a(\hat{\psi}, \hat{s}; z, a) \neq \frac{\rho \mathbb{1}(hO \in J)}{\rho \mathbb{1}(hO \in K) + (1 - \rho)\mathbb{1}(lO \in K)}.$$

In words, Requirements 1 and 2 say that there exists a strategy in PBE-2 that is not played in PBE-1, and such a strategy provides strictly higher payoff for some consumers. Requirement 3 says that the reason why the strategy $(q, \hat{z}, \hat{a})$ is not played in PBE-1 is that the off-equlibrium path beliefs of producers in PBE-1 do not take it into account. However, because this alternative equilibrium is profitable for some consumers, producers should have expected it. In other words, producers' beliefs in PBE-1 are arguably “unreasonable”. A PBE is undefeated if there does not exist another PBE that satisfies Requirements 1-3.

In addition to Undefeated Equilibrium refinement, another popular refinement method used in similar environments is the Intuitive Criterion (Cho and Kreps, 1987). For example, Rocheteau (2011) shows that a proposed equilibrium fails the Intuitive Criterion if there exists an unsent offer such that

(1) The unsent offer brings l-type consumers strictly lower payoff regardless of the inference the producer draws from the unsent offer. As a result, the producer should believe that the offer comes from h-types.

(2) Conditional on the producer believing that the offer comes from h-type consumers, the offer provides strictly higher payoff to h-type consumers.

The main difference between the Undefeated Equilibrium refinement and the Intuitive Criterion is that the latter replies on off-equilibrium messages as signals. As pointed out by Mailath et al. (1993), if the producer does believe that the unsent offer only comes from h-type consumers, then the original equilibrium will no longer exist, since h-type consumers will always deviate. Then, for l-type consumers, their payoff in the current equilibrium should *not* be relevant – instead, they should consider their payoff in the *new* equilibrium given the producer's belief. Undefeated Equilibrium refinement eliminates such a concern by considering alternative PBE (as opposed to an off-equilibrium message) when deciding whether an equilibrium passes the refinement. In this simplified model, a separating equilibrium that passes the Intuitive Criterion also satisfies the Undefeated Equilibrium refinement. However, the Undefeated Equilibrium refinement allows pooling equilibria if it is Pareto-optimal. For more details on the comparison between Undefeated Equilibrium and the Intuitive Criterion, see Bajaj (2018).

Part II. Modified Undefeated Equilibrium Refinement

Now, we show that the version of Undefeated Equilibrium refinement introduced in this paper (Definition 2) shares the same basic logic as the original version defined in Mailath et al. (1993). To see this, first note that there are two important features of the signaling game in this paper.

(1) Both asset buyers and producers must form beliefs regarding asset quality based on the offer they receive and the asset portfolios of the consumers they are matched with.

(2) The game appears to have “two stages”, i.e., the interactions between asset sellers and buyers in the AM, and the interactions between consumers and producers in the DM. However, the two stages cannot be solved separately. This is because, firstly, in the AM, asset sellers' strategies and (consequently) asset buyers' beliefs depend on what they think producers' beliefs will be like in the following DM, since producers' beliefs affect asset's continuation value and by extension asset sellers' payoff in the *AM*. Secondly, producers' beliefs in the DM also depend on what they think asset buyers' beliefs were like in the preceding AM, since asset buyers' beliefs affect asset sellers' strategies and asset portfolios that the producers observe at the beginning of the DM.

This two features mean that we need to solve agents' problems in the AM and DM together. Furthermore, we need to take into account the fact that both asset buyers and producers form beliefs. We copy our definition of the Undefeated Equilibrium refinement (Definition 2) below.

A Perfect Bayesian Equilibrium in the AM and the DM (PBE-1) is defeated by another Perfect Bayesian Equilibrium in the AM and the DM (PBE-2) if there exists $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ that satisfies

Requirement 1: There exist $K \subseteq \{lO, hO, lT, hT\}$ such that $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ is played by type-K consumers in PBE-2 but not in PBE-1.

Requirement 2: There exist $J \subseteq K$ such that type-J consumers play $(\hat{\psi}, \hat{s}; q, \hat{z}, \hat{a})$ and obtain strictly higher surplus in PBE-2.

Requirement 3: In PBE-1, at least one of the following conditions is satisfied:

$$\gamma^a(\hat{\psi}', \hat{s}'; z, a) \neq \frac{\rho \mathbb{1}(hO \in J)}{\rho \mathbb{1}(hO \in K) + (1-\rho)\mathbb{1}(lO \in K)}, \tag{A.1}$$

$$\gamma^g(q', \hat{z}', \hat{a}'; z + \hat{\psi}'\hat{s}', a - \hat{s}', z, a) \neq \frac{\rho(1-\tau)\mathbb{1}(hO \in J) + \rho\tau\mathbb{1}(hT \in J)}{\rho(1-\tau)\mathbb{1}(hO \in K) + (1-\rho)(1-\tau)\mathbb{1}(lO \in K) + \rho\tau\mathbb{1}(hT \in K) + (1-\rho)\tau\mathbb{1}(lT \in K)}. \tag{A.2}$$

A Perfect Bayesian Equilibrium in the AM and the DM is undefeated if and only if there does not exist $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ that satisfies conditions (1)-(3).

If we compare the above definition with the definition of Undefeated Equilibrium in **Part I**, we can see the main change is in Requirement 3, while Requirements 1 and 2 are straightforward adaptions of their counterparts in the original definition of Undefeated Equilibrium. Regarding Requirement 3,

The right hand side of Condition (A.1) is the share of asset sellers with high-quality assets that play $(\hat{\psi}', \hat{s}')$ in the AM. Condition (A.1) therefore says that asset buyers, in their off-equilibrium path beliefs in PBE-1, fails to take into account the part of the strategy $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ that is played in the AM.

The right hand side of Condition (A.2) the share of asset sellers with high-quality assets that play $(\hat{\psi}', \hat{s}')$ in the AM and play $q', \hat{z}', \hat{a}'$ in the DM. Condition (A.2) says that producers, in their off-equilibrium path beliefs in PBE-1, fails to take into account the strategy $(\hat{\psi}', \hat{s}'; q', \hat{z}', \hat{a}')$ that is played in the AM and DM.

In other words, the new Requirement 3 says that "unreasonable" beliefs from asset buyers and/or producers must be the reason why a strategy profitable for some consumers is not played in PBE-1. This captures the fact that the two stages (i.e., the AM and the DM) of the game are interconnected and that "unreasonable beliefs" from *either asset buyers and producers* can be why certain equilibrium outcomes exist. Such equilibria then eliminated from our consideration (i.e., "defeated") since they are based on unreasonable beliefs. As discussed in Part I of this section, this is the same logic behind the original definition of Undefeated Equilibrium proposed by Mailath et al. (1993).

Appendix B Proofs

Proof of Proposition 1: Firstly, condition (1) ensures that it is optimal for asset buyers in the AM and producers in the DM to accept C-type consumers' offers conditional on asset buyers' and producers' beliefs, thus ensuring that all asset buyers' and producers' strategies are optimal . Second, condition (2) guarantees that asset buyers' beliefs on the equilibrium path, $\gamma^a(\psi^{jk}, s^{jk}; z, a)$, are consistent with C-type consumers' strategies in the AM. Specifically, if an offer provides C-types with high-quality assets higher surplus compared $\underline{v}^{hO}$ but provides C-types with low-quality assets lower surplus compared $\underline{v}^{lO}$, then asset buyers believe that this offer comes from a C-type with high-quality assets. If the opposite is true, asset buyers believe that the offer comes from a C-type with high-quality assets. If an offer provides C-types with high-quality assets higher surplus compared $\underline{v}^{hO}$, and it also provides C-types with low-quality assets higher surplus compared $\underline{v}^{lO}$, asset buyers believe that both types are making the same offer. Note that since we focus on only pure strategy equilibria, in the AM, the offers can either be pooling (i.e., $(\psi^{lO}, s^{lO}) = (\psi^{hO}, s^{hO})$) or separating (i.e., $(\psi^{lO}, s^{lO}) \neq (\psi^{hO}, s^{hO})$).

Condition (3) requires $\gamma^g(q, \hat{z}, \hat{a}; \tilde{z}, \tilde{a}, z, a)$ to be consistent with C-type consumers' strategies in the DM. Firstly, similar to condition (2), producers believe that an offer comes from a C-type with high-quality assets if and only if C-types with high-quality assets are the sole type that have the incentive to make such an offer. Secondly, because producers can observe consumers' portfolios both before and after the AM, they can infer the quality of their assets.

(a) Because asset buyers' beliefs must satisfy Bayes' rule, in the AM only C-types with high-quality assets (either in transparent meetings or opaque meetings) may sell at a price that is strictly higher than $\rho\delta_h + (1-\rho)\delta_l$. To see why, note that C-types with low-quality assets in transparent meetings can at most sell at $\psi = \delta_l$. Next, if C-types in opaque meetings make pooling offers, they can at most sell at $\psi = \rho\delta_h + (1-\rho)\delta_l$, which is the average asset quality in opaque meetings. If C-types in opaque meetings make separating offers, those with low-quality assets can at most sell at $\psi = \delta_l$.
(b) The belief in (a) ensures that C-types with high-quality assets in transparent meetings will not sell at a price less that $\rho\delta_h + (1-\rho)\delta_l$. This in turn ensures that only C-types in opaque meetings may sell at a price that is strictly higher than δ_l but less than $\rho\delta_h + (1-\rho)\delta_l$, which happens only when they make pooling offers. Hence, if $\tilde{\psi} > \delta_l$, the consumers must have been in opaque meetings in the AM and made pooling offers.
(c) The belief in (b) ensures that C-types in opaque meetings will not sell at price less that δ_l when they choose to make pooling offers. Hence, if $\tilde{\psi} \leq \delta_l$, then the consumer must have low-quality assets.

Finally, it is clear that under condition (4), no consumers have the strict incentive to deviate and make an offer not in the set of equilibrium offers $\{(\psi^{jk}, s^{jk}; q^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$. In other words, the condition on off-equilibrium is sufficient to ensure that $\{(\psi^{jk}, s^{jk}; q^{jk}, \hat{z}^{jk}, \hat{a}^{jk})\}$ is part of a PBE. $\square$

Proof of Proposition 2: We show how the proposed strategies are derived, and why they constitute a unique set of undefeated equilibria. Consider the following problem:

$$\max_{\psi,s,q,\hat{z},\hat{a}} \{u(q) + \psi s - \hat{z} - \delta_h(\hat{a} + s)\} \tag{B.1}$$

$$\begin{aligned} \text{s.t. } & q \leq \hat{z} + \mathbb{1}(\hat{z} > z)\delta_h \hat{a}; \\ & \hat{z} \leq z + \psi s; \quad \psi \leq \delta_h; \\ & \psi s \leq z_b; \quad s \leq a; \quad \hat{a} \leq a - s. \end{aligned} \tag{B.2}$$

Denote the solution as (a) $\psi^{hT} = \delta_h$, (b) $s^{hT} = (0, \min\{z_b/\delta_h, (q^* - z)/\delta_h, a\}]$, (c) $q^{hT} = \min\{q^*, z + \delta_h a\}$, (d) $\hat{z}^{hT} = z + \psi^{hT} s^{hT}$, and (e) $\hat{a}^{hT} = \frac{q^{hT} - \hat{z}^{hT}}{\delta_h}$. Next, consider the following problem:

$$\max_{\psi,s,q,\hat{z},\hat{a}} \{u(q) + \psi s - \hat{z} - \delta_h(\hat{a} + s)\} \tag{B.3}$$

$$\begin{aligned} \text{s.t. } & q \leq \hat{z} + \mathbb{1}(\hat{z} > z)\rho\delta_h \hat{a}; \\ & \hat{z} \leq z + \psi s; \quad \psi \leq \rho\delta_h; \\ & \psi s \leq z_b; \quad s \leq a; \hat{a} \leq a - s, \end{aligned} \tag{B.4}$$

where

$$\tilde{\rho}^g = \frac{(1-\tau^a)\rho}{(1-\tau^a)\rho + 1 - \rho}.$$

Denote the solution as (a) $\psi_p = \rho\delta_h$; (b) $q_p = \max\{z, \min\{z + \rho\delta_h a, \tilde{q}\}\}$, where $\tilde{q}$ solves $u'(\tilde{q}) = \frac{1}{\rho}$; (c) $s_p = (0, \min\{z_b/(\rho\delta_h), (q_p - z)/(\rho\delta_h)\}]$, (d) $\hat{z}_p = z + \psi_p s_p$, and (e) $\hat{a}_p = \frac{q_p - \hat{z}_p}{\rho\delta_h}$.

Now, we show $(\psi_p, s_p, q_p, \hat{z}_p, \hat{a}_p)$ and $(\psi^{hT}, s^{hT}, q^{hT}, \hat{z}^{hT}, \hat{a}^{hT})$ constitute a unique set of undefeated equilibria. First, type-lO and type-hO consumers must pool in the AM and DM, because otherwise producers will recognize that type-lO consumers' assets are worthless. Second, type-lO and type-hO consumers cannot mimic type-hT consumers because $\psi^{hT} \neq \psi_p$, so

$$(z + \psi_p s_p, a - s_p) \neq (z + \psi^{hT} s^{hT}, a - s^{hT})$$

for any choices of s_p and s^{hT}. Then, there does not exist an equilibrium where type-hT and/or type-hO consumers are strictly better off since problems (B.1) and (B.3) maximize type-hT and type-hO consumers' surpluses, respectively. This also means that any other equilibria are defeated by the proposed equilibrium, because consumers with high-quality assets are better off in the proposed equilibrium. Type-lT can only use money in the DM since they are the only agents who enter the DM with a portfolio of (z, a). Finally, to understand constraint ((B.2)), note that given the equilibrium strategies, if type-hT consumers do not sell in the AM (i.e., $\hat{z} = z$), then they will be treated as type-lT consumers. Similarly, constraint (B.4) says that if type-lO and type-hO consumers do not sell in the AM, they will be treated as type-lT consumers. $\square$

Proof of Proposition 3: First, define

$$\begin{aligned}
G_1(z) &= \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\}) - 1] + (1-\tau)[u'(z+\rho\delta_h a) - 1]\}; \\
G_2(z) &= \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\}) - 1]\}; \\
G_3(z) &= \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\}) - 1] + (1-\tau)[u'(z) - 1]\}.
\end{aligned}$$

Based on earlier analysis, we know that $G_1(z)$ is the marginal value of real balances for all $z \le \tilde{q} - \rho\delta_h a$; $G_2(z)$ is the marginal value of real balances for all $\tilde{q} - \rho\delta_h a < z \le \tilde{q}$; and $G_3(z)$ is the marginal value of real balances for all $z > \tilde{q}$. Now, define $i_1 = G_1(\tilde{q} - \rho\delta_h a)$. It is clear that for all $z > \tilde{q} - \rho\delta_h a$, we have $G_2(z) < i_1$ and $G_3(z) < i_1$. In other words, for all $i \ge i_1$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\}) - 1] + (1-\tau)[u'(z+\rho\delta_h a) - 1]\},$$

which has a unique solution.

Next, consider $i < i_1$. Define v_1 to be the surplus from holding $z = \tilde{q} - \rho\delta_h a$ units of real balances.

$$v_1(i) = \int_0^{\tilde{q}-\rho\delta_h a} [G_1(z) - i]\,\mathrm{d}z.$$

Define $v_2(i)$ to be the surplus from holding $\tilde{q} - \rho\delta_h a < z_2(i) \le \tilde{q}$ units of real balances where $z_2(i)$ solves $G_2(z_2) = i$ for some $i \in [G_2(\tilde{q}), G_2(\tilde{q} - \rho\delta_h a)]$.

$$v_2(i) = \int_0^{z_2(i)} \{[G_1(z) - i]\mathbf{1}(z \le \tilde{q} - \rho\delta_h a) + [G_2(z) - i]\mathbf{1}(z > \tilde{q} - \rho\delta_h a)\}\,\mathrm{d}z.$$

Similarly, define $v_3(i)$ to be the surplus from holding $z_3(i) > \tilde{q}$ units of real balances where $z_3(i)$ solves $G_3(z_3) = i$ for some $i \in (0, G_3(\tilde{q})]$.

$$v_3(i) = \int_0^{z_3(i)} \{[G_1(z) - i]\mathbf{1}(z \le \tilde{q} - \rho\delta_h a) + [G_2(z) - i]\mathbf{1}(\tilde{q} - \rho\delta_h a < z \le \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\}\,\mathrm{d}z.$$

We have two cases to discuss.

Case (1): suppose $G_3(\tilde{q}) \ge G_2(\tilde{q} - \rho\delta_h a)$. This implies that $G_2(z) < G_3(\tilde{q})$ for all $\tilde{q} - \rho\delta_h a < z \le \tilde{q}$.

Then, $v_3(G_3(\tilde{q})) < v_1(G_3(\tilde{q}))$, because the marginal value of money is less than i for all $\tilde{q}-\rho\delta_h a < z \leq \tilde{q}$. Now, note that

$$v_3(i) - v_1(i) = \int_{\tilde{q}-\rho\delta_h a}^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z \leq \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z,$$

which is decreasing in i. Hence, there exists $\tilde{i} < i_1$ such that $v_3(\tilde{i}) = v_1(\tilde{i})$. Suppose $\tilde{i} \geq G_2(\tilde{q} - \rho\delta_h a)$. Then $v_2(i) < v_3(i)$ for all $i \in [G_2(\tilde{q}), G_2(\tilde{q} - \rho\delta_h a))$, because $v_2(G_2(\tilde{q} - \rho\delta_h a)) = v_1(G_2(\tilde{q} - \rho\delta_h a))$, and

$$v_3(i) - v_2(i) = \int_{z_2(i)}^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z_2(i) \leq z \leq \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z$$

is also decreasing in i since $G_2(z) - i < 0$ for all $z > z_2(i)$. In this case, let $i_2 = i_3 = \tilde{i}$. Then, for all $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$. For all $i \leq i_3$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z) - 1] + \rho\tau[u'(\min\{z + \delta_h a, q^*\}) - 1] + (1-\tau)[u'(z) - 1]\}. \tag{B.5}$$

It is clear that a solution exists and is unique. If $\tilde{i} < G_2(\tilde{q} - \rho\delta_h a)$, then there exists $i_3 < G_2(\tilde{q} - \rho\delta_h a)$ such that $v_3(i_3) = v_2(i_3)$. Let $i_2 = G_2(\tilde{q} - \rho\delta_h a)$. Then for all $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$. For all $i_3 < i < i_2$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z) - 1] + \rho\tau[u'(\min\{z + \delta_h a, q^*\}) - 1]\}, \tag{B.6}$$

which also has a unique solution. Lastly, for all $i \leq i_3$, z solves (B.5).

Case (2): suppose $G_3(\tilde{q}) < G_2(\tilde{q}-\rho\delta_h a)$. Then, for all $i \in [G_3(\tilde{q}), G_2(\tilde{q}-\rho\delta_h a)]$, we have $v_2(i) > v_1(i)$. Now, consider $i < G_3(\tilde{q})$. Note that $v_3(G_3(\tilde{q})) < v_2(G_3(\tilde{q}))$, and that

$$v_3(i) - v_2(i) = \int_{z_2(i)}^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z_2(i) \leq z \leq \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z,$$

which is decreasing in i. Hence, there must exist $i_3 < G_2(\tilde{q} - \rho\delta_h a)$ such that $v_3(i_3) = v_2(i_3)$. Now, let $i_2 = G_2(\tilde{q} - \rho\delta_h a)$, then for all $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$. For all $i_3 < i < i_2$, z solves (B.6). Lastly, for all $i \leq i_3$, z solves (B.5).

Finally, we prove the case where $\rho\delta_h a \geq \tilde{q}$. Since we assume $u'(0) = \infty$, we have $G_2(0) = \infty$. Following the discussion above, we know there exists $i^\dagger < \infty$ such that $\tilde{v}_3(i^\dagger) = \tilde{v}_2(i^\dagger)$, where

$$\tilde{v}_2(i) = \int_0^{z_2(i)} [G_2(z) - i] \, \mathrm{d}z,$$

and

$$\tilde{v}_3(i) = \int_0^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z \leq \tilde{q}) + [G_3(z) - i]\mathbf{1}(z > \tilde{q})\} \, \mathrm{d}z.$$

It is straightforward to see that for all $i > i^\dagger$, z solves (B.5), and for all $i \leq i^\dagger$, z solves (B.5). $\square$

Proof of Proposition 4: I prove the comparative statics of z with respect to i and τ. The rest of the comparative statics follows directly from Proposition 2. First, assume $\rho\delta_h a < \tilde{q}$. If $i \geq i_1$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1] + (1-\tau)[u'(z+\rho\delta_h a)-1]\}.$$

It is straightforward to see that z is decreasing in i. Now, take derivative with respect to τ to get

$$\frac{\partial z}{\partial \tau} = -\frac{(1-\rho)[u'(z)-1] + \rho[u'(\min\{z+\delta_h a, q^*\})-1] - [u'(z+\rho\delta_h a)-1]}{(1-\rho)\tau u''(z) + \rho\tau u''(z+\delta_h a)\mathbf{1}(z+\delta_h a < q^*) + (1-\tau)u''(z+\rho\delta_h a)} > 0,$$

because $(1-\rho)[u'(z)-1] + \rho[u'(\min\{z+\delta_h a, q^*\})-1] \geq (1-\rho)[u'(z)-1] + \rho[u'(z+\delta_h a)-1] \geq u'(z+\rho\delta_h a)-1$ as long as $u'''(.) > 0$. Next, if $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$. Hence, z is unaffected by changes in i and τ. If $i_3 < i < i_2$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1]\}.$$

Again, z is decreasing in i. The derivative with respect to τ is

$$\frac{\partial z}{\partial \tau} = -\frac{(1-\rho)[u'(z)-1] + \rho[u'(\min\{z+\delta_h a, q^*\})-1]}{(1-\rho)\tau u''(z) + \rho\tau u''(z+\delta_h a)\mathbf{1}(z+\delta_h a < q^*)} > 0.$$

Finally, if $i \leq i_3$, z solves

$$i = \lambda\{(1-\rho)\tau[u'(z)-1] + \rho\tau[u'(\min\{z+\delta_h a, q^*\})-1] + (1-\tau)[u'(z)-1]\},$$

so z is decreasing in i. The derivative with respect to τ is

$$\frac{\partial z}{\partial \tau} = -\frac{(1-\rho)[u'(z)-1] + \rho[u'(\min\{z+\delta_h a, q^*\})-1] - [u'(z)-1]}{(1-\rho)\tau u''(z) + \rho\tau u''(z+\delta_h a)\mathbf{1}(z+\delta_h a < q^*) + (1-\tau)u''(z)} < 0.$$

Now, assume $\rho\delta_h a \geq \tilde{q}$. Recall that for all $i > i^\dagger$, z solves (4.4), and for all $i \leq i^\dagger$, z solves (4.5). Hence, the results follow from the above arguments.□

Proof of Proposition 5: We start with the DM. For the moment, let us assume that $\tau = 0$. In this case, there are two types of equilibrium in the DM.
Case 1: separating equilibrium. Suppose that the DM equilibrium is separating. Let the offer be $(q, \hat{z}, \hat{a})$. Consumers with high-quality assets solve

$$\begin{aligned}
&\max_{q,\hat{z},\hat{a}} u(q) - \hat{z} - \delta_h \hat{a} \\
\text{s.t. } &\hat{z} + \delta_h \hat{a} \geq q, && \text{(B.7)}\\
&v^* \geq u(q) - \hat{z} - \delta_l \hat{a}, && \text{(B.8)}\\
&\hat{z} \leq \tilde{z},\ \hat{a} \leq \tilde{a},
\end{aligned}$$

where v^* is given by

$$\begin{aligned}
v^* = &\max_{q,\hat{z},\hat{a}} u(q) - \hat{z} - \delta_l \hat{a} \\
\text{s.t. } &\hat{z} + \delta_l \hat{a} = q, \\
&\hat{z} \leq \tilde{z},\ \hat{a} \leq \tilde{a}.
\end{aligned}$$

Let the solutions to the first problem and the second problem be (q_h, z_h, a_h) and (q_l, z_l, a_l), respectively. We have $q_l = \min\{\tilde{z} + \delta_l \tilde{a}, q^*\}$. Next, we show that (B.7) and (B.8) must bind at (q_h, z_h, a_h). Suppose (B.7) is strict, let $(q', \hat{z}', \hat{a}')$ be such that $\hat{z}' = z_h$, $\hat{a}' < a_h$, and

$$u(q_h) - z_h - \delta_l a_h = u(q') - \hat{z}' - \delta_l \hat{a}'.$$

Then, it is easy to show that $u(q_h) - z_h - \delta_h a_h < u(q') - \hat{z}' - \delta_h \hat{a}'$, a contradiction. Next, suppose (B.8) is strict, then because $q_h < q_l$, the consumer can increase a_h and obtain higher utility without giving consumers with low-quality assets the incentive to deviate. Hence, it must be that (B.8) also binds. We can then rewrite the first problem as

$$\begin{aligned}
&\max_{\hat{z},\hat{a}} u(\hat{z} + \delta_h \hat{a}) - \hat{z} - \delta_h \hat{a} \\
\text{s.t. } & u(q_l) - q_l = u(\hat{z} + \delta_h \hat{a}) - \hat{z} - \delta_l \hat{a}, \\
& \hat{z} \le \tilde{z}.
\end{aligned}$$

Since $\tilde{z} < q^*$ by assumption, we can conclude that $z_h = \tilde{z}$. And a_h solves

$$u(q_l) - q_l = u(\tilde{z} + \delta_h a_h) - \tilde{z} - \delta_l a_h.$$

It is straightforward to show that a_h is decreasing in $\tilde{z}$, and that $\tilde{z} + \delta_h a_h$ is increasing in $\tilde{z}$.
Case 2: pooling DM equilibrium. Consumers with high-quality assets solve

$$\begin{aligned}
&\max_{q,\hat{z},\hat{a}} u(q) - \hat{z} - \delta_h \hat{a} \\
\text{s.t. } & \hat{z} + \bar{\delta}\hat{a} = q, \\
& u(q) - \hat{z} - \delta_l \hat{a} \ge u(q^*) - q^*, \qquad \text{(B.9)} \\
& \hat{z} \le \tilde{z}, \quad \hat{a} \le \tilde{a},
\end{aligned}$$

where $\bar{\delta} = \rho\delta_h + (1-\rho)\delta_l$ is the average asset quality. Note that (B.9) is the participation constraint for consumers with low-quality assets. Let the solution be (q_p, z_p, a_p). Define $\tilde{q}$ to be such that $u'(\tilde{q}) = \frac{\delta_h}{\bar{\delta}}$. If $\tilde{z} \ge \tilde{q}$, pooling cannot happen because consumers with high-quality assets will prefer not selling any assets. Hence, (B.9) is not satisfied. Now, suppose $\tilde{z} < \tilde{q}$. If (B.9) does not bind, we have $q_p = \tilde{q}$, $z_p = \tilde{z}$, and $a_p = (q_p - z_p)/\bar{\delta}$. Since a_p is decreasing in $\tilde{z}$, (B.9) binds when $\tilde{z}$ is sufficiently large. In such case, it is clear that consumers with high-quality assets will prefer separating equilibrium.

Finally, we solve for the equilibrium in the AM. Consider a consumer with high-quality assets. She may sell some assets at the pooling price in the AM and make a separating offer in the DM, or she may make a pooling offer in the AM. In the latter case, she does not have the incentive to sell in the AM. This is because that selling at the pooling price does not increase her surplus. However, unless consumers sell all of their assets in the AM, selling in the AM makes (B.9) more likely to bind because $\tilde{z}$ is larger.

Now, consider the first case and let $s \ge 0$ denote the assets sold in the AM. Consider a consumer with high-quality assets. In the DM, the consumer will have $z + \bar{\delta}s$ units of real balances. We can conclude that the consumer will offer $\hat{z} = z + \bar{\delta}s$ in the DM, because otherwise the consumer can increase her utility by lowering s. For now, let us assume that trade in the AM is not constrained by asset buyers' money holdings z_b. The consumer solves

$$\max_{s,\hat{a}} u(z + \bar{\delta}s + \delta_h \hat{a}) - z - \delta_h s - \delta_h \hat{a} \qquad \text{(B.10)}$$

$$\begin{aligned} \text{s.t. } & u(\min\{z+\bar{\delta}s+\delta_l(a-s),q^*\})-\min\{z+\bar{\delta}s+\delta_l(a-s),q^*\} \\ & \quad = u(z+\bar{\delta}s+\delta_h\hat{a})-z-\bar{\delta}s-\delta_l\hat{a}, \\ & s+\hat{a}\leq a. \end{aligned} \tag{B.11}$$

Note that as long as $s < a$, then $\hat{a} > 0$. To see why, note that it must be that $z + \bar{\delta}s < q^*$, otherwise consumers' with high-quality assets can reduce s and increase their utility. If $s < a$, consumers with high-quality assets will have assets available to use directly as payment in the DM. Because $z+\bar{\delta}s < q^*$, a separating offer in the DM that includes both money and assets (i.e., $\hat{a} > 0$) increases the utility of consumers with high-quality assets. If $s = 0$, consumers makes separating offers in the DM but do not sell in the AM. To conclude, consumers make separating offers in the DM unless $s = a$.

Using (B.11), we can define $\hat{a}$ as a function of s, $\hat{a}(s)$. We have

$$\hat{a}'(s) = \frac{(\bar{\delta}-\delta_l)[u'(q_l)-1]-\bar{\delta}[u'(q_h)-1]}{\delta_h u'(q_h)-\delta_l},$$

where $q_l = \min\{z+\bar{\delta}s+\delta_l(a-s),q^*\}$ and $q_h = z+\bar{\delta}s+\delta_h\hat{a}(s)$. Then, the first derivative of (B.10) with respect to s is given by

$$\bar{\delta}u'(q_h)-\delta_h+\delta_h[u'(q_h)-1]\hat{a}'(s) \propto \frac{[(1-\rho)\delta_l+(2\rho-1)\delta_h]u'(q_h)}{\rho\delta_h}+[u'(q_l)-1][u'(q_h)-1]-1 \tag{B.12}$$

We assume $(1-\rho)\delta_l+(2\rho-1)\delta_h > 0$. Note that q_l is increasing in s if $q_l < q^*$. In addition,

$$\frac{\mathrm{d}(\bar{\delta}s+\delta_h a)}{\mathrm{d}s} = \frac{\bar{\delta}\left(1-\frac{\delta_l}{\delta_h}\right)+(\bar{\delta}-\delta_l)[u'(q_l)-1]}{u'(q_h)-\frac{\delta_l}{\delta_h}} > 0,$$

so q_h is increasing in s as well. Hence, we can conclude that (B.12) is decreasing in s. Then, there is a unique solution to (B.10). Furthermore, $s = 0$ if

$$\frac{[(1-\rho)\delta_l+(2\rho-1)\delta_h]u'(z+\delta_h\hat{a}(0))}{\rho\delta_h}+[u'(\min\{z+\delta_l a,q^*\})-1][u'(z+\delta_h\hat{a}(0))-1]-1 \leq 0 \tag{B.13}$$

and $s = a$ if

$$\frac{[(1-\rho)\delta_l+(2\rho-1)\delta_h]u'(z+\bar{\delta}a)}{\rho\delta_h}+[u'(z+\bar{\delta}a)-1]^2-1 \geq 0. \tag{B.14}$$

Now define z'' to be such that (B.13) holds at equality and z' to be such that (B.14) holds at equality. We can then conclude that $s = a$ if $z < z'$ and $s = 0$ if $z \geq z''$. Note that for (B.14) to hold, it must be that $u'(z+\bar{\delta}a) > \frac{\delta_h}{\bar{\delta}}$, otherwise $z' = 0$, and consumers do not make pooling offers in the DM. If $z' \leq z < z''$, s solves

$$\frac{[(1-\rho)\delta_l+(2\rho-1)\delta_h]u'(q_h)}{\rho\delta_h}+[u'(q_l)-1][u'(q_h)-1]-1 = 0. \tag{B.15}$$

In this case, an increase in z will have no effect on q_l and q_h, because otherwise (B.11) and (B.15) will not hold at the same time. This implies that $z + \bar{\delta}s$ will remain unchanged. Finally, suppose $z' \leq z < z''$. Consumers may be constrained in the AM by asset buyers' money holdings z_b. In other words,

$$\frac{[(1-\rho)\delta_l + (2\rho-1)\delta_h]u'(q_h)}{\rho\delta_h} + [u'(q_l)-1][u'(q_h)-1] - 1 > 0$$

if $\bar{\delta}s = z_b$.

Finally, if $\tau > 0$, so some consumers are in transparent meetings. If such a consumer has high-quality assets, she will be able to sell at a price equal to δ_h. If such a consumer has low-quality assets, she has to sell at a price equal to δ_l. Consumers in opaque meetings will also sell a positive amount of assets at price $\bar{\delta}$. This means that in the DM, the latter cannot mimic the former or consumers in opaque meetings. □

Proof of Proposition 6: Consider the marginal value of real balances at the beginning of the AM for consumers in opaque meetings. First, consider consumers in transparent meetings. If they have high-quality assets, then the marginal value is given by $u'(\max\{z+\delta_h a_s, q^*\})$. If they have low-quality assets, then the marginal value is given by $u'(\max\{z+\delta_l a_s, q^*\})$. Next, consider consumers in opaque meetings. We have several cases to discuss.
(1) Suppose that $z \geq z''$. The marginal value of real balances is one for consumers with low-quality assets. The marginal value of real balances for consumers with high-quality assets is given by

$$\frac{\{(\delta_h - \delta_l) + \delta_h[u'(\min\{z+\delta_l a, q^*\}) - 1]\}[u'(z+\delta_h a) - 1]}{\delta_h u'(z+\delta_h a) - \delta_l} + 1.$$

(2) Suppose that $z' \leq z < z''$. The marginal value of real balances is one for consumers with low-quality assets. For consumers with high-quality assets, the marginal value of real balances may depend on z_b as well. For simplicity, we assume that $z' \geq \bar{\delta}a$. This will guarantee that $z'' < 2z'$, which means that in a symmetric equilibrium, consumers are not constrained by z_b. Note that $z' > \bar{\delta}a$ implies $z'' < 2z'$ because $z' + \bar{\delta}a > z'' + \delta_h a(0)$. Finally, if consumers are not constrained by z_b, the marginal value of real balances will be one.
(3) Suppose that $z < z'$. The marginal value of real balances for both types of consumers is given by $u'(z+\bar{\delta}a)$.

In what follows, we focus only on symmetric equilibrium where all agents choose the same money holding in the CM. That is, $z = z_b$. We show how the marginal value of real balances in this scenario depends on z in Figure 13. Then, by following the proof of Proposition 3, it is straightforward to show that there exist μ' and μ'' such that if $\mu > \mu''$, the optimal z will be smaller than z'. If $\mu' < \mu \leq \mu''$, the optimal z will be between z' and z''. Finally, if $\mu \leq \mu'$, the optimal z will be larger than z''. □

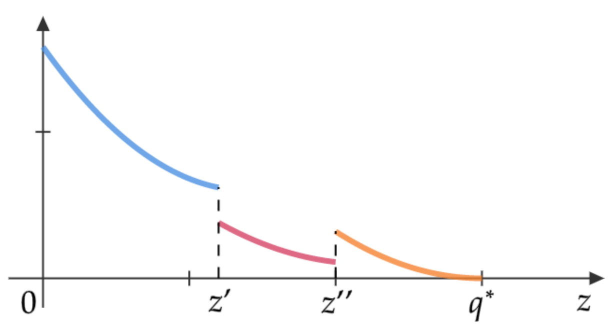


Figure 13: Marginal value of z

Proof of Lemma 2: First, assume $a < \frac{\tilde{q}}{\rho\delta_h}$. Suppose $z < \tilde{q} - \rho\delta_h a$. We have

$$\mathcal{B}_1'(z) = (1-\rho)u'(z) + \rho u'(\min\{z + \delta_h a, q^*\}) - u'(z + \rho\delta_h a).$$

If $z + \delta_h a \leq q^*$, then it is clear that $\mathcal{B}_1'(z) > 0$ since $u'''(.) > 0$. Now, suppose $z + \delta_h a > q^*$. Note that $q^* - \delta_h a - (\tilde{q} - \rho\delta_h a) = q^* - \tilde{q} - (1-\rho)\delta_h a$ is decreasing in a. Suppose $\tilde{q} = \rho\delta_h a$, then $q^* - \delta_h a - (\tilde{q} - \rho\delta_h a) = q^* - \delta_h a > 0$ because $\delta_h a < q^*$ by assumption. In other words, $q^* - \delta_h a - (\tilde{q} - \rho\delta_h a) > 0$ when $a < \min\left\{\frac{\tilde{q}}{\rho\delta_h}, \frac{q^*}{\delta_h}\right\}$. This implies that if $z + \delta_h a > q^*$, then $z + \rho\delta_h a > \tilde{q}$. In other words, for all $q^* - \delta_h a < z \leq \tilde{q}$, we have

$$\mathcal{B}_1'(z) = (1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1] > 0.$$

Note also that when $z = \tilde{q}$, $\mathcal{B}_1(z) > 0$. However, if $z \leq \tilde{q} - \rho\delta_h a$, $\mathcal{B}_1(z) < 0$ because $u''(.) < 0$. Hence, there exists $\tilde{q} - \rho\delta_h a < \tilde{z} < \tilde{q}$ such that $\mathcal{B}_1(\tilde{z}) = 0$. Finally, suppose $\tilde{q} < z < q^*$. We have

$$\mathcal{B}_1'(z) = \rho[u'(\min\{z + \delta_h a, q^*\}) - u'(z)] < 0.$$

Second, suppose $a \geq \frac{\tilde{q}}{\rho\delta_h}$. We have

$$\mathcal{B}_1'(z) = (1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1] > 0$$

for all $z \leq \tilde{q}$, and

$$\mathcal{B}_1'(z) = \rho[u'(\min\{z + \delta_h a, q^*\}) - u'(z)] < 0$$

for all $\tilde{q} < z < q^*$. Note that $\mathcal{B}_1(z) < 0$ when $z = 0$, but $\mathcal{B}_1(z) > 0$ when $z = \tilde{q}$. Hence, there exists $0 < \tilde{z} < \tilde{q}$ such that $\mathcal{B}_1(\tilde{z}) = 0$. $\square$

Proof of Proposition 7: Assume $a < \frac{\tilde{q}}{\rho\delta_h}$. From Proposition 2, when $z < \tilde{q} - \rho\delta_h a$, the marginal value of money is $u'(z + \rho\delta_h a)$. Since $\kappa < \mathcal{B}_1(\tilde{z})$, there exists $\tilde{z} < z^\dagger < \tilde{q}$ such that $\mathcal{B}_1(z^\dagger) = \kappa$. Then, when $\tilde{q} - \rho\delta_h a \leq z \leq z^\dagger$, the marginal value of money is 1. Since $\mathcal{B}_1'(z) < 0$ for all $\tilde{q} < z < q^*$ and $\mathcal{B}_1(q^*) = 0$, there exists $z^\ddagger > \tilde{q}$ such that $\mathcal{B}_1(z^\ddagger) = \kappa$. Hence, when $z^\dagger < z < z^\ddagger$, agents pay κ, and the marginal value of money is $(1-\rho)u'(z) + \rho u'(\min\{z + \delta_h a, q^*\})$. Finally, when $z^\ddagger \leq z \leq q^*$, the marginal value of money is $u'(z)$.

Now, define $G_1(z) = \lambda[u'(z + \rho\delta_h a) - 1]$, $G_2(z) = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1]\}$, and $G_3(z) = \lambda[u'(z) - 1]$. Next, let $i_1 = G_1(\tilde{q} - \rho\delta_h a)$. It is clear that for all $z > \tilde{q} - \rho\delta_h a$, we have $G_2(z) < i_1$ and $G_3(z) < i_1$. In other words, for all $i < i_1$, z solves $G_1(z) = i$ where $i = \frac{1+\mu}{\beta} - 1$.

Next, consider $i \leq i_1$. Define v_1 to be the surplus from holding $z = \tilde{q} - \rho\delta_h a$ units of real balances.

$$v_1(i) = \int_0^{\tilde{q} - \rho\delta_h a} [G_1(z) - i] \, \mathrm{d}z.$$

Define $v_2(i)$ to be the surplus from holding $\tilde{q} - \rho\delta_h a < z_2(i) < z^\ddagger$ units of real balances, where $z_2(i)$ solves $G_2(z) = i$ for some $i \in [0, G_2(z^\dagger))$.

$$v_2(i) = \int_0^{z_2(i)} \{[G_1(z) - i]\mathbf{1}(z < \tilde{q} - \rho\delta_h a) - i\mathbf{1}(\tilde{q} - \rho\delta_h a \leq z \leq z^\dagger) + [G_2(z) - i]\mathbf{1}(z^\dagger < z < z^\ddagger)\} \, \mathrm{d}z.$$

Similarly, define $v_3(i)$ to be the surplus from holding $z_3(i) > z^{\ddagger}$ units of real balances where $z_3(i)$ solves $G_3(z_3) = i$ for some $i \in [0, G_3(\tilde{q})]$.

$$v_3(i) = \int_0^{z_3(i)} \{[G_1(z) - i]\mathbf{1}(z < \tilde{q} - \rho\delta_h a) - i\mathbf{1}(\tilde{q} - \rho\delta_h a \leq z \leq z^{\dagger}) \\ + [G_2(z) - i]\mathbf{1}(z^{\dagger} < z < z^{\ddagger}) + [G_3(z) - i]\mathbf{1}(z \geq z^{\ddagger})\}\, \mathrm{d}z.$$

Now, consider $v_2(i) - v_1(i)$ for all $i \in [0, G_2(z^{\dagger})]$.

$$v_2(i) - v_1(i) = \int_{\tilde{q}-\rho\delta_h a}^{z_2(i)} \{-i\mathbf{1}(\tilde{q} - \rho\delta_h a \leq z \leq z^{\dagger}) + [G_2(z) - i]\mathbf{1}(z^{\dagger} < z < z^{\ddagger})\}\, \mathrm{d}z.$$

It is clear that $v_2(i) - v_1(i)$ is decreasing in i and there exists some $\tilde{i}_2 \in (0, G_2(z^{\dagger}))$ such that $v_2(\tilde{i}_2) - v_1(\tilde{i}_2) = 0$. Next, consider $v_3(i) - v_2(i)$ for all $i \in [G_2(z^{\ddagger}), \min\{G_2(z^{\dagger}), G_3(z^{\ddagger})\}]$.

$$v_3(i) - v_2(i) = \int_{z_2(i)}^{z_3(i)} \{[G_2(z) - i]\mathbf{1}(z_2(i) < z < z^{\ddagger}) + [G_3(z) - i]\mathbf{1}(z \geq z^{\ddagger})\}\, \mathrm{d}z,$$

which is decreasing in i, and there exists some $\tilde{i}_3 \in (G_2(z^{\ddagger}), \min\{G_2(z^{\dagger}), G_3(z^{\ddagger})\})$ such that $v_3(\tilde{i}_3) - v_2(\tilde{i}_3) = 0$ provided that $v_3(\min\{G_2(z^{\dagger}), G_3(z^{\ddagger})\}) - v_2(\min\{G_2(z^{\dagger}), G_3(z^{\ddagger})\}) < 0$. We have two cases.
Case I: Suppose $\tilde{i}_2 \geq \tilde{i}_3 > G_2(z^{\ddagger})$. Then, let $i_2 = \tilde{i}_2$ and $i_3 = \tilde{i}_3$. For all $i_2 \leq i \leq i_1$, $z = \tilde{q} - \rho\delta_h a$. For all $i_3 < i < i_2$, z solves $G_2(z) = i$. For all $i \leq i_3$, z solves $G_3(z) = i$.
Case II: Suppose $\tilde{i}_3 > \tilde{i}_2$ or $v_3(\min\{G_2(z^{\dagger}), G_3(z^{\ddagger})\}) - v_2(\min\{G_2(z^{\dagger}), G_3(z^{\ddagger})\}) \geq 0$. Consider $v_3(i) - v_1(i)$ for all $i \leq G_3(z^{\ddagger})$.

$$v_3(i) - v_1(i) = \int_{\tilde{q}-\rho\delta_h a}^{z_3(i)} \{-i\mathbf{1}(\tilde{q} - \rho\delta_h a \leq z \leq z^{\dagger}) + [G_2(z) - i]\mathbf{1}(z^{\dagger} < z < z^{\ddagger}) + [G_3(z) - i]\mathbf{1}(z \geq z^{\ddagger})\}\, \mathrm{d}z,$$

which is decreasing in i, and there exists some $\tilde{i}_4 \in (0, G_3(z^{\ddagger}))$ such that $v_3(\tilde{i}_4) - v_1(\tilde{i}_4) = 0$. In this case, let $i_2 = i_3 = \tilde{i}_4$. Then, for all $i_2 \leq i \leq i_1$, $z = \tilde{q} - \rho\delta_h a$. For all, $i \leq i_3$, z solves $G_3(z) = i$. $\square$

Proof of Proposition 8: I prove the comparative statics of z with respect to i. The rest of the comparative statics follows directly from Proposition 2. If $i \geq i_1$, z solves $i = \lambda[u'(z + \rho\delta_h a) - 1]$. Then it is clear that z is decreasing in i. If $i_2 \leq i < i_1$, $z = \tilde{q} - \rho\delta_h a$, then z is unaffected by i. If $i_3 < i < i_2$, z solves $i = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{z + \delta_h a, q^*\}) - 1]\}$, then it is clear that z is decreasing in i. Finally, if $i \leq i_3$, z solves $i = \lambda[u'(z) - 1]$, so z is decreasing in i. $\square$

Proof of Lemma 3: Suppose $z < q^{\dagger}(\tilde{\tau}) - \xi(\tilde{\tau})\delta_h a$, we have

$$\frac{\partial \mathcal{B}_2(z, \tilde{\tau})}{\partial z} = u'(z + \delta_h a) - 1 - [u'(z + \xi(\tilde{\tau})\delta_h a) - 1] < 0.$$

Suppose $z > q^{\dagger}(\tilde{\tau})$, we have

$$\frac{\partial \mathcal{B}_2(z, \tilde{\tau})}{\partial z} = u'(q^{hT}) - 1 - [u'(z) - 1] < 0.$$

Suppose $z \in [q^\dagger(\tilde{\tau}) - \xi(\tilde{\tau})\delta_h a, q^\dagger(\tilde{\tau})]$, we have

$$\frac{\partial \mathcal{B}_2(z,\tilde{\tau})}{\partial z} = u'(z+\delta_h a) - 1 > 0.$$

Finally, $\mathcal{B}_2(z,\tilde{\tau})$ is increasing in $\tilde{\tau}$ because $u(q(\tilde{\tau})) + z + \rho\delta_h a - q(\tilde{\tau})$ is decreasing in $\tilde{\tau}$. □

Proof of Lemma 4: Suppose $\tilde{\tau} \in (0,1)$. Let z^* denote the optimal choice of z. We have

$$u(q^{hT}) - q^{hT} - [u(q(\tilde{\tau}'(z^*))) - q(\tilde{\tau}'(z^*))] = \kappa,$$

where $q^{hT} = \min\{z^*+\delta_h a, q^*\}$, and $q(\tilde{\tau}) = \max\{z^*, \min\{z^*+\xi(\tilde{\tau})\delta_h a, q^\dagger(\tilde{\tau})\}\}$ where $u'(q^\dagger(\tilde{\tau})) = 1/\xi(\tilde{\tau})$ and $\xi(\tilde{\tau}) = \frac{\rho(1-\tilde{\tau})}{\rho(1-\tilde{\tau})+1-\rho}$. In other words, conditional on z^* and $\tilde{\tau}$, consumers with high-quality assets are indifferent between paying κ or not. However, ex ante, the surplus when choosing to pay κ is

$$V_1 \equiv \lambda\{(1-\rho)[u(q^{lT}) + z^* - q^{lT}] + \rho[u(q^{hT}) + z^* + \delta_h a - q^{hT} - \kappa]\} + (1-\lambda)(z^* + \rho\delta_h a) - (i+1)z^*,$$

where $q^{lT} = z^*$, while the ex ante surplus when choosing not to pay κ later is given by

$$V_2 \equiv \lambda[u(q(\tilde{\tau}'(z^*))) + z^* + \rho\delta_h a - q(\tilde{\tau}'(z^*))] + (1-\lambda)(z^* + \rho\delta_h a) - (i+1)z^*.$$

It is straightforward to show that $V_1 = V_2$ if and only if $q(\tilde{\tau}'(z^*)) = z^*$. Take the derivatives of V_1 and V_2 with respect to z^* and get

$$\begin{aligned}&\lambda\{(1-\rho)[u'(z^*) - 1] + \rho[u'(\min\{q^*, z^* + \delta_h a\}) - 1]\} - i,\\ &\lambda[u'(z^*) - 1] - i.\end{aligned}$$

Notice that $(1-\rho)[u'(z^*) - 1] + \rho[u(\min\{q^*, z^* + \delta_h a\}) - 1] = \lambda[u'(z^*) - 1]$ iff $z^* = q^*$. This implies that a consumer considering not paying κ later in the AM and a consumer considering paying κ would not have chosen the same z. In other words, $\tilde{\tau} \in (0,1)$ is not possible if z is chosen optimally. □

Proof of Proposition 9: Define $G_1(z) = \lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$ and

$$G_2(z) = \begin{cases}\lambda[u'(\max\{z, \min\{z+\rho\delta_h a, \tilde{q}\}\}) - 1], \text{ if } z < \tilde{q} - \rho\delta_h a,\\ 0, \text{ if } \tilde{q} - \rho\delta_h a \le z \le \tilde{q},\\ \lambda[u'(z) - 1], \text{ if } z > \tilde{q}.\end{cases}$$

Then, $G_1(z)$ is the marginal value of holding money when a consumer later pays κ in the AM, and $G_2(z)$ is the marginal value of holding money when a consumer later does not κ in the AM There are three scenarios to consider.

(1) Assume that $\kappa \ge \bar{\mathcal{B}}_2(\tilde{q})$. Define z_1 and z_2 to be such that $\underline{\mathcal{B}}_2(z_1) = \kappa$ and $\bar{\mathcal{B}}_2(z_2) = \kappa$. Then, for all $z \le z_1$, the marginal value of money is given by $G_1(z)$. For all $z \ge z_2$, the marginal value of money is given by $G_2(z)$. For all $z \in (z_1, z_2)$, the marginal value of money is given by $G_1(z)$ if consumers expect $\tilde{\tau} = 0$, and $G_2(z)$ if consumers expect $\tilde{\tau} = 1$.

(2) Assume that $\underline{\mathcal{B}}_2(\tilde{q} - \rho\delta_h a) < \kappa < \bar{\mathcal{B}}_2(\tilde{q})$. Define z_2 to be such that $\bar{\mathcal{B}}_2(z_2) = \kappa$. Then, there exist z_1 and $z_1 < z_1' < z_2$ such that $\underline{\mathcal{B}}_2(z_1) = \kappa$ and $\underline{\mathcal{B}}_2(z_1') = \kappa$. Then, for all $z \le z_1$, the marginal value of money is $\lambda\{(1-\rho)[u'(z) - 1] + \rho[u'(\min\{q^*, z + \delta_h a\}) - 1]\}$. For all $z \ge z_1'$, the marginal value of money is $\lambda[u'(\max\{z, \min\{z + \rho\delta_h a, \tilde{q}\}\}) - 1]$. For all $z \in (z_1, z_1')$, the marginal value of

money is $\lambda[u'(\max\{z, \min\{z+\rho\delta_h a, \tilde{q}\}\}) - 1]$ if consumers expect $\tilde{\tau} = 0$, and $\lambda\{(1-\rho)[u'(z)-1] + \rho[u'(\min\{q^*, z+\delta_h a\}) - 1]\}$ if consumers expect $\tilde{\tau} = 1$.
(3) Assume that $\kappa \leq \underline{\mathcal{B}}_2(\tilde{q} - \rho\delta_h a)$. Define z_2 to be such that $\bar{\mathcal{B}}_2(z_2) = \kappa$. Then, for all $z < z_2$, the marginal value of money is $\lambda\{(1-\rho)[u'(z)-1] + \rho[u'(\min\{q^*, z+\delta_h a\}) - 1]\}$. For all $z \geq z_2$, the marginal value of money is $\lambda[u'(\max\{z, \min\{z+\rho\delta_h a, \tilde{q}\}\}) - 1]$.

In all cases, the marginal value of money switches from $G_1(z)$ to $G_2(z)$ when z is sufficiently large, with the only difference being the thresholds. By following the argument in the proof of Proposition 7, it is straightforward to show that there exist $i_1 > i_2 \geq i_3 \geq i_4 > 0$ such that for all $i \geq i_1$, z solves $i = \lambda\{(1-\rho)[u'(z)-1] + \rho[u'(\min\{q^*, z+\delta_h a\}) - 1]\}$. For all $i_2 \leq i < i_1$, $z = z_1$ where z_1 solves $i_1 = \lambda\{(1-\rho)[u'(z)-1] + \rho[u'(\min\{q^*, z+\delta_h a\}) - 1]\}$. For all $i_3 \leq i < i_2$, z solves $i = \lambda[u'(z+\rho\delta_h a) - 1]$. For all $i_4 \leq i < i_3$, $z = z_2$ where z_2 solves $i_3 = \lambda[u'(z+\rho\delta_h a) - 1]$. Finally, for all $z \leq i_4$, z solves $i = \lambda[u'(z) - 1]$. $\square$

Proof of Proposition 10: I prove the comparative statics of z with respect to i. The rest of the comparative statics follows directly from Proposition 2. If $i \geq i'_1$, z solves $i = \lambda\{(1-\rho)[u'(z)-1] + \rho[u'(\min\{q^*, z+\delta_h a\}) - 1]\}$. Then it is clear that z is decreasing in i. If $i'_2 \leq i < i'_1$, $z = z_1$ where z_1 solves $i'_1 = \lambda\{(1-\rho)[u'(z)-1] + \rho[u'(\min\{q^*, z+\delta_h a\}) - 1]\}$, then z is unaffected by i. If $i'_3 < i < i'_2$, z solves $i = \lambda[u'(z+\rho\delta_h a) - 1]$, then it is clear that z is decreasing in i. If $i'_4 \leq i < i'_3$, $z = z_2$ where z_2 solves $i'_3 = \lambda[u'(z+\rho\delta_h a) - 1]$, then z is unaffected by i. Finally, if $i \leq i'_4$, z solves $i = \lambda[u'(z) - 1]$, so z is decreasing in i. $\square$

Appendix C Additional Results

C.1 Transparent meetings in the DM

In this section, we assume that a fraction τ^g of the DM meetings are "transparent" in the sense that producers can observe the quality of the real assets. The remaining $1-\tau^g$ of the meetings are "opaque" in the sense that the quality of the assets, δ, is consumers' private information. We maintain the assumption that a fraction, τ^a, of the AM meetings are "transparent", and the remaining $1-\tau^a$ of the meetings are "opaque". Whether a consumer is in transparent or opaque meetings in the AM and DM is determined by two idiosyncratic shocks that are independent of each other. We maintain the assumption that consumers learn which type of AM meetings they will be in at the beginning of the AM. In what follows, we consider two possibilities for when the consumers learn about their DM meetings: at the beginning of the AM and at the beginning of the DM.

For simplicity, we assume that $\delta_l = 0$ throughout this section. We find that the main mechanism of the benchmark model without DM transparency holds: consumers who *(1) have high-quality assets, (2) are in opaque AM meetings, and (3) may be or will be in opaque DM meetings* choose to sell some assets in order to prevent type-lT consumers from mimicking them in the DM. More importantly, this holds for any value of $\tau^g < 1$, i.e., regardless of whether the DM is equally transparent as the AM, more transparent, or less so.[12]

[12]It is also worth mentioning that although we endogenize τ^a in the paper, we do not endogenize τ^g for two reasons. First, we think of the information cost κ as standing in for a fee to access an intermediary/specialist who can guarantee the quality of the asset. Frictions-reducing intermediaries make sense in an asset market where buyers and sellers *come to trade assets*, and are therefore interested in selecting the *right* platform or intermediary, more than in a goods market where the 'asset buyer' is busy producing goods. Second, it is reasonable to assume that the information cost is lower in the AM than in the DM: asset buyers in the AM are better equipped with the knowledge of the assets and therefore require less information to be convinced of the quality (especially given that these buyers visit the secondary market with the intention to purchase this specific type of assets).

C.1.1 Consumers learn about their DM meetings at the beginning of AM

We solve for equilibrium by first considering the solutions to several problems. Then, we show that the solutions constitute the unique equilibrium in the AM and DM.

First, denote $\boldsymbol{a}^{hT} \equiv (\psi^{hT}, s^{hT}, q^{hT}, \hat{z}^{hT}, \hat{a}^{hT})$ as the solution to the following problem:

$$\begin{aligned} &\max_{\psi,s,q,\hat{z},\hat{a}} \{u(q) + \psi s - \hat{z} - \delta_h(\hat{a} + s)\} \\ \text{s.t. } &q \leq \hat{z} + \mathbb{1}(\hat{z} > z)\delta_h \hat{a}; \\ &\hat{z} \leq z + \psi s; \quad \psi \leq \delta_h; \quad \psi s \leq z_b; \quad s \leq a; \quad \hat{a} \leq a - s. \end{aligned} \tag{C.1}$$

We have (a) $\psi^{hT} = \delta_h$, (b) $s^{hT} = (0, \min\{z_b/\delta_h, (q^* - z)/\delta_h, a\}]$, (c) $q^{hT} = \min\{q^*, z + \delta_h a\}$, (d) $\hat{z}^{hT} = z + \psi^{hT} s^{hT}$, and (e) $\hat{a}^{hT} = \frac{q^{hT} - \hat{z}^{hT}}{\delta_h}$.

Next, define

$$\tilde{\rho} = \frac{(1-\tau^a)(1-\tau^g)\rho}{(1-\tau^a)(1-\tau^g) + (1-\rho)(1-\tau^a)\tau^g},$$

which is the average asset quality if *all type-lO consumers* pool with *type-hO consumers who will be in opaque DM meetings*. Denote $\boldsymbol{a}_p \equiv (\psi_p, s_p, q_p, \hat{z}_p, \hat{a}_p)$ as the solution to the following problem:

$$\begin{aligned} &\max_{\psi,s,q,\hat{z},\hat{a}} \{u(q) + \psi s - \hat{z} - \delta_h(\hat{a} + s)\} \\ \text{s.t. } &q \leq \hat{z} + \mathbb{1}(\hat{z} > z)\rho\delta_h \hat{a}; \\ &\hat{z} \leq z + \psi s; \quad \psi \leq \tilde{\rho}\delta_h; \quad \psi s \leq z_b; \quad s \leq a; \quad \hat{a} \leq a - s. \end{aligned} \tag{C.2}$$

We have $\psi_p = \tilde{\rho}\delta_h$. Note that if $s = 0$, then $\hat{z} = z$, and consumers' assets cannot be used as payment in the DM. However, since $\tilde{\rho} < \rho$, for any choice of $s' > 0$, there exists $0 < s'' < s'$ that increases the consumer's surplus. To ensure a solution exists, we assume that either $s \geq \epsilon$ or $s = 0$, where the exogenous parameter $\epsilon > 0$ is assumed to be sufficiently small. We interpret ϵ as the minimum sale quantity when a consumer decides to sell assets. Then, we have (a) $\psi_p = \tilde{\rho}\delta_h$; (b) $q_p = \max\{z, \min\{z + \tilde{\rho}^a\delta_h\epsilon + \rho\delta_h(a-\epsilon), \tilde{q}\}\}$; (c) $s_p = \min\{q_p - z, \epsilon\}$, (d) $\hat{z}_p = z + \psi_p s_p$, and (e) $\hat{a}_p = \frac{q_p - \hat{z}_p}{\rho\delta_h}$.

Finally, denote $\boldsymbol{a}^{hO} \equiv (\psi^{hO}, s^{hO}, q^{hO}, \hat{z}^{hO}, \hat{a}^{hO})$ as the solution to the following problem:

$$\begin{aligned} &\max_{\psi,s,q,\hat{z},\hat{a}} \{u(q) + \psi s - \hat{z} - \delta_h(\hat{a} + s)\} \\ \text{s.t. } &q \leq \hat{z} + \delta_h \hat{a}; \quad \hat{z} \leq z + \psi s; \quad \psi \leq \tilde{\rho}\delta_h; \\ &\psi s \leq z_b; \quad s \leq a; \quad \hat{a} \leq a - s. \end{aligned}$$

We have (a) $\psi^{hO} = \tilde{\rho}\delta_h$, (b) $s^{hO} = 0$, (c) $q^{hO} = \min\{q^*, z + \delta_h a\}$, (d) $\hat{z}^{hT} = z$, and (e) $\hat{a}^{hT} = \frac{q^{hT} - \hat{z}^{hT}}{\delta_h}$.

The unique equilibrium in the AM and DM is as follows:

(1) Type-hT consumers who will be in opaque DM meetings play strategy $\boldsymbol{a}^{hT}$.

(2) All type-lO consumers as well as type-hO consumers who will be in opaque DM meetings play strategy $\boldsymbol{a}_p$.

(3) Type-hO who will be in transparent DM meetings play strategy $\boldsymbol{a}^{hO}$.

(4) All type-lT consumers, regardless of the type of their DM meetings, do not sell in the AM and use only money in the DM.

To see why this is the unique equilibrium, first note that type-hO consumers who will be in transparent

DM meetings cannot do worse than waiting until the DM to use their assets. If they sell in the AM, they will have to accept a price discount due to information asymmetry (i.e., $\psi^{hO} = \tilde{\rho}\delta_h$). Second, in the AM, all type-lO consumers must pool with the type-hO consumers who will be in opaque DM meetings, because otherwise type-lO consumers will not be able to sell assets, and producers will recognize that their assets are worthless regardless of the types of the DM meetings. If type-hO consumers who will be in opaque DM meetings choose not to sell in the AM, they will treated as type-lT consumers in the DM, just like in the benchmark (hence constraint (C.2)). Selling some assets in the AM can therefore lead to a discontinuous jump in the average asset quality in opaque DM meetings for type-hO consumers. Finally, for type-hT consumers, selling the AM at $\psi^{hT} = \delta_h$ allows them to differentiate themselves from the other consumers in opaque DM meetings. If they do not sell in the AM, they will treated as type-lT consumers in the DM (hence constraint (C.1)).

C.1.2 Consumers learn about their DM meetings at the beginning of DM

First, define $V^T(\tilde{z}, \tilde{a})$ to be the value of a consumer with high-quality assets entering a transparent meetings in the DM. We have

$$\begin{aligned} V^T(\tilde{z}, \tilde{a}) = &\max_{q,\hat{z},\hat{a}} \{u(q) - \hat{z} - \delta_h \hat{a}\} \\ &\text{s.t. } q \le \hat{z} + \delta_h \hat{a}; \quad \hat{z} \le \tilde{z}; \quad \hat{a} \le \tilde{a}. \end{aligned}$$

Denote the solution as (a) $q^T = \min\{q^*, \tilde{z} + \delta_h \tilde{a}\}$, (b) $\hat{z}^T = \tilde{z}$, and (c) $\hat{a}^T = \frac{q^T - \hat{z}^T}{\delta_h}$.

Define $V^{TO}(\tilde{z}, \tilde{a})$ to be such that

$$\begin{aligned} V^{TO}(\tilde{z}, \tilde{a}) = &\max_{q,\hat{z},\hat{a}} \{u(q) - \hat{z} - \delta_h \hat{a}\} \\ &\text{s.t. } q \le \hat{z} + \mathbb{1}(\hat{z} > z)\delta_h \hat{a}; \\ &\quad\;\; \hat{z} \le \tilde{z}; \quad \hat{a} \le \tilde{a}. \end{aligned} \tag{C.3}$$

If $\hat{z} > z$, then the solution is given by $q^O = \min\{q^*, \tilde{z} + \delta_h \tilde{a}\}$, (b) $\hat{z}^O = \tilde{z}$, and (c) $\hat{a}^O = \frac{q^O - \hat{z}^O}{\delta_h}$. Let (ψ^{hT}, s^{hT}) be the solution to the following problem

$$\begin{aligned} &\max_{\psi,s} \{\tau^g V^T(z + \psi s, a - s) + (1 - \tau^g) V^{TO}(z + \psi s, a - s)\}, \\ &\text{s.t. } \psi \le \delta_h; \quad \psi s \le z_b; \quad s \le a. \end{aligned}$$

Then, we have (a) $\psi^{hT} = \delta_h$, and (b) $s^{hT} = (0, \min\{z_b/\delta_h, (q^* - z)/\delta_h, a\}]$.

Next, define $V^{OO}(\tilde{z}, \tilde{a})$ to be such that

$$\begin{aligned} V^{OO}(\tilde{z}, \tilde{a}) = &\max_{q,\hat{z},\hat{a}} \{u(q) - \hat{z} - \delta_h \hat{a}\} \\ &\text{s.t. } q \le \hat{z} + \rho\mathbb{1}(\hat{z} > z)\delta_h \hat{a}; \\ &\quad\;\; \hat{z} \le \tilde{z}; \quad \hat{a} \le \tilde{a}. \end{aligned} \tag{C.4}$$

Denote the solution as (a) $q_p = \max\{z, \min\{\tilde{z} + \rho\delta_h \tilde{a}, \tilde{q}\}\}$; (b) $\hat{z}_p = \tilde{z}$, and (c) $\hat{a}_p = \frac{q_p - \hat{z}_p}{\rho\delta_h}$. Let (ψ_p, s_p) be the solution to the following problem

$$\max_{\psi,s} \{\tau^g V^T(z + \psi s, a - s) + (1 - \tau^g) V^{OO}(z + \psi s, a - s)\},$$

$$\text{s.t. } \psi \le \rho\delta_h; \quad \psi s \le z_b; \quad s \le a.$$

We have (a) $\psi_p = \rho\delta_h$, and (b) $s_p = \min\{q_p - z, \epsilon\}$.

The unique equilibrium in the AM and DM is as follows:

(1) Type-hT consumers play (ψ^{hT}, s^{hT}) in the AM. They play $(q^T, \hat{z}^T, \hat{a}^T)$ if they are in transparent DM meetings, and $(q^O, \hat{z}^O, \hat{a}^O)$ if they are in opaque DM meetings.

(2) Type-hO consumers play (ψ_p, s_p) in the AM. Type-hO consumers play $(q^T, \hat{z}^T, \hat{a}^T)$ if they are in transparent DM meetings, and $(q_p, \hat{z}_p, \hat{a}_p)$ if they are in opaque DM meetings.

(3) Type-lO consumers play (ψ_p, s_p) in the AM. Type-hO consumers use only money they are in transparent DM meetings, and they play $(q_p, \hat{z}_p, \hat{a}_p)$ if they are in opaque DM meetings.

(4) All type-lT consumers do not sell in the AM and use only money in the DM.

To see why this is the unique equilibrium, first note that if type-hO consumers choose not to sell in the AM, they will they will treated as type-lT consumers in the DM, just like in the benchmark (hence constraint (C.4)). So long as $\tau^g < 1$, it is never optimal to not sell assets in the AM *and* then use assets as payment in opaque DM meetings. For type-hT consumers, selling the AM at $\psi^{hT} = \delta_h$ allows them to differentiate themselves from the other consumers in opaque DM meetings. If they do not sell in the AM, they will treated as type-lT consumers in the DM (hence constraint (C.3)).

C.2 Endogenizing τ when $\delta_l > 0$

In this section, we discuss endogenizing τ when δ_l is assumed to be strictly positive. First, we consider the case where the cost κ must be paid after the realization of the consumption shock but before the realization of the quality shock. Let $\hat{a}$ denote the the asset holding of a consumer at the beginning of the AM. Let $B_1(z)$ denote the benefit of paying κ. We have several cases to discuss.
(1) Suppose that $z \ge z''$. We have

$$B_1(z) = \rho[u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h] - \rho[u(q_h) + z + \delta_h\hat{a} - q_h],$$

where $\tilde{q}_h = \max\{z + \delta_h\hat{a}, q^*\}$, and $q^h = z + \delta_h a$ solves (B.11) when $s = 0$. It is straightforward to show that in this case, $B_1'(z) < 0$.
(2) Suppose that $z' \le z < z''$. We have

$$\begin{aligned} B_1(z) =& \rho[u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h] + (1-\rho)[u(\tilde{q}_l) + z + \delta_l\hat{a} - \tilde{q}_l] \\ & - \rho[u(q_h) + \delta_h(\hat{a} - a - s)] - (1-\rho)[u(q_l) + \delta_l(\hat{a} - a - s)], \end{aligned}$$

where $\tilde{q}_l = \max\{z + \delta_l\hat{a}, q^*\}$, $q_h = z + \bar{\delta}s + \delta_h a$, $q_l = \min\{z + \bar{\delta}s + \delta_l a, q^*\}$, and s and a solve (B.15) and (B.11). Recall that z does not affect q_h or q_l in this case. Hence, $B_1'(z) > 0$ if $\tilde{q}_h$ and/or $\tilde{q}_l$ are less than q^*, and $B_1'(z) = 0$ if otherwise.
(3) Suppose that $z < z'$. We have

$$B_1(z) = \rho[u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h] + (1-\rho)[u(\tilde{q}_l) + z + \delta_l\hat{a} - \tilde{q}_l] - u(z + \bar{\delta}\hat{a})$$

If $u'''(.) > 0$, then $B_1(z) > 0$ as long as $\tilde{q}_h < q^*$. Under the assumption that $a < \min\left\{\frac{\tilde{q}}{\bar{\delta}}, \frac{q^*}{\delta_h}\right\}$, we have $q^* - \delta_h a - (\tilde{q} - \rho\delta_h a) = q^* - \delta_h a > 0$. Hence, $z + \delta_h a > q^*$ implies that $z + \rho\delta_h a > \tilde{q}$. In other words, $\tilde{q}_h < q^*$ when $z < z'$.

We summarize the equilibrium in the following proposition.

Proposition 11 *There exist $\mu^\dagger$ and $\mu^\ddagger$ such that*
(1) *If $\mu > \mu^\ddagger$, consumers do not pay κ, and they pool in the DM.*
(2) *If $\mu^\dagger < \mu \leq \mu^\ddagger$, consumers pay κ.*
(3) *If $\mu \leq \mu^\dagger$, consumers do not pay κ, and they separate in the DM.*

Now, we assume that consumers pay κ after they learn the quality of assets. In this case, only consumers with high-quality assets will pay the cost. Let $\tilde{\tau}$ denote the proportional of consumers who pay κ. Then, either $\tilde{\tau} = 1$ or $\tilde{\tau} = 0$. Intuitively, this is because the marginal value of real balances depends on whether consumers pay κ or not. Therefore, even if consumers are indifferent regarding paying κ or not ex post, they will strictly prefer different holdings of real balances ex ante, which means consumers will not be indifferent ex post, a contradiction. Now, let $\bar{B}_2(z)$ and $\underline{B}_2(z)$ denote the benefits of paying κ given $\tilde{\tau} = 1$ and $\tilde{\tau} = 0$, respectively. We have several cases.
(1) Suppose that $z \geq z''$. We have

$$\bar{B}_2(z) = \underline{B}_2(z) = u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h - [u(q_h) + z + \delta_h\hat{a} - q_h],$$

where $\tilde{q}_h = \max\{z + \delta_h\hat{a}, q^*\}$, and $q_h = z + \delta_h a$ solves (B.11)) when $s = 0$.
(2) Suppose that $z' \leq z < z''$. We have

$$\begin{aligned}\bar{B}_2(z) &= u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h - [u(q_h) + z + \delta_h\hat{a} - q_h],\\ \underline{B}_2(z) &= u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h - [u(\tilde{q}_h) + \delta_h(\hat{a} - a - s)],\end{aligned}$$

where $\tilde{q}_h = z + \bar{\delta}s + \delta_h a$, and s and a solve (B.15) and (B.11). Recall that z does not affect $\tilde{q}_h$ in this case. When $\tilde{\tau} = 1$, if a consumer with high-quality assets chooses not to pay κ, then her optimal strategy is to offer a separating offer.
(3) Suppose that $z < z'$. We have

$$\begin{aligned}\bar{B}_2(z) &= u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h - [u(q_h) + z + \delta_h\hat{a} - q_h],\\ \underline{B}_2(z) &= u(\tilde{q}_h) + z + \delta_h\hat{a} - \tilde{q}_h - u(z + \bar{\delta}\hat{a}).\end{aligned}$$

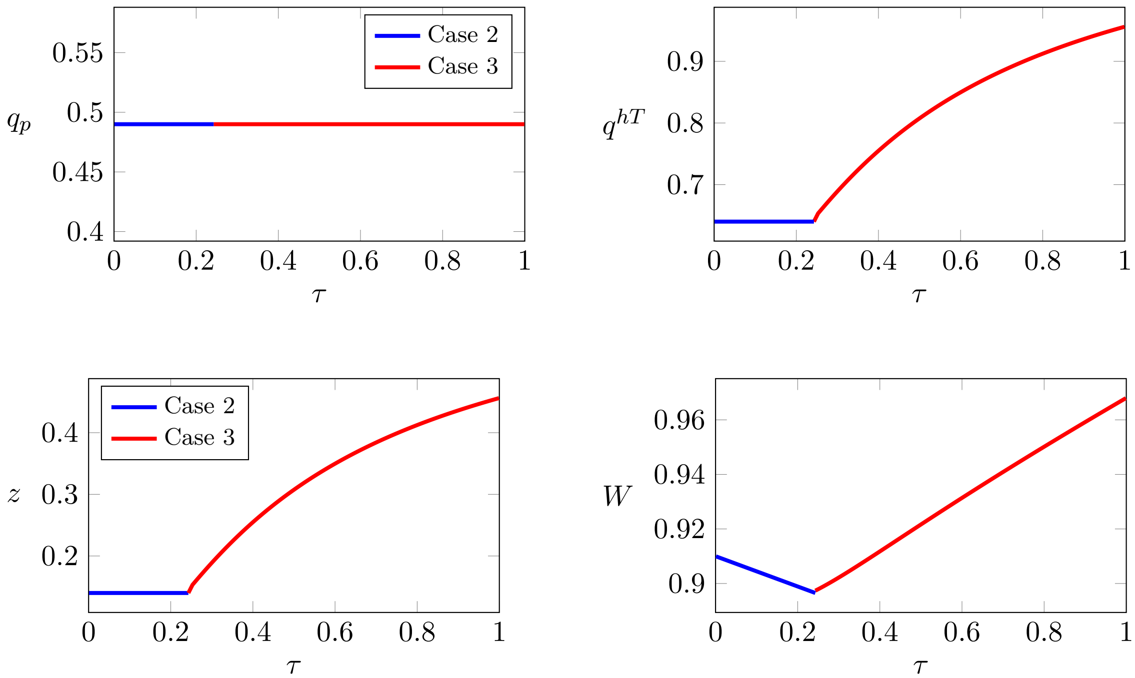
Similar to Case (2), when $\tilde{\tau} = 1$, if a consumer with high-quality assets chooses not to pay κ, then her optimal strategy is to offer a separating offer.

We summarize the equilibrium in the following proposition.

Proposition 12 *There exists $\mu^\diamond$ such that consumers with high-quality assets will pay κ if and only if $\mu \leq \mu^\diamond$.*

C.3 Numerical Examples with an Exogenous τ

In this appendix, we provide some more numerical examples when τ is assumed to be exogenous (see Section 3). In the following examples, $u(q) = \frac{q^{1-\sigma}}{1-\sigma}$, $\sigma = 0.5$, $\lambda = 0.5$, $\rho = 0.7$, $\tau = 0.5$, $a = 0.5$, $\delta_h = 1$, and $i = 0.08$.



In the following example, $i = 0.25$.