Incentivizing Banks as Conduits: Evidence from the Paycheck Protection Program

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Abstract

How does the policy motivate banks to transfer government welfare to small businesses? This study delves into this question within the context of the Paycheck Protection Program (PPP), which assists small businesses affected by the COVID-19 pandemic. Using the bunching estimation approach and leveraging the incentive discontinuity in fee schemes, this paper examines the SBA PPP loan policies by analyzing loan supply elasticities w.r.t. commissions and loan demand elasticities w.r.t. potential interest rates. I find evidence that expectations of loan forgiveness policies influenced borrowers' actions, while commissions had no discernible impact on lender behavior.

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1 Introduction

In contemporary public economics, discussions invariably revolve around two central questions: the efficiency and equity of government policies. This discourse assumes heightened significance during societal crises, compelling governments to take swift action to achieve critical social rescue objectives. In such scenarios, a fundamental question emerges: How can we ensure, to a reasonable extent, that equitable outcomes are maintained when speed is essential?

The emergence of a social experiment like the COVID-19 pandemic posed a formidable challenge for the Small Business Administration (SBA). The agency provides financial resources to stabilize the economy, particularly safeguarding small businesses from the imminent risk of bankruptcy and the resulting decline in employment opportunities. Established regulatory mechanisms for U.S. small business loans traditionally included government guarantees and interest rate caps, such as the previous 7(a) loan program. Nevertheless, the SBA re-envisioned these conventional tools for PPP loans to facilitate loan disbursement in the pandemic by offering lenders a one hundred percent guarantee, loan size-dependent commission fees contingent upon their involvement in the disbursement process, and forgiveness policies for borrowers who can maintain employment relationships and salary levels as pre-pandemic. Kinks and notches emerged in the processing fee schedule and forgiveness requirements, generating incentive distortions around the policy thresholds. This naturally raises the question: When a government agency relies on banks as conduits to execute its policies, what are the impacts of discontinuous incentives on banks' disbursement? What is the elasticity of loan supply w.r.t. commissions? [1]

[1] Since the establishment of the PPP program, researchers explored the effectiveness of the PPP Program: Hubbard and Strain (2020) examined the effectiveness of the PPP program in increasing small business employment, financial well-being, and overall survival; Bartik et al. (2020) estimated the employment effects of PPP loans and found that the program helped to reduce layoffs; Balyuk et al. (2021) examined the impact of PPP on small businesses and found that while it served as a positive financing supply shock, not all firms could take advantage of it. Also, the equitable Distribution of PPP Funds has been examined by Humphries et al. (2020), showing that small businesses had less awareness and access to information about government assistance programs than larger firms. This led to smaller firms missing out on initial PPP funds, highlighting

Prior research in the field of public economics has explored the effects of incentive schemes on individuals' behavioral responses. A recently developed empirical method in the public economics field is the bunching estimation. Under the assumption that individuals respond to marginal price, kinks or notches in incentive schemes lead private sector actions to make significant adjustments. The original example is the tax scheme, in which tax rate structures encourage high-ability individuals to declare their incomes just below bracket thresholds. This leads to a concentration of reporting below the tax scheme thresholds and a corresponding absence of reporting above them. Saez (2010) and Chetty et al. (2011) developed frameworks for estimating labor supply elasticities for kinks, Kleven and Waseem (2013) developed the framework for elasticities for notches. [2] Cox et al. (2020) proposed a novel bunching estimation to quantify banks' market power and evaluate the effectiveness of policy interventions by analyzing the SBA Express lending program. Adapting and developing the traditional bunching estimation method, Cox et al. (2020) also estimated loan size w.r.t. interest rates in a two-dimensional contract space, explaining and predicting the bunching responses in government guarantee loan and rate cap programs.

Previous bunching approach estimations have explored various scenarios but often needed more estimations specific to loan disbursement contexts. The primary objective of this study is to examine the effects of discontinuity schemes within PPP loan policies on the disbursement process of PPP loans. I utilize the PPP loan data released from the SBA and implement the bunching approach for quantity estimate elasticities of loan amounts w.r.t fee

the need for improved outreach and accessibility of such programs for all businesses. Cororaton and Rosen (2021) found that the firms that received PPP funds were typically smaller, with more employees, fewer investment opportunities, and preexisting debt balances. However, Granja et al. (2022) suggested that they did not find evidence to support the claim that PPP funds flowed to areas more adversely affected by the pandemic. Instead, they found significant heterogeneity across banks regarding disbursing PPP funds.

[2] Ito (2014) didn't observe any bunching responses but demonstrated that individuals respond to the average price. Researchers have broadened the implication of the bunching approach as public economics continues developing. This expansion encompasses diverse areas such as labor supply, environmental policy, education, healthcare, etc. Best et al. (2015b) studied inefficient tax policies that fight evasion in developing countries and highlighted that mortgage debt responses to interest rates were related to a more fundamental structural parameter. Harju et al. (2019) followed the bunching window selection method developed by Kleven and Waseem (2013) to study the mechanisms behind entrepreneurs' responses to size-dependent tax regulation.

structure at each threshold. This paper employs a theoretical model based on the framework presented in Cox et al. (2020) to explain and predict bunching responses at thresholds within the PPP fee structure.

The empirical results coincide with the model's predictions. The findings indicate limited behavioral responses at the notch points in the processing fee schedule but significant bunching responses at the loan thresholds associated with forgiveness regulations. This indicates that lenders were generally unaffected by the incentive distortion schemes, leading to the disbursement of loans without distortion loan sizes. Consequently, small businesses could secure loans in the expected amounts they qualified for. However, small borrowers were influenced by their expectations regarding the distortion forgiveness approval probabilities. The findings in this paper shed light on future emergency funds programs. The SBA could expect to incorporate kinks or notches in the commission scheme to incentivize banks as conduits without concerns about distorted loan sizes. Additionally, the SBA needed to monitor policies related to borrower incentives closely, as borrowers often make substantial adjustments to avoid non-forgiveness by the SBA, resulting in borrowing smaller amounts than their eligible loan size.

This paper is organized as follows: Section 2 provides institutional details on PPP loans, SBA regulations, and the design of processing fee schedules. Section 3 presents the theoretical model used to predict the behavioral responses of market participants. Section 4 offers a data description and presents summary statistics results. Section 5 outlines the empirical framework and the bunching estimation approach. Section 6 presents the empirical results for PPP lenders and borrowers, and Section 7 interprets the findings and concludes.

2 Background

Small businesses are a crucial source of employment in the United States, with $31.7 million such businesses employing 60.6 million workers as of 2020. These enterprises account for 47.1% of the private-sector labor force, pivotal in the country's economic landscape. However, the COVID-19 pandemic precipitated a widespread economic downturn and an unprecedented surge in unemployment rates. The pandemic's impact was compounded by government-issued lockdowns that effectively halted economic activity. As such, policymakers required effective and targeted policy tools to support employment and ensure small business continuity throughout this tumultuous period.

As government action responses toward the COVID-19 pandemic, the U.S. government deployed the Coronavirus Aid, Relief, and Economic Security Act (CARES Act) as a policy tool to alleviate the pandemic's adverse effects. The CARES Act, which amounted to $2.2 trillion, was signed into law by President Trump on March 27, 2020, and provided a massive economic stimulus package. The Paycheck Protection Program (PPP) was one of the CARES Act programs, which received a budget of $953 billion and issued approximately 12 million loans.

The PPP program has multiple objectives, primarily ensuring small business continuity and preserving employee-firm relationships during the COVID-19 pandemic. The primary objective of this program was to provide forgivable loans to small businesses with fewer than 500 employees to support their monthly payments, including payroll expenses, rent, utilities, mortgage interest, healthcare, and debt. Because small and medium-sized businesses rely heavily on traditional bank lending to sustain their balance sheets, governments tend to transmit small business policies through conventional lending channels (Mills & McCarthy, 2014). The PPP was the most extensive and innovative fiscal policy intervention, with short- and medium-term goals of preserving the economy's productive capacity and supporting labor demand. The funds acted as a revenue-replacement program for helping small businesses that could not access institutional debt markets or equity financing. This program achieved the policy objectives with the government backing the loans and allowing banks to charge processing fees (Hubbard & Strain, 2020). The efficient implementation of the revenue replacement program was crucial to the success of supporting small businesses during the pandemic and avoiding the need for the government to assist in rebuilding employee-firm relationships as the economy recovers.

The SBA leveraged its existing 7(a) loan programs to administer loans through the PPP. The unique feature in PPP's piecewise linear commission schedule allows for implementing the bunching analysis developed by Henrik J Kleven & Waseem (2013) and Cox et al. (2022). This study's methodology was based on the traditional bunching method while incorporating the insights from the two-dimensional bunching model.

2.1 SBA Regulations

The SBA utilizes various regulatory methods, including government guarantees and interest rate caps, to ensure the successful execution of its loans. Loan terms, such as guaranteed percentage and loan amount, may differ across different types of loans. The SBA's primary loan program, the 7(a) loan program, provides small businesses with financial assistance through various types of loans.

The PPP is another type of 7(a) loan extension to support small businesses during the pandemic. The funds have two main objectives: to preserve employment relationships and to help small businesses survive the economic shutdown. The SBA has designed policies for lenders and borrowers to achieve these goals. It is generally intended to be non-collateralized, forgivable, and 100% guaranteed with commissions paid to lenders.

For PPP lenders, as per the SBA Interim Final Rule, the PPP loans are guaranteed 100% by the government. Lenders can apply processing fees from the SBA once the loans have been fully disbursed. (Once the SBA approves a PPP loan application, lenders must disburse funds within ten business days). One significant policy feature of PPP loans is splitting potential borrowers' interest rates into two parts. One part is fixed at a minimal rate of one percent for borrowers if their loans cannot be forgiven. The other part is loan size-dependent and reflects the risk level of regular loans. The government will pay this interest portion to incentivize lenders to participate in loan disbursement. As government funding channels quickly disburse funds, lenders leverage existing relationships with small businesses, thereby assisting these businesses that have been adversely affected, preventing bankruptcy and averting defaults on previous debts. This commission design incentivizes lenders to approve loan applications and mitigates the risk of a wave of loan defaults.

Small businesses that meet the requirements outlined in Table B.11 can apply for PPP loans through the SBA E-Tran platform. After obtaining funds, small businesses have 8 to 24 weeks as their covered period for using funds in eligible costs. During the covered period, businesses were required to maintain their employment positions and employee salary levels as of the pre-shutdown period in 2019 or 2020. The utilization of funds had to follow the SBA 60/40 spending rule, which meant spending 60% of PPP loans on payroll payments and 40% on other eligible costs, such as rent, utilities, and mortgage interest. Priority weight has been put on the payroll cost. The payroll costs for each employee were capped at $100,000 annually. Businesses could borrow up to 2.5 times their average monthly payroll costs[3]. Therefore, borrowers can calculate the maximum loan amount for payroll cost, that with a cap of $15,385 per employee for those who received a PPP loan before June 5, 2020, and elected to use an eight-week covered period. For those who borrowed a PPP loan after this date, the covered period could be 24 weeks, and the cap loan amount per employee increased to $20,833. This number was also the maximum amount for self-employed individuals or owner-employees for their salary. There is also a maximum number for the total loan amount. That was capped at $2 million in 2020 or $10 million in 2021.

After the covered period, small businesses have a ten-month period to apply for loan forgiveness.[4] Otherwise, the PPP loan payment will no longer be deferred, and small busi-

[3]Businesses in the accommodation and food services sector (NAICS code 72) could apply for loans up to 3.5 times their average monthly payroll costs, reflecting the unique challenges faced by this industry during the pandemic.

[4]For loans issued after June 5, 2020, the maturity of the loans was increased from two to five years,

nesses will be responsible for the fixed one percent interest. Nevertheless, the probability of forgiveness is very high. Therefore, the forgivable requirement motivates borrowers to allocate the funds toward maintaining payroll and other necessary costs, effectively minimizing the risk of permanent layoffs and bankruptcies.

2.2 PPP Processing Fee Scheme

The first round of PPP was disbursed from April 3, 2020, to April 16, 2020, then extended to June 30, 2020, with an additional extension to August 8, 2020. The second round of PPP was disbursed from January 11, 2021, to May 31, 2021. The SBA has established a particular commission scheme to incentivize lenders to participate in PPP lending. SBA Procedure Notice 5000-20028, which became effective on May 21, 2020, represents the original procedure notice that guides PPP lenders regarding reporting loan disbursement and collecting processing fees. For the first round of PPP loans, lenders are eligible to receive commissions at a rate of 5% for loans up to $350,000, 3% on loans in the range between $350,000 and $2 million, and 1% on loans in the range between $2 million up to $10 million. The SBA has subsequently issued several procedure notices to update the reporting and payment process. Notably, the SBA Procedure Notice 5000-20036, which became effective on July 13, 2020, provides additional guidance on lenders' fee payment and reporting process. The SBA Procedure Notice 5000-20091, which became effective on February 8, 2021, introduced certain modifications to the fee rates. The commission rates for the First Draw of PPP loans made before December 27, 2020, remained the same as previously announced. However, the commission rates were adjusted for loans made on or after December 27, 2020. Lenders will receive 50% or a maximum of $2,500 for loans up to $50,000, 5% for loans in the range between $50,000 and $350,000, 3% for loans in the range between $350,000 and $2 million, and 1% for loans above $2 million up to $10 million. For Second Draw PPP loans, the SBA adjusted the maximum loan amount at $2 million, and commission rates at 50%

providing small businesses with a longer-term repayment plan.

or a maximum of \$2,500 for loans up to \$50,000, 5% for loans in the range between \$50,000 and \$350,000, and 3% for loans above \$350,000. Table 1 presents the PPP funds Processing Fee Rates in two rounds. The processing fee structures and adjustments are summarized in Figures 1 and 2.

Although the commission rates have undergone minor changes, the rates for loans around \$350k remained unchanged in these two rounds. If small businesses borrow just below \$350k, PPP lenders can earn up to \$17,500 as processing fees. In contrast, if a lender processed loans just above \$350k, the processing fee will drop to \$10,500. By lending just above the threshold, fees decrease by 40%: (\$17,500 - \$10,500) / \$17,500 = 0.4. These make lending below the threshold the optimal strategy for lenders, dominating the loan range from \$350,000 to \$583,333. A similar scenario applied to the threshold at \$2 million, where the dominant loan range was \$2 million to \$6 million.

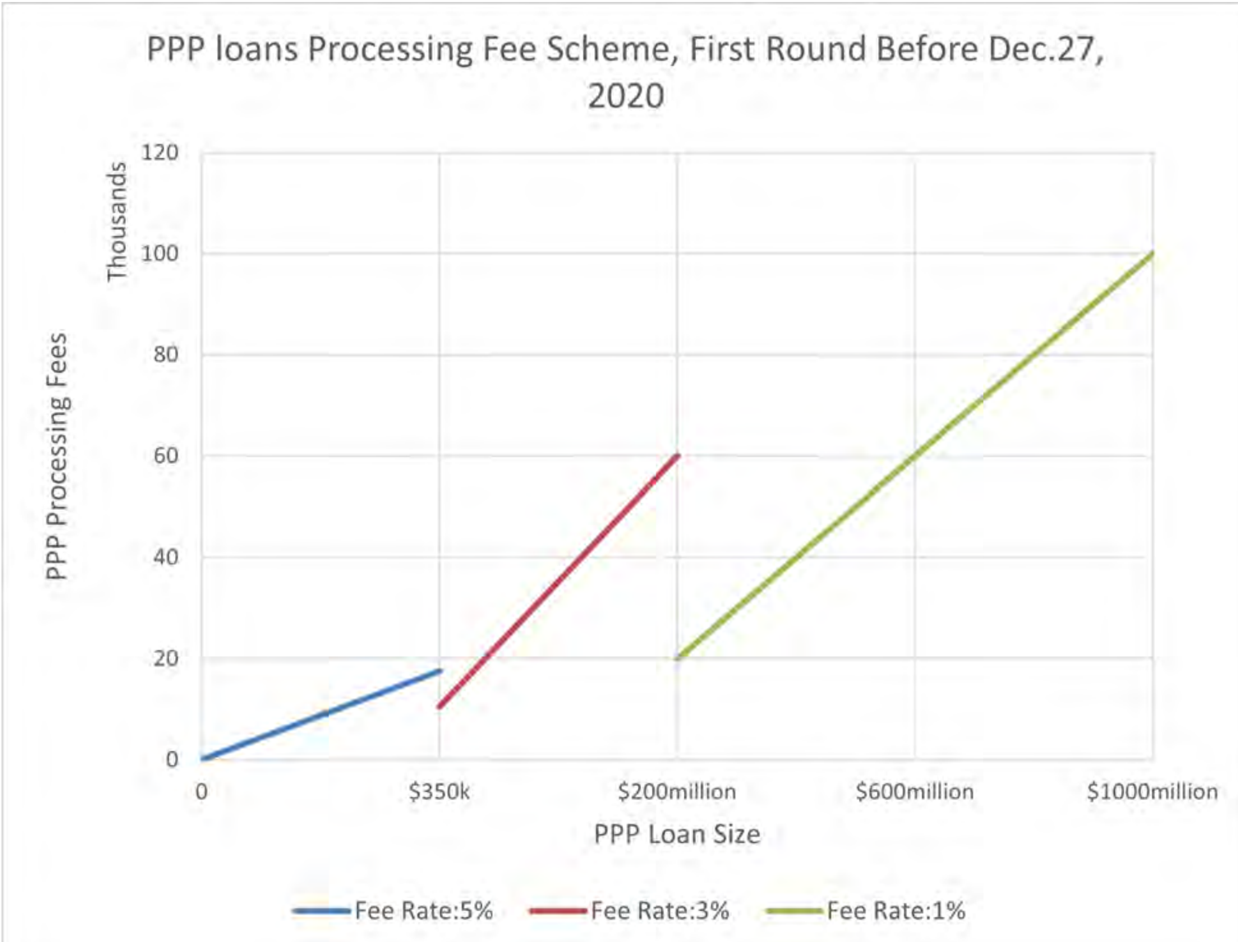
In the second round of PPP loans, the fee rate for loans under \$50k is set at 50% or fixed at \$2,500, whichever is less. This can be interpreted as equivalent to setting the fee rate at 50% for loans under \$5k, and fixing the fee at \$2,500 for loans between \$5k and \$50k.

Table 1: The SBA Processing Fee Rates for the PPP Lenders

PPP Processing Fee Rates for Lenders			
	2020	1st round after 12/27/2020	2021
Fee Rates or Price	Cut-off Points		
50%		$\leq \$5k$	$\leq \$5k$
\$2,500		\$5k-\$50k	\$5k-\$50k
5%	$\leq \$350k$	\$50k - \$350k	\$50k - \$350k
3%	\$350k-\$2m	\$350k-\$2m	> \$350k
1%	>\$2m	>\$2m	

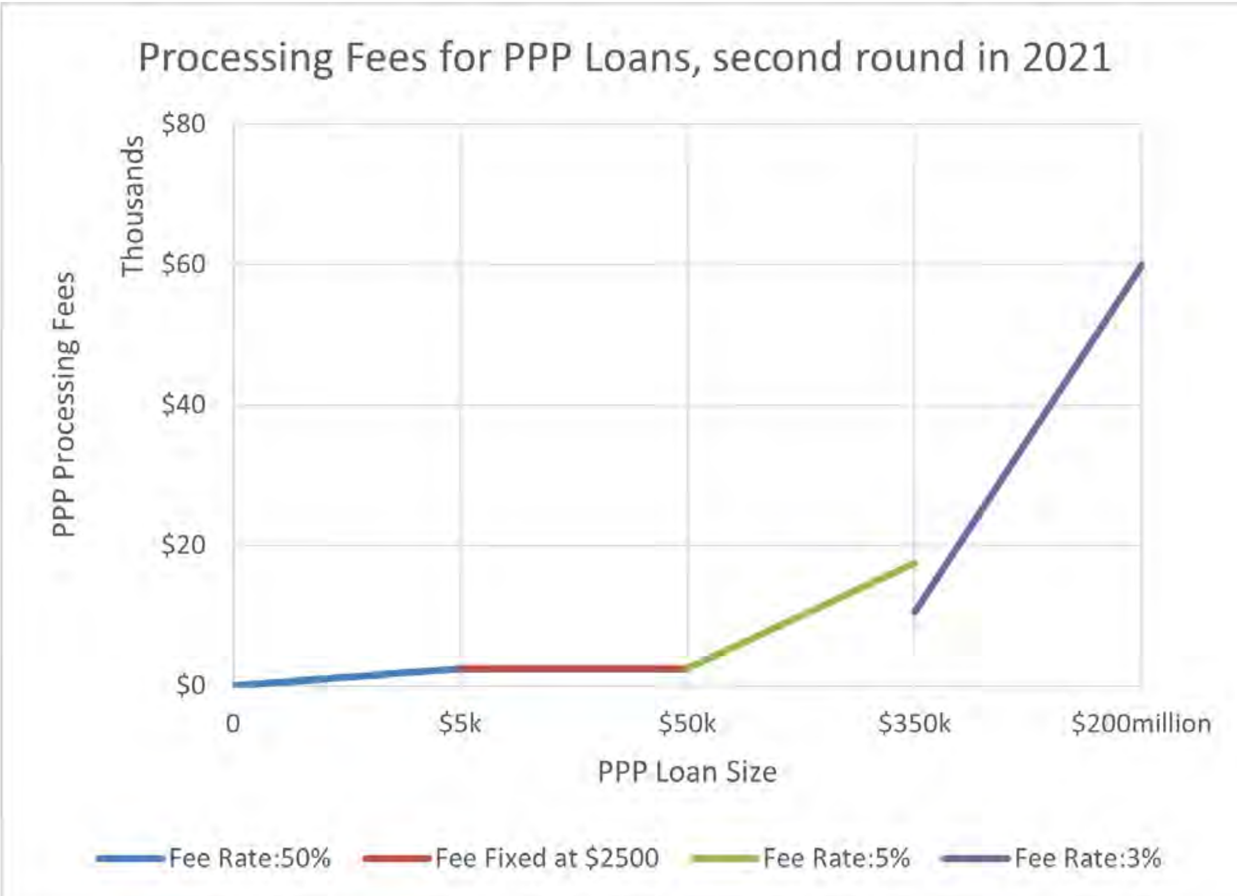
Notes: Table 1 The SBA Processing Fee Rates for PPP Lenders. The fee rates and cut-off points presented in this table were sourced from three SBA Procedure Notices: 5000-20028 (effective May 21, 2020), 5000-20036 (effective July 13, 2020), and 5000-20091 (effective February 8, 2021). Notably, the processing fee scheme features two primary loan thresholds, which are \$350,000 and \$2,000,000.

Figure 1: PPP loans Processing Fee Structure



Notes: Fig 1 displays the processing fee structure for the first round of PPP loans made before December 27, 2020. The fee rates were collected from SBA Procedure Notice 5000-20028, effective May 21, 2020. The primary loan thresholds in the processing fee scheme are \$350,000 and \$2,000,000. The x-axis is not on a numerical scale for a better illustrate of the processing fees for smaller loan sizes. The original fee structure in 2020 involved a 5% commission fee for loans up to \$350,000, a 3% fee for loans between \$350,000 and \$2 million, and a 1% for loans in the range between \$2 million up to \$10 million. The solid lines indicate the potential processing fees lenders could earn, while the dotted lines represent the optimal loan ranges. The optimal strategy at \$350,000 dominates the loan range between \$350,000 to \$583,333, while the optimal strategy at \$2 million dominates the loan range between \$2 million to \$6 million.

Figure 2: Adjusted Processing Fee Structure for the PPP loans at second round



Notes: Fig 2 shows the Processing Fee scheme for the second round of PPP loans in 2021, where the x-axis is not in numerical scale for better visualization of the smaller loan sizes. The fees for PPP loans have undergone adjustments for both the first round of loans after December 27, 2020 and the second round in 2021. In SBA Procedure Notice, the SBA updated the fee scheme for loans under $50k to be 50% or $2500, whichever is less. This adjustment creates a concave kink at $5k and a convex kink at $50k.

3 Model

Cox et al. (2020) have proposed a lending market model that offers a comprehensive framework for analyzing the impact of imperfect competition on distributional distortions within the market. Their study utilizes a sophisticated two-dimensional bunching estimation approach, considering both interest rate and loan size, to assess the degree of market power held by banks and evaluate the effectiveness of policy interventions.

The model developed by Cox et al. (2020) enables the analysis to capture the unique characteristics of the PPP, such as the flat but discontinuous processing fee rates, 100% guarantees, and forgiveness provisions. In the following model setup, I adapted the base model proposed by Cox et al. (2020) and made slight adjustments to align it with the regulations imposed by the SBA for the PPP funds. It is important to note that this adapted model primarily focuses on analyzing loan contracts at the fee scheme threshold, while the welfare analysis will be addressed in future work.

Consider a market with finite K banks and a continuum of borrowers of finite measure. Both parties are risk-neutral. Let k represent banks, and i represent borrowers.

3.1 Investment Technology

Each borrower i has a stochastic investment technology that produces output as a function of investment size L. The project generates output L^{α}. The parameter α captures the concavity of the production function. The term z_i is a productivity shifter. Suppose not consider fraud case in this model, every borrower uses their funds to produce. Suppose the PPP funds succeed in approving forgiven at probability p_{fi}

$$\text{Project succeed or failed} = \begin{cases} p_{fi}\text{: succeed, loan has been forgiven} \\ (1-p_{fi})\text{: failed, loan has not been forgiven} \end{cases} \tag{1}$$

The borrower's forgiveness probability is p_{fi}. The loan will be successfully forgiven if the investment meets the SBA forgiveness requirement, including reserving employment positions and salary levels. With probability, $1 - p_{fi}$, the investment fails to meet the SBA requirement for forgiveness. The $\delta_i \in [0, 1]$ is the fraction of the PPP loan that can be recovered by borrower i.

3.2 Loan Contracts

The interest rate r that borrowers originally paid in normal times has been split into two parts: r_b and r_p in this program. $r_b = ar$ stands for the risk-free interest rate, which borrowers will pay. $r_p = (1-a)r$ stands for the risk premium that the government will pay. The PPP policies were further fixed r_b to 1% if the loan has not been forgiven by the SBA, and r_p to be loan size-dependent capped interest rates. Borrowers' decisions are only impacted by the government's fixed repay interest rate, $\bar{r_b}$. This interest rate would be effective only if the SBA has not approval forgiven. In contrast, lenders' decisions are impacted by the government loan size-dependent processing fee rate $\bar{r_p}$ and the borrower's fixed repay interest rate $\bar{r_b}$. The critical assumption made in this model setup is that lenders could not choose these two interest rates at the same time in an optimization process.

This study first unconstrained the processing fee rate r_p, to obtain the optimal loan size. Then, this study analyzes the loan size and processing fee rate under the scenario of size-dependent restrictions. If a borrower cannot meet the SBA requirement for forgiveness, the borrower should also pay the loan principle and $r_b = 1\%$ interest fee. They don't have to pay the loan principle and the $r_b = 0$ if they meet the forgiveness requirement. Borrowers know about the interest rate and forgiveness policy and observe loan contract terms (r_b, L), while lenders observe loan contract terms (r_p, r_b, L). If the contract offered by bank k is accepted by borrower i, it generates contractual value $v_i(L)$ to borrower i and expects profit $\pi_{ik}(L)$ for bank k.

When unconstrained by policies, borrowers value function v_i:

$$\begin{aligned} v_i &\equiv p_{fi} z_i L^\alpha + (1 - p_{fi})[z_i L^\alpha - (1 + r_b)\delta_i L] \\ &= z_i L^\alpha - (1 - p_{fi})(1 + r_b)\delta_i L \end{aligned} \tag{2}$$

and banks' profit function π_{ik}

$$\pi_{ik} \equiv p_{fi}(1 + r_p)L + (1 - p_{fi})(1 + r_p + r_b)\delta_i^G L - c_k L \tag{3}$$

When the project meets the forgiveness requirements, the borrower gets paid $z_i\ L^\alpha$ from the project, and the bank gets paid $(1 + r_p)L$ from the government; when the project fails to meet the forgiveness requirement, the borrower repayment -$(1 + r_b)\delta_i L$ and the bank gets paid $(1 + r_b)\delta L$ from the borrower and $r_p\ L$ from the government.

Since the PPP is one hundred percent guaranteed by the SBA. Following (Cox et al., 2020), I define λ percent to represent the loan guarantee. In the PPP, the SBA promises to pay the lender λ percent of the unpaid principal of the loan when the loan is not being forgiven and the borrower defaults. And define $\delta_i^G = \delta_i + \lambda(1 - \delta_i)$ as the fraction of principal lenders recover from the borrower or the SBA. Then, the lenders' profit function becomes:

$$
\begin{aligned}
\pi_{ik} &= p_{fi}(1+r_p)L + (1-p_{fi})(1+r_p+r_b)\delta_i^G L - c_k L \\
&= p_{fi}(1+r_p)L + (1-p_{fi})(1+r_p+r_b)(\delta_i + \lambda(1-\delta_i))L - c_k L \\
&= [p_{fi}(1+r_p) + (1-p_{fi})(1+r_p+r_b) - c_k]L \\
&= [(1+r_p) + (1-p_{fi})r_b - c_k]L
\end{aligned} \tag{4}
$$

The expected utility that borrower i obtains from selecting contract (L) from bank k is

$$u_{ik}(r_b, L) \equiv \xi_{ik} v_i(L) \tag{5}$$

The term $\xi_{ik} \geq 0$ is a random taste shock i.i.d. across borrowers and banks, independent of the borrower and bank characteristics.

3.3 Bank Competition

Borrowers apply the PPP funds on the SBA online platform, where they can find plenty of lenders. Banks $k = 1, \ldots, K$ compete for borrowers by simultaneously offering contracts. Each bank k offers one contract (L_{ik}) to each borrower i. This study also assumes that each borrower can always walk away from the investment opportunity if loan terms are too unattractive and only accept the contract that generates the highest and non-negative expected utility. The probability that the borrower i chooses the contract offered by bank k is a logit choice probability function:

$$q_{ik} \equiv Pr(\text{i chooses k}) = Pr(u_{ik} \geq \max\left\{0, \max_{k'} u_{ik}\right\}) \tag{6}$$

3.4 Distribution of Taste Shock

As the assumption made in the literature, assume the log of idiosyncratic taste shocks $ln\xi_{ik}$ are drawn from Type-I extreme value (Gumbel) distribution, with CDF $G(\xi;\sigma) = e^{-\gamma e^{-\sigma\xi}}$ (Cox et al., 2020), where γ is a normalizing constant. As the author suggested, $\sigma > 0$ is the critical parameter that captures the substitutability of loans across banks and relates inversely to the variance of borrowers' idiosyncratic taste shocks. Under the distribution assumption, the choice probability for any given bank becomes (Cox et al., 2020):

$$q_{ik}(\{v_{ik'}\}_{K'=1}^{K}) = \frac{\max\left\{0, \quad v_{ik}^{\sigma}\right\}}{\sum_{K'=1}^{K} \max\left\{0, \quad v_{ik'}^{\sigma}\right\}} \tag{7}$$

3.5 Equilibrium Loan Contract

There are two significant features of the PPP. First, the "risk premium" of the interest fees that borrowers would typically pay in a normal loan is paid through government processing fees in the PPP. Second, the processing fee rate has been designed to be flat and capped, with the cap being dependent on the loan size.

There are two types of notches in the PPP regulation policies. The first type of notches in the lender's processing fee scheme. At this type of threshold, lenders' and borrowers' decisions may both matter in loan contracts. The forgiveness regulation policies created the second type of notch. Because different expected probabilities of forgiveness—the efforts that required borrowers to meet requirements are different—the expected fee rates are different on each side of the threshold.

Each bank's profit maximization problem can be written as follows:

I. at the threshold of the lender's processing fee scheme let $1 + r_p = R_p$. If the lender gets $p_{fi}R_{1ik} + (1 - p_{fi})(R_{1ik} + r_b) - c_k > 0$ the PPP loan will be disbursed, otherwise, the bank could not cover the lending cost in this process.

$$\begin{aligned} &\text{Max}_{R,L}\left\{[p_{fi}R_{pik} + (1-p_{fi})(R_{pik}+r_b) - c_k]\right\} L_{ik}\frac{v_{ik}^{\sigma}}{\sum_j v_{ij}^{\sigma}} \\ &\qquad\qquad \text{s.t. } v_{ik} = z_i L_{ik}^{\alpha} - (1-p_{fi})(1+r_b)\delta_i L_{ik} \end{aligned} \tag{8}$$

$$\begin{aligned} &\Leftrightarrow \text{Max}_{R,L}\left\{R_{pik} + (1-p_{fi})r_b - c_k\right\} L_{ik}\frac{v_{ik}^{\sigma}}{\sum_j v_{ij}^{\sigma}} \\ &\qquad\qquad \text{s.t. } v_{ik} = z_i L_{ik}^{\alpha} - (1-p_{fi})(1+r_b)\delta_i L_{ik} \end{aligned} \tag{9}$$

F.O.C with respect to L

$$\begin{aligned} &\frac{v_{ik}^{\sigma}}{\sum_j v_{ij}^{\sigma}} + L_{ik}\sigma v_{ik}^{\sigma-1}\left[\frac{\sum_j v_{ij}^{\sigma} - v_{ik}^{\sigma}}{(\sum_j v_{ij}^{\sigma})^2}\right]\frac{\partial v_{ik}}{\partial L_{ik}} = 0 \\ &\Leftrightarrow v_{ik} + \sigma(1-1/K)\left\{z_i\alpha L_{ik}^{\alpha} - (1-p_{fi})(1+r_b)\delta_i L_{ik}\right\} = 0 \end{aligned} \tag{10}$$

substitute v_{ik}

$$\begin{aligned} &z_i L_{ik}^{\alpha} - (1-p_{fi})(1+r_b)\delta_i L_{ik} + \sigma(1-1/K)\left\{z_i\alpha L_{ik}^{\alpha} - (1-p_{fi})(1+r_b)\delta_i L_{ik}\right\} = 0 \\ &\Leftrightarrow [1+\alpha\sigma(1-1/K)]z_i L_{ik}^{\alpha-1} = [1+\sigma(1-1/K)](1-p_{fi})(1+r_b)\delta_i \\ &L_{ik} = \left[\frac{[1+\alpha\sigma(1-1/K)]z_i}{[1+\sigma(1-1/K)](1-p_{fi})(1+r_b)\delta_i}\right]^{\frac{1}{1-\alpha}} \end{aligned} \tag{11}$$

Since the variable R_p only appears in the lender's profit function and not in the borrower's optimization problem, it is not feasible to solve for the optimal loan size L_{ik} by taking the first-order condition with respect to R_p. Based on equation 11, we can interpret that the SBA processing fee rate r_p does not directly impact the optimal loan size.

Furthermore, the interest rate r_b only becomes relevant when the SBA has not forgiven the loan, and it remains fixed at any given processing fee rate threshold. Therefore, the impact of the interest rate r_b on the loan size at the processing fee rate threshold is likely to be limited.

II. Suppose banks solve the optimization problem by choosing loan size and the borrowers' payback interest rate. Firstly, assume unconstrained r_b, and let $1 + r_b = R_b$

$$\begin{aligned} \text{Max}_{R,L} \left\{ p_{fi} + r_p + (1 - p_{fi}) R_{bik} - c_k \right\} L_{ik} \frac{v_{ik}^{\sigma}}{\sum_j v_{ij}^{\sigma}} \\ \text{s.t. } v_{ik} = z_i L_{ik}^{\alpha} - (1 - p_{fi}) R_{bik} \delta_i L_{ik} \end{aligned} \tag{12}$$

let $\tilde{c_i} = \frac{c_k - p_{fi} - r_p}{1 - p_{fi}}$, then the optimization problem become:

$$\begin{aligned} \text{Max}_{R,L} \left\{ R_{bik} - \tilde{c_i} \right\} L_{ik} \frac{v_{ik}^{\sigma}}{\sum_j v_{ij}^{\sigma}} \\ \text{s.t. } v_{ik} = z_i L_{ik}^{\alpha} - (1 - p_{fi}) R_{bik} \delta_i L_{ik} \end{aligned} \tag{13}$$

take the First order condition with respect to R and L:

$$\begin{aligned} &\begin{cases} \{R\}\, L_{ik} \frac{v_{ik}^{\sigma}}{\sum_j v_{ij}^{\sigma}} + (R_{bik} - \tilde{c_i}) L_{ik} \sigma v_{ik}^{\sigma - 1} \left[\frac{\sum_j v_{ij}^{\sigma} - v_{ik}^{\sigma}}{(\sum_j v_{ij}^{\sigma})^2} \right] \frac{\partial v_{ik}}{\partial R_{ik}} = 0 \\ \{L\}\, \frac{v_{ik}^{\sigma}}{\sum_j v_{ij}^{\sigma}} + L_{ik} \sigma v_{ik}^{\sigma - 1} \left[\frac{\sum_j v_{ij}^{\sigma} - v_{ik}^{\sigma}}{(\sum_j v_{ij}^{\sigma})^2} \right] \frac{\partial v_{ik}}{\partial L_{ik}} = 0 \end{cases} \\ \Leftrightarrow &\begin{cases} v_{ik} = (R_{bik} - \tilde{c_i}) \sigma (1 - 1/K)(1 - p_{fi}) \delta_i L_{ik} \\ v_{ik} + \sigma (1 - 1/K) \left\{ z_i \alpha L_{ik}^{\alpha} - (1 - p_{fi})(1 + r_b) \delta_i L_{ik} \right\} = 0 \end{cases} \end{aligned} \tag{14}$$

substitute the first equation into the second to eliminate v_{ik} and re-arrange

$$
\begin{aligned}
z_i \alpha L_{ik}^{\alpha-1} &= (1-p_{fi})\delta_i \tilde{c}_i \\
\Leftrightarrow L_{ik}^{1-\alpha} &= \frac{z_i \alpha}{(1-p_{fi})\delta_i \tilde{c}_i} \\
\Leftrightarrow L_{ik}^{1-\alpha} &= \frac{z_i \alpha}{\delta_i (c_k - p_{fi} - r_p)}
\end{aligned}
\tag{15}
$$

substitute the definition of v_{ik} into the F.O.C. with respect to R

$$
\begin{aligned}
z_i L_{ik}^{\alpha} - (1-p_{fi}) R_{bik} \delta_i L_{ik} &= (R_{bik} - \tilde{c}_i)\sigma(1-1/K)(1-p_{fi})\delta_i L_{ik} \\
z_i L_{ik}^{\alpha-1} - (1-p_{fi}) R_{bik} \delta_i &= (R_{bik} - \tilde{c}_i)\sigma(1-1/K)(1-p_{fi})\delta_i
\end{aligned}
\tag{16}
$$

using eq. 15 to substitute for $L_{ik}^{\alpha-1}$:

$$
\begin{aligned}
z_i \frac{(1-p_{fi})\delta_i \tilde{c}_i}{z_i \alpha} - (1-p_{fi}) R_{bik} \delta_i &= (R_{bik} - \tilde{c}_i)\sigma(1-1/K)(1-p_{fi})\delta_i \\
\tilde{c}_i - \alpha R_{bik} &= \alpha (R_{bik} - \tilde{c}_i)\sigma(1-1/K)
\end{aligned}
\tag{17}
$$

Then, it obtains the same expression of the relationship between R_{ik} and $\tilde{c}_i$ as in the literature. After rearranging the equation, I get:

$$
R_{bik}(\alpha + \alpha\sigma(1-1/K)) = \tilde{c}_i(1 + \alpha\sigma(1-1/K))
\tag{18}
$$

Therefore it obtains the same expression for the profit margin as in the literature:

$$
\mu_{ik} \equiv \frac{R_{bik}}{\tilde{c}_i} - 1 = \frac{1-\alpha}{(\alpha(1+\sigma(1-1/K)))}
\tag{19}
$$

3.6 Bank's Response to Policy Interventions

I. In the first type of notches scenario, the equation 11 shows that only the interest rate $R_b = 1 + r_b$ is present in the denominator, while the processing fee rate r_p is absent from this equation. As a result, the variance of the processing fee rate would not impact the optimal loan size, and its influence at the processing fee rate threshold is even less likely.

Let's denote (r_i^H, L_i^H) as the loan contract on the left side of the processing fee threshold, and (r_i^L, L_i^L) as the loan contract on the right side of the processing fee threshold.

$$(r_i^H, L_i^H) \equiv (\bar{r_b}, \min(\bar{L}, \left[\frac{[1+\alpha\sigma(1-1/K)]z_i}{[1+\sigma(1-1/K)](1-p_{fi})(1+\bar{r_b})\delta_i}\right]^{\frac{1}{1-\alpha}})) \tag{20}$$

$$(r_i^L, L_i^L) \equiv (\bar{r_b}, \min(\bar{L}, \left[\frac{[1+\alpha\sigma(1-1/K)]z_i}{[1+\sigma(1-1/K)](1-p_{fi})(1+\bar{r_b})\delta_i}\right]^{\frac{1}{1-\alpha}})) \tag{21}$$

From the definition of (r_i^H, L_i^H) and (r_i^L, L_i^L), show that the interest rates for the loan contracts on the two sides of the processing fee threshold are the same. The loan size contributes the only possible variance in the loan contract. At the processing fee rate threshold, the equilibrium loan contract is (r_i^H, L_i^H) if the inequality 22 holds, and is (r_p^L, L_i^L) if the inequality 22 not hold.

$$\begin{aligned}
L_i^H \left\{1 + r_p^H + (1-p_{fi})r_i^H - c_k\right\} q_{ik}(\bar{p_{fi}}, r_i^H, L_i^H) \geq \\
L_i^L \left\{1 + r_p^L + (1-p_{fi})r_i^L - c_k\right\} q_{ik}(\bar{p_{fi}}, r_i^L, L_i^L) \\
\Leftrightarrow \\
L_i^H \left\{1 + r_p^H + (1-p_{fi})\bar{r_b} - c_k\right\} q_{ik}(\bar{p_{fi}}, \bar{r_b}, L_i^H) \geq \\
L_i^L \left\{1 + r_p^L + (1-p_{fi})\bar{r_b} - c_k\right\} q_{ik}(\bar{p_{fi}}, \bar{r_b}, L_i^L)
\end{aligned} \tag{22}$$

If this inequality 22 holds, we could observe bunching in the empirical approach.

Whether the PPP lenders choose to scale back loan size depends on borrowers' choice probability, which in turn depends on what contracts the lenders provide and the forgiveness rates. The comparison of the logit choice probability $q_{ik}(\bar{p_{fi}}, \bar{r_b}, L^H)$ with $q_{ik}(\bar{p_{fi}}, \bar{r_b}, L^L)$ already taken into consideration of the comparison of the loan value $v_{ik}(\bar{p_{fi}}, \bar{r_b}, L^H)$ with $v_{ik}(\bar{p_{fi}}, \bar{r_b}, L^L)$. Since the borrower's value function increases with the loan size, given a fixed forgiveness rate and interest rate at the processing fee threshold, borrowers care about the loan size, determining whether they take the loan contracts on the processing fee threshold. Borrowers always prefer larger loan sizes, which means that the inequality 22 is likely not satisfied. If so, no bunching is predicted at the processing fee threshold in this model.

II. Optimization by choosing the borrower's interest rate and loan size. Considering the special scenario of under policy intervention, the interest rate has been constrained, following the literature (Cox et al., 2020):

let $\tilde{c_i} = \frac{c_k - p_{fi} - r_p}{1 - p_{fi}}$

the lender's unconstrained profit maximization is :

$$\text{Max}_{R,L}(R_b - \tilde{c_i})L\frac{v_i^\sigma}{\sum_j v_{ij}^\sigma} \tag{23}$$

F.O.C with respect to R and L

$$\begin{aligned} \{R\}\, v_i &= (R_b - \tilde{c_i})\sigma(1 - 1/K)(1 - p_{fi})\delta_i L \\ \{L\}\, v_i &= \sigma(1 - 1/K)((1 - p_{fi})R_b\delta_i L - z_i\alpha L^\alpha) \end{aligned} \tag{24}$$

using the equation of the first order condition with respect to L, substitute the expression of v_i:

$$\begin{aligned} z_i L^{\alpha-1}[1+\sigma\alpha(1-1/K)] &= \sigma(1-1/K)(1-p_{fi})R_b\delta_i + (1-p_{fi})R_b\delta_i \\ L &= \left(\frac{(1-p_{fi})R_b\delta_i(1+\sigma(1-1/K)}{z_i(1+\sigma\alpha(1-1/K))}\right)^{\frac{1}{\alpha-1}} \end{aligned} \tag{25}$$

from the F.O.C, solving the unconstrained optimal loan size L^* and interest rate R^*:

$$\begin{aligned} (R_b-\tilde{c}_i)(1-p_{fi})\delta_i L &= (1-p_{fi})R_b\delta_i L - z_i\alpha L^\alpha \\ L^* &= \left(\frac{z_i\alpha}{\tilde{c}_i(1-p_{fi})\delta_i}\right)^{\frac{1}{1-\alpha}} \end{aligned} \tag{26}$$

$$\begin{aligned} R_{bik}(\alpha+\alpha\sigma(1-1/K)) &= \tilde{c}_i(1+\alpha\sigma(1-1/K)) \\ R_b^* &= \left(\frac{1+\alpha\sigma(1-1/K)}{\alpha+\alpha\sigma(1-1/K)}\right)\tilde{c}_i \end{aligned} \tag{27}$$

The $R_b^* = 1+r_b^*$ represents the unconstrained interest rate that borrowers would pay to banks in normal loans, which could not be observed in the PPP loans.

Then suppose R_b is given exogenously, for example, under policy intervention, but L is optimally chosen, to express L in terms of unconstrained L^* and exogenous R_b.

$$\begin{aligned} L &= \left(\frac{z_i(1+\sigma\alpha(1-1/K))}{(1-p_{fi})R_b\delta_i(1+\sigma(1-1/K)}\right)^{\frac{1}{1-\alpha}} \\ \left(\text{substitute } (L^*)^{1-\alpha} = \left(\frac{z_i\alpha}{\tilde{c}_i(1-p_{fi})\delta_i}\right)\rightarrow\right) &= \left(\frac{(L^*)^{1-\alpha}\tilde{c}_i(1-p_{fi})\delta_i(1+\sigma\alpha(1-1/K))}{\alpha(1-p_{fi})R_b\delta_i(1+\sigma(1-1/K)}\right)^{\frac{1}{1-\alpha}} \\ \left(\text{substitute } \tilde{c}_i = \left[\frac{\alpha+\alpha\sigma(1-1/K)}{1+\alpha\sigma(1-1/K)}\right]R_b^*\rightarrow\right) &= \left(\frac{(L^*)^{1-\alpha}R_b^*}{R_b}\right)^{\frac{1}{1-\alpha}} \\ &= L^*\left(\frac{R_b^*}{R_b}\right)^{\frac{1}{1-\alpha}} \end{aligned} \tag{28}$$

Suppose the unconstrained contract (r_i^*, L_i^*) is infeasible under the policy environment with rate cap $\bar{r}^L$ for $L < \bar{L}$ and $\bar{r}^H$ for $L > \bar{L}$. Let (r_i^H, L_i^H) represent the loan contract on the right side of the policy regulation threshold: $(r_i^H, L_i^H) \equiv (\bar{r}^H, L_i^*(\frac{1+r^*}{1+r^H})^{\frac{1}{1-\alpha}})$. let (r_i^L, L_i^L) represent the loan contract on the left side of the policy regulation threshold:

$$(r_i^L, L_i^L) \equiv \begin{cases} \left(\bar{r}^L, \bar{L}\right) \text{ if} L_i^* \geq \bar{L} \\ \left(\bar{r}^L, \min\left\{\bar{L}, L_i^* \left(\frac{1+r^*}{1+\bar{r}^L}\right)^{\frac{1}{1-\alpha}}\right\}\right) \text{ if} L_i^* < \bar{L} \end{cases} \tag{29}$$

The loan will be rationed if $p_{fi} + r_p + (1 - p_{fi})(1 + \bar{r}^H) - c_k < 0$. Otherwise, the equilibrium contract is (r_i^L, L_i^L) if inequality 30 hold, and is (r_i^H, L_i^H) if the inequality 30 fails to hold.

$$\begin{aligned} L_i^L \left\{p_{fi}^H + \bar{r_p} + (1 - p_{fi}^H)(1 + \bar{r}^L) - c_k\right\} q_{ik}(p_{fi}^H, r_i^L, L_i^L) \geq \\ L_i^H \left\{p_{fi}^L + \bar{r_p} + (1 - p_{fi}^L)(1 + \bar{r}^H) - c_k\right\} q_{ik}(p_{fi}^L, r_i^H, L_i^H) \\ \Leftrightarrow \\ L_i^L \left\{1 + \bar{r_p} + (1 - p_{fi}^H)\bar{r}^L - c_k\right\} q_{ik}(p_{fi}^H, r_i^L, L_i^L) \geq \\ L_i^H \left\{1 + \bar{r_p} + (1 - p_{fi}^L)\bar{r}^H - c_k\right\} q_{ik}(p_{fi}^L, r_i^H, L_i^H) \end{aligned} \tag{30}$$

If the inequality 30 holds, we would expect to observe a bunching response at the forgiveness policy regulation threshold for PPP loan borrowers. Whether borrowers accept the loan contracts at the threshold depends on their choice probability q_{ik}, which already considers their value function v_{ik}. Around the policy regulation threshold, the expected probability of loan forgiveness p_{fi} approval by the SBA can vary due to the different burdens of requirements. If we consider this variance in the borrower's value function, the choice probability $q_{ik}(p_{fi}, r_b, L)$ would be higher on the left side of the threshold, which means they are likely to choose to borrow just below the threshold.

Above all, this model did not consider fraud loans and analyzed the discontinuity incentives on different types of thresholds. The inequality 22 shows that borrowers are less likely to take loan contracts below the processing fee rate threshold if they originally could borrow larger loans. The inequality 30 indicates that even though the model does not include fraud cases, it still predicts that borrowers are likely to respond at the policy regulation threshold.

4 Data

In this study, I utilize the PPP dataset released by the SBA in June 2021. This dataset includes the two draws of PPP loans disbursed by U.S. lenders during the pandemic recession from 2020 to 2021. The summary statistics of the dataset are presented in Tables 2 and 3.

This dataset provides comprehensive information about PPP loan borrowers, including their small business location and characteristics, lenders' locations, and loan status. The dataset contains information about the borrower's name, address, city, state, zip code, business type, jobs reported, gender and ethnicity of the business owner, age of the business, NAICS Code, and the allocation of funds for various purposes such as payroll, utilities, mortgage interest, rent, refinance EIDL, health care, and debt interest. Additionally, the dataset also provides information about loan identification numbers, loan status, maturity period, SBA guarantee percentage, initial and current approval amounts, un-disbursed amounts, and district numbers. Information about lenders, such as originating and servicing lenders, lender location ID, name, address, city, state, and zip, is also available in the dataset.

4.1 Basic Statistic Results

Table 2 and 3 present summary statistics of the main variables in the PPP loans dataset. The dataset provides information about small businesses' characteristics, loan information, and lenders' information. The average PPP loan size for the two rounds of loans is approximately \$68k. As of the end of the second round, the average un-disbursed loan amount was \$23k, and the average number of jobs saved per loan was 7.6. PPP loans have been primarily used for payroll and other utility costs. The average spending on payroll was $65,801, while the second-largest spending was on rent costs, with an average spending of $668. Most borrowers had been in business for over two years. The majority of borrowers were located in urban areas.

Figure 3 illustrates the distribution of PPP loans in both loan size and logarithm scales, including data from 2020 and 2021. Panels a and b show the distributions of loan size in logarithm scale in normal density plots and kernel density plots, respectively. Panels c and d display the loan distributions separately in normal and kernel density plots. The PPP loan histograms exhibit log-normal distributions. Notably, the distributions show significant bunching around the loan size of $20,833 (log(loan)=9.94), which is the cap forgiveness amount per employee or for owner compensation, as observed from Figure 3 panels a and b.

Table 2: PPP Loans Summary statistics, 2020 & 2021.

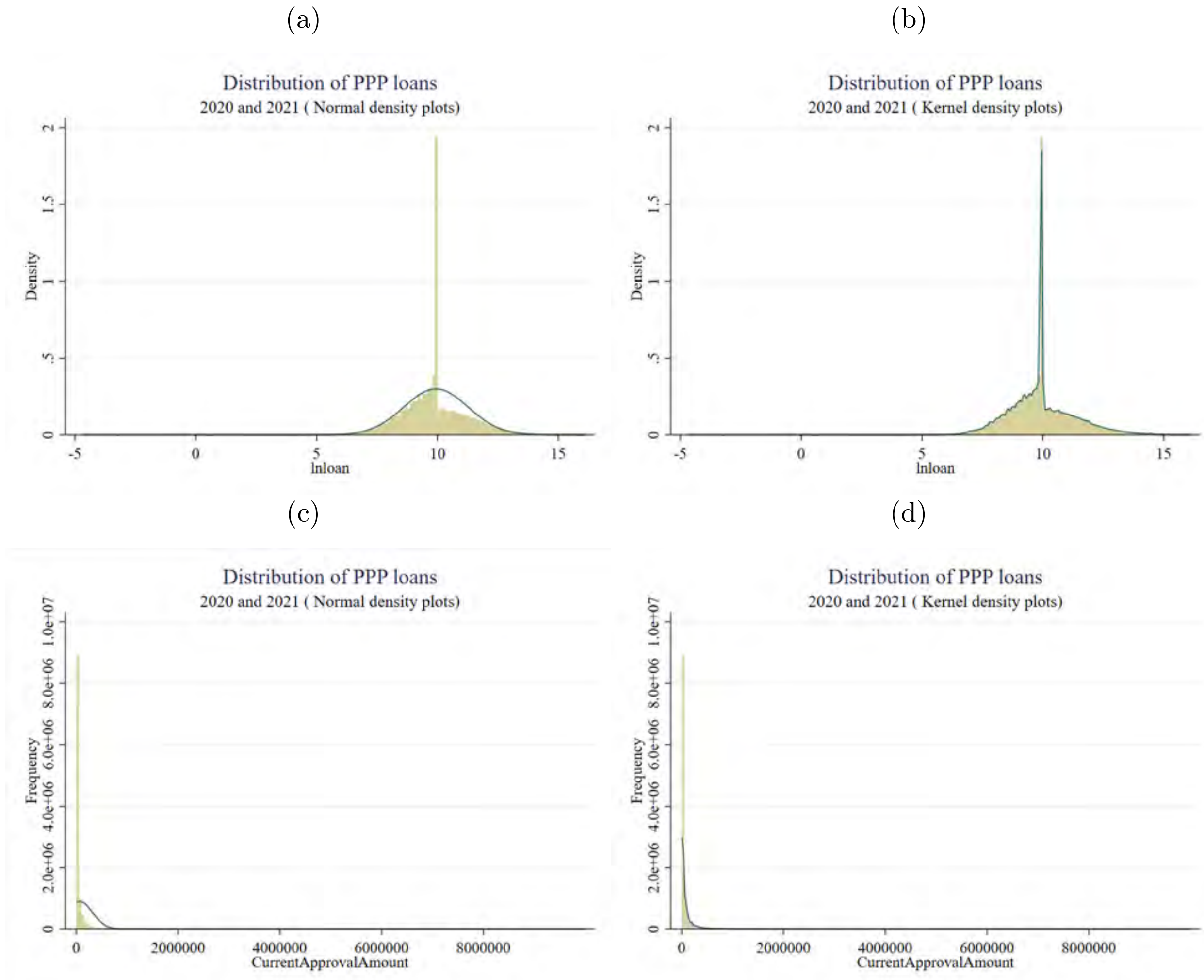
PPP Loans Summary Statistics				
	All Data			
All rounds of loans:	N	Mean	Median	S.D.
Initial approval amount ($)	11,823,594	67989.17	20680	256702.1
Current approval amount ($)	11,823,594	67647.31	20677	254095.8
Undisbursed amount ($)	11,823,594	2346.866	0	26973.69
Utilities proceed ($)	11,823,594	595.9758	0	14688.34
Payrolls proceed ($)	11,823,594	65801.49	20553	245870
Mortgage interest proceeds ($)	11,823,594	241.0344	0	10425.6
Rent proceeds ($)	11,823,594	668.397	0	11536.7
Mortgage interest proceeds ($)	11,823,594	241.0344	0	10425.6
Refinance EIDL proceed ($)	11,823,594	25.61878	0	2824.904
Health care proceeds ($)	11,823,594	265.0746	0	7904.919
Debt interest proceeds ($)	11,823,594	48.76153	0	2990.472
Demographics				
Jobs reported	11,823,594	7.642795	1	24.52318

Notes: Tables 2 and 3 show the summary statistic results that are calculated from the PPP loan dataset released by the SBA in May 2021. Including all the loan information at that time. In the process of estimations, I cleaned the extreme observations in this dataset.

Table 3: PPP Loans Summary statistics, 2020 & 2021. (Continue)

PPP Summary Statistic (continue)			
Demographics Characteristic variables:	Freq.	Percent	Cum.
Business Age Description			
Change of Ownership	2,200	0.02	0.02
Existing or more than two years old	10,583,763	89.51	89.53
New Business or two years or less	702,912	5.94	95.48
Startup, Loan Funds will Open Business	3,318	0.03	95.51
Unanswered	531,401	4.49	100
Rural-Urban Indicator			
Rural	2,494,234	21.1	21.1
Urban	9,329,360	78.9	100
Race			
American Indian or Alaska Native	85,631	0.72	0.72
Asian	295,164	2.5	3.22
Black or African American	901,966	7.63	10.85
Eskimo & Aleut	22	0	10.85
Multi Group	57	0	10.85
Native Hawaiian or Other Pacific Island	10,783	0.09	10.94
Puerto Rican	711	0.01	10.95
Unanswered	8,961,776	75.8	86.74
White	1,567,484	13.26	100
Ethnicity			
Hispanic or Latino	372,616	3.15	3.15
Not Hispanic or Latino	2,991,799	25.3	28.46
Unknown/Not Stated	8,459,179	71.54	100
N	11,823,594	100	

Figure 3: Histogram of PPP loans



Notes: Figure 3 depicts the histograms of PPP loans, both in loan size and logarithmic scales, for the years 2020 and 2021. Panel a and b present the distributions of loan size in logarithmic scale using normal density plots and kernel density plots, respectively. Panels c and d illustrate the loan distributions separately using normal and kernel density plots. The histograms of PPP loans exhibit log-normal distributions. Notably, a significant bunching is observed around ln(loan size)=10, which corresponds to the cap forgiveness or owner compensation amount of \$20,833.

5 Empirical Strategy

The field of public economics has developed a theoretical framework for estimating agents' responses to incentives. Specifically, this framework is designed to model situations where a continuous distribution of agents faces a piecewise linear schedule of incentives that changes at certain points. The agents' characteristic and starting point employed by most of the literature is that agents maximize an iso-elastic quasi-linear utility function under a tax rate fee scheme. The location of mass points where the slope or intercept of the fee scheme changes generates a data-generating process for optimal reported income, as evidenced in previous literature (Bertanha et al., 2021).

The width of the bunching segment is determined by two key factors: the notch (incentives change in intercept) and elasticity (how an individual responds to this discontinuity in incentives). By analyzing the notch and individual responses, the elasticity of individuals can be recovered. The empirical method relies on comparing the actual distribution to a counterfactual distribution (what the distribution would look like without the notch). By comparison, measure the extent of bunching and the missing mass, enabling the estimation of individual responses to the incentives (Kleven and Waseem, 2013).

5.1 Empirical Model Setup

In the scenario of PPP loans, the lender with the greatest market power (referred to as H) offers the largest loans before introducing the fee scheme notch, choose loan size $z^* + \Delta z^*$ before introducing the fee scheme notch. After introducing a fee notch, these lenders will be indifferent between locating at the notch point z^* and the interior point z^I. Meanwhile, the lender with the least market power (L) offers the smallest loans before the introduction of the notch and continues to choose a loan size of z^* after the introduction of the fee notch. Lenders with market power between H and L choose to locate at the notch point z^*. If there is homogeneity in elasticities, there will be a density hole in the post-notch distribution because no one is willing to choose a loan size between z^* and z^I. However, the density hole will be diminished if there is heterogeneity in elasticities or frictions to bunching.

The main objective is to construct a counterfactual distribution and use it to estimate the loan responses around the fee scheme threshold by comparing the fitted distributions to the empirical distribution. This estimation involves analyzing the data at the bin level and applying polynomial regression, as detailed in (Kleven and Waseem, 2013) in equation 31:

$$b_j = \Sigma_{i=0}^{p} \beta_i * (z_j)^i + \Sigma_{i=zL}^{zU} \gamma_i * \mathbf{1}[z_j = i] + v_j \tag{31}$$

Where b_j is the number of borrowers in bin j; z_j is the loan level in bin j; p is the polynomial order. Standard errors are bootstrapped by random resampling from the estimated residuals in 31. The basic idea is to fit counterfactual distribution from empirical distribution by excluding bins located in the bunching interval, which is conducted by excluding the second term $\Sigma_{i=zL}^{zU} \gamma_i * 1[z_j = i]$ in this equation to fit the regression. The counterfactual distribution is estimated as the predicted values from Eq 31 omitting the contribution of the dummies in the excluded range, that is,

$$\widehat{b_j} = \sum_{i=0}^{p} \widehat{\beta}_i * (z_j)^i \tag{32}$$

The use of counterfactual distribution enables the computation of the bunching mass and missing mass within the bunching interval, allowing for the derivation of the bunching estimator and the recovery of the elasticities of loan size with respect to fees. Specifically, the excess bunching (B) and missing mass (M) can be estimated by computing the discrepancy between the observed and counterfactual bin counts in the relevant loan ranges.

$$\widehat{B} = \sum_{j=ZL}^{Z^*} (b_j - \widehat{b_j}) \tag{33}$$

$$\widehat{M} = \sum_{j>Z^*}^{ZU} (\widehat{b_j} - b_j) \tag{34}$$

If the population exhibits heterogeneity in elasticities or experiences response frictions, then the proportion of loans in the dominated region D which are unresponsive can be estimated as follows:

$$\widehat{a^*} = \sum_{j \in D} b_j / \sum_{j \in D} \widehat{b_j} \tag{35}$$

The reduced form elasticities of loan supply with respect to the marginal processing fee rate are estimated using the approaches developed by (Kleven and Waseem, 2013) and by (Best et al., 2015a), which have been applied in the context of tax and labor supply. In the case of small business loans, the reduced-form elasticity of loan supply with respect to the processing fee rate is defined as follows:

$$\varepsilon_R \equiv \frac{\Delta z^*/z^*}{(R^* - R)/R} \tag{36}$$

Here, z^* represents the loan threshold for the schedule of incentives, and Δz^* is the loan amount response to the notch. $R^* \equiv 1 + r^*$ denotes the gross marginal rate of return, while $R \equiv 1 + r$ represents the gross interest rate below the notch.

The introduction of a processing fee structure and monitoring policies for borrowers in the PPP loans market piecewise-linear incentives schemes for market participants separately. This study hypothesizes that the notches in the SBA's PPP processing fee structure create potential motivations for lenders to originate smaller loans just below the thresholds. Higher-ability lenders may strategically work with their borrowers, leveraging existing relationships to assist them and prevent defaulting on previous loans.

To examine the impact of notches on PPP loan disbursement and identify loan distortions, this study employs the bunching estimation method developed by Kleven and Waseem (2013), Best et al. (2015a), and Harju et al. (2019) by determining bunching estimators and elasticities of loan size in relation to the incentive scheme.

5.2 Empirical Strategy

This section estimated the bunching estimators and elasticities of loan size with respect to commission fee rates at the loan thresholds of $350k and $2m. Firstly, the loan observations needed to be grouped into bin-level data and then run polynomial regression at the notches. However, the bunching approach is sensitive to the choice of several parameters, including the bunching window (or bunching interval), bin size, and polynomial order (Adam et al., 2015, as cited in Bosch et al., 2020). To address this, I followed the bunching window guidance provided by Harju et al. (2019) and used the Freedman-Diaconis rule to determine the bin size and number. The order of polynomials in Eq 31 was chosen using the Bayesian Information Criteria (BIC) method following the method provided by Bosch et al. (2020). The F.D. rule is a commonly used method for determining the bin size in a histogram. It is based on the interquartile range of the data and provides a way to estimate an appropriate bin size that considers the sample size and the variability of the data. The BIC is a criterion for model selection that balances model complexity and goodness of fit. In the case of polynomial regression, the BIC can be used to choose the polynomial order that best fits the data. Overall, these methods help to ensure that the estimation of bunching estimators and elasticities is robust to the choice of parameters and provides reliable results.

The counterfactual distribution refers to what the loan distribution would look like in the absence of bunching. It is obtained by distributing the fitted values in Eq 31, excluding the area affected by the notch point. The bunching area is estimated based on the bunching window, and it is the difference between the empirical density distribution and the counterfactual distribution in the excluded range. The bunching mass is then calculated using the following formula:

$$\hat{B} = \sum_{j=ZL}^{Z^*} (b_j - \hat{b_j})/\hat{b_j} \tag{37}$$

The bunching mass measures the extent to which the observed loan distribution deviates from the counterfactual distribution due to bunching behavior. The equation indicates that the estimated bunching mass is the percentage of the empirical density that exceeds the average height of the counterfactual density distribution in the bunching area. Once the bunching mass has been estimated using Eq 31, the elasticity of loan size with respect to the commission fee was calculated using the reduced form formula given in Eq 36.

This study distinguishes between two sources of notches: the nonlinear processing fee scheme designed for lenders and the discontinuity incentives designed for borrowers in the forgiveness policies.

This estimation used the subsample of loans borrowed only once, including 6,880,574 observations. To analyze counterfactual distributions in different periods, the data has been divided into two groups based on the borrowing time: before and after May 21, 2020. The reason for splitting the data into two groups is that May 21, 2020, was a significant date as it marked the announcement of the SBA's first Procedure Notice.

6 Results

6.1 Empirical Results for PPP Lenders

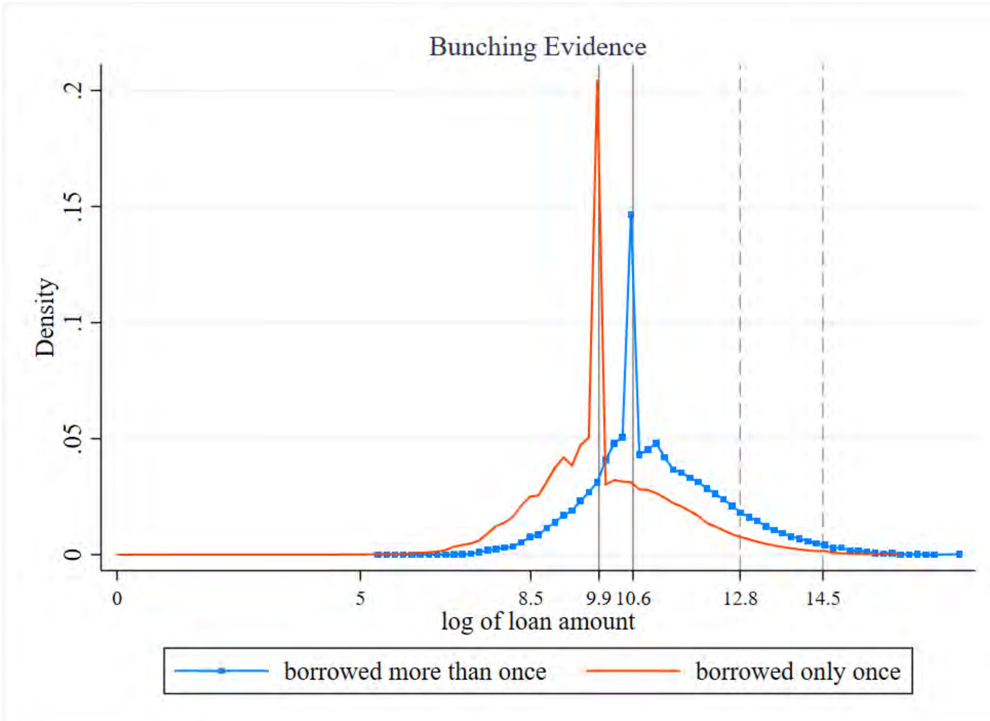
This study mainly explores the loan responses to the notch points at \$350k and \$2m, using the polynomial regression as shown in Equation 31, while leaving the thresholds of \$5k and \$50k for further analysis.

Figure 4 displays the empirical density distribution for loans borrowed once and more than once. The vertical solid lines show significant mass points at ln(loan) = 9.94 and

10.6, corresponding to the loan size of \$20,833 and around \$41,666, where small businesses borrowed once and twice at \$20,833. The vertical dash lines also indicate the cutoff points at ln(loan) = 12.8 and 14.5, which correspond to the natural logarithm of loan sizes of \$350k and \$2m, respectively. However, no significant bunching was observed at these threshold points. This study focuses on estimating borrowers who only received PPP loans once.

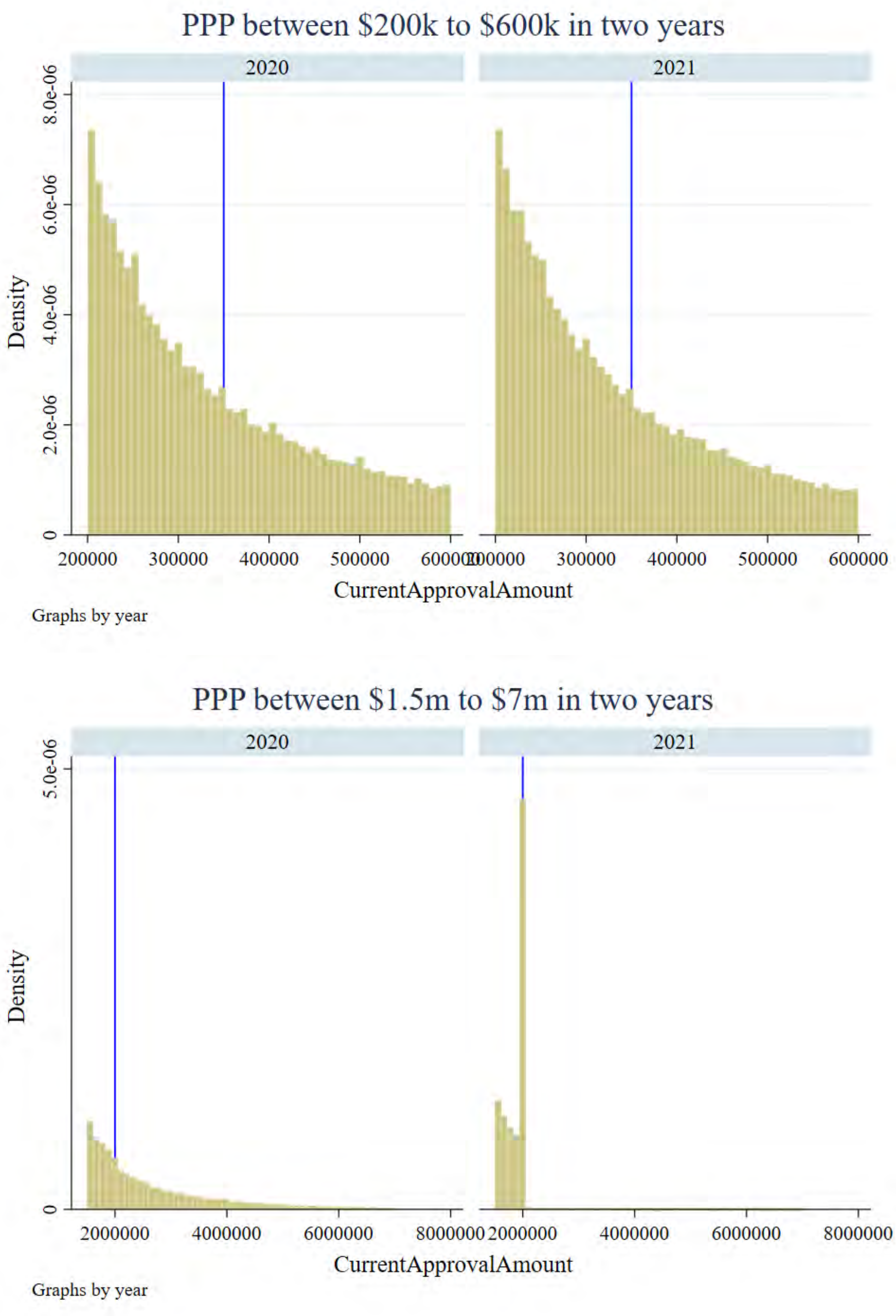
Figure 5 displays the histograms for PPP loans separately for 2020 and 2021 at loan ranges \$200k to \$600k and \$1.5m to \$7m that cover the primary thresholds of \$350k and \$2m generated from the processing fee scheme. The blue vertical lines in each panel show the cutoff points. There is a small number of loans at the threshold of \$350,000 in panels a and b, while a significant number appears at the threshold of \$2 million in 2021, as shown in panel d. The large bunch in panel d could be attributed to the policy changes that set the total loan amount to this loan size.

Figure 4: Bunching Evidence



Notes: Figure 4 displays the distribution of PPP loans in natural logarithm scale, separating the entire sample into two groups: firms that borrowed only once and those that borrowed more than once. The x-axis is presented in logarithmic scale, with 9.96 representing the logarithm of a loan size of $20,833, which is the cap forgiveness amount per employee or owner compensation, and 10.6 representing the logarithm of a loan size of twice that amount, or $41,666. The vertical dotted lines indicate the processing fee notches at loan sizes of $350k and $2m, whose logarithmic equivalents are 12.8 and 14.5, respectively. Notably, the figure shows significant bunching at loan sizes of ln(loan)=9.94 and 10.6, which may be explained by small businesses borrowing at $20,833 once and twice. Conversely, the expected significant bunching mass at the thresholds of $350k and $2m is not clearly evident in this figure.

Figure 5: Histograms of PPP Loans around two thresholds in Two Years.

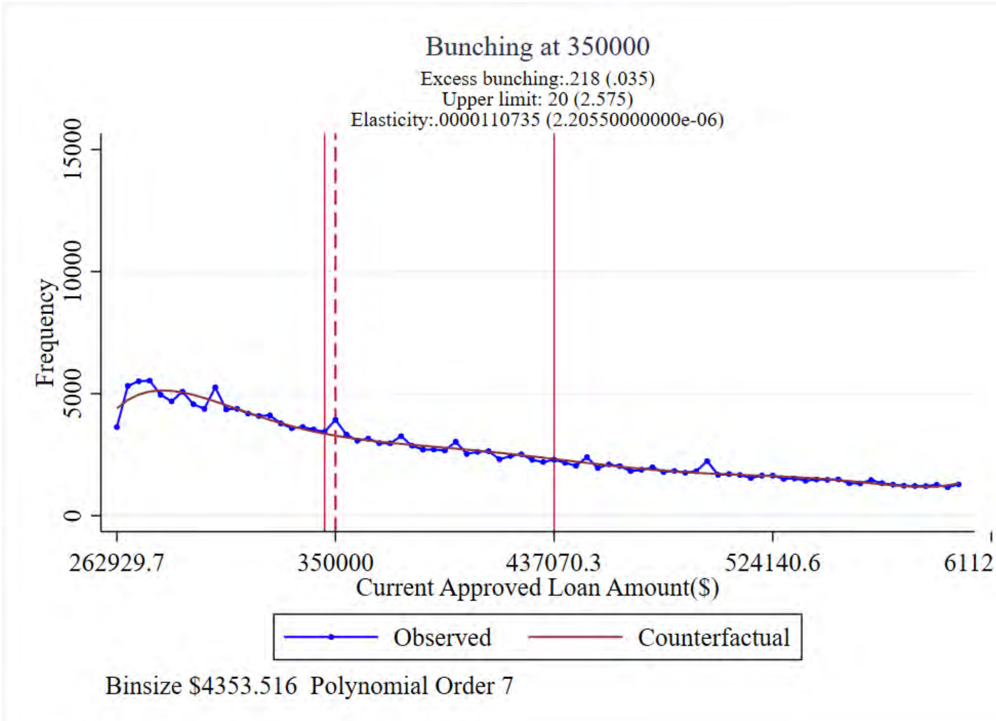


Notes: Figure 5 displays four panels labeled as a, b, c, and d, respectively. Panel a and b show PPP loans ranging from $300k to $600k, covering the threshold of $350k. Panels c and d show PPP loans ranging from $1.5m to $7m, covering the threshold of $2 million. The data from 2020 are on the left panels, while the data from 2021 are on the right panels. Solid lines mark the main notch points. The solid lines in panels a and b show the threshold at $350,000, where the notch in the discontinuity in fee rates generates a dominant loan range from $350,000 to $583,000. The solid lines in panels c and d show the threshold at $2 million, where the dominant loan range is between $2 million to $6 million. In 2021, the SBA changed the maximum loan amount from $10 million to $2 million. Thus, the bunching mass observed at $2 million was not in response to the discontinuity fee scheme.

6.1.1 Bunching Estimation Results, whole sample at Loan Scale and Logarithm Scale

Figure 6 shows the estimation results at the threshold of $350k, using sub-sample data covering this threshold. Each panel displays an empirical density distribution of the loans and a fitted counterfactual distribution, which is estimated from a polynomial regression for each threshold. Precisely, the counterfactual distribution is estimated by running polynomial regression at the bin level, separately obtaining the fitted value by excluding the bins in the range impacted by the notch point. Figures 6 illustrate the threshold point at $350k. Each panel adjusts the bin size according to the Freedman-Diaconis rule to ensure that the notch point is bin-centered. Specifically, Figure 6 shows that bin width is $4,353. Bayesian Information Criterion (BIC) has been used to select the polynomial order for each regression. The estimated loan ranges were defined by +/-15% of the threshold.

Figure 6: Bunching Estimation at loan size of $350k



Notes: Figure 6 shows the bunching estimation results at loan size of $350k. The figure shows the empirical density distribution of the loan amounts for borrowers clustered in bins (dotted blue graph) and the estimated counterfactual density (solid red graph). The X-axis is the current PPP-approved loan amount; the Y-axis is the counts of observations in each bin. The bunching estimator $\hat{b}$ is the excess mass in the excluded range. Bin widths have been selected by the Freedman-Diaconis rule (F.D. rule). The bin size for the bunching estimation is $4,353. The notch point is at the bin center. BIC has been used to select the polynomial order in polynomial regression. Here, the optimal order is seven. The counterfactual density distribution is estimated by fitting a seventh-order polynomial. The lower bound of the excluded range (or, say, the bunching window) is defined by two bins on the left side of the notch point, while the upper bound of the excluded range is defined as 20 bins on the right side of the threshold point. This is determined by the iteration process together with the polynomial regression. The bunching window for this threshold has been selected by the iteration process proposed at (Kleven and Waseem, 2013) and (Harju et al., 2019). Vertical dashed lines mark notch point; vertical solid lines mark excluded ranges' lower and upper bounds. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

Figures 6 illustrate the estimation results using the whole dataset at the threshold of \$350k. The threshold of \$2m is not included because the SBA lowered the maximum loan amount from \$10 million to \$2 million in 2021; the estimation results would be biased if the whole sample were taken \$2 million as a threshold. As displayed in the graph, PPP lenders did not take significant responses to the incentives created by the thresholds at \$350k.

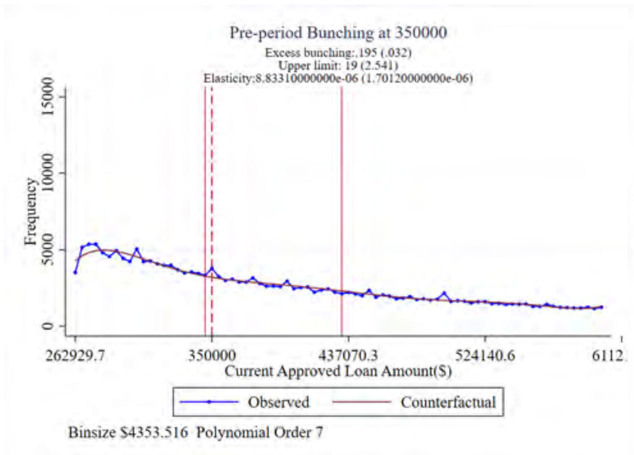
6.1.2 Bunching Estimation Results, counterfactual distribution fitted from pre-period data

Policy regulations were unclear in the early stage of this small business revenue replacement program, as rules varied several times. The lending market participators, including lenders, received incomplete information about the program. The first SBA procedure notice that informed PPP lenders of the reporting PPP loans process and collected the processing fees on fully disbursed loans was announced on May 21, 2020. That could be considered a positive shock to the PPP lenders. The follow estimation use the the dataset been splitted at May 21, 2020, and further estimated the counterfactual density distributions by fitting them with the data from the pre- and post-announcement periods. The results are shown in Figure 8 and Figure 9 (loans in logarithm). The left side panels display the empirical density distribution with the fitted counterfactual density distribution from the pre-announcement period dataset. In contrast, the right-side panels display the empirical density distribution with the fitted counterfactual density distribution from the post-announcement period dataset. Lenders take fewer responses to this loan size in the pre-and post-announcement periods. The threshold results at \$2m in 2021 not included. Because the threshold of \$2m is the maximum loan amount in 2021, the bunching estimation result would be biased for 2021.

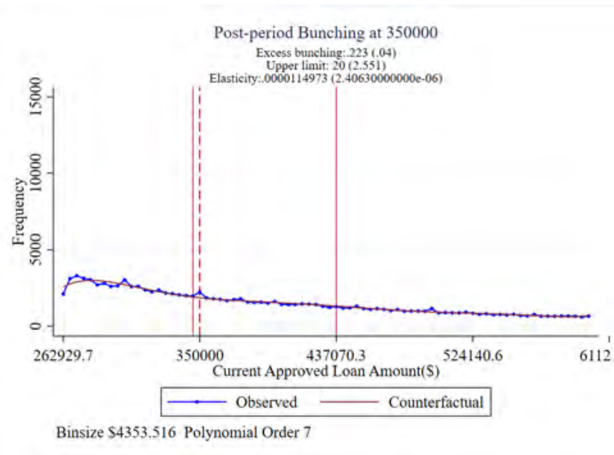
Notes: Panels a. and c. display the empirical and counterfactual density distributions at each threshold before May 21, 2020, when the SBA procedure notice was announced. These corresponding subsamples fit counterfactual density distribution; Panel b is the empirical density distributions and counterfactual density distributions at a threshold of \$350k after May 21, 2020. The Blue dotted line is the empirical density distribution of loans; the dark red line is the fitted counterfactual distribution. The only policy shock on May 21, 2020, was the announcement of the SBA procedure notice of the lender's processing

Figure 7: Bunching estimation at each threshold for different periods

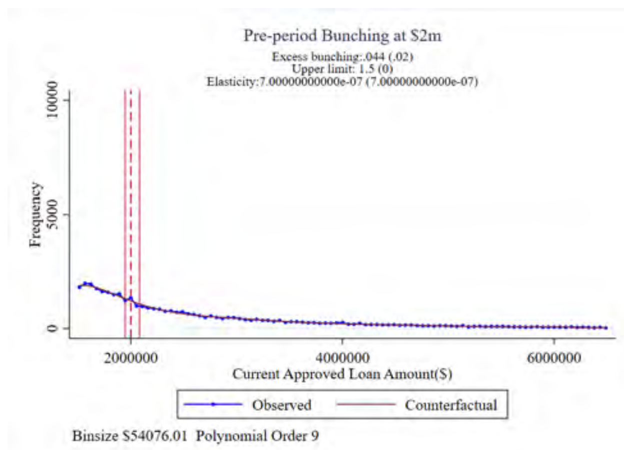
(a)



(b)



(c)



fee guidance. The bunching window for each panel has been selected by the iteration process proposed by (Kleven and Waseem, 2013) and (Harju et al., 2019). The Freedman-Diaconis rule (F.D. rule) has selected the Bin width for each panel. BIC has selected the order of polynomials in each regression. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

Table 4: Estimating bunching responses at each threshold.

Bunching Estimation at Each Threshold						
	Full sample responses		Pre-period responses		Post-period responses	
	(1) Bunching(b)	(2) Elasticity(ε)	(3) Bunching(b)	(4) Elasticity(ε)	(5) Bunching(b)	(6) Elasticity(ε)
At threshold of $350,000	0.218	0.000011	0.195	8.83e-06	0.223	0.000011
	(0.034)	0	(0.033)	0	(0.035)	0
At threshold of $2000,000	3.196	0.004	0.044	7.00e-07	-	-
	(0.079)	0	(0.021)	0	-	-

Notes: Table 4 presents the estimated bunching mass and the elasticities of loan size with respect to fees at each threshold for different periods. Columns 1 to 2 display the estimation results from the total sample; columns 3 to 4 display the estimation results from the pre-period sample (before May 2020); columns 5-6 display the results from the post-period sample (after May 2020). Columns 1, 3, and 5 show the bunching estimator b hat, based on Eq 37. Bunching estimator b hat is the excess mass in the excluded range around the notch point, in proportion to the average counterfactual density in the excluded range. Columns 2, 4, and 6 present an estimate of the elasticities of loan size with respect to the fee structure. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine

Tables 4 show the bunching estimation results from Figure 6 to Figure 7. In Table 4, the bunching estimation results were collected from all the panels of Figure 6, including the bunching estimation for the whole sample; and from all the panels of Figure 7, including the bunching estimation at each threshold in pre- and post-period. We could observe the dynamically changed behavioral responses to the SBA regulation policies by comparing the estimation results from two periods. No significant bunching mass has been observed at the loan size at $350k and $2m.

Lenders lacking of responses to the discontinuity incentive scheme for possible several reasons. The first possible explanation is that small business borrowers applied for PPP loans on the SBA E-tran platform, where they can apply for loans from many different lenders. This means the PPP funds market is highly competitive. Thus, none of the lenders has the monopoly or negotiation power on the loan contracts. Additionally, if they tried to make more commission fees by adopting the optimal strategy, their customers would reject the loan contract and turn to other lenders immediately. Thus, their optimal strategies were not feasible due to the competition. The second possible explanation is that lenders were not sensitive to the processing fee scheme because they did not take the processing fees as an essential profit component. The third possible reason is that those loans were disbursed quickly, and the limited time window did not allow lenders to respond precisely to loan thresholds to increase their profits. Alternatively, they might care more about their whole lending amount. To disburse loans quickly to small businesses adversely affected by the economic shutdown, they have minimal time to make optimal choices. Besides, banks probably have built previous relationships with existed borrowers and would like to issue as much as their borrowers wants for keeping the good relationships in the future. Furthermore, their optimal strategies are contrary to policy goals. The last possible reason is that those lenders decided to respond to the average price instead of the marginal price at each threshold.

Following the analysis of the two-dimensional model by Cox et al. (2020), the empirical results reflect that inequality 14 does not hold. Banks offer loan contracts, but whether small borrowers accept these contracts depends on the borrower's choice probability. Borrowers only observe their expected payback interest rate, which is fixed at 1% if the loan cannot be forgiven. The fixed interest rate can be interpreted as their expected marginal price. They can be aware of the surplus changes for different sizes of loans. This feature determines that borrowers first would consider if the SBA can forgive them. The borrower's expected loan value increases in loan size. In contrast, banks' processing fee rates can only be observed by lenders. Therefore, borrowers' choice probability should only be determined by their loan size. Thus, they would reject contracts $(\bar{r_p}^H, L_i^H)$ with smaller loan sizes and prefer to take contracts $(\bar{r_p}^L, L_i^L)$ with larger loan sizes. If all lenders' pure Nash equilibrium strategies offer the contract $(\bar{r_p}^L, L_i^L)$, then we could not observe a significant bunch at the threshold of processing fee rates.

As the analysis proposed by (Almunia and Lopez-Rodriguez, 2018), the average bunching estimator $\hat{b}$ measures the average loan response of the bunchers and the lack of response of non-bunchers to the regulatory policies, by multiplying the bunching estimator $\hat{b}$ by the bin width w. The bunching estimators in table 4 can be interpreted as follow: At the loan threshold of \$350k, the bunching estimator $\widehat{b_{350k}} = 0.218$ (with standard error 0.034, statistically different from zero at the 0.1% level). Given that the estimation process set bin width at \$4,353 at this threshold, that indicates the average loan size is $0.218 \times \$4,353 = \$948.954$ which is approximately 0.27% of their borrowed loan size in response to the discontinuity commission at the threshold of \$350k. In the year 2020, the bunching estimator $\widehat{b_{350k}} = 0.195$ (with standard error 0.033, statistically different from zero at the 0.1% level), the average loan response is $0.195 \times \$4353 = \$849$ in response to the notch. In 2021, the bunching estimator $\widehat{b_{350k}} = 0.223$, and the average loan response is $0.223 \times \$4353 = \$971$ in response to the notch. The average loan response each year is approximately 0.2% of the borrowed loan size.

This study only estimated the bunching responses at \$2 m at 2020. The estimated bunching estimator $\widehat{b_{2m}}$ is 0.044 (with a standard error of 0.021, statistically significant at the 0.1% level), indicating a significant response to the notch point. The bin width used in estimation around \$2 million is \$54,076, so the average loan response is estimated to be $\$0.044 \times 54{,}076 = 2,379.344$ in response to the notch point.

By estimating the PPP loan "bunching" responses, the government can have an integrated understanding of the SBA's regulation policies' efficiency and can find room for better policies in the future. By understanding different behavioral responses at different policy thresholds, government policymakers could adjust their anticipation at variant policy thresholds.

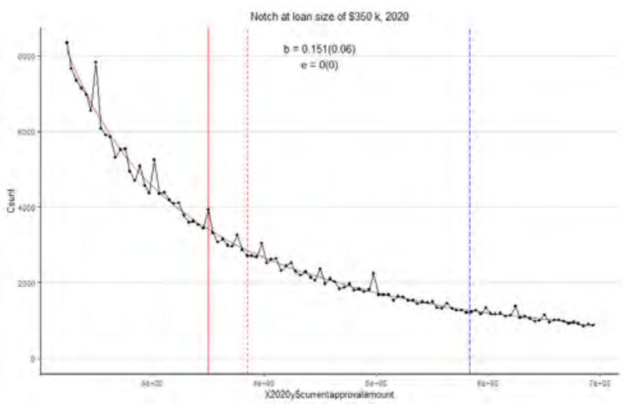
6.1.3 Robustness Results

In this section, I re-estimate the main bunching estimation results using a slightly adjusted bunching method proposed by (Mavrokonstantis, 2019). Besides applying the traditional bunching approach, this method can exclude the round number bunching and minor bunching near the central bunching region. This method is estimated on a loan scale and counted amount.

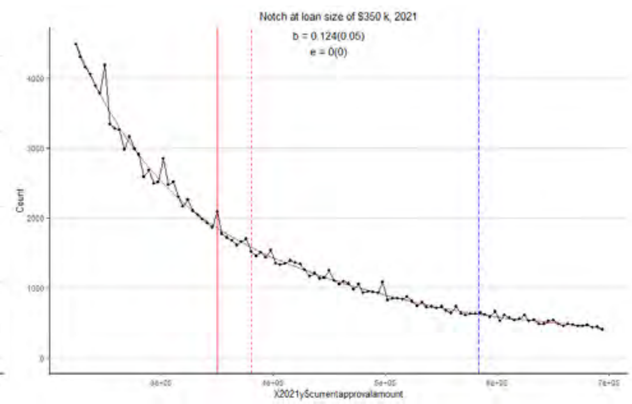
Figure 8 and Table 5 display the robustness bunching estimation at each threshold for different periods. The results did not include the threshold of \$2m in 2021 because \$2m is the maximum lending loan amount in 2021. After controlling for round number bunching and minor bunching near the primary thresholds, the robustness estimation results coincide with the previous results. These bunching results show that bunching estimators at the loan size of \$350k are $\widehat{b_{350k}} = 0.151$ (standard error is 0.06) in the year 2020, and $\widehat{b_{350k}} = 0.124$ (standard error is 0.05) in the year of 2021. These results are slightly smaller than the previous estimation, which could be the robustness estimation controlled for the round number bunching and minor bunching around the main threshold. The bin width in this estimation is chosen the same amount as the previous estimation because the FD rule has chosen it.

Figure 8: Robustness estimation for each loan threshold in 2020 and 2021.

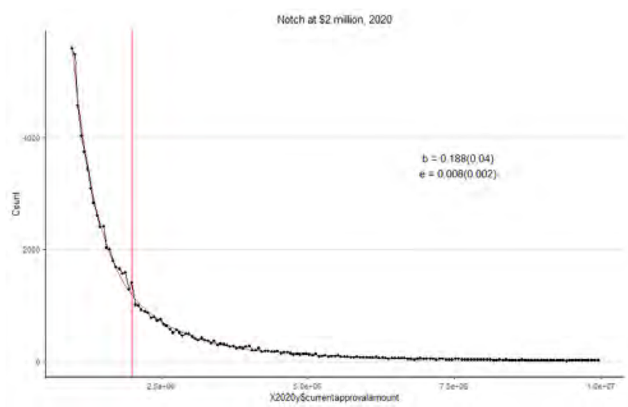
(a)



(b)



(c)



Notes: Figure 8 displays the robustness estimation at $350k and $2m. Panels a, and c, display the empirical density distributions and counterfactual density distributions at each threshold in 2020. The X-axis shows the loan scale. The Y-axis shows the counted loan number. The accordingly subsample fits counterfactual density distributions. Panel b displays empirical and counterfactual density distributions at the threshold of $350k in 2021. The black dotted line is the empirical density distribution of loans; the dark red line is the fitted counterfactual distribution. The iteration process has selected the bunching windows for each threshold and the order of polynomials in each regression. The standard errors shown in parentheses are obtained by bootstrapping the estimation routine 500 times.

Table 5: Robustness bunching estimation at each threshold for the years 2020 and 2021

Robustness estimation at each threshold				
	2020 bunching responses		2021 bunching responses	
	(1) Bunching(b)	(2) Elasticity(ε)	(3) Bunching(b)	(4) Elasticity(ε)
At threshold of \$350,000	0.151	0	0.124	0
	(0.06)	0	(0.05)	0
At threshold of \$2000,000	0.188	0.008	-	-
	(0.04)	(0.002)	-	-

Notes: Table 5 shows the bunching robustness estimation results at thresholds of \$350k and \$2m. The result of \$2m in 2021 did not include in this table because the \$2m is the maximum loan amount in 2021.

6.2 Empirical Results for PPP borrowers

As described previously, there are two types of fee notches. One exists in the lender's processing fee scheme, and another in the borrower's loan size-dependent monitoring policies. The second one can have further implications on borrowers' loan forgiveness applications. Empirical estimation results show that borrowers might play a vital role in the behavioral responses incentivized by regulations in this PPP lending market. As an entirely forgivable government relief program, the PPP could either be forgiven or repaid with a 1% interest rate. Thus, the probability of taking different interest rates, or the borrower's expected forgiveness rate, plays a crucial role in the borrower's decision-making process when borrowing loan size at monitoring policy thresholds. At the lender's processing fee rates thresholds, even though the fee notch can incentivize lenders to scale back loan size, the probability of whether to accept the loan contract depends on the borrowers. Lenders adopt a pure Nash Equilibrium Strategy. The inequality in the two-dimensional bunching model can more accurately explain the behavioral responses of these two market participants when borrowing happens at the lender's processing fee thresholds.

Although the government has guaranteed the PPP funds to be fully forgivable, small firms still must meet certain policy requirements for applying forgiveness. Their understanding of the policies would affect the borrowing loan amount. The $20,833 and $150,000 are two essential thresholds because the policy requirements differ between the two sides of the thresholds.

The PPP funds were established to maintain employee and firm relationships. Therefore, this revenue replacement program aims to cover 60% of the borrower's payroll payments. The maximum forgiveness amount per employee or owner compensation is determined by the maximum annual salary level, set at $100,000. This means that the maximum forgiveness loan amount per employee is calculated as follows: $100,000 per year divided by 12 months multiplied by 2.5 months (if the borrower chooses 24 weeks of covered period), resulting in \$20,833. If a firm employed more than one person, it would calculate the total covered payroll payment based on this amount. Thus, the loan amount of \$20,833 represents the maximum forgiveness amount per employee or owner compensation but does not necessarily indicate the maximum loan amount for total borrowing.

The calculation for the maximum forgivable loan amount per employee exhibits slight variations across different periods and industries. Before June 5, 2020, when the covered period was eight weeks, the maximum forgiveness amount was \$100,000 per year divided by 52 weeks and multiplied by 8 weeks, resulting in \$15,385 instead of \$20,833. Additionally, the multiplier for the "Accommodation and Foodservice" sector, identified by NAICS code 72, differs from all other sectors. In this sector, the multiplier is 3.5, instead of the standard 2.5, leading to a maximum loan amount per employee of \$100,000 per year divided by 12 months and multiplied by 3.5 months, resulting in \$29,167 instead of \$20,833. Furthermore, it's noteworthy that the portion of the loan allocated for Owner Compensation does not contribute to the payroll payment and, consequently, is exempt from the SBA's 60/40 spending rule for forgiveness. Small businesses borrowing below this threshold can easily secure forgiveness from the SBA. Upon examining loan distribution at various sizes, including \$15,385, \$20,833, and \$29,167, a notable concentration of loans was observed specifically at the \$20,833 level.

According to the SBA's policy, the maximum forgiveness amount is determined by the owner-employee and self-employed individual's 2.5 months' worth of 2019 or 2020 net profit, known as the Owner Compensation Replacement. Because self-employed individuals do not pay themselves through payroll, this concept allows them to claim a portion of the PPP loan to compensate for lost income due to the pandemic. Anyone who files with a Form 1040 Schedule C can claim the Owner Compensation Replacement, regardless of whether they have employees. Given that this cap amount applies only to a portion of the total loan, whether small business borrowers borrowed for their employees or themselves, the total loan amounts were not necessarily restricted to \$20,833.

Policy requirements diverge for loans at the $150k threshold. Small borrowers below this limit may seek loan forgiveness without extensive paperwork by utilizing SBA form 3508S for their First or Second draw PPP loans. Conversely, those surpassing this threshold must opt for forgiveness via SBA form 3508 or 3508EZ. Notably, form 3508S demands fewer calculations and less documentation, streamlining the process for eligible borrowers who stay under the $150k threshold. Unlike the other forms, 3508S does not mandate revealing the calculations behind the loan forgiveness determination, resulting in varying monitoring levels on either side of the threshold. The forgiveness application process is simplified for loans under $150k.

I implement the traditional bunching approach to estimate loan responses and elasticities of loan size with respect to the interest rate at these forgiveness policy thresholds for borrowers.

The method for the bin size choosing and the optimal polynomials' determination follow (Bosch et al., 2020). On each threshold, I keep +/-25% of the sub-sample. That is, for estimating the threshold at the loan size of $20,833, the data set was kept for loan size from $15,624 to $26,041; for estimating the threshold at $150,000, the data set was kept for loan size from $112,500 to $187,500. I implement the Freedman-Diaconis rule to determine the bin width in each loan range. The optimal bin width is $14.12966 for the threshold at $20,833 (loan range from $15,624 to $26,041, with optimal bin number 738), and the optimal bin width is $881.977 for the threshold at $150,000 (loan range from $112,500 to $187,500, with optimal bin number 85), separately.

Furthermore, I apply the BIC criterion to determine the optimal order of polynomials in polynomial regressions to obtain the counterfactual distribution for the loan range separately. The optimal polynomial order for the polynomial regression is q=1 for the loan range which includes the threshold at $20,833, and q=9 for the loan range which includes the threshold at $150,000, separately.

These results provide insights into the behavior of small business borrowers and how they responded to the PPP during the pandemic.

Figure 9 intuitively shows histograms of the PPP loans at ranges from \$0 to \$60,000 and from \$100,000 to \$300,000 in 2020 and 2021. If comparing the distributions in 2020 and 2021, it is clear that there is a growing pattern for the loans borrowed at each threshold, which blue lines have marked. This might be evidence that the optimal borrowing strategy requires time to figure out.

Figure 10 illustrates the bunching responses at the notch points \$20,833 and \$150k. In contrast, figure 11 shows the logarithm scale estimation of these two points. When creating the bins level data, it is necessary to ensure that the notch point is bin-centered at each threshold and that the bin size is adjusted accordingly in each panel. All the bin widths were selected by the Freedman-Diaconis (FD rule). Figure 10 defines bin width equals \$10 for panel a, \$882 for panel b. In Figure 11, the bin width in each panel is 0.00726. BIC has been used to choose the optimal order of polynomials in each corresponding polynomial regression.

Panel a in Figures 10 and 11 show a significant bunching mass. Panel a illustrates borrowers' responses to the SBA regulation at the loan size of \$20,833 (the logarithm of this loan amount approximately equals 9.94). That is the maximum forgivable amount per employee and the Owner's Compensation. If the borrowed loan size per employee or Owner Compensation is just below this amount, this loan amount is automatically eligible for forgiveness. If the total loan size exceeds this amount, the exceeded portion needs to meet some requirements to be forgiven by the SBA. This means the application burden differs on two sides of this threshold. Thus, the probability of small firms potentially being forgiven and, thus, zero interest rates below the point is higher. Small business borrowers might consider adjusting their loan application depending on their expectation of the potential interest rate around the threshold.

Panel b in both Figures 10 and 11 estimate the responses from borrowers to the SBA regulation at the loan size of $150k. The burden of the application process varies on either side of this threshold, as it necessitates different paperwork. Opting for a loan below this specified size makes the forgiveness process considerably more straightforward.

Amidst the pandemic, many small businesses opted to increase their debt levels to capitalize on available financial opportunities, often seeking to access grants or funds without the immediate intent of repayment. As a consequence, they sought to sidestep further financial strains. Specifically, numerous businesses borrowed below the threshold with PPP loans to ensure forgiveness.

The interpretation of bunching estimators as an average response stems from the assumption of heterogeneity among small businesses. Following the analysis in (Almunia and Lopez-Rodriguez, 2018), the average bunching estimator $\hat{b}$ is a weighted average of the bunchers' loan response and the non-bunchers' lack of response to regulatory policies. I multiply the bunching estimator $\hat{b}$ by the bin width w to derive a monetary metric. The bunching estimators presented in Figure 10 can be understood based on these calculations.

At the threshold of $20,833, $\widehat{b_{20833}} = 595.436$ (standard error 21.851, statistically different from zero at the 0.1% level) with a bin width of $10. This estimator suggests that small businesses initially within the bunching interval, on average, decrease their loan size by approximately $5,954.36 (roughly 28 percent of their borrowed loan size) in response to the regulatory policies.

At the threshold of $150k, $\widehat{b_{150k}} = 1.808$ (standard error 0.248, statistically different from zero at the 0.1% level) with a bin width of $882. This implies that borrowers within the bunching interval initially reduce their loan size by an average of $1,594.656 (approximately 1 percent of their borrowed loan size) in response to the SBA small business application regulation policies.

In the initial phase of this small business revenue replacement program, policy regulations needed to be clearly defined for many new participants and underwent several changes. Participants in the lending market required more detailed information about the program. The first notice outlining SBA procedures was issued on May 21, 2020. This notice guided how lenders could apply for processing fees from the SBA upon completing loan disbursement. The announcement could be perceived as a positive policy shock for lenders participating in the PPP.

Lenders and borrowers may require time to familiarize themselves with the details of this program. Additionally, I proceed to estimate the counterfactual density distributions by fitting them with data from both pre- and post-announcement periods, illustrated in Figure 12 and Figure 13 (depicting loans in logarithm). Panels a and c present the empirical density distribution alongside the fitted counterfactual density distribution during the pre-announcement period.

On the contrary, panels b and d illustrate the empirical density distribution along with the correspondingly fitted counterfactual density distribution in the post-announcement period. These figures reveal significant variations in loan response at the thresholds of $20,833 and 150$k between the two periods.

The trend in loan bunching response appears to be on the rise over time. This observation suggests that participants took time to grasp and adopt optimal strategies. Despite being capped, some eligible small businesses borrowed larger loan amounts (as demonstrated in Figure 15, where I estimated bunching responses for different firm sizes). This suggests that once participants became familiar with the policies, they strategically responded to the discontinuous incentives created by the SBA regulation. Consequently, the bunching responses in the pooled sample were primarily influenced by the post-announcement period of PPP loans.

To assess the policy impacts between the two periods, Figure 14 employs pre-announcement period data to fit the counterfactual density distribution. Subsequently, the bunching mass, the elasticity of loan size w.r.t. commissions, and the change in loan size are calculated by comparing the empirical density distribution of the post-announcement period data with the counterfactual density distribution fitted by pre-period data. As depicted in this figure, the substantial bunching mass and the significant elasticity of loan size w.r.t. fee schemes suggest robust behavioral responses in the post-announcement period.

To better understand the loan bunching phenomenon around $20,833, I examined job reports submitted by small businesses that borrowed loans at this threshold. The examination revealed that most of these businesses reported having either zero or one employee, while some indicated having up to five hundred employees. This indicates that the eligibility for this specific loan amount extends to smaller entities and larger firms or their branches.Larger firms were not precluded from borrowing at this loan size based on per-employee or owner compensation. The responses from different groups of firms in 2020 are illustrated in Figure 15.

In Panel A of Figure 15, the borrowers reporting one or fewer employees constitute a significant portion of the excess mass at this threshold. As previously examined, even though this group reported one or fewer employees, the loan funds attributed to owner compensation or per employee represented only a fraction of the total loan. Nevertheless, for these small firms to secure full forgiveness of their loans from the SBA in the subsequent months, it became apparent that considering this loan size as the total amount would be their optimal strategy after they gained insights from the policies.

Panel B showcases small businesses reporting more than one employee. Small businesses with more than one employee were apparently not constrained by this low amount of $20,833. However, despite not being restricted by this threshold, this graph exhibited a significant bunching, contributing to the total excess mass at this threshold.

These panels collectively illustrate that \$20,833 is not a universal cap for small businesses with up to one employee. Furthermore, businesses with more than one employee, ranging up to five hundred employees, were not restricted to borrowing exclusively at \$20,833. However, it is surprising that many of them opted to borrow precisely this amount.

The allowance for owner's compensation enabled firms to claim a portion of the PPP loan to offset lost income for the business owner due to the pandemic. Consequently, the substantial excess mass at this threshold indicates that small business borrowers strategically opted to reduce their loan size. The model analyzed in Chapter 2 demonstrates that small borrowers choose a loan contract based on their highest choice probability. If they opt to borrow just below the threshold, then Inequality 30 holds for these borrowers at this loan size.

Several small business borrowers recognized that borrowing at this threshold could represent an optimal strategy for their financial situation. The estimation results for bunching at each threshold and across different periods are detailed in Tables 7 and 8, encapsulating insights from Figure 10 to Figure 13.

The bunching estimation results were aggregated from all the panels of Figure 10, including the bunching estimation for the entire sample, as well as all the panels of Figure 12, encompassing the bunching estimation at each threshold in the pre-period and post-period.

Significant bunching is observed, especially at the loan thresholds of \$20,833 and \$150k, particularly in the post-period. The elasticities of loan size w.r.t. the fee scheme at the \$20,833 threshold ranged from 0.062 to 0.356, approximately six times higher than in the pre-period. Similarly, though the magnitude of elasticity of loan size w.r.t. the fee scheme at \$150k is less than that at \$20,833, ranging from 0.004 to 0.019, the elasticity in the post-period is around five times that in the pre-period. These findings indicate that small business borrowers experienced delayed and amplified responses as they learned about the SBA's regulatory policies, highlighting the presence of information frictions in adopting their optimal borrowing strategies.

Table 6 compiles the findings from panel A of Figure 10 to Figure 14. These results provide insights into the behavioral responses of market participants to the discontinuous potential interest rates at the $20,833 threshold. A comparison between the pre- and post-periods reveals a gradual intensification of loan bunching at this threshold over time. This suggests that borrowers have seized the opportunity to learn and adjust their optimal borrowing strategies. Notably, both small businesses with a single employee and firms with multiple employees demonstrate bunching responses at this loan size.

Table 7 presents bunching estimators for loan sizes of 20, 833*and*150,000. Columns 1 and 2 involve the entire sample, columns 3 and 4 focus on the pre-period subsample, and columns 5 and 6 analyze the post-period subsample. The results highlight significant bunching responses at the $20,833 threshold. However, it's important to note that this estimation doesn't differentiate between reactions due to the capped loan amount and those resulting from choosing an optimal strategy. Additionally, the analysis reveals notable bunching responses at the $150,000 threshold. As discussed in Chapter 2, borrowers are motivated to reduce their loan size due to the discontinuity paperwork and requirement burden imposed by the policy.

Following the approach proposed by Almunia and Lopez-Rodriguez (2018), I multiply the bunching estimator $\hat{b}$ by the bin width w to interpret the average response, assuming heterogeneity among small businesses. The bunching estimators in Table 7 can be interpreted using these calculations.

At the loan threshold of $20,833, the estimated bunching estimator $\widehat{b_{21k}}$ is 595.436 (with a standard error of 21.851), statistically significant at the 0.1% level. Using a bin width of $10.093 at this loan range, the average loan response is estimated to be 595.436×10.093

$= 6,009.736$ (approximately 28.85% of the borrowed loan size) in response to the policy regulation threshold at $20,833. In 2020, the estimated bunching estimator $\widehat{b_{21k}}$ is 128.715 (with a standard error of 5.038), statistically significant at the 0.1% level. The average loan response is estimated at $128.715 \times 10.093 = 1,299.12$ in response to the notch. This year's average loan response is approximately 6.24% of the borrowed loan size. 2021 the estimated bunching estimator $\widehat{b_{21k}}$ increased to 742.929. The average loan size response is estimated at $742.929 \times 10.093 = 7,498.38$ in response to this notch. This year's average loan response is approximately 36% of the borrowed loan size.

Similarly, the bunching estimators at the $150,000 notch point can be analyzed using a similar approach. In the full sample, the estimated bunching estimator $\widehat{b_{150k}}$ is 1.808 (with a standard error of 0.248), statistically significant at the 0.1% level. The bin width is $881,977, indicating that the average loan size is estimated to be $1.808 \times 881{,}977 = 1,594.61$ (approximately 1% of the borrowed loan size) in response to the notch at $150,000. In 2020, the estimated bunching estimator was $\widehat{b_{150k}} = 0.661$ (with a standard error of 0.085), statistically significant at the 0.1% level. The average loan size is estimated to be $0.661 \times 881{,}977 = 582.987$ (approximately 0.39% of the borrowed loan size) in response to the notch at $150,000. In 2021, the estimated bunching estimator increased to $\widehat{b_{150k}} = 3.266$ (with a standard error of 0.321), statistically significant at the 0.1% level. The average loan size is estimated at $3.266 \times 881{,}977 = 2,880.54$ (approximately 1.92% of the borrowed loan size) in response to the notch point at $150,000.

Table 8 displays the bunching estimators at the loan sizes of $20,833 and $150,000 on a logarithmic scale. Similar to the previous table, this one reveals a consistent pattern. Significant bunching responses are observed at the loan size of $20,833, and these responses appear to intensify over time. Notably, when using the logarithmic scale, the bunching estimator at $150,000 surpasses the estimates obtained using the regular loan scale for both the whole sample and the post-period subsample.

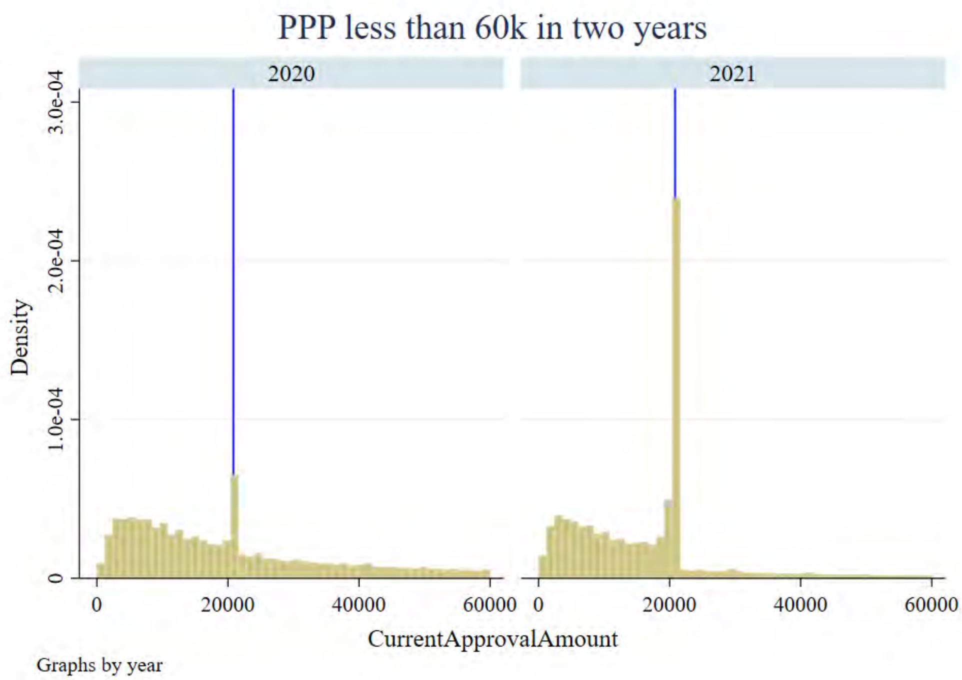
Table 8 presents the bunching estimators at loan sizes of $20, 833and150$,000 on a logarithmic scale. Similar to the previous table, this one reveals a consistent pattern. Significant bunching responses are observed at the loan size of \$20,833, which appear to intensify over time. Notably, when using the logarithmic scale, the bunching estimator at \$150,000 surpasses the estimates obtained using the regular loan scale for both the whole sample and the post-period subsample.

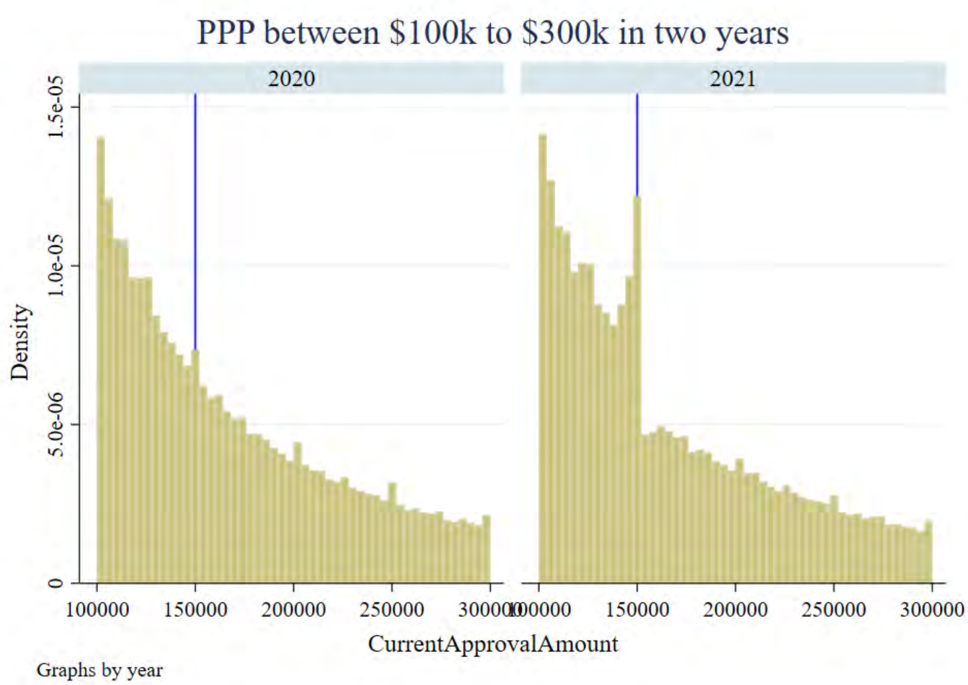
As previously discussed, there are two types of fee notches—one in the lender's processing fee scheme and another created by forgiveness application policies. Empirical estimation results indicate that borrowers play a vital role in the behavioral responses incentivized by government policies in this PPP lending market. As a fully forgivable government relief program, the PPP could either be forgiven or repaid at a 1% interest rate. Thus, the probability of accepting different interest rates, the borrower's expected forgiveness rate, plays a crucial role in the borrower's decision-making process.

At the lender's processing fee rate thresholds, although the fee notch can incentivize lenders to scale back loan size, the probability of whether borrowers accept the loan contract depends on the borrowers. Inequalities 22 and 30 in Chapter 2 can explain the loan bunching responses when borrowing occurs at the lender's processing fee thresholds and at the forgiveness regulation thresholds.

By analyzing the "bunching" responses in PPP loans, the government can gain a comprehensive understanding of the effectiveness of the SBA's regulatory policies and identify opportunities for future policy improvements. This insight into diverse responses at thresholds set by various policies allows government policymakers to refine their expectations at different policy thresholds.

Figure 9: Histograms of PPP Loans in Two Loan Ranges in Two Years.

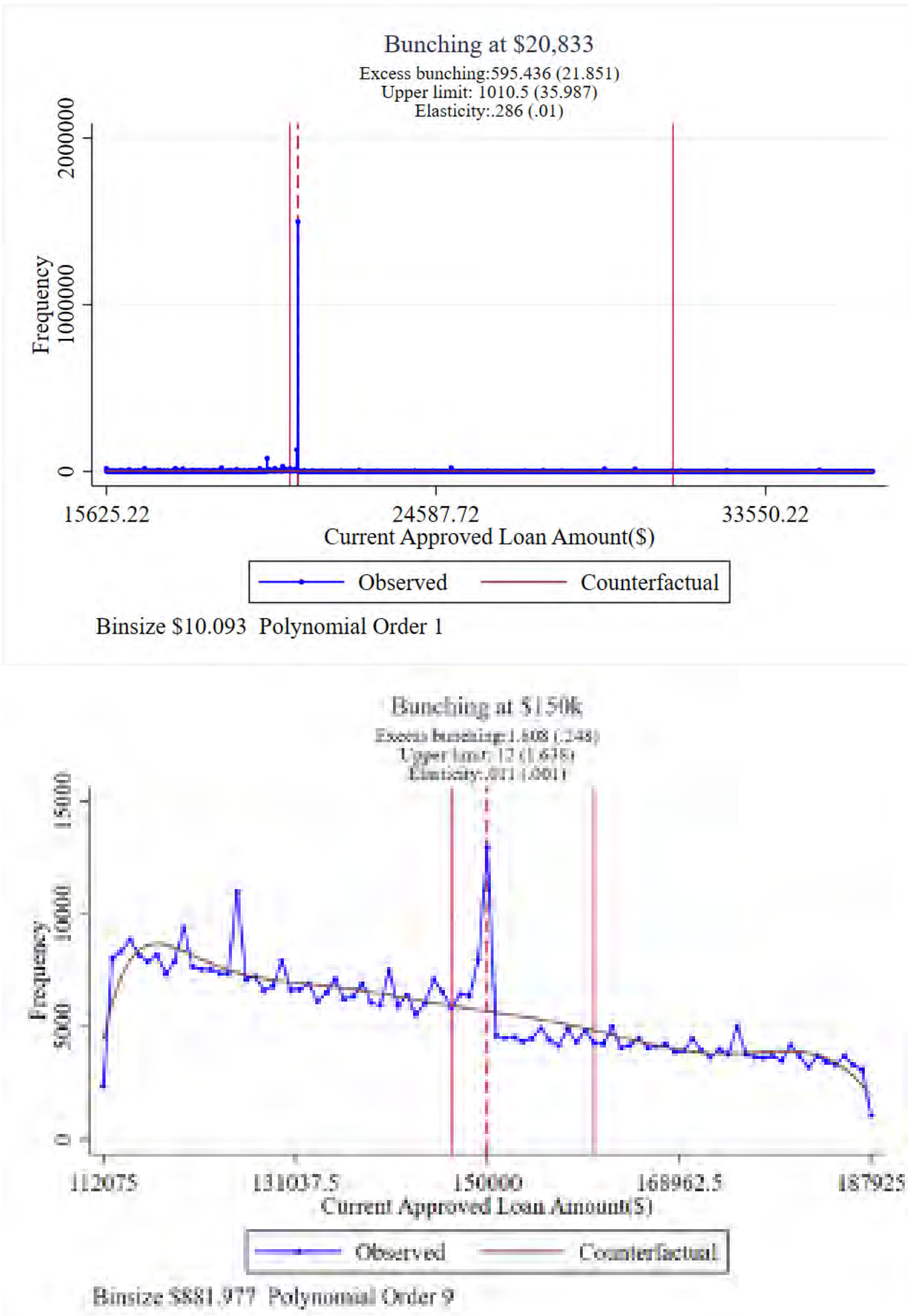




Notes: Figure 9 Panel a.PPP Loans, range under $60k; Panel b. PPP Loans range from $100k to $300k. The figure shows the histogram of PPP loans for two different regions separately for 2020 and 2021. The data distribution from 2020 is on the left of each panel, while the data distribution from 2021 is on the right of each panel. Solid lines marked the main notches points. Panel a. shows the loans under loan size of

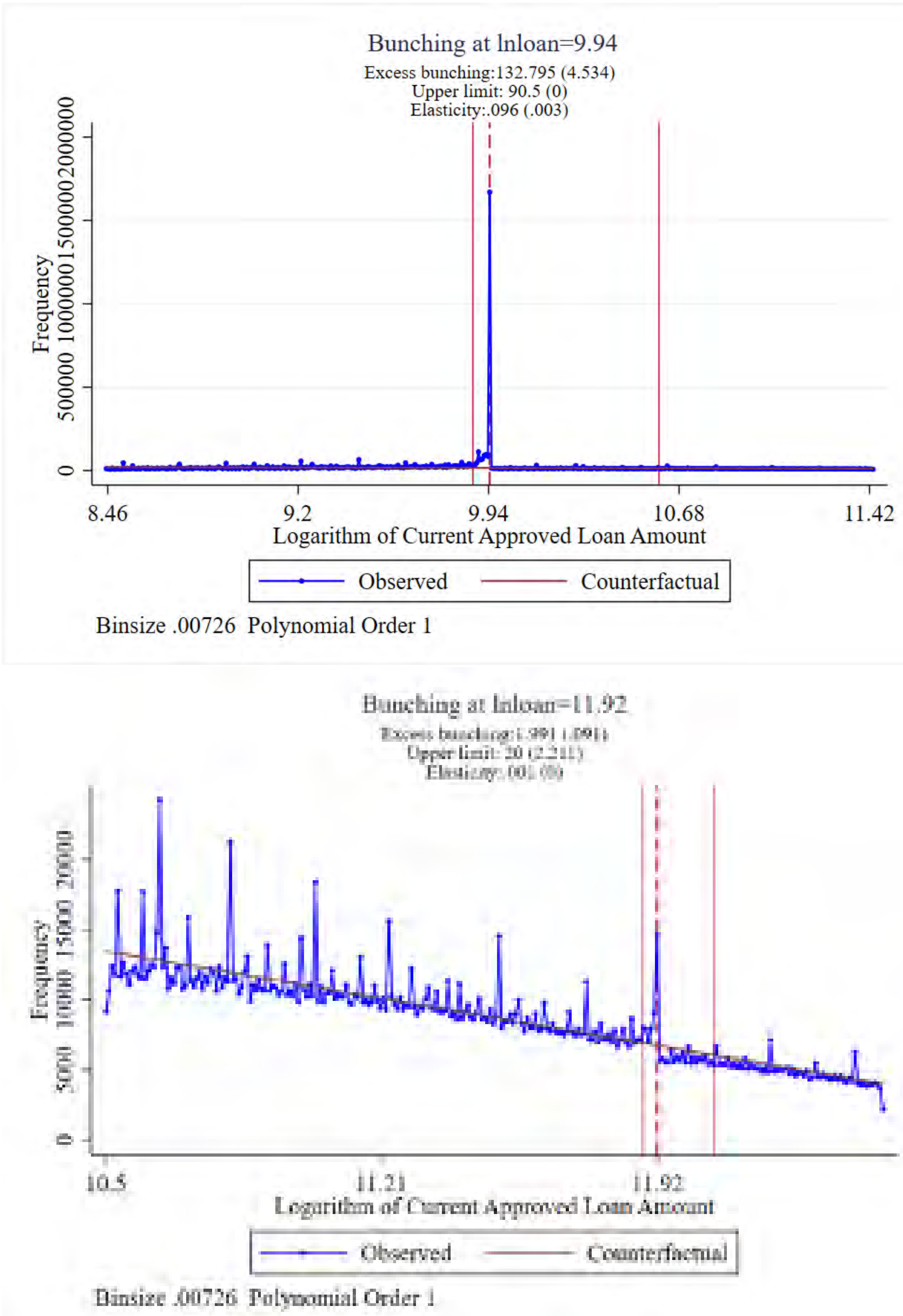
$60k. The solid lines indicate a threshold of $20,833. The processing fees were adjusted on the second round of PPP loans for loans Between $5k and $50k, and the $20,833 is the maximum loan amount per employee for forgiveness. Panel b shows the loans between $100,000 and $300,000, with the solid lines showing the loans at $150,000.

Figure 10: Bunching Estimation at each regulation threshold



Notes: Figure 10 The bunching windows for each threshold has been selected by the iteration process proposed by (Kleven and Waseem, 2013) and (Harju et al., 2019). Bin widths have been selected by the Freedman-Diaconis rule (FD rule). BIC has selected the order of the polynomial in each regression. The figure shows the empirical density distribution of the natural logarithm of currently approved loan amounts for borrowers clustered in bins (dotted blue graph) and the estimated counterfactual density (solid red graph). The X-axis is the current PPP-approved loan amount; Y-axis is the counts of observations in each bin. Panel a. Bunching estimated the loan amount at the policy threshold at $20833. The lower bound of the excluded range is defined by nine bins on the left side of the notch point. In contrast, the upper bound of the excluded range is defined by the iteration process and the polynomial regression at 1010.5 bins on the right side of the threshold; Panel b. Bunching at the threshold at $150k. The lower bound of the excluded range is defined by five bins on the left side of the notch point. In comparison, the upper bound of the excluded range is defined by the iteration process and the polynomial regression at 12 bins upper to the threshold point. The counterfactual density in each panel is separately estimated by fitting a first-order polynomial for panel a and a ninth-order for panel b. Data only includes The group of borrowers who borrowed once. Vertical dashed lines mark notch points; vertical solid lines mark excluded ranges' lower and upper bounds. The bin size for the empirical density $10 and $881 separately. Notch points are at bin centers. Bunching estimator b is the excess mass in the excluded range. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

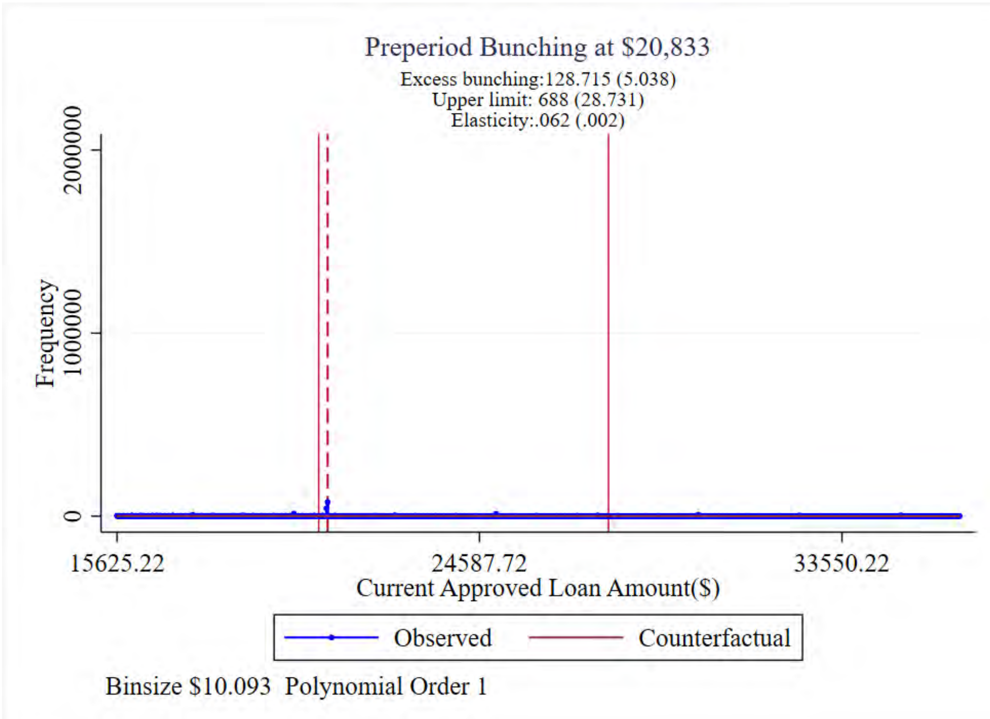
Figure 11: Bunching Estimation at each regulation threshold (loan size in logarithm scale)



Notes: Figure 11 The bunching windows for each threshold has been selected by the iteration process proposed by (Kleven and Waseem, 2013) and (Harju et al., 2019). Bin widths have been selected by the Freedman-Diaconis rule (FD rule). BIC has selected the order of polynomials in each regression. The figure shows the empirical density distribution of the natural logarithm of currently approved loan amounts for borrowers clustered in bins (dotted blue graph) and the estimated counterfactual density (solid red graph). Panel a. The logarithm value of $20,833 equals 9.94. The lower bound of the excluded range is defined by nine bins on the left side of the notch point. In comparison, the upper bound of the excluded range is defined by the iteration process and the polynomial regression at 90 bins on the right side around the threshold point, Panel b. The logarithm value of $150k equals 11.92. The lower bound of the excluded range is defined by five bins on the left side of the notch point. In contrast, the upper bound of the excluded range is defined by the iteration process and the polynomial regression at 20 bins on the right side around the threshold point. The counterfactual density is estimated from the group of borrowers borrowed once by fitting a first-order polynomial for panel a and first-order for panel b. Vertical dashed lines mark notch points; vertical solid lines mark excluded ranges' lower and upper bounds. The bin size for the empirical density is 0.00726 for each panel. Notch points are bin centered. Bunching b is the excess mass in the excluded range. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

Figure 12: Bunching estimation at each threshold for different periods(continued on next page)

(a) Pre-period data at \$21k



(b) Post-period data at \$21k

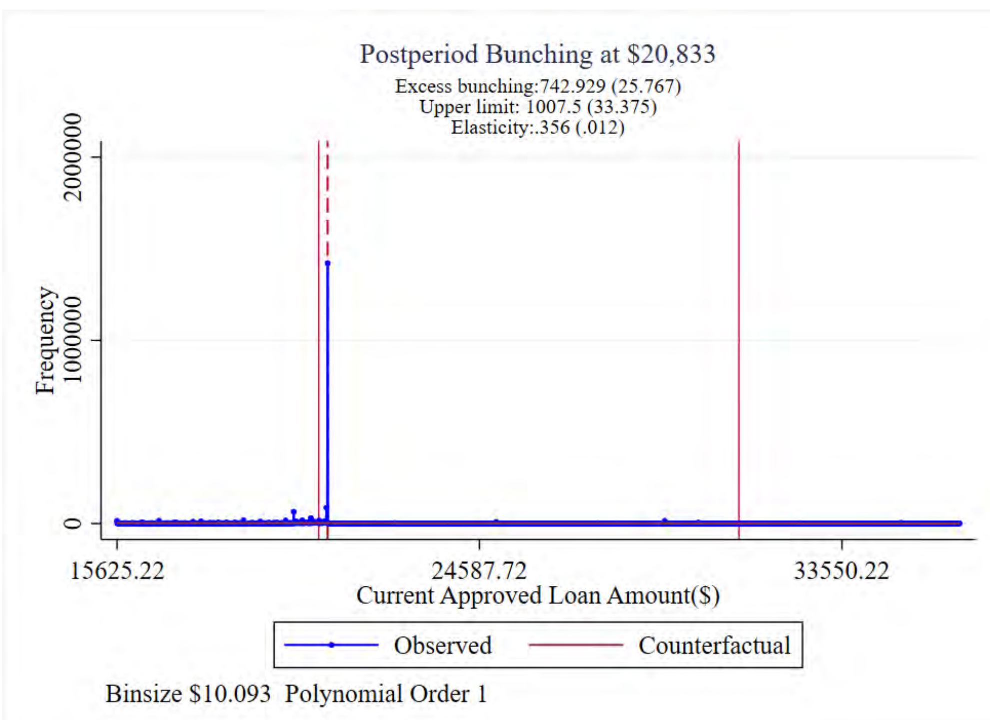
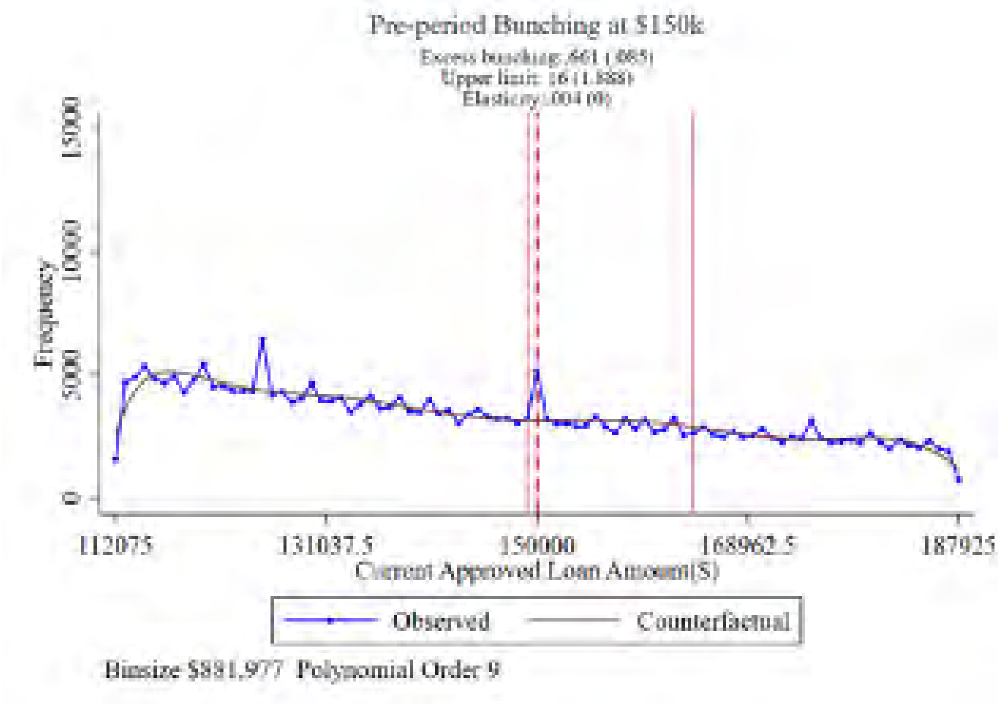
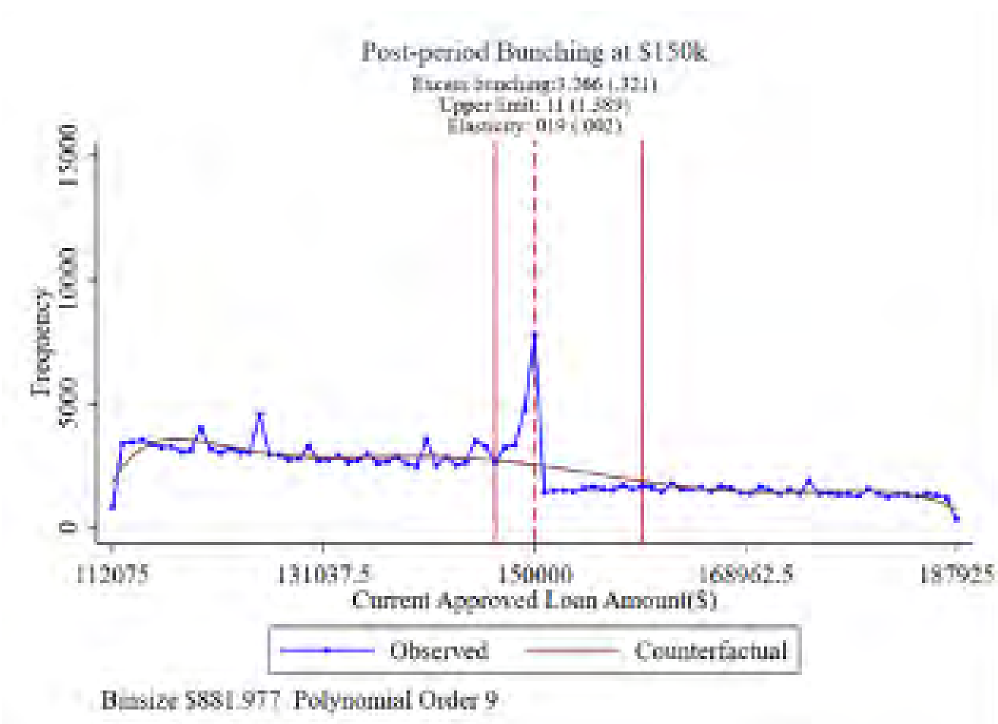


Figure 12: Bunching estimation at each threshold for different periods(continued from previous page)

(c) Pre-period data at $150k



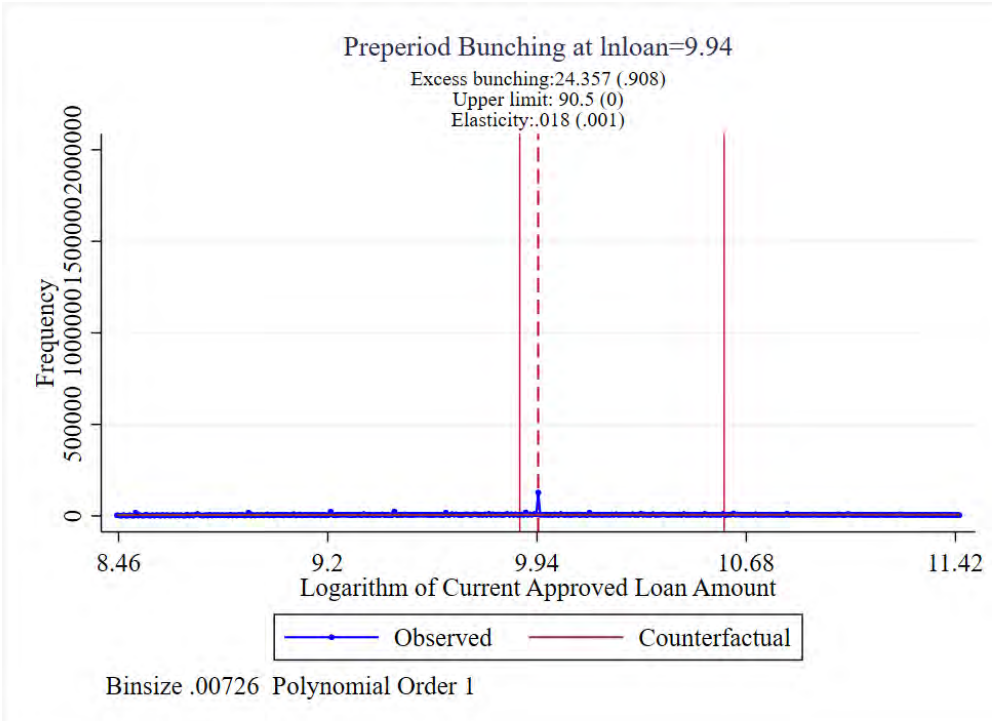
(d) post-period data at $150k



Notes: Figure 12 Panels a, c display the empirical and counterfactual density distributions at each threshold before May 21, 2020, when the SBA procedure notice was announced. These corresponding subsamples fit counterfactual density distribution; Panel b, d are the empirical density distributions and counterfactual density distributions at each threshold after May 21, 2020, and the accordingly fitted counterfactual density distributions. The Blue dotted line is the empirical density distribution of loans; the dark red line is the fitted counterfactual distribution. The policy around the loan size of $20,833 and $150k was not changed after May 21, 2020. The only policy shock on May 21, 2020, was the announcement of the SBA procedure notice of the lender's processing fee guidance. There is a significant change in the responses in panels a and b. After May 21, 2020, the elasticity is around seven times as large as that in the pre-period. The results indicate that borrowers or lenders took responses to policies. The bunching window for each panel has been selected by the iteration process proposed by (Kleven and Waseem, 2013) and (Harju et al., 2019). Bin widths for each panel has been selected by the Freedman-Diaconis rule (FD). BIC has selected the order of polynomials in each regression. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

Figure 13: Bunching estimation at each threshold for different periods (loan size in logarithm scale)(continued on next page)

(a) Pre-period bunching at ln(loan)=9.94



(b) Post-period bunching at ln(loan)=9.94

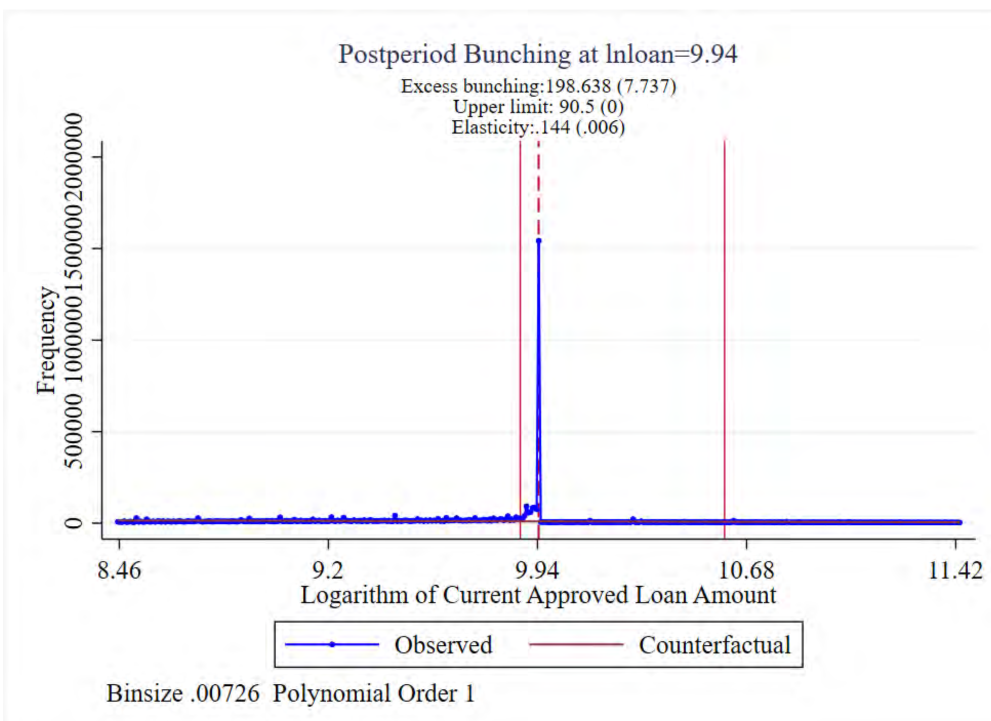
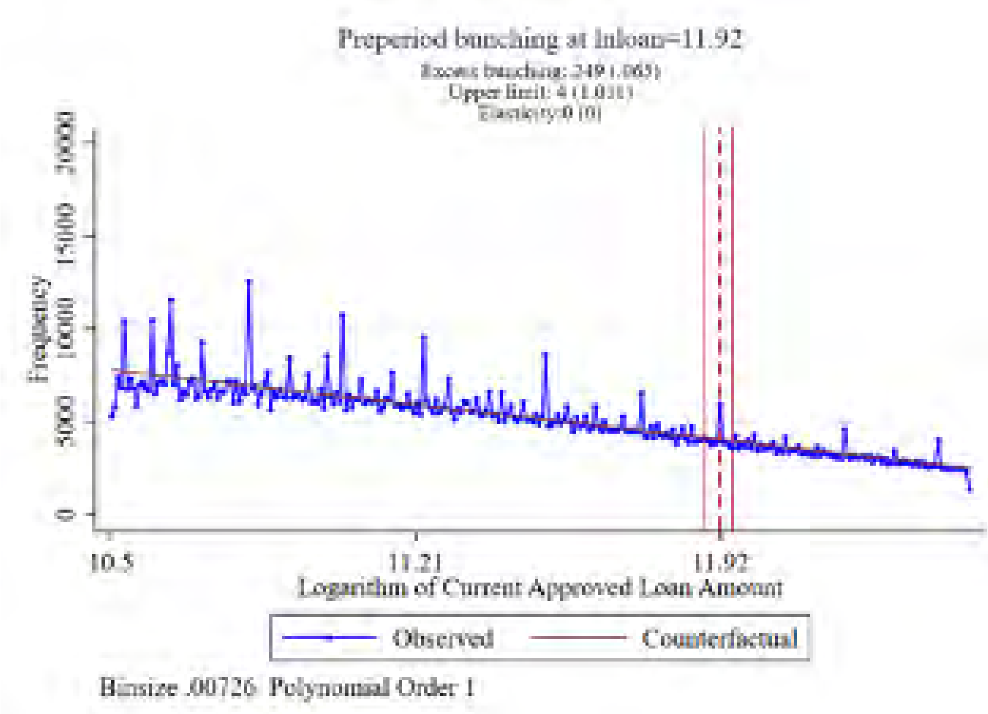
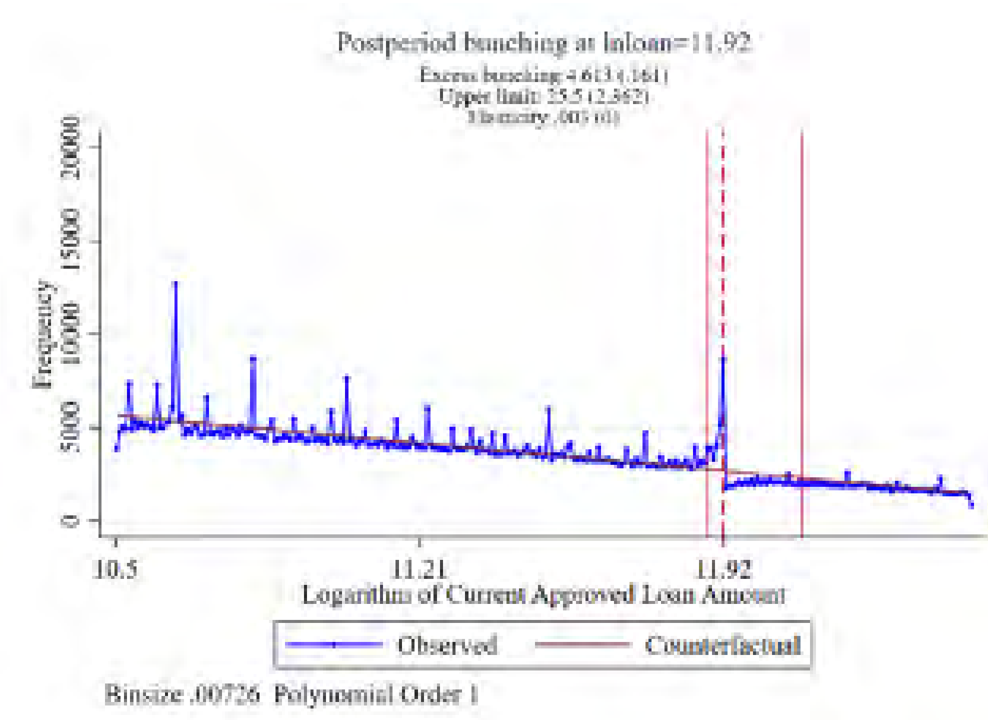


Figure 13: Bunching estimation at each threshold for different periods (loan size in logarithm scale)(continued from the previous page)

(c) Pre-period bunching at ln(loan)=11.92



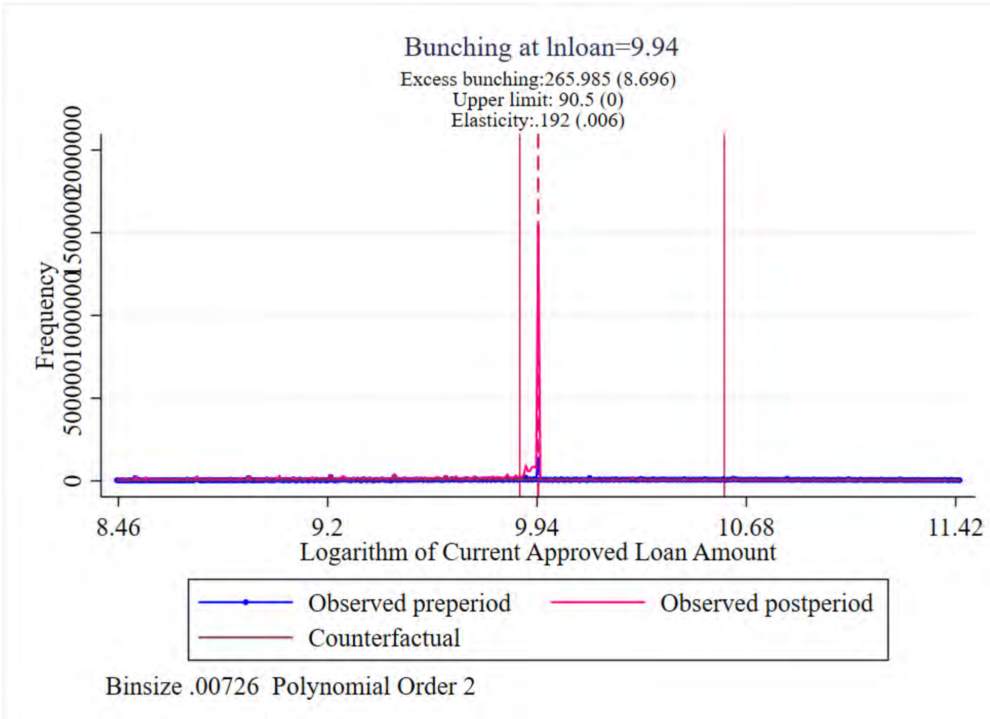
(d) Post-period bunching at ln(loan)=11.92



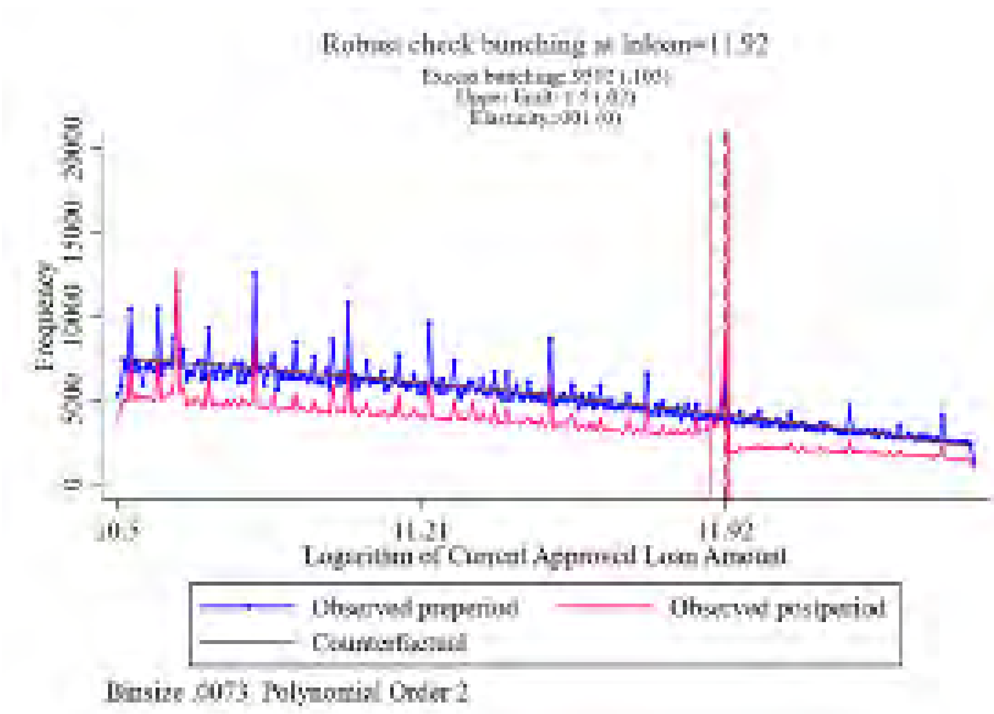
Notes: Figure 13 Panel a and c are for the empirical density distributions at each threshold before May 21, 2020, when the SBA procedure notice has been announced. The corresponding subsamples fit counterfactual density distribution; Panel b, d are for the empirical density distributions at each threshold after May 21, 2020, and the accordingly fitted counterfactual density distributions. The Blue dotted line is the empirical density distribution of loans; the dark red line is the fitted counterfactual distribution. The policy around the loan size of $20,833 and $150k was not changed after May 21, 2020. The only policy shock on May 21, 2020, was the announcement of the SBA procedure notice to guide the lender's processing fee. After May 21, 2020, the elasticity was around ten times as large as that in the pre-period, indicating that borrowers or lenders responded to policies. The bunching window for each threshold has been selected by the iteration process proposed by (Kleven and Waseem, 2013) and (Harju et al., 2019). Bin widths has been selected by the Freedman-Diaconis rule (FD rule). BIC has selected the order of polynomials in each regression. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

Figure 14: Bunching estimation, counterfactual distribution fitted by pre-period data (loan size in logarithm scale).

(a) Counterfactual fitted by pre-period data, ln(loan)=9.94



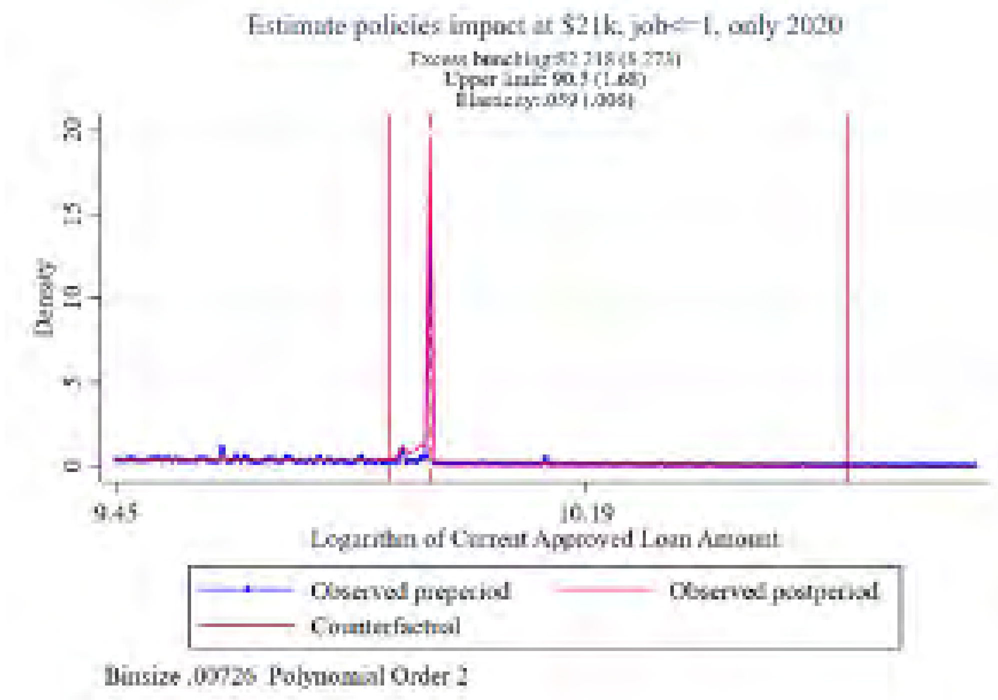
(b) Counterfactual fitted by pre-period data, ln(loan)=11.92



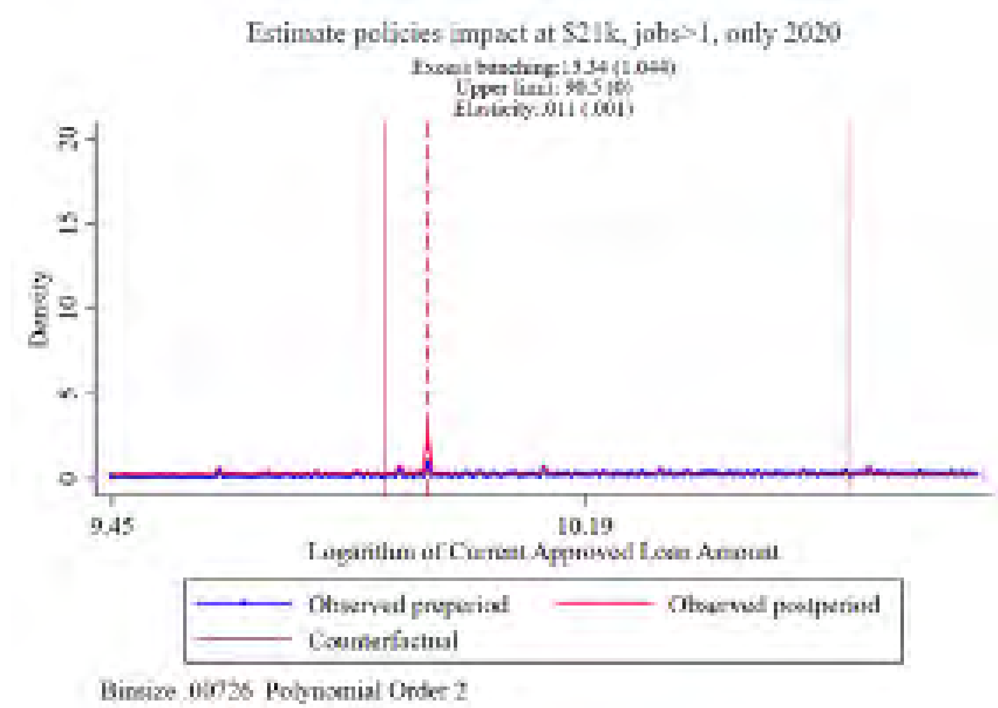
Notes: Figure 14 Pre-period data fit the counterfactual distribution shown in each panel, and bunching mass is calculated by the difference between the fitted counterfactual distribution and the post-period empirical distribution. The polynomial degree has been chosen separately, at 2 for panels a and b. The bin size is chosen at 0.00726. polynomial and the excluded range were chosen to fit best the counterfactual distribution between the empirical distribution of the pre-period and post-period. The Blue dotted line is the empirical density distribution of loans borrowed before May 21, 2020; the pink dotted line is the empirical density distribution of loans borrowed after May 21, 2020. Bin widths has been selected by the Freedman-Diaconis rule (FD rule). BIC has selected the order of polynomials in each regression. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

Figure 15: Bunching estimation at the threshold of $20,833 for different sizes of firms.

(a) Firms have less or equal to one employee



(b) Firms have more than one employees



Notes: Figure 15 Panel a. displays the group of small businesses whose employee positions is equal to or less than one; Panel b. displays the group of small businesses whose employee position is more significant than one. The Blue dotted line is the empirical density distribution of loans borrowed before May 21, 2020; the pink dotted line is the empirical density distribution of loans borrowed after May 21, 2020; The dark red line is the counterfactual distribution fitted by the pre-period data. Bin widths have been selected by the Freedman-Diaconis rule (FD rule). BIC has selected the order of polynomials in each regression. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

Table 6: Bunching Estimation Results at the threshold of $20,833

Bunching Estimation at the threshold of $20,833		
	Bunching (b)	Elasticity (ε)
Total single loan	595.436	0.286
	(21.851)	(0.01)
Pre-period single loan	128.715	0.062
	(5.038)	(0.002)
Post-period single loan	742.929	0.356
	(25.767)	(0.012)
The group with up to one employee	82.218	0.59
	(8.273)	(0.006)
The group with more than one employee	15.34	0.011
	(1.044)	(0.001)
Fitted by pre-period data	265.985	0.192
	(8.696)	(0.006)

Notes: Table 6 The group of Total single loans is using the total loans that the borrower only borrowed once; b. the group of pre-period single loans is using the loans borrowed only once and before May 21, 2020; c. the group of post-period single loans is using the loans borrowed only once and after May 21, 2020; d. is estimated by using the data in the pre-period to fit the counterfactual distribution and calculate the bunching mass and elasticity by comparing it with the post-period data.

Table 7: Estimating Bunching Responses at Thresholds of \$20,833 and \$150k

	Bunching Estimation at each Borrowers' Threshold					
	Full sample responses		Pre-period responses		Post-period responses	
	(1) Bunching(b)	(2) Elasticity(ε)	(3) Bunching(b)	(4) Elasticity(ε)	(5) Bunching(b)	(6) Elasticity(ε)
At threshold of \$20,833	595.436	0.286	128.715	0.062	742.929	0.356
	(21.851)	(0.01)	(5.038)	(0.002)	(25.767)	(0.012)
At threshold of \$150,000	1.808	0.011	0.661	0.004	3.266	0.019
	(0.248)	(0.001)	(0.085)	0	(0.321)	(0.002)

Notes: This table 7 presents the estimated bunching mass and the elasticities of loan size with respect to fees at each threshold for the different periods. Columns 1 to 2 display the estimation results from the total sample; columns 3 to 4 display the estimation results from the pre-period sample (before May 2020); columns 5-6 display the results from the post-period sample (after May 2020). Columns 1, 3, and 5 show the bunching estimator b hat, based on Eq 37. Bunching estimator $\hat{b}$ is the excess mass in the excluded range around the notch point, in proportion to the average counterfactual density in the excluded range. Columns 2, 4, and 6 present an estimate of the elasticities of loan size with respect to the fee structure. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine

Table 8: Estimating Bunching Responses at Thresholds of \$20,833 and \$150k (in logarithm scale)

Bunching Estimation at each Borrowers' Threshold						
	Full sample responses		Pre-period responses		Post-period responses	
	(1) Bunching(b)	(2) Elasticity(ε)	(3) Bunching(b)	(4) Elasticity(ε)	(5) Bunching(b)	(6) Elasticity(ε)
At threshold of \$20,833	132.795***	0.096***	24.357***	0.018***	198.638***	0.144***
	(4.534)	(0.003)	(0.908)	(0.001)	(7.737)	(0.006)
At threshold of \$150,000	1.991***	0.001***	0.249***	0	4.613***	0.003***
	(0.091)	0	(0.065)	0	(0.161)	0

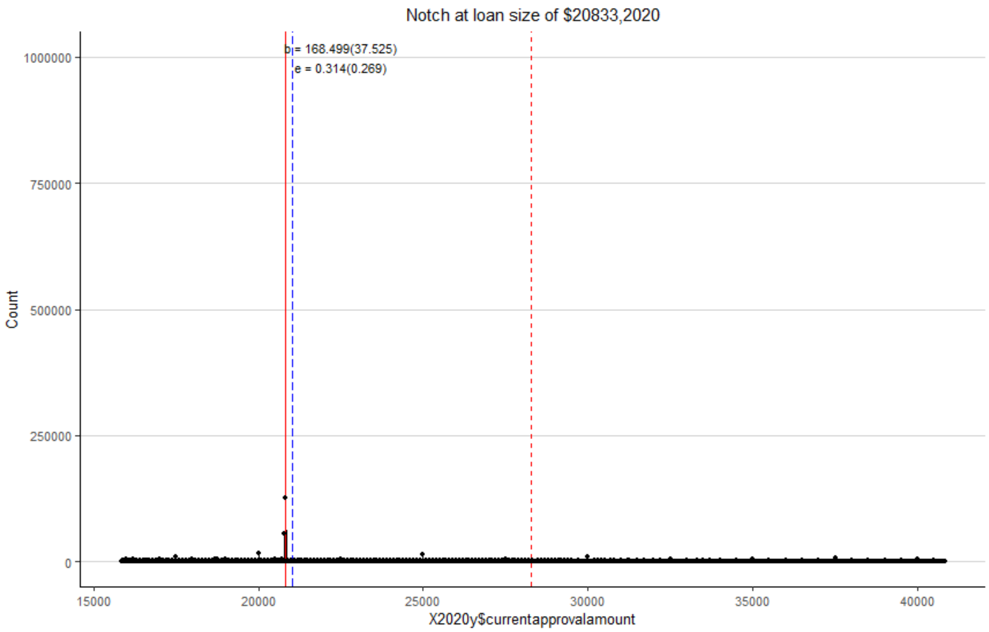
Notes: This table 8 presents the estimated bunching mass and the elasticity of loan size with respect to fees at each threshold point at different times. Columns 1 to 2 display the estimation results from the total sample; columns 3 to 4 display the estimation results from the pre-period sample (before May 2020); columns 5-6 display the results from the post-period sample (after May 2020). Columns 1, 3, and 5 reproduce the bunching estimate b, based on estimating Equation 31. Bunching b is the excess mass in the excluded range around the kink or notch in proportion to the average counterfactual density in the excluded range. Columns 2, 4, and 6 present an estimate of the elasticities of loan size with respect to the fee structure. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine, In this section, I re-estimate the main bunching estimation results using an R package proposed by (Mavrokonstantis, 2019). Besides applying the traditional bunching approach, this estimation can exclude the round number bunching and minor bunching near the central bunching region. From the empirical distribution shown in Figure 9, a minor bunching area is near the significant bunching area at the threshold of $20,833. In the robustness check, the R package allowed this estimation to control this minor bunching.

Figure 16 presents the robustness analysis of the bunching estimations at loan sizes of $20,833 and $150,000, separately for the years 2020 and 2021. The R package was utilized to control for minor bunching near the central excess mass. The analysis reveals a significant bunching area at the loan threshold of $20,833. Additionally, a small but statistically significant bunching estimator is observed at the $150,000 threshold. The loan bunching responses at both thresholds became more pronounced over time.

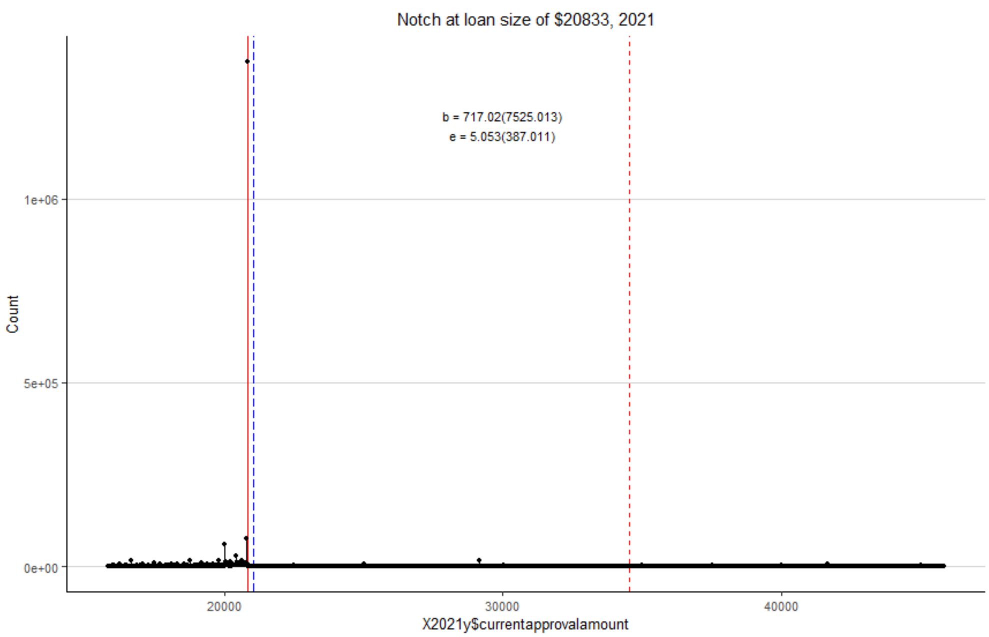
Table 9 shows the collected bunching estimators at each threshold.

Figure 16: Robustness Check bunching at each threshold in two periods(continue in the next page)

(a) At $20833 in 2020



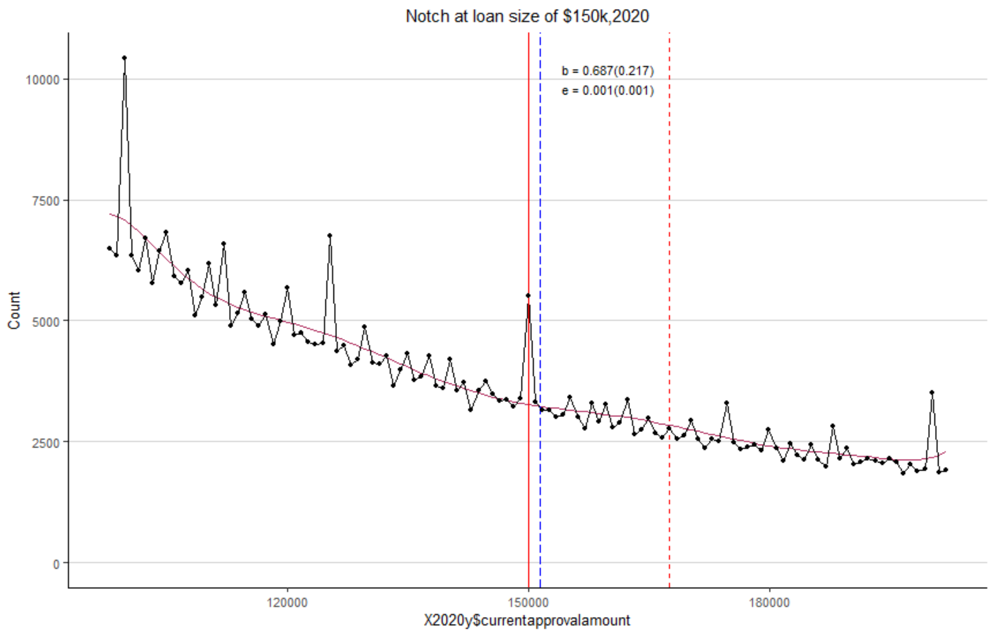
(b) At $20833 in 2021



Notes: Figure 16 displays the robustness estimation on the loan size of $20,833 and $150k, separately in 2020 and 2021.

Figure 16: Robustness Check bunching at each threshold in two periods(continue with the previous page)

(c) At $150k in 2020



(d) At $150k in 2021

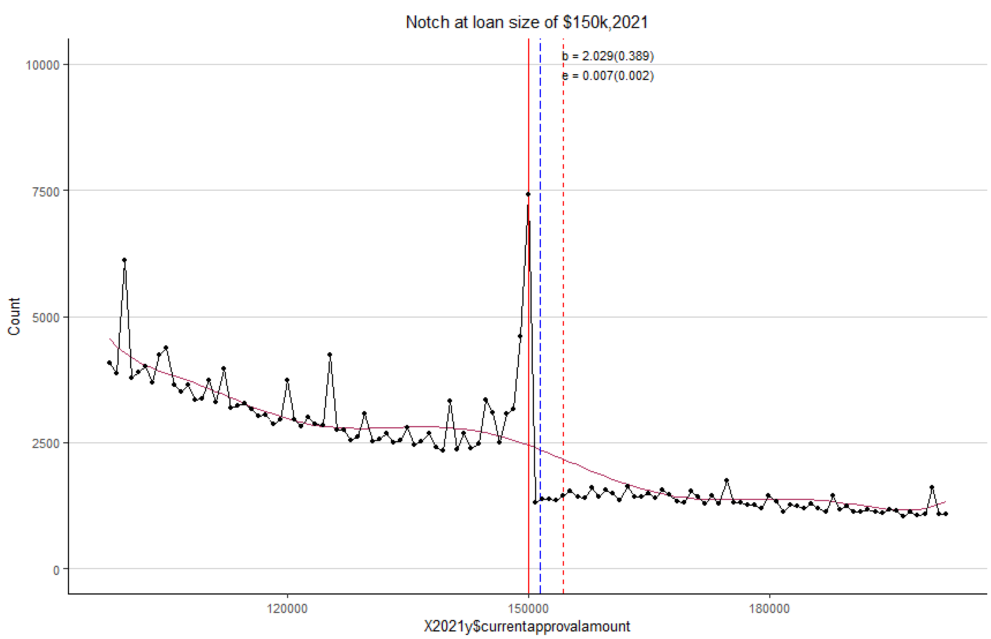


Table 9: Robustness Check Bunching Responses at Each Threshold in Two periods

Robustness Bunching Estimation at each Threshold				
	Pre-period responses		Post-period responses	
	(1) Bunching(b)	(2) Elasticity(\var epsilon)	(3) Bunching(b)	(4) Elasticity(\var epsilon)
At threshold of $20833	168.499(37.525)	0.314(0.269)	717.02(7525.013)	5.053(387.011)
At threshold of $150000	0.687(0.217)	0.001(0.001)	2.029(0.389)	0.007(0.002)

Notes: Table 9 shows the robustness estimators on the loan size of $20,833 and $150k, separately in 2020 and 2021.

In October 2020, the SBA and Treasury introduced a simplified PPP forgiveness process for loans of $50,000 or less. The previous forgiveness requirements primarily focused on employment positions and compensation without specifying whether an exemption threshold existed. When the SBA announced the Interim Final Rule for the streamlined forgiveness process for PPP loans under $50,000, approximately 3.7 million loans fell within this category. Among these loans, 1.7 million were granted to small businesses that reported having either zero or just one employee. Consequently, the Interim Final Rule exempted PPP loans under $50,000 from forgiveness reduction due to employment or salary changes. Small businesses applying for forgiveness under this threshold will utilize SBA Form 3508S.

Therefore, the incentives at the thresholds of $20,833 and $150,000 may differ slightly. At the loan threshold of $20,833, small business borrowers are not penalized for reductions in employment or salary. Loans above $50,000 do not have exemptions for employment or salary reductions, but $150,000 represents another policy cutoff point. As per the SBA forgiveness policy, different monitoring efforts are applied below and above this $150,000 threshold.

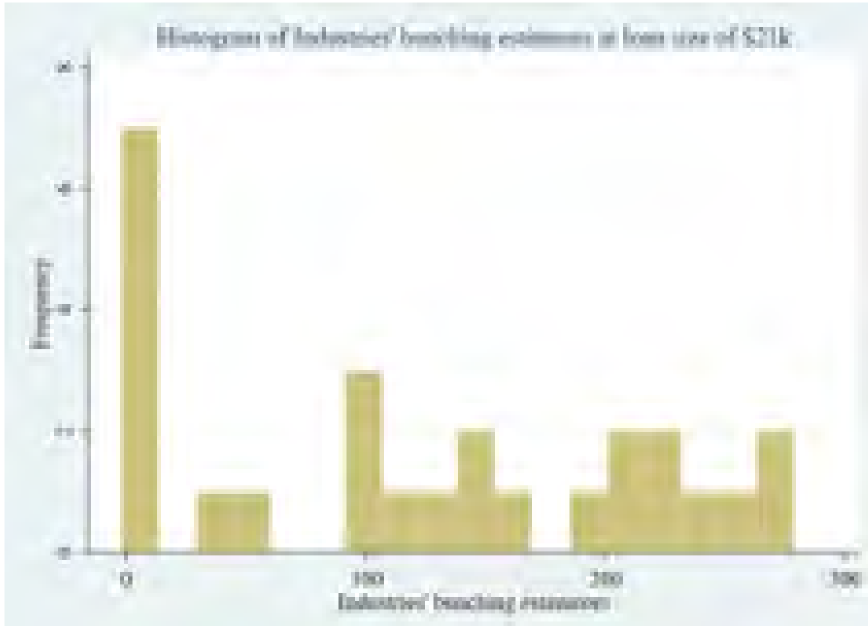
Figure 17 presents the histogram of bunching estimators at the loan thresholds of $20,833 and $150,000, categorized by sectors defined by two digits of the NAICS code, showcasing significant variation across these industry panels. Similar to the previous figure, considerable variation is observed within these industrial panels.

Figure A.19 displays the bunching estimators for loan borrowed at \$20,833, highlighting the variations among different sectors. It is evident that the bunching estimators differ across these industries. Specifically, industries with NAICS codes 21, 22, 49, 55, and 92 exhibit distinct bunching loan sizes, that are below \$20,833. The presence of different bunching points across industries warrants further investigation to understand the underlying reasons for these responses.

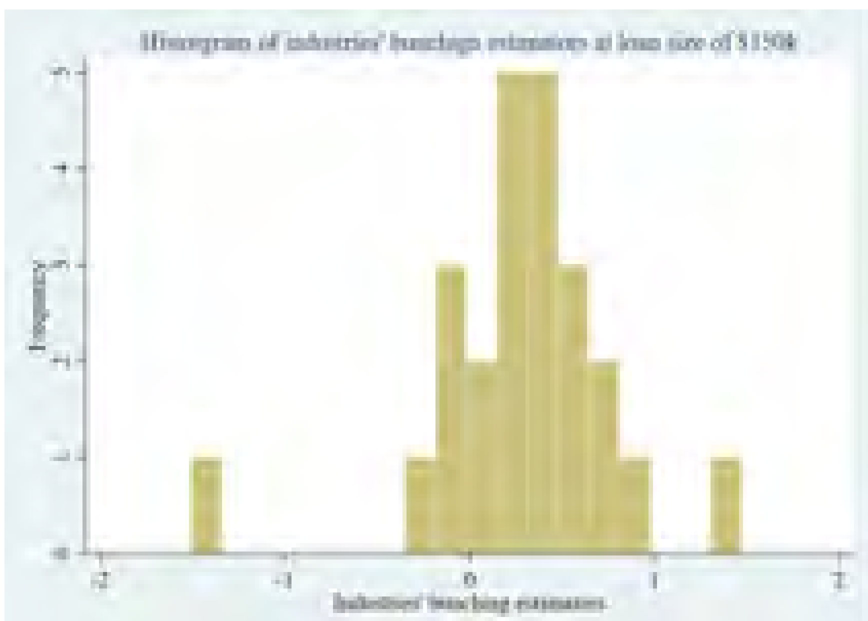
Additionally, Figure A.20 displays the bunching estimators for loans borrowed at \$150,000. Loan bunching responses variance among different industries as well.

Figure 17: histogram in industry

(a) At $150k in 2020



(b) At $150k in 2021



Notes: Figure 17 Panel a. shows the histogram of the bunching estimators estimated at the loan size of $21k by sectors. The bunching estimators are collected from the industries panels in Figure 15. Panel b. shows the histogram of the bunching estimators estimated at the loan size of $150k by sectors. The bunching estimators are collected from the industries panels in Figure 16.

Table 10: Bunching Estimation: Heterogeneity Across Different Industries

Bunching Estimation: Heterogeneity Across Different Industries					
		At $20,833		At $150,000	
NAICS Code	Industry	Bunching estimator	Elasticity	Bunching Estimator	Elasticity
11	Agriculture, Forestry, Fishing and Hunting	271.971(11.676)	0.13(0.011)	0.472(0.082)	0.003(0.001)
21	mining, Quarrying, and Oil and Gas Extraction	-1.951(0.198)	0.001(0)	0.478(0.12)	0.003(0.001)
22	utilities	-0.940(0.206)	0 0.689(0.172)	0.004(0.002)	
23	Construction	117.465(3.721)	0.056(0.004)	0.336(0.069)	0.002(0.001)
31	Manufacturing	-1.955(0.058)	-0.001(0)	0.072(0.101)	0
32	Manufacturing	2.604(0.288)	0.001(0)	0.313(0.087)	0.002(0.001)
33	Manufacturing	92.317(2.822)	0.044(0.003)	0.233(0.074)	0.001(0.001)
42	Wholesale Trade	136.867(5.105)	0.066(0.005)	0.255(0.066)	0.001(0.001)
44	Retail Trade	57.394(1.294)	0.028(0.001)	-0.039(0.113)	0
45	Retail Trade	93.196(3.126)	0.045(0.003)	0.572(0.102)	0.003(0.001)
48	Transportation and Warehousing	230.19(9.155)	0.11(0.009)	1.285(0.157)	0.007(0.002)
49	Transportation and Warehousing	-0.368(0.258)	0	-0.113(0.141)	-0.001(0.002)
51	Information	217.256(8.156)	0.104(0.008)	0.830(0.105)	0.005(0.001)
52	Finance and Insurance	235.401(11.076)	0.113(0.11)	0.214(0.104)	0.001(0.001)
53	Real Estate and Rental and Leasing	278.298(14.772)	0.133(0.014)	-0.191(0.089)	-0.001(0.001)
54	Professional, Scientific, and Technical Services	255.984(11.915)	0.123(0.011)	0.196(0.068)	0.001(0.001)
55	Management of Companies and Enterprises	1.886(0.447)	0.001(0)	-1.712(0.21)	-0.01(0.002)
56	Administrative and Support and Waste Management and Remediation Services	150.665(5.034)	0.072(0.005)	0.356(0.079)	0.002(0.001)
61	Educational Services	153.423(4.395)	0.074(0.004)	-0.237(0.135)	-0.001(0.002)
62	Health Care and Social Assistance	203.264(8.868)	0.097(0.009)	0.209(0.069)	0.001(0.001)
71	Arts, Entertainment, and Recreation	185.66(5.984)	0.089(0.006)	0.015(0.126)	0
72	Accommodation and Food Services	36.956(0.837)	0.018(0.001)	0.371(0.083)	0.002(0.001)
81	Other Services(except public Administration)	104.325(2.544)	0.05(0.002)	0.454(0.065)	0.003(0.001)
92	Public Administration (not covered in economic census)	-2.14(0.085)	-0.001(0)	-0.092(0.231)	-0.001(0.003)
99	Missing code	161.288(4.479)	0.077(0.004)	0.944(0.112)	0.005(0.001)
missing code		212.184(7.383)	0.102(0.007)	0.058(0.125)	0

Notes: Table 10 displays the bunching estimators for different industries defined by NAICS two digits code, separately estimated at $20833 and $150k.

7 Conclusion

As the pivotal component of the social relief fiscal program during the COVID-19 pandemic, the PPP has undergone various evaluations regarding achieving policy goals. With a discontinuous processing fee structure, lenders can strategically lend just below the loan thresholds to maximize commission fees. If they react to these discontinued incentives, it could impact the policy's effectiveness. Therefore, estimating lenders' loan supply elasticities is crucial in the context of discontinued incentives in commission rates.

From a traditional bunching model perspective, the PPP lenders' optimal strategy was lending just below the loan threshold to obtain higher commission fees. Higher-ability lenders can adopt this optimal strategy because they have sufficient market power to negotiate loan size with borrowers. Therefore, according to the theoretical prescription, it was expected that lenders would scale back the loans initially located in the dominant loan region. This loan bunching response requires that lenders have market or negotiation power in the lending process. However, because the one-dimensional model only contains one market participant, the interaction between lender and borrower could not be addressed. The two-dimensional model derived following the Cox et al. (2020) gives a more accurate explanation of the empirical findings in this study. The two-dimensional model is able to address the lender-borrowers' interaction process. This model predicts lenders' decision-making inequality and indicates whether lenders adopt their optimal strategy by comparing their expected benefits when lending at the loan threshold and lending without restriction to loan size. At the same time, the optimal strategy depends on borrowers' choice probability, which in turn depends on lenders' loan contracts. This two-dimensional model better explained why the lenders lacked responses to loan thresholds.

In this study, I employ the bunching approach to scrutinize the effectiveness of this fiscal policy by assessing market participants' behavioral responses to the PPP fee scheme. The empirical findings indicate that lenders are almost lacking in response to the discontinuous incentives of reducing loan size to increase processing fees in the PPP loan lending process. Consequently, loan sizes have not been distorted at these critical thresholds. From this angle, the SBA's funds are transferred through bank conduits to small businesses without distortion in loan sizes.

For the PPP loan borrowers, the empirical results show significant loan bunching responses at the thresholds created by the SBA's regulations. These thresholds were created by the forgiveness policies that imposed various requirements on two sides of loan thresholds. Even though the government guarantees PPP loans are entirely forgivable, small businesses must meet specific criteria to qualify for forgiveness. However, during the challenging revenue crisis caused by the pandemic, many small businesses struggled to meet these requirements. As a result, they were reluctant to take on extra debt unless they believed that loans would be one hundred percent forgiven. Consequently, they chose to borrow below the policy threshold, guaranteeing a higher probability of forgiveness approval. Furthermore, the intensity of these responses grows over time, suggesting that market participants take time to learn and adapt their optimal borrowing strategies.

Appendix

A Figures

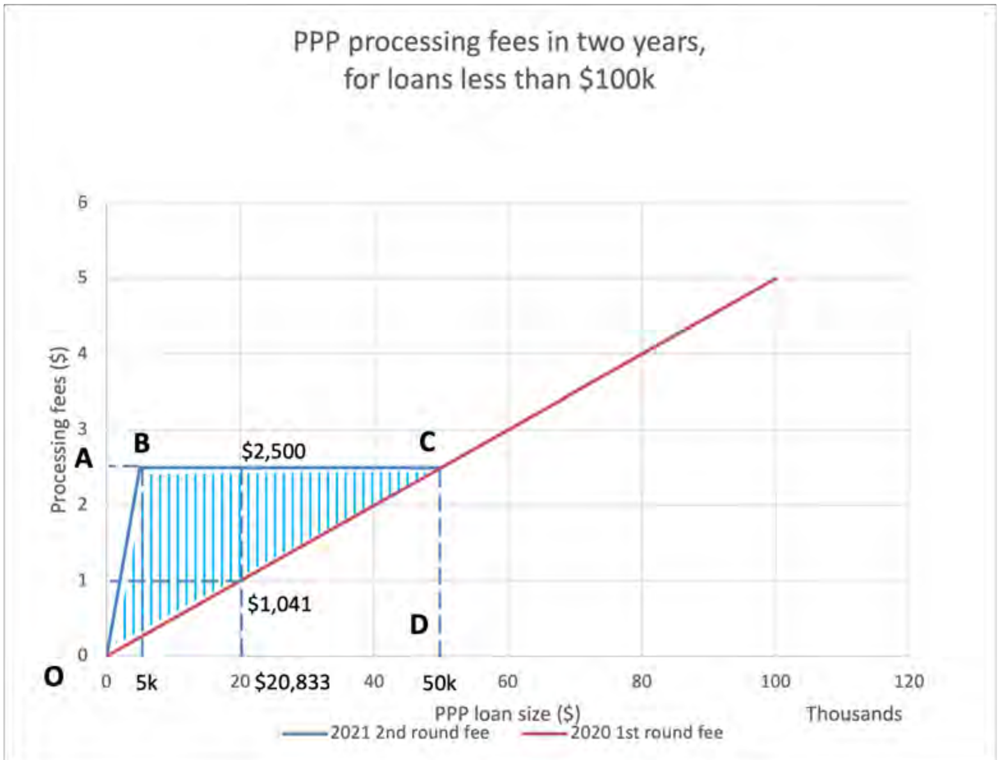
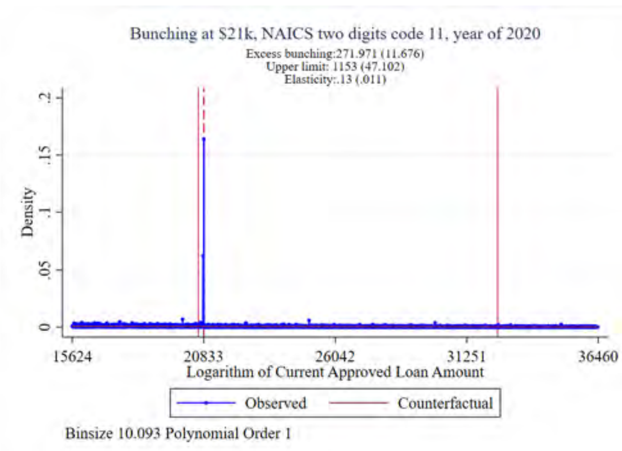


Figure A.18: PPP processing fees in two years, for loans less than $100k

Notes: At the end of 2020 and the beginning of 2021, in the SBA Procedure Notice, SBA adjusted the fees for loans under $50k, that is, 50% or $2,500, whichever is less for loans under $50k. This adjustment creates a concave kink at $5k and a convex kink at $50k. The potential processing fees lenders can charge are marked by solid lines, and dotted lines mark the kink and notch points.

Figure A.19: Heterogeneity at loan size of $21k

(a)



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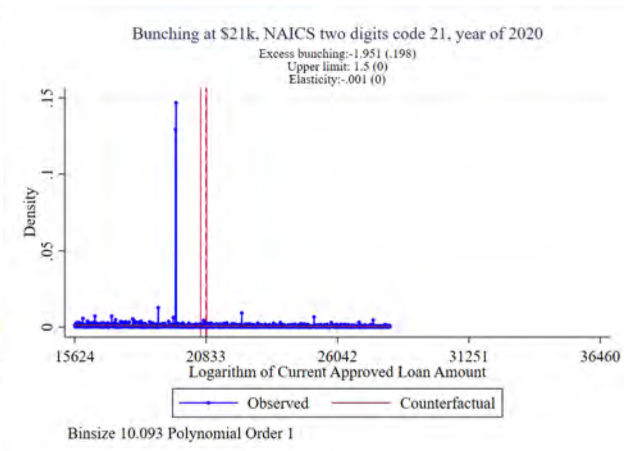

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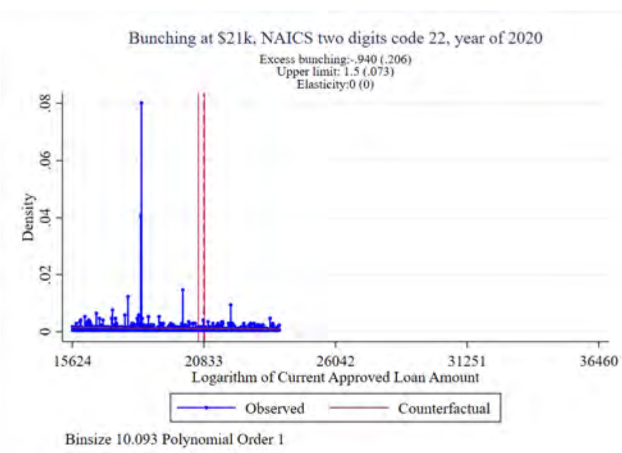

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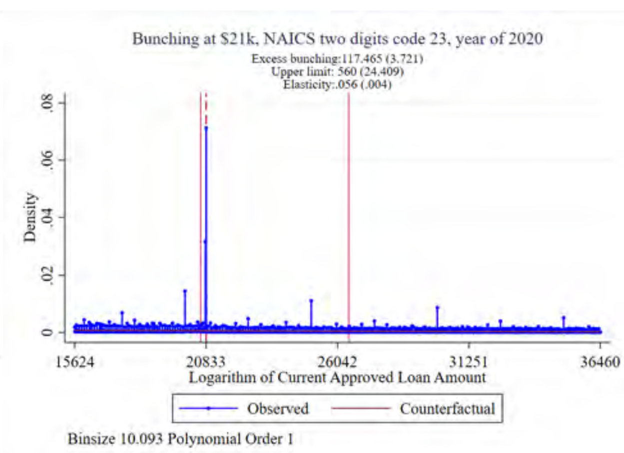
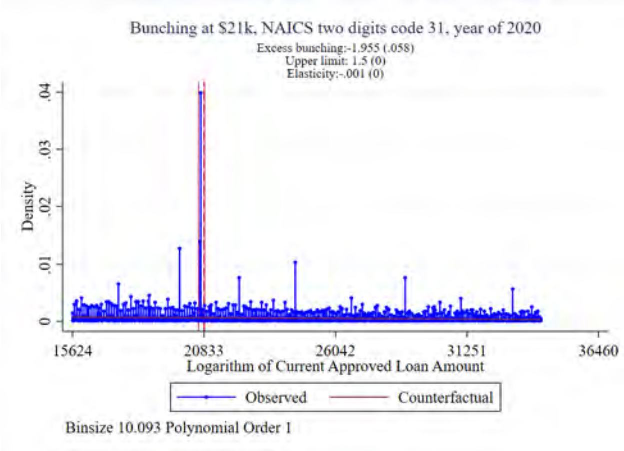


Figure A.19: Heterogeneity at loan size of $21k(continue from the previous page)

(e)



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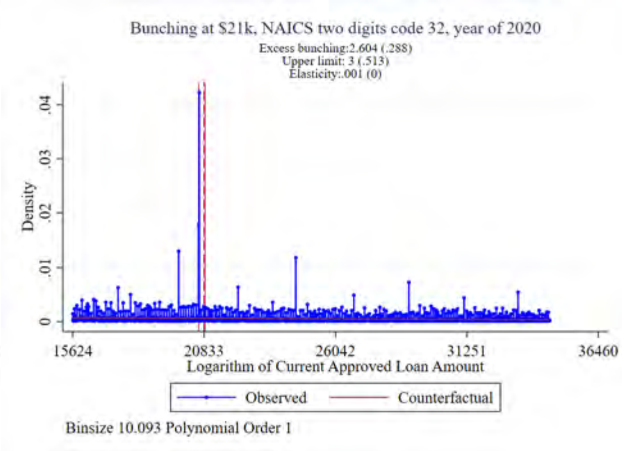

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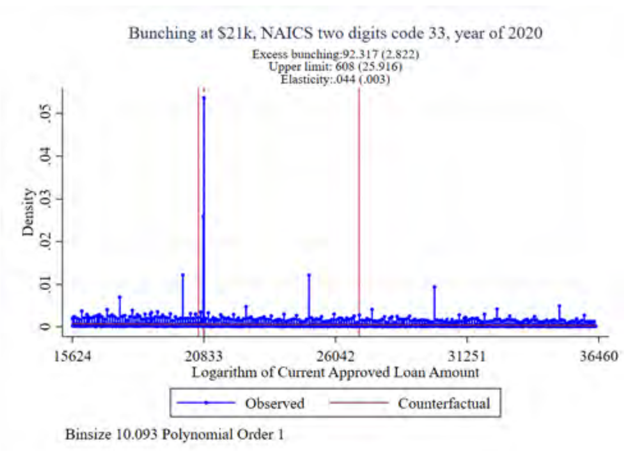

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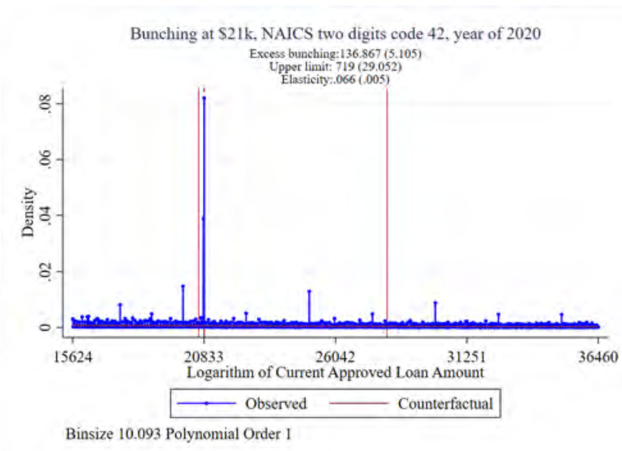

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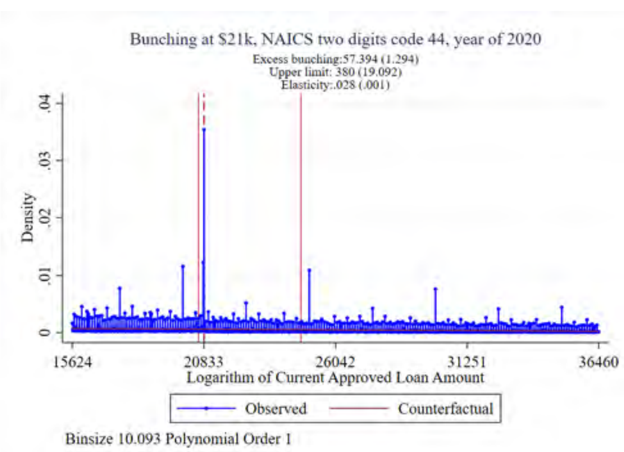

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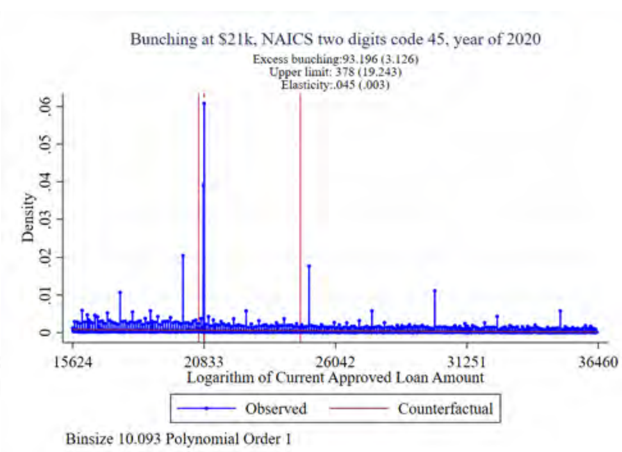

Figure A.19: Heterogeneity at loan size of $21k(continue from the previous page)

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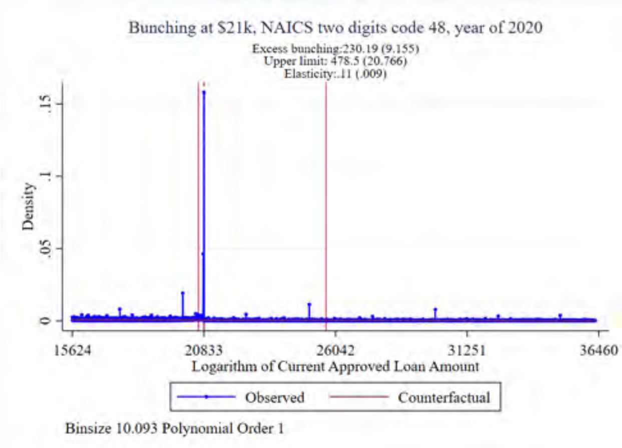

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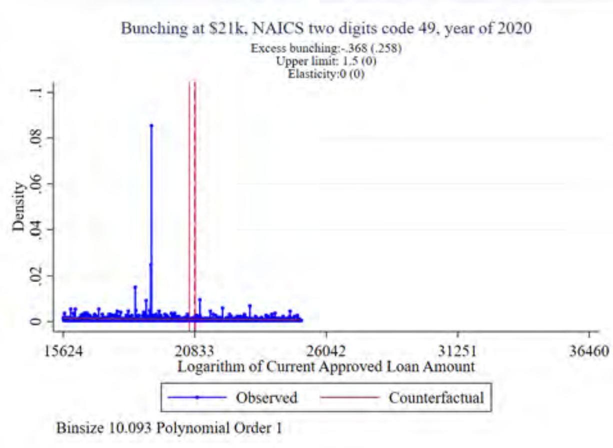

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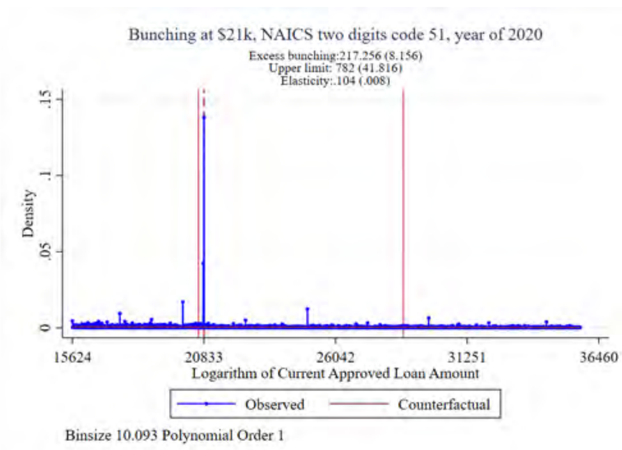

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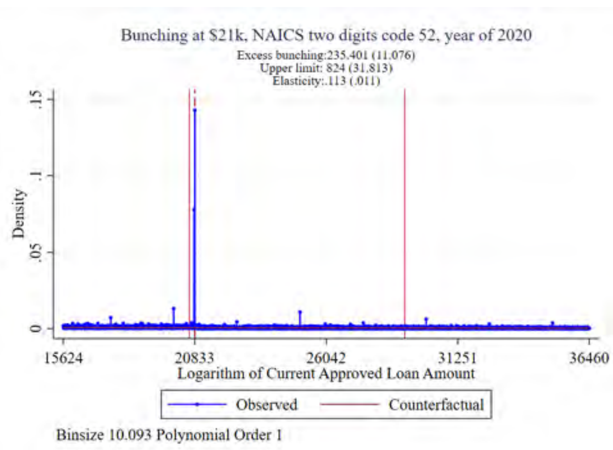

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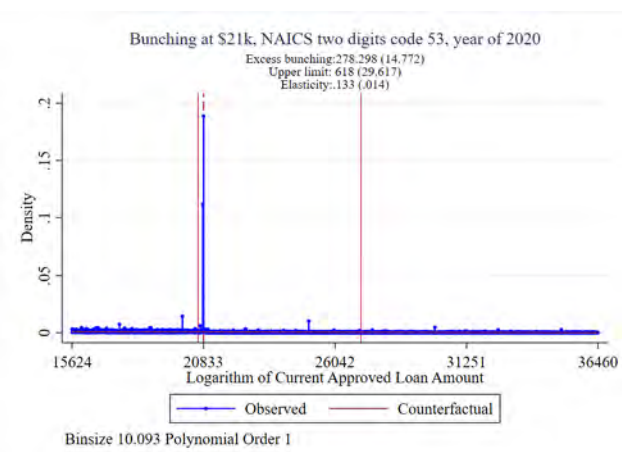

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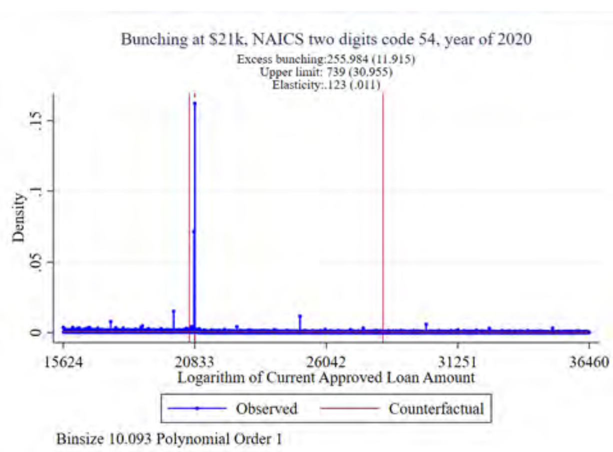

Figure A.19: Heterogeneity at loan size of $21k(continue from the previous page)

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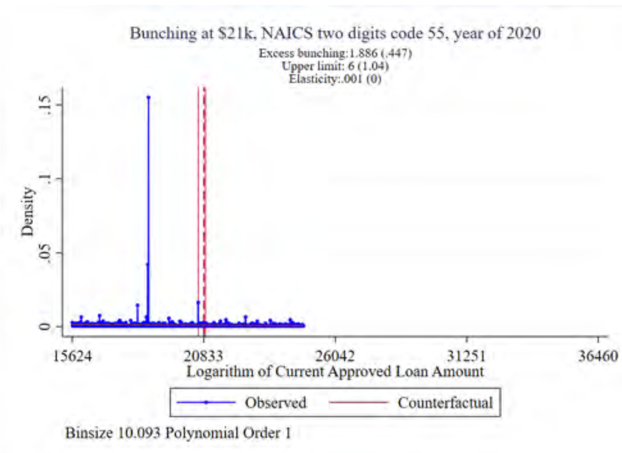

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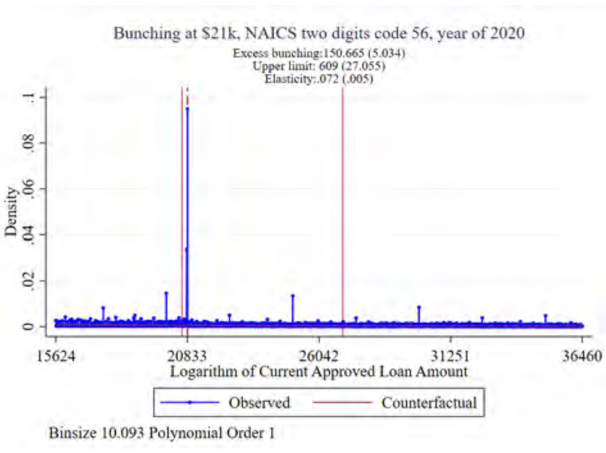

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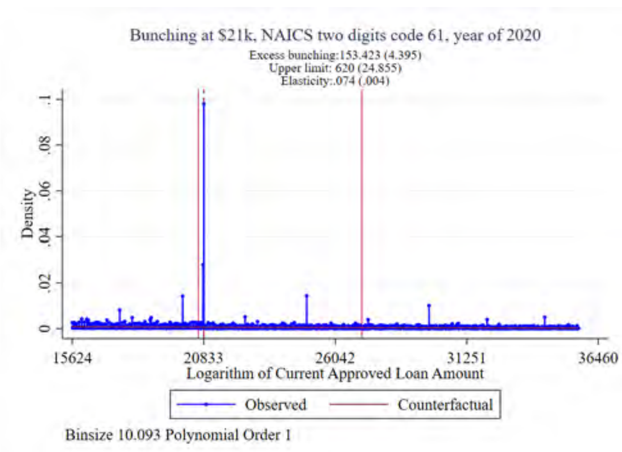

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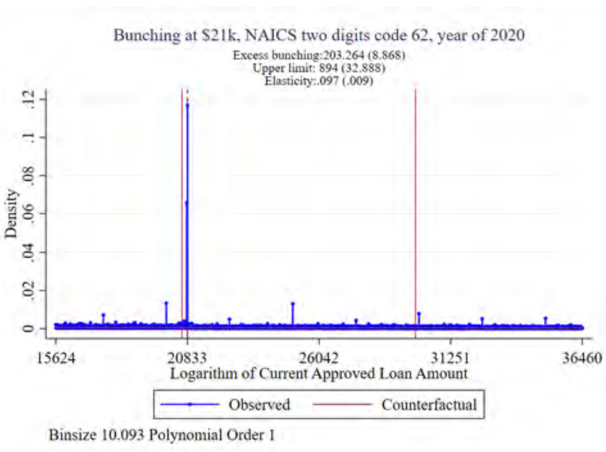

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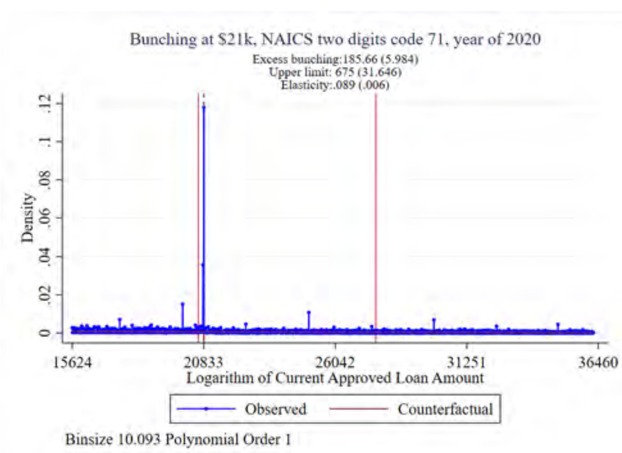

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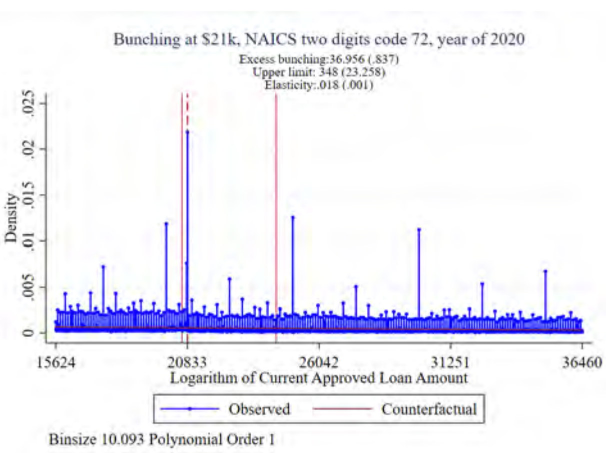

Figure A.19: Heterogeneity at loan size of $21k(continue from the previous page)

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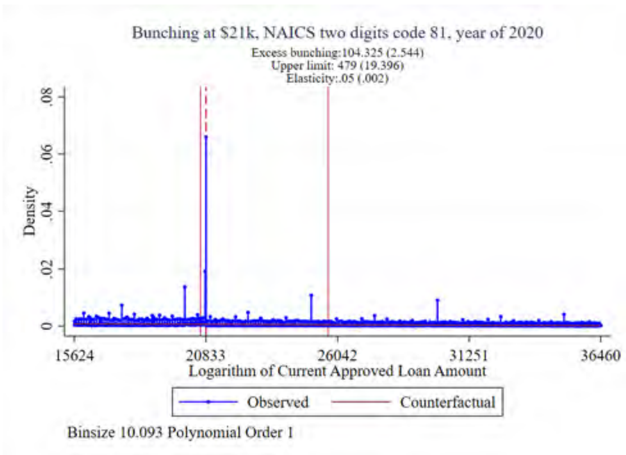

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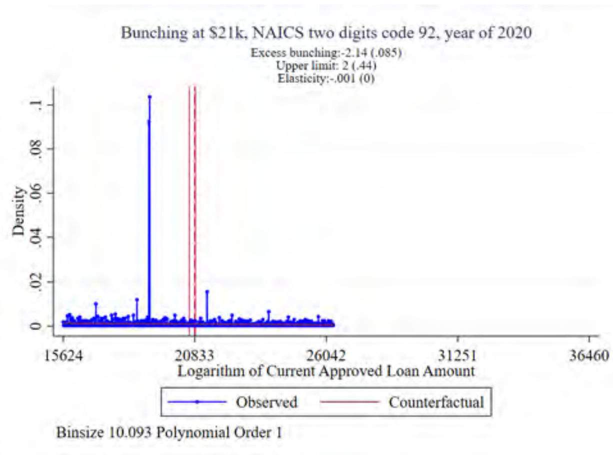

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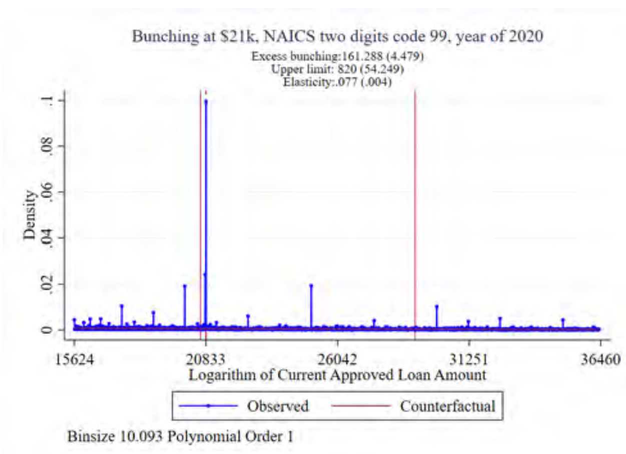

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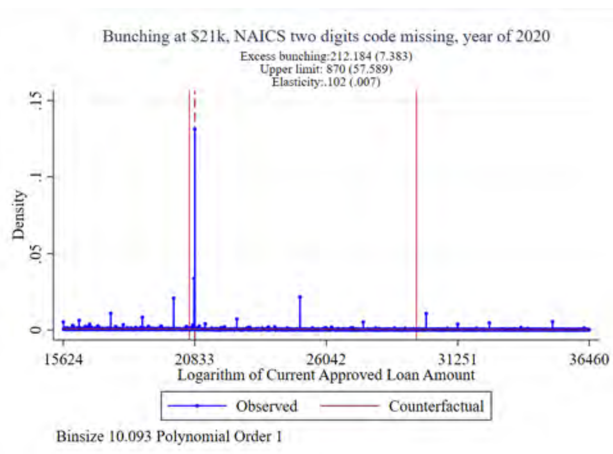
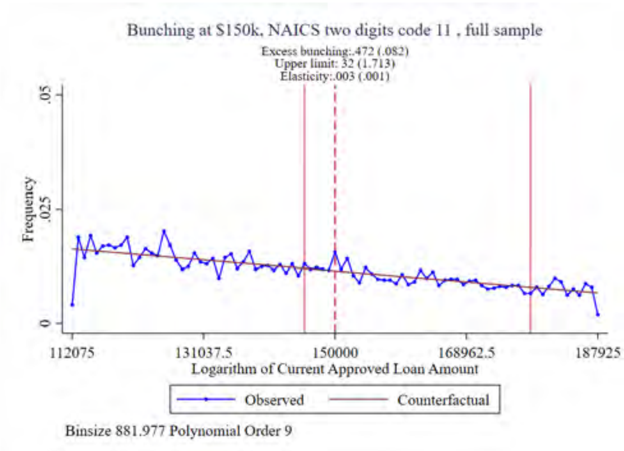


Notes: This figure A.19 displays the bunching loan responses at $21k by industries. Industries are defined by the two- digits NAICS code. Each of the panels displays one industry defined by the NAICS code. The bunching window in each panel has been selected by the iteration process proposed by (Kleven and Waseem, 2013) and (Harju et al., 2019). Bin width has been selected by the Freedman-Diaconis rule (FD rule). BIC has selected the order of polynomials in each regression. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

Figure A.20: Heterogeneity in the industry at loan size of $150k

(a)



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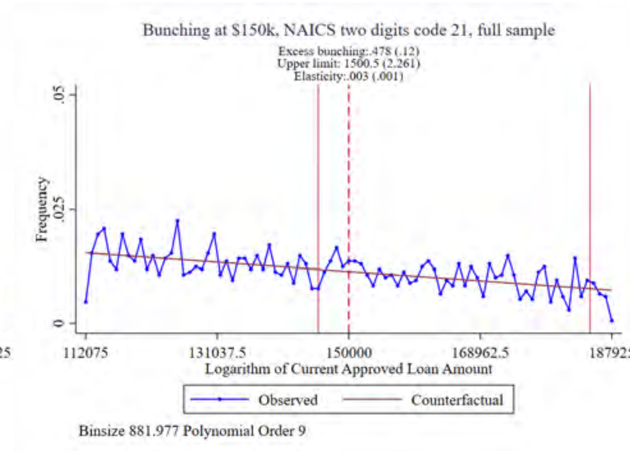

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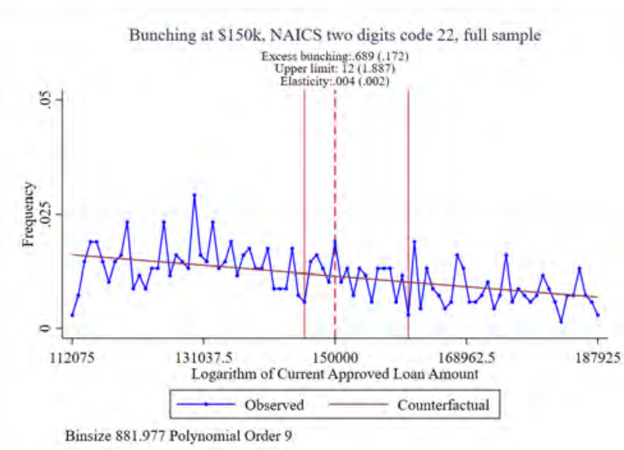

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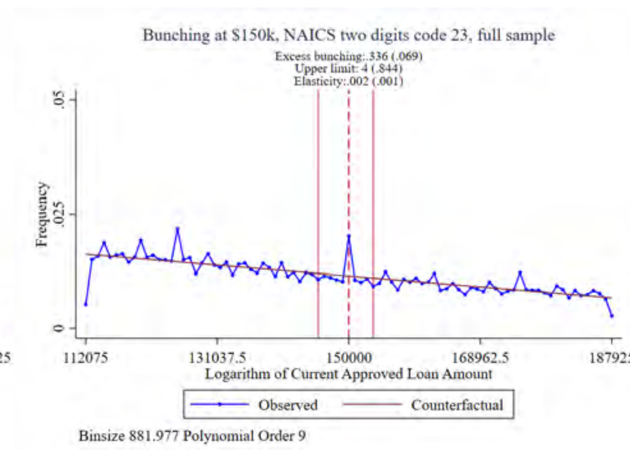

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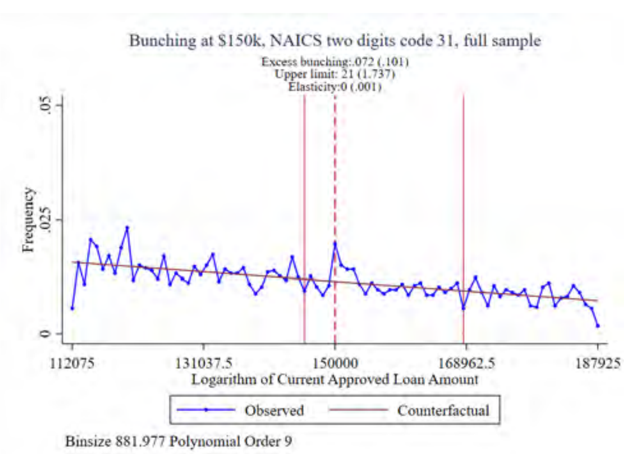

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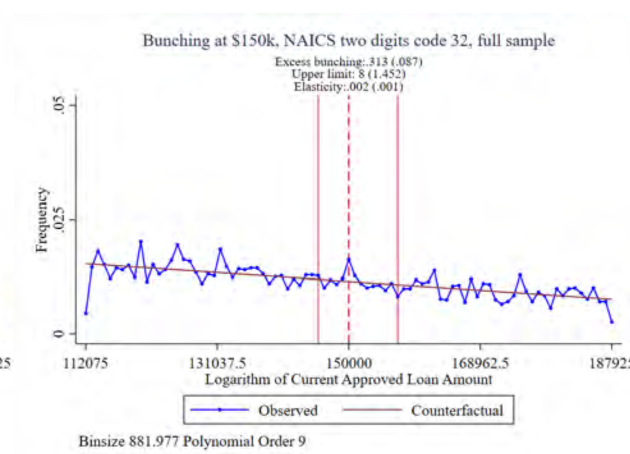
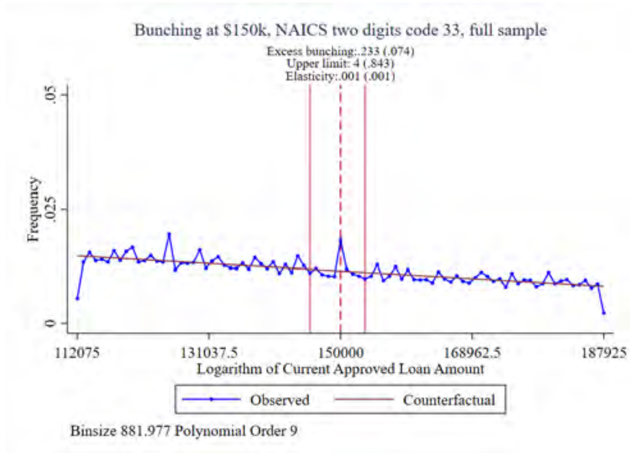


Figure A.20: heterogeneity in industry 150k(continue from the previous page)

(g)



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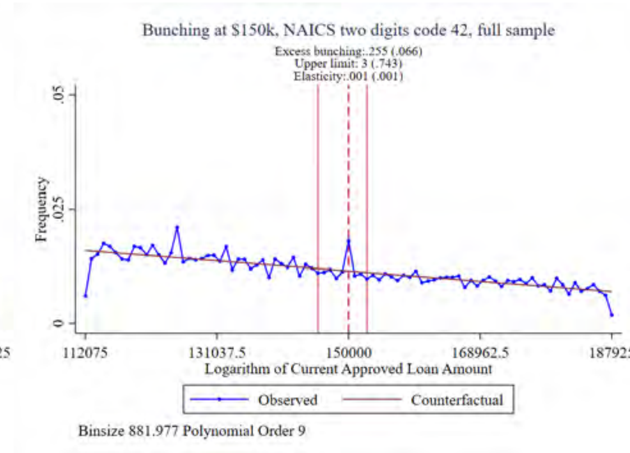

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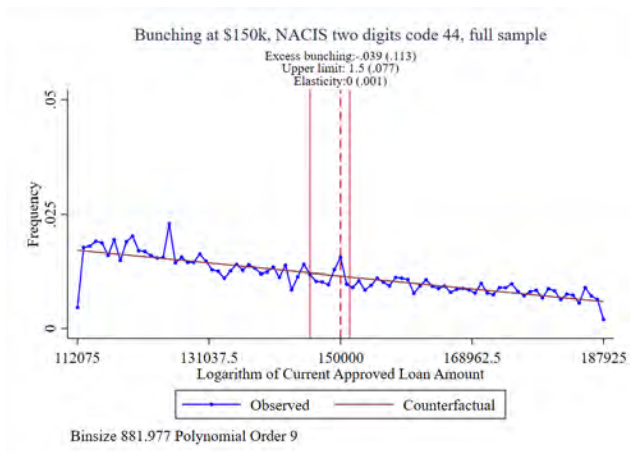

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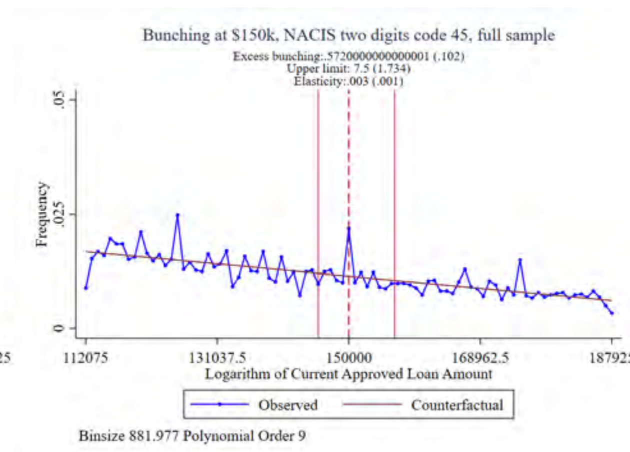

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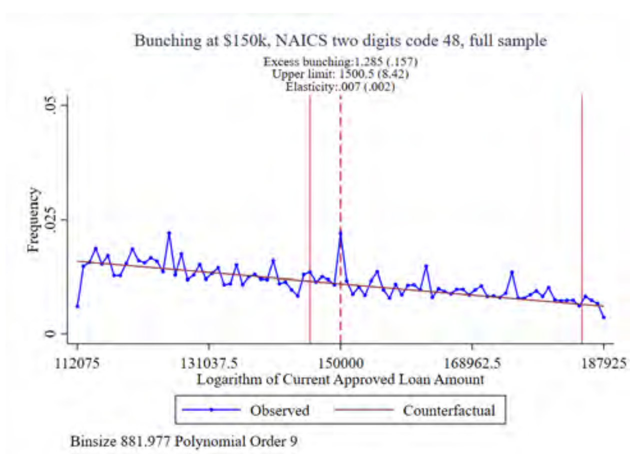

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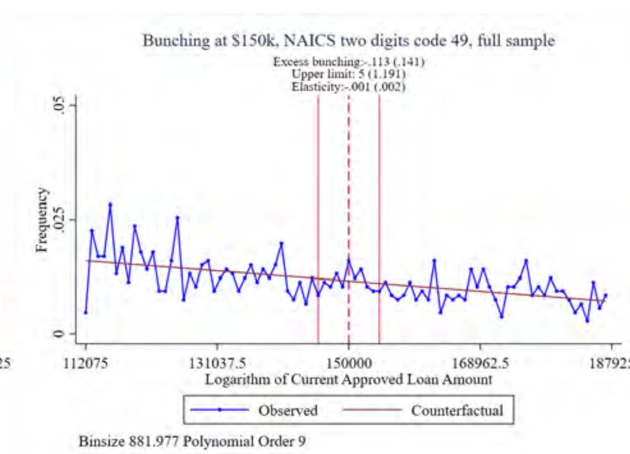
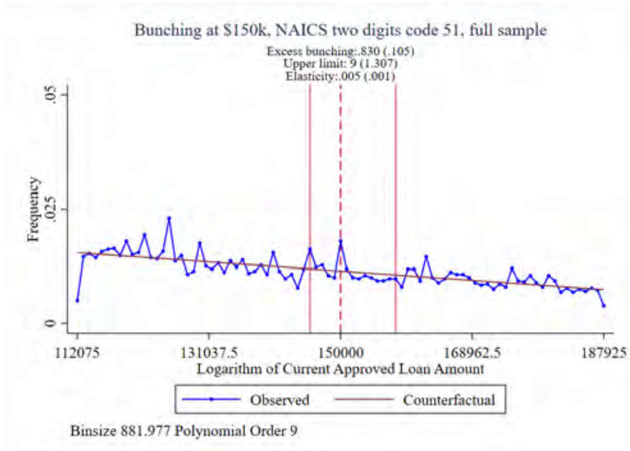


Figure A.20: heterogeneity in industry 150k(continue from the previous page)

(m)



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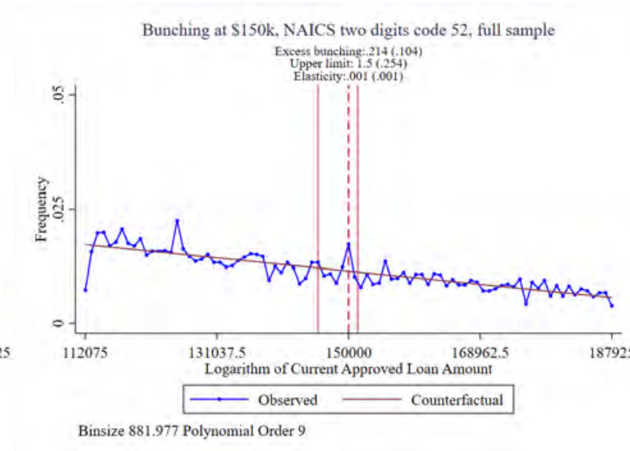

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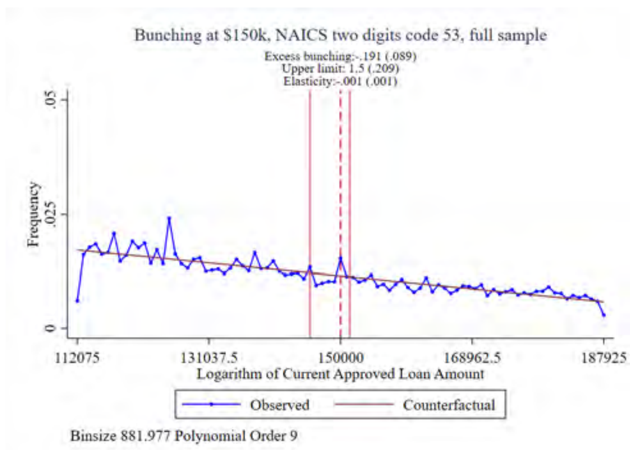

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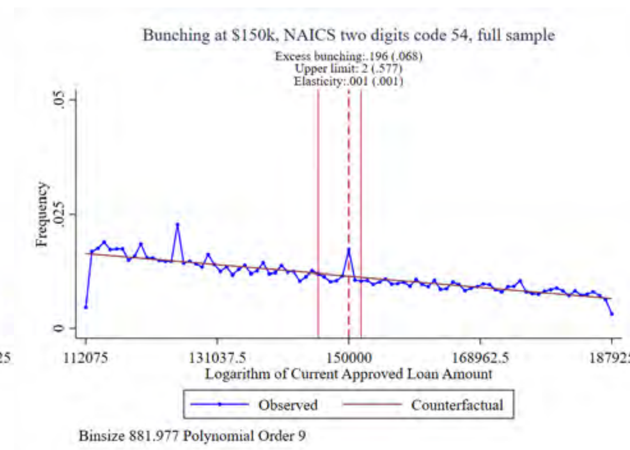

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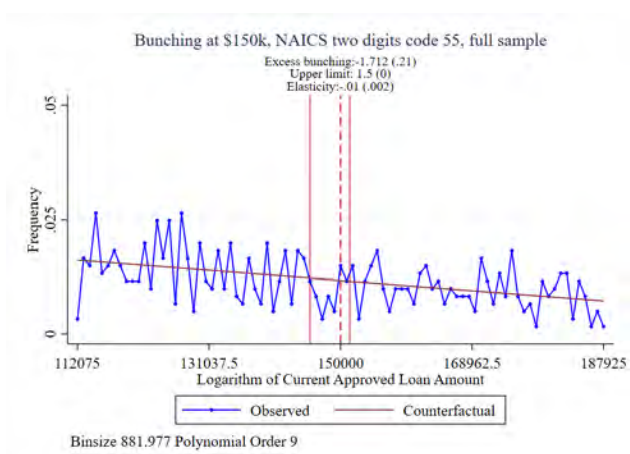

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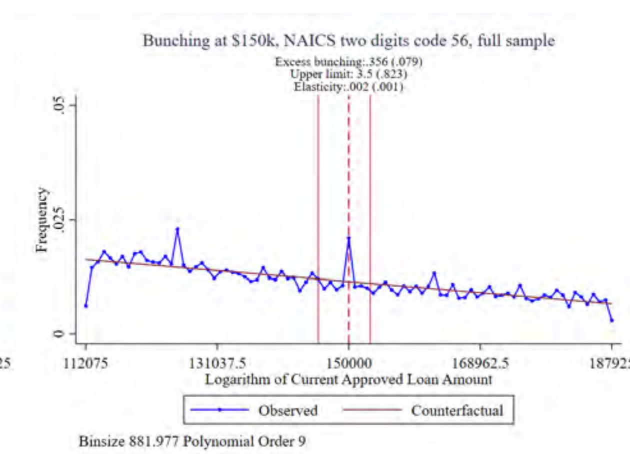

Figure A.20: heterogeneity in industry 150k(continue from the previous page)

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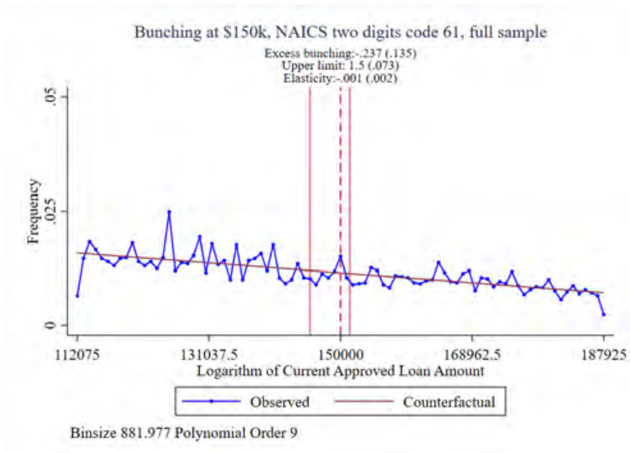

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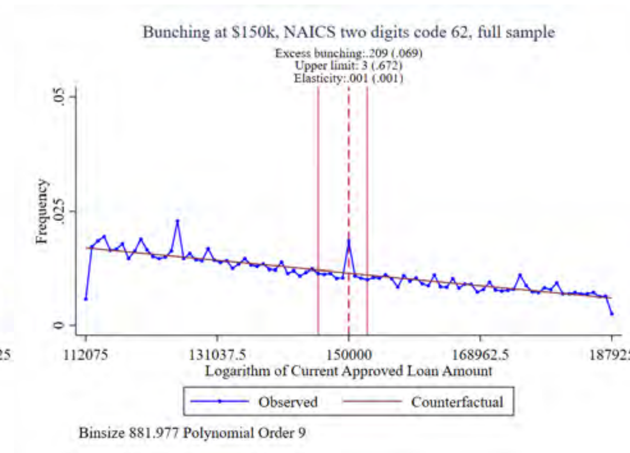
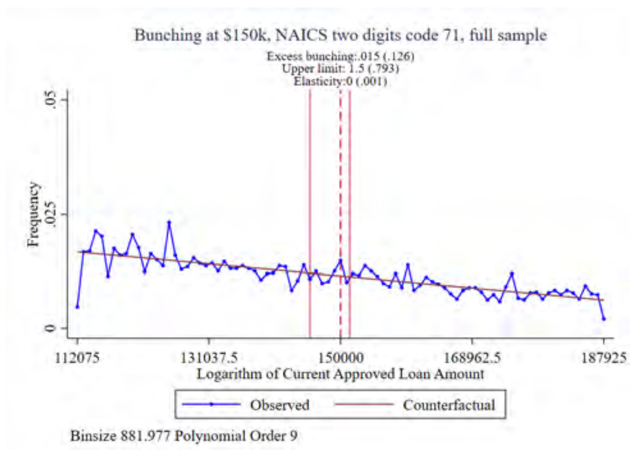
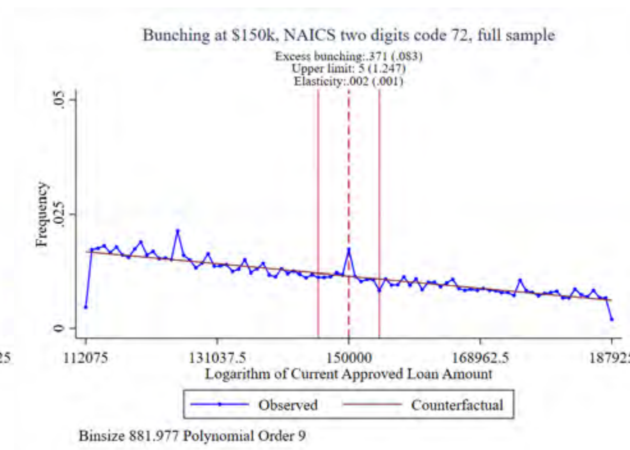


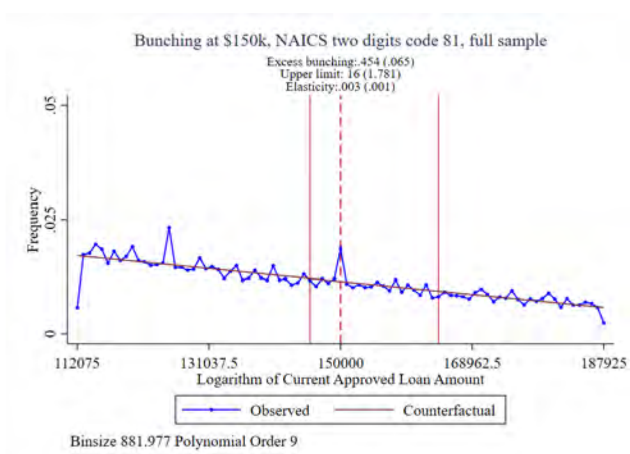
(u)



(v)



(w)



(x)

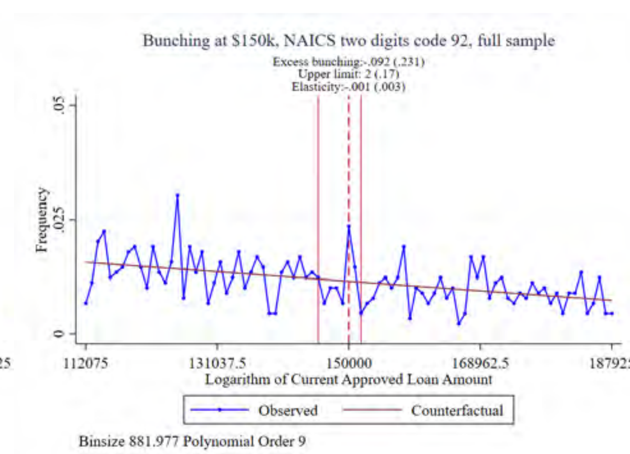
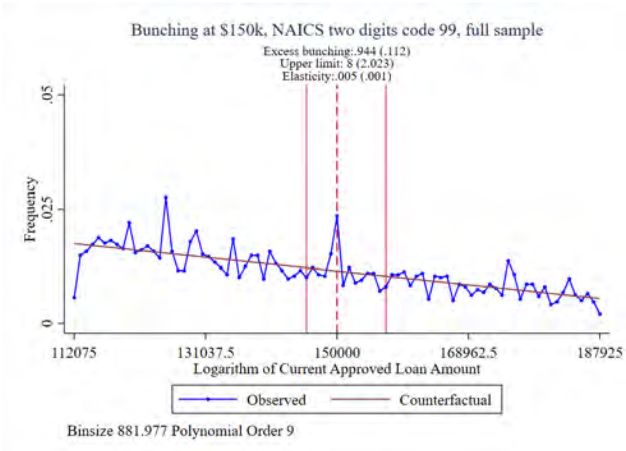
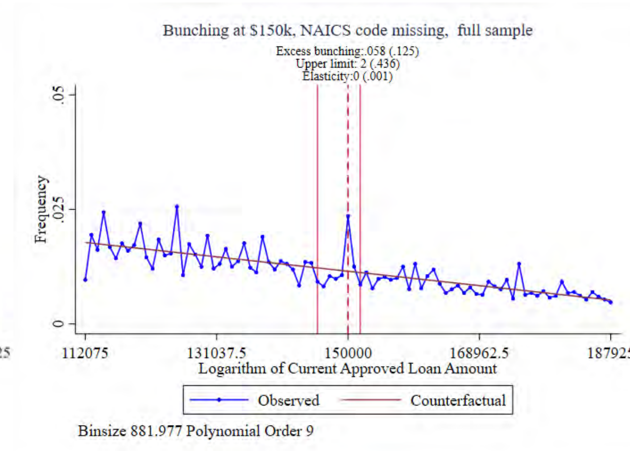


Figure A.20: heterogeneity in industry 150k(continue from the previous page)

(y)



(z)



Notes: Figure A.20 displays the bunching loan responses at $150k by industries. Industries are defined by the two- digits NAICS code. Each of the panels displays one industry defined by the NAICS code. The bunching window in each panel has been selected by the iteration process proposed by (Kleven and Waseem, 2013) and (Harju et al., 2019). Bin width has been selected by the Freedman-Diaconis rule (FD rule). BIC has selected the order of polynomials in each regression. The standard errors, shown in parentheses, are obtained by bootstrapping the estimation routine 500 times.

B Tables

Table B.11: Eligibility requirements for Small Businesses

	Eligibility requirements for Small Businesses
First Draw of PPP loans	. Sole proprietors, independent contractors, and self-employed persons . Any small business concern that meets SBA's size standards (either the industry size standard or the alternative size standard) . Any business, nonprofit organization, 501(c)(19) veterans' organization, or tribal business concern (sec. 31(b)(2)(C) of the Small Business Act) with 500 employees or that meets the SBA industry size standard if more than 500 . Any business with a NAICS code that begins with 72 (Accommodations and Food Services) that has more than one physical location and employs less than 500 per location
Second Draw of PPP loans	. Previously received a First Draw PPP loan and will or has used the total amount only for authorized uses, . It has no more than 300 employees, and . Can demonstrate at least a 25% reduction in gross receipts between comparable quarters in 2019 and 2020.

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