SUPPLEMENT TO "UNCERTAINTY SHOCKS IN A MODEL OF EFFECTIVE DEMAND": TECHNICAL APPENDIX
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BY SUSANTO BASU AND BRENT BUNDICK[1]

APPENDIX A: ADDITIONAL DETAILS CONCERNING EMPIRICAL EVIDENCE

A.1. *Data Construction and Estimation*

THIS SECTION PROVIDES additional details on the data construction and estimation proce-dure for the empirical evidence from Section 2 of the main text. We estimate our baseline VAR using data on the VXO, GDP, consumption, investment, hours worked, the GDP de-flator, the M2 money stock, and the Wu and Xia (2016) shadow rate. To match the concept in the model, we measure consumption in the data as the sum of non-durable and services consumption. Then, we use the sum of consumer durables and private fixed investment as a measure of investment in our baseline empirical model. To match the quarterly fre-quency of the macroeconomic data, we average a weekly VXO series for each quarter. Thus, our measure of uncertainty captures the average implied stock market volatility within a quarter. We convert output, consumption, investment, and hours worked to per-capita terms by dividing by population. Except for the shadow rate, all other variables enter the VAR in log levels.

We include four lags in the estimation of the VAR and generate our confidence intervals using the Bayesian method outlined in Sims and Zha (1999).[2] Figure A.1 plots the VXO over time as well as the series of identified, structural uncertainty shocks from the VAR. The empirical model identifies large uncertainty shocks after the 1987 stock market crash, the failure of Lehman Brothers, and the euro area sovereign debt crisis.

To generate the unconditional moments in Table II of the main text, we detrend the log of each empirical data series using the HP filter with a smoothing parameter of 1600. We measure the unconditional volatility using the sample standard deviation of the detrended variable. We compute the empirical moments over the 1986–2014 sample period, which is the same time frame used in our empirical VAR. In Appendix Section D.3, we provide further analysis of the unconditional moments predicted by the model.

We estimate stochastic volatility using a simple model-free and nonparametric method based on rolling sample standard deviations. Given an empirical data series, we estimate a rolling 5-year standard deviation. This procedure provides a time-series of realized volatility estimates for the given data series. Then, we compute the standard deviation of this time-series of estimates. This simple measure provides an estimate of the stochas-tic volatility in the data series. If the actual data were homoscedastic, the estimates of the 5-year rolling standard deviations should show little volatility and the resulting statistic would be near zero.
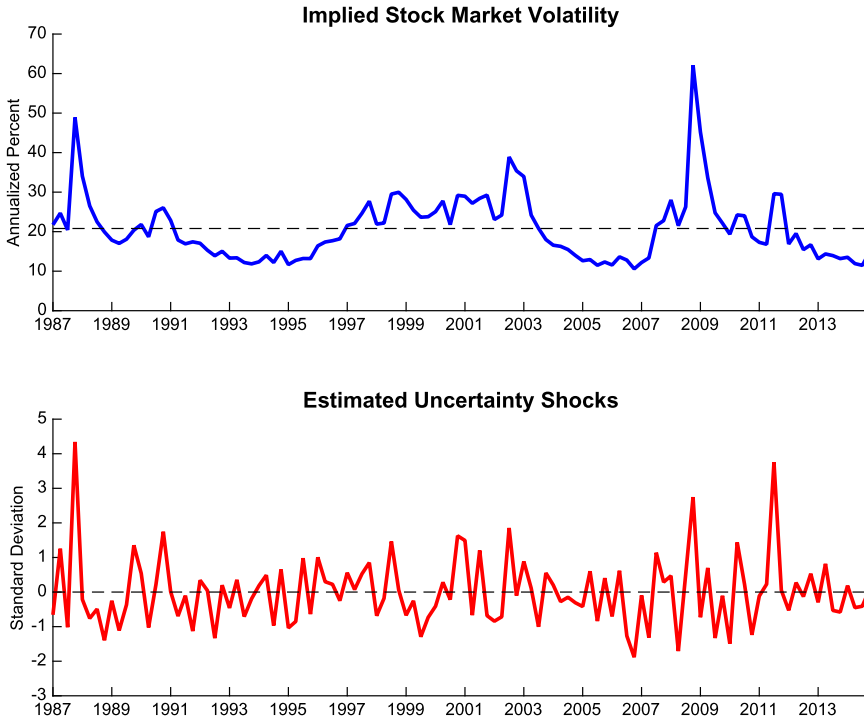

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FIGURE A.1.—VXO and estimated uncertainty shocks.

A.2. *Robustness of Macroeconomic Comovement*

We argue that macroeconomic comovement between output, consumption, investment, and hours worked is a key stylized fact after an identified uncertainty shock. In this section, we show that our key empirical result is robust along several dimensions.

We now re-estimate our baseline specification but include the Standard & Poor's 500 Stock Price Index in the VAR. Many other studies, such as Bloom (2009), commonly include stock prices in their empirical specifications. According to our theoretical model, first- and second-moment shocks both affect equity prices at impact, but only second-moment shocks affect the expected volatility of equity returns. To be consistent with our model, we keep the VXO ordered first, followed by stock prices and the other macroeconomic variables. This ordering allows both uncertainty and non-uncertainty shocks to affect stock prices at impact. Consistent with our model, Figure A.2 shows that stock prices decline after an identified uncertainty shock. Including stock prices in the VAR produces a slightly larger decline in investment, but overall, the results are very similar to our baseline specification.

In our baseline specification, we use the VXO as our measure of uncertainty, which measures the expected volatility of the Standard & Poor's 100 Stock Price Index. However, the VIX, which is the implied volatility on the Standard & Poor's 500 Stock Price Index, is a more well-known measure of expected stock market volatility. We use the VXO in our baseline model because it is available beginning in 1986, whereas the VIX is only available beginning in 1990. Figure A.3 re-estimates our baseline model using the VIX, rather than the VXO. While the confidence intervals for some variables are slightly larger
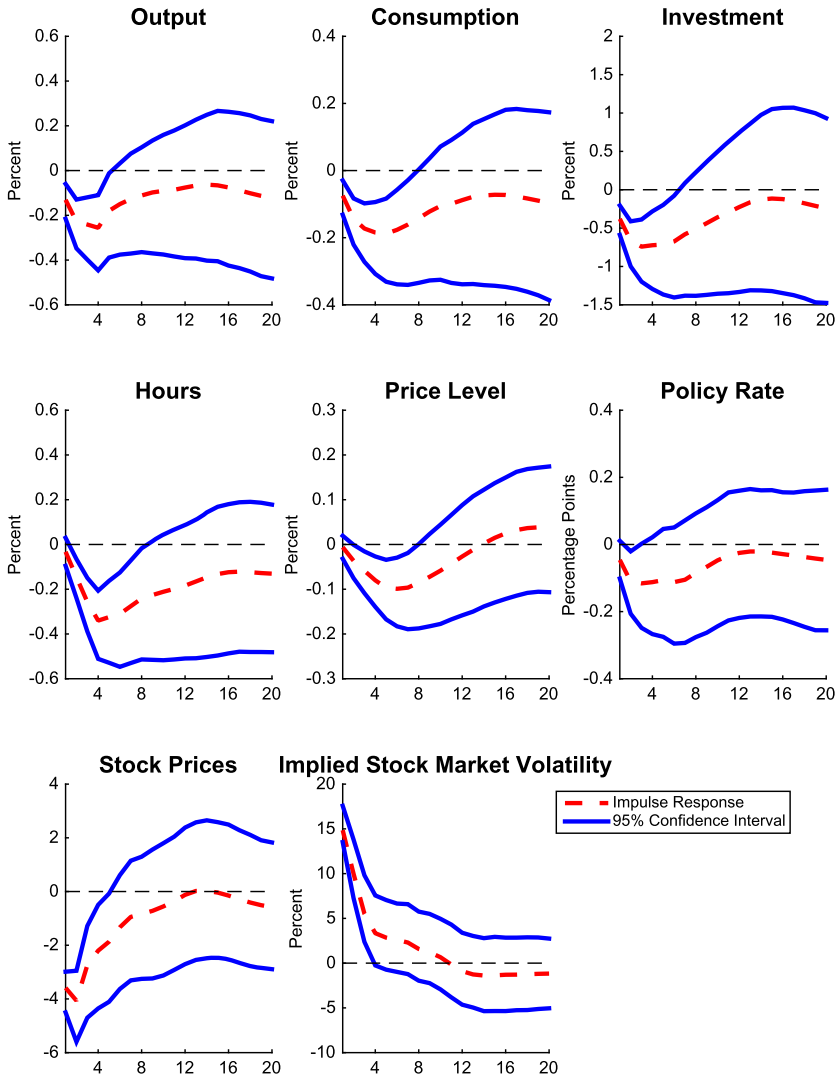
FIGURE A.2.—Including stock prices in the baseline empirical specification.

(owing to the shorter data sample), we continue to observe macroeconomic comovement following an uncertainty shock in the data.

In our main empirical model, we treated consumer durables as a form of investment. If we instead use the standard National Income and Product Accounts definitions of consumption and investment, Figure A.4 shows a larger impact effect on consumption with a slight over-shoot after three years. The response of investment, however, remains similar to our baseline results.

Our baseline results are also robust to using higher frequency estimation. In our baseline model, we aggregate a weekly VXO series to quarterly frequency. However, the VXO reflects the expected S&P 100 volatility over the next 30 days, not over the next quarter. To ensure our results are robust to this aggregation strategy, we estimate a version of our empirical model using monthly frequency data on the VXO, output, non-durable plus
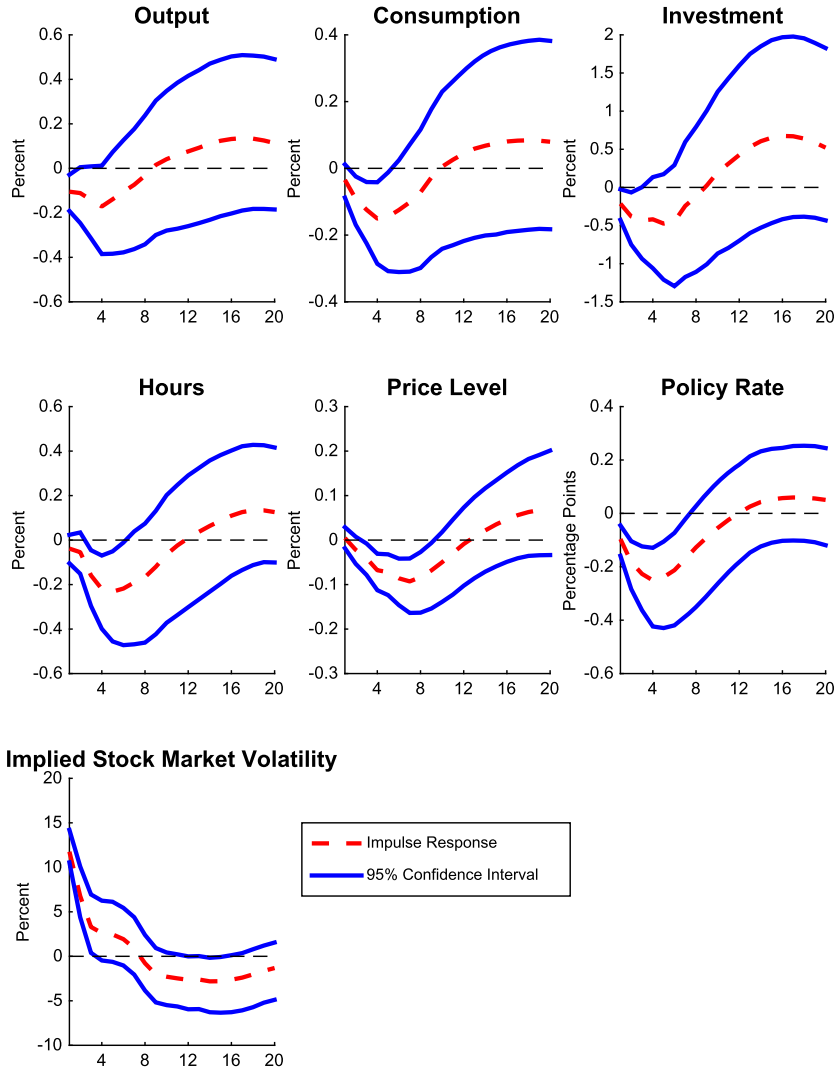
FIGURE A.3.—Measuring uncertainty with the VIX.

services consumption, durable consumption, hours worked, the personal consumption expenditure price index, the M2 money stock, and the shadow rate. To construct a monthly GDP series, we splice together monthly GDP from Macroeconomic Advisers beginning in 1992 with Stock and Watson's (2010) monthly GDP estimates from the NBER Business Cycle Dating committee website.[3] Figure A.4 shows that our results are nearly unchanged if we use this higher frequency data. However, data on investment are not available at a monthly frequency. Therefore, we rely on the aggregated quarterly data for our baseline empirical results.

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[3]Since 2010, the Bureau of Economic Analysis now includes intellectual property as a form of investment. Thus, we splice the two series together using the growth rates from the Stock and Watson estimates to fix issues with the actual level of the data series.
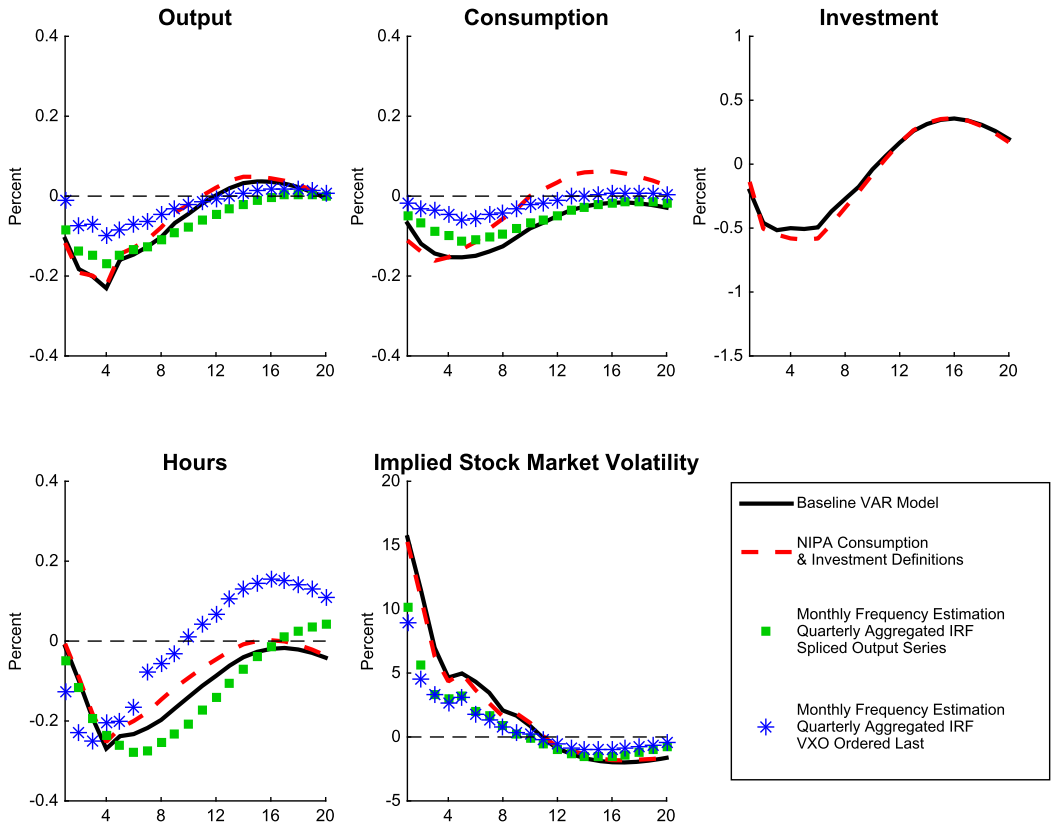
FIGURE A.4.—Alternative empirical specifications. Note: Each series represents point estimates from a different empirical specification.

In addition, we compute the impulse response to an uncertainty shock with the VXO ordered last in our structural VAR. Figure A.4 also shows that our main stylized fact regarding macroeconomic comovement remains under this alternative identification scheme, which allows contemporaneous macroeconomic events to affect the level of uncertainty. While this ordering is not consistent with our theoretical model, it shows that our baseline identification scheme alone is not crucial for our main result. We compute this robustness check using monthly data, to match the interpretation of the VXO as closely as possible. However, the results with quarterly data produce similar findings.

Figure A.5 contains three additional specifications, which examine alternative assumptions about monetary policy. As we discuss in the main text, the Federal Reserve hit the zero lower bound on nominal interest rates at the end of 2008. While we model this outcome rigorously using our theoretical model, it is less clear how to model the stance of monetary policy during our 1986–2014 sample period econometrically. In our baseline VAR results, we used the Wu and Xia (2016) shadow rate as our indicator of monetary policy. Away from the zero lower bound, this series equals the federal funds rate. But at the zero lower bound, the shadow rate uses information from the entire yield curve to summarize the stance of monetary policy. However, this modeling choice is clearly not the only reasonable one.
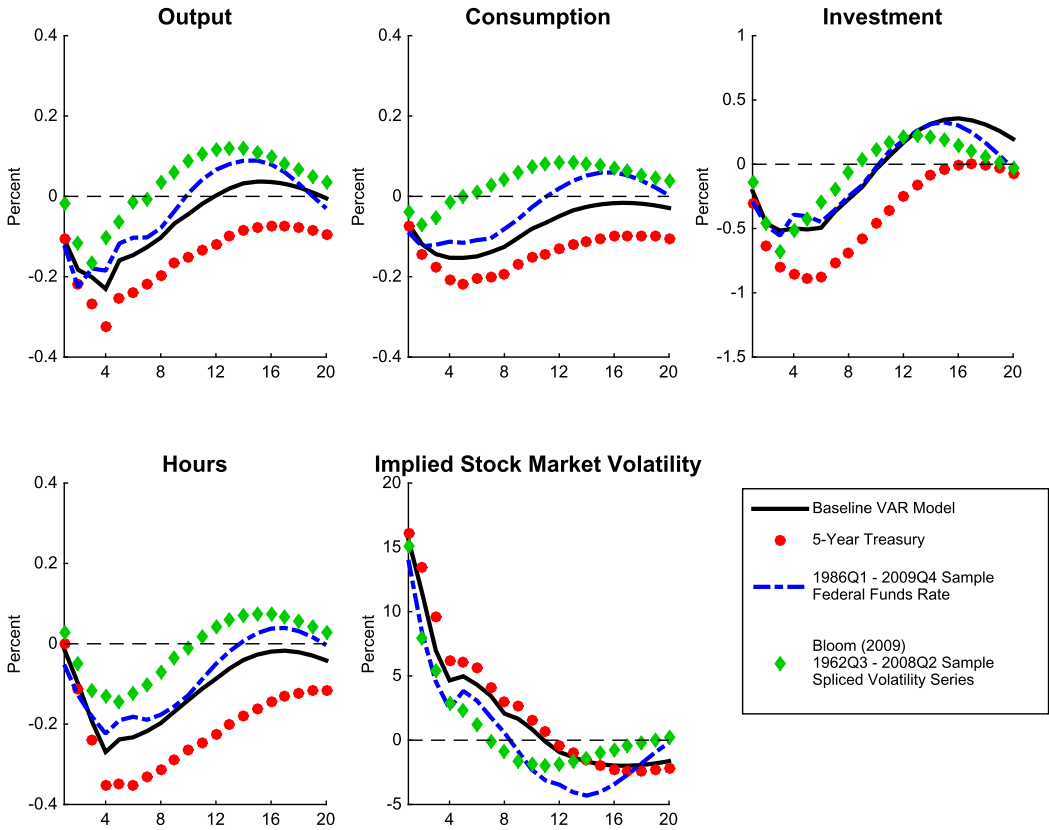
FIGURE A.5.—Alternative specifications for monetary policy. Note: Each series represents point estimates from a different structural vector autoregression.

As an alternative, we can use the 5-year Treasury rate as a control for monetary policy. Since hitting the zero lower bound, the Federal Reserve used a variety of large-scale asset purchases and forward guidance to help stabilize the economy. Longer-term Treasury rates reflect the effects of these unconventional policies. An alternative modeling assumption is to use the federal funds rate but end the sample period before the zero lower bound binds for too long. Figure A.5 shows that either of these alternative assumptions actually produces responses that are larger than our baseline model. A different modeling assumption is to remove the post-2008 period altogether. If we use the 1962Q3–2008Q2 sample of Bloom (2009) with the federal funds rate as the measure of monetary policy, our stylized fact remains: Higher uncertainty generates declines in output, consumption, investment, and hours worked.[4]

APPENDIX B: MODEL

This section provides a detailed derivation and discussion of the baseline dynamic, stochastic general-equilibrium model that we use in our analysis of uncertainty shocks.

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[4]Since the VXO is first measured in 1986, we use the spliced volatility series constructed by Bloom (2009). This series splices together predicted volatility from a time-series model in the pre-1986 period with the ex ante VXO measure of implied volatility after 1986.

The baseline model shares many features of the models of Ireland (2003), Ireland (2011), and Jermann (1998). The model features optimizing households and firms and a central bank that follows a Taylor rule to stabilize inflation and offset adverse shocks. We allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). Our baseline model considers technology and household discount rate shocks. The discount rate shocks have a time-varying second moment, which we interpret as the degree of uncertainty about future demand.

B.1. *Households*

In our model, the representative household maximizes lifetime utility given Epstein–Zin preferences over streams of consumption C_t and leisure $1 - N_t$. The key parameters governing household decisions are its risk aversion σ over the consumption-leisure basket and its intertemporal elasticity of substitution ψ. The parameter $\theta_V \triangleq (1 - \sigma)(1 - 1/\psi)^{-1}$ controls the household's preference for the resolution of uncertainty. The household receives labor income W_t for each unit of labor N_t supplied to the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and holds equity shares S_t and one-period risk-less bonds B_t issued by representative intermediate goods firm. Equity shares have a price of P_t^E and pay dividends D_t^E for each share S_t owned. The risk-less bonds return the gross one-period risk-free interest rate R_t^R. The household divides its income from labor and its financial assets between consumption C_t and holdings of financial assets S_{t+1} and B_{t+1} to carry into next period. The discount rate of the household β is subject to shocks via the stochastic process a_t.

The representative household maximizes lifetime utility by choosing C_{t+s}, N_{t+s}, B_{t+s+1}, and S_{t+s+1} for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$V_t = \max\left[a_t\left(C_t^\eta(1 - N_t)^{1-\eta}\right)^{(1-\sigma)/\theta_V} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\sigma}\right)^{1/\theta_V}\right]^{\theta_V/(1-\sigma)}$$

subject to its intertemporal household budget constraint each period,

$$C_t + \frac{P_t^E}{P_t}S_{t+1} + \frac{1}{R_t^R}B_{t+1} \le \frac{W_t}{P_t}N_t + \left(\frac{D_t^E}{P_t} + \frac{P_t^E}{P_t}\right)S_t + B_t.$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\frac{\partial V_t}{\partial C_t} = \lambda_t, \tag{S.1}$$

$$\frac{\partial V_t}{\partial N_t} = \lambda_t \frac{W_t}{P_t}, \tag{S.2}$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t\left\{\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}}\right)\right\}, \tag{S.3}$$

$$1 = R_t^R \mathbb{E}_t\left\{\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\right\}, \tag{S.4}$$

where λ_t denotes the Lagrange multiplier on the household budget constraint. Epstein–Zin preferences imply the following relationships:

$$\frac{\partial V_t}{\partial C_t} = a_t V_t^{1-(1-\sigma)/\theta_V} \eta \frac{\left(C_t^\eta (1-N_t)^{1-\eta}\right)^{(1-\sigma)/\theta_V}}{C_t},$$

$$\frac{\partial V_{t+1}}{\partial C_{t+1}} = a_{t+1} V_{t+1}^{1-(1-\sigma)/\theta_V} \eta \frac{\left(C_{t+1}^\eta (1-N_{t+1})^{1-\eta}\right)^{(1-\sigma)/\theta_V}}{C_{t+1}},$$

$$\frac{\partial V_t}{\partial C_{t+1}} = \beta V_t^{1-(1-\sigma)/\theta_V} \left(\mathbb{E}_t V_{t+1}^{1-\sigma}\right)^{1/\theta_V - 1} \mathbb{E}_t \left\{ V_{t+1}^{-\sigma} \left(\frac{\partial V_{t+1}}{\partial C_{t+1}}\right) \right\}$$

$$= \beta V_t^{1-(1-\sigma)/\theta_V} \left(\mathbb{E}_t V_{t+1}^{1-\sigma}\right)^{1/\theta_V - 1} \mathbb{E}_t \left\{ V_{t+1}^{-\sigma} a_{t+1} V_{t+1}^{1-(1-\sigma)/\theta_V} \eta \frac{\left(C_{t+1}^\eta (1-N_{t+1})^{1-\eta}\right)^{(1-\sigma)/\theta_V}}{C_{t+1}} \right\}.$$

Thus, we define the household stochastic discount factor M between periods t and $t+1$:

$$M_{t+1} \triangleq \left(\frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t}\right) = \left(\beta \frac{a_{t+1}}{a_t}\right) \left(\frac{C_{t+1}^\eta (1-N_{t+1})^{1-\eta}}{C_t^\eta (1-N_t)^{1-\eta}}\right)^{(1-\sigma)/\theta_V} \left(\frac{C_t}{C_{t+1}}\right) \left(\frac{V_{t+1}^{1-\sigma}}{\mathbb{E}_t \left[V_{t+1}^{1-\sigma}\right]}\right)^{1-1/\theta_V}.$$

Using the stochastic discount factor, we can eliminate λ and simplify Equations (S.1)–(S.4):

$$\frac{1-\eta}{\eta} \frac{C_t}{1-N_t} = \frac{W_t}{P_t}, \tag{S.5}$$

$$\frac{P_t^E}{P_t} = \mathbb{E}_t \left\{ M_{t+1} \left(\frac{D_{t+1}^E}{P_{t+1}} + \frac{P_{t+1}^E}{P_{t+1}}\right) \right\}, \tag{S.6}$$

$$1 = R_t^R \mathbb{E}_t \{M_{t+1}\}. \tag{S.7}$$

Equation (S.5) represents the household intratemporal optimality condition with respect to consumption and leisure, and Equations (S.6) and (S.7) represent the Euler equations for equity shares and one-period risk-less firm bonds.

B.2. *Intermediate Goods Producers*

Each intermediate goods-producing firm i rents labor $N_t(i)$ from the representative household to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. The intermediate goods firms own their capital stocks $K_t(i)$, and face convex costs of changing the quantity of installed capital. Firms also choose the rate of utilization of their installed physical capital $U_t(i)$, which affects its depreciation rate. Each firm issues equity shares $S_t(i)$ and one-period risk-less bonds $B_t(i)$. Firm i chooses $N_t(i)$, $I_t(i)$, $U_t(i)$, and $P_t(i)$ to maximize firm cash flows $D_t(i)/P_t(i)$ given aggregate demand Y_t and price P_t of the finished goods sector. The intermediate goods firms all have the same constant returns to scale Cobb–Douglas production function, subject to a fixed cost of production Φ and their level of productivity Z_t.

Each firm producing intermediate goods maximizes discounted cash flows using the household's stochastic discount factor:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \left(\frac{\partial V_t / \partial C_{t+s}}{\partial V_t / \partial C_t} \right) \left[\frac{D_{t+s}(i)}{P_{t+s}} \right]$$

subject to the production function:

$$\left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} Y_t \leq \left[K_t(i) U_t(i) \right]^\alpha \left[Z_t N_t(i) \right]^{1-\alpha} - \Phi,$$

and subject to the capital accumulation equation:

$$K_{t+1}(i) = \left(1 - \delta(U_t(i)) - \frac{\phi_K}{2} \left(\frac{I_t(i)}{K_t(i)} - \delta \right)^2 \right) K_t(i) + I_t(i),$$

where

$$\frac{D_t(i)}{P_t} = \left[\frac{P_t(i)}{P_t} \right]^{1-\theta_\mu} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi_P}{2} \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

and depreciation depends on utilization via the following functional form:

$$\delta(U_t(i)) = \delta + \delta_1(U_t(i) - U) + \left(\frac{\delta_2}{2} \right)(U_t(i) - U)^2.$$

The behavior of each firm i satisfies the following first-order conditions:

$$\frac{W_t}{P_t} N_t(i) = (1-\alpha) \Xi_t \left[K_t(i) U_t(i) \right]^\alpha \left[Z_t N_t(i) \right]^{1-\alpha},$$

$$\frac{R_t^K}{P_t} U_t(i) K_t(i) = \alpha \Xi_t \left[K_t(i) U_t(i) \right]^\alpha \left[Z_t N_t(i) \right]^{1-\alpha},$$

$$q_t \delta'(U_t(i)) U_t(i) K_t(i) = \alpha \Xi_t \left[K_t(i) U_t(i) \right]^\alpha \left[Z_t N_t(i) \right]^{1-\alpha},$$

$$\begin{aligned}
\phi_P \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] & \left[\frac{P_t}{\Pi P_{t-1}(i)} \right] \\
&= (1 - \theta_\mu) \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu} + \theta_\mu \Xi_t \left[\frac{P_t(i)}{P_t} \right]^{-\theta_\mu - 1} \\
&\quad + \phi_P \mathbb{E}_t \left\{ M_{t+1} \frac{Y_{t+1}}{Y_t} \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} - 1 \right] \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} \frac{P_t}{P_t(i)} \right] \right\},
\end{aligned}$$
(S.8)

$$\begin{aligned}
q_t = \mathbb{E}_t \Bigg\{ M_{t+1} \Bigg(& U_{t+1}(i) \frac{R_{t+1}^K}{P_{t+1}} + q_{t+1} \Bigg(1 - \delta(U_{t+1}(i)) - \frac{\phi_K}{2} \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} - \delta \right)^2 \\
&+ \phi_K \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} - \delta \right) \left(\frac{I_{t+1}(i)}{K_{t+1}(i)} \right) \Bigg) \Bigg) \Bigg\},
\end{aligned}$$

$$\frac{1}{q_t} = 1 - \phi_K\left(\frac{I_t(i)}{K_t(i)} - \delta\right),$$

where Ξ_t is the marginal cost of producing one additional unit of intermediate good i, and q_t is the price of a marginal unit of installed capital. R_t^K/P_t is the marginal revenue product per unit of capital services $K_t U_t$, which is paid to the owners of the capital stock. Our adjustment cost specification is similar to the specification used by Jermann (1998) and allows Tobin's q to vary over time.

Each intermediate goods firm finances a percentage ν of its capital stock each period with one-period risk-less bonds. The bonds pay the one-period real risk-free interest rate. Thus, the quantity of bonds $B_t(i) = \nu K_t(i)$. Total firm cash flows are divided between payments to bond holders and equity holders as follows:

$$\frac{D_t^E(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu\left(K_t(i) - \frac{1}{R_t^R}K_{t+1}(i)\right).$$

Since the Modigliani and Miller (1958) theorem holds in our model, leverage does not affect firm value or optimal firm decisions. Leverage makes the payouts and price of equity more volatile and allows us to define a concept of equity returns in the model. We use the volatility of equity returns implied by the model to calibrate our uncertainty shock processes in Section 6.

B.3. *Final Goods Producers*

The representative final goods producer uses $Y_t(i)$ units of each intermediate good produced by the intermediate goods-producing firm $i \in [0, 1]$. The intermediate output is transformed into final output Y_t using the following constant returns to scale technology:

$$\left[\int_0^1 Y_t(i)^{(\theta_\mu - 1)/\theta_\mu}\, di\right]^{\theta_\mu/(\theta_\mu - 1)} \geq Y_t.$$

Each intermediate good $Y_t(i)$ sells at nominal price $P_t(i)$ and each final good sells at nominal price P_t. The finished goods producer chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i)\, di,$$

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta_\mu} Y_t.$$

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index P_t can be written as follows:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta_\mu}\, di\right]^{1/(1-\theta_\mu)}.$$

B.4. *Equilibrium*

The assumption of Rotemberg (1982) (as opposed to Calvo (1983)) pricing implies that we can model our production sector as a single representative intermediate goods-producing firm. In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$, employ the same amount of labor $N_t(i) = N_t$, and choose the same level of capital and utilization rate $K_t(i) = K_t$ and $U_t(i) = U_t$. Thus, all firms have the same cash flows and payout structure between bonds and equity. With a representative firm, we can define the unique markup of price over marginal cost as $\mu_t = 1/\Xi_t$ and gross inflation as $\Pi_t = P_t/P_{t-1}$.

B.5. *Monetary Policy*

We assume a cashless economy where the monetary authority sets the net nominal interest rate r_t to stabilize inflation and output growth. Monetary policy adjusts the nominal interest rate in accordance with the following rule:

$$r_t = r + \rho_\pi(\pi_t - \pi) + \rho_y \Delta y_t, \tag{S.9}$$

where $r_t = \ln(R_t)$, $\pi_t = \ln(\Pi_t)$, and $\Delta y_t = \ln(Y_t/Y_{t-1})$. Changes in the nominal interest rate affect expected inflation and the real interest rate. Thus, we include the following Euler equation for a zero net supply nominal bond in our equilibrium conditions:

$$1 = R_t \mathbb{E}_t \left\{ M_{t+1} \left(\frac{1}{\Pi_{t+1}} \right) \right\}. \tag{S.10}$$

B.6. *Shock Processes*

The demand and technology shock processes are parameterized as follows:

$$a_t = (1 - \rho_a)a + \rho_a a_{t-1} + \sigma_{t-1}^a \varepsilon_t^a,$$
$$\sigma_t^a = (1 - \rho_{\sigma^a})\sigma^a + \rho_{\sigma^a}\sigma_{t-1}^a + \sigma^{\sigma^a}\varepsilon_t^{\sigma^a},$$
$$Z_t = (1 - \rho_Z)Z + \rho_Z Z_{t-1} + \sigma^Z \varepsilon_t^Z.$$

ε_t^a and ε_t^Z are first-moment shocks that capture innovations to the level of the stochastic process for technology and household discount factors. We refer to $\varepsilon_t^{\sigma^a}$ as second-moment or "uncertainty" shock since it captures innovations to the volatility of the exogenous process for household discount factors. An increase in the volatility of the shock process increases the uncertainty about the future time path of household demand. All three stochastic shocks are independent, standard normal random variables.

B.7. *Solution Method*

Our primary focus is examining the effect of an increase in the second moment of the preference shock process. Using a standard first-order or log-linear approximation to the equilibrium conditions of our model would not allow us to examine second-moment shocks, since the approximated policy functions are invariant to the volatility of the shock processes. Similarly, second-moment shocks would only enter as cross-products with the other state variables in a second-order approximation, and thus we could not study the

effects of shocks to the second moments alone. In a third-order approximation, however, second-moment shocks enter independently in the approximated policy functions. Thus, a third-order approximation allows us to compute an impulse response to an increase in the volatility of the discount rate shocks, while holding constant the levels of those variables.

To solve the baseline model, we use the Dynare software package developed by Adjemian, Bastani, Juillard, Karamé, Mihoubi, Perendia, Pfeifer, Ratto, and Villemot (2011). Dynare computes the rational expectations solution to the model using third-order Taylor series approximation around the deterministic steady state of the model. Section B.8 contains all the equilibrium conditions for the baseline model. To assist in numerically calibrating and solving the model, we introduce constants into the period utility function and the production function to normalize the value function V and output Y to both equal 1 at the deterministic steady state. We use Dynare version 4.4.3 in Matlab 2014b to solve and simulate the baseline model.

As discussed in Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011), approximations higher than first-order move the ergodic distributions of the model endogenous variables away from their deterministic steady-state values. With the exception of our simulation exercises in Sections 5.3 and 6.2 of the main text, we always analyze a traditional impulse response in percent deviation from the stochastic steady state of the model. To construct these responses, we set the exogenous shocks in the model to zero and iterate our third-order solution forward. After a sufficient number of periods, the endogenous variables of the model converge to a fixed point, which we denote the stochastic steady state. We then hit the economy with a one standard deviation uncertainty shock but assume the economy is hit by no further shocks. We compute the impulse response as the percent deviation between the equilibrium responses and the pre-shock stochastic steady state.

By default, Dynare uses an alternative simulation-based procedure to construct impulse responses for second-order and higher model solutions. This method is based on the generalized impulse response of Koop, Pesaran, and Potter (1996). As opposed to being centered around the stochastic steady state, these alternative responses are computed in deviations from the ergodic mean of the endogenous variables. In addition, these responses combine both the effects of higher uncertainty about future shocks with higher realized volatility of the actual shocks hitting the economy. We choose to compute the traditional impulse responses at the stochastic steady state for two reasons. First, Figure B.1 shows that these two procedures produce nearly identical results, yet the computational time is significantly less for the traditional impulse response. This computational advantage is particularly helpful when we estimate some of the model parameters using impulse response matching, which requires us to repeatedly solve the model under various parameterizations. Second, the traditional impulse response around the stochastic steady state allows us to analyze an increase in uncertainty about the future without any change in the realized volatility of the shock processes.

B.8. *Complete Model*

In the symmetric equilibrium, the baseline model in Dynare notation is as follows:

```
y + fixedcost = productionconstant*(z*n)^(1 - alpha)
                *(u*k(-1))^(alpha);

c + leverageratio*k/rr = w*n + de + leverageratio*k(-1);
```
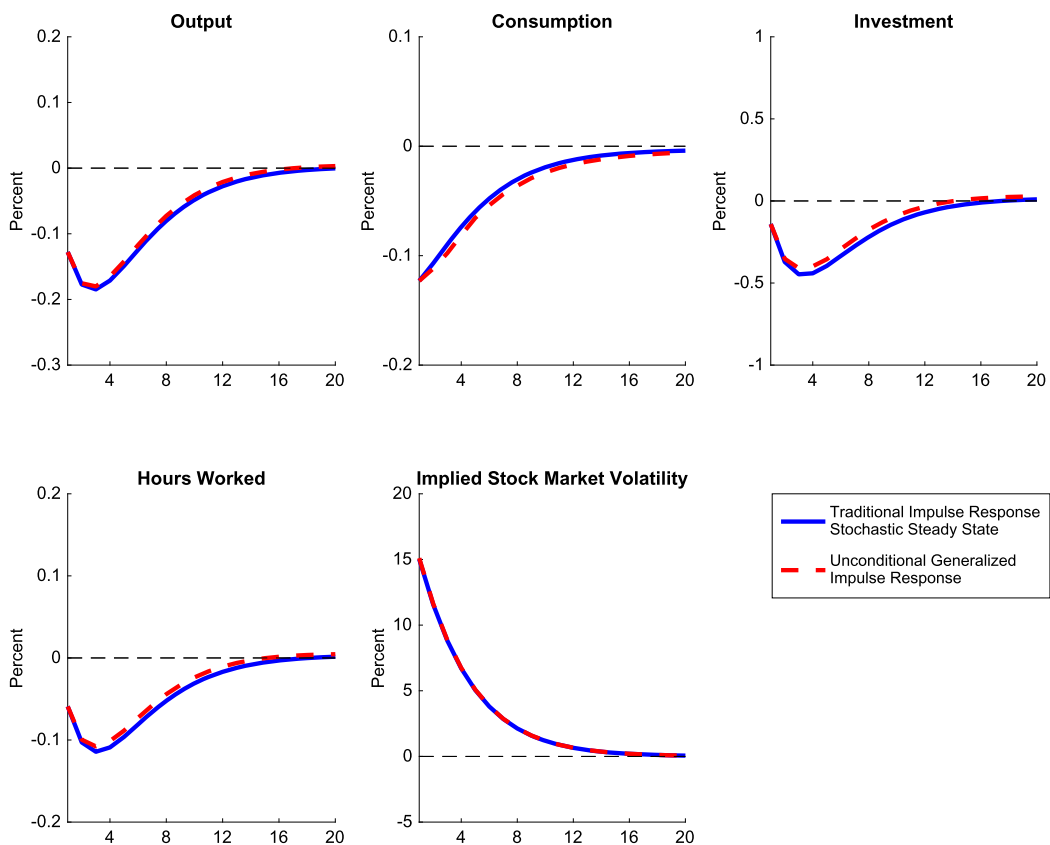
FIGURE B.1.—Alternative impulse response construction. Note: Impulse responses are plotted in percent deviations from either the stochastic steady state or their ergodic mean.

```
w = ((1 - eta)/eta)*c/(1 - l);

vf = (utilityconstant*a*(c^(eta)*(1 - n)^(1 - eta))^((1
     - sigma)/thetavf)
     + beta*expvfsigma^(1/thetavf))^(thetavf/(1 - sigma));

expvfsigma = vf(+1)^(1 - sigma);

w*n = (1 - alpha)*(y + fixedcost)/mu;

rrk*u*k(-1) = alpha*(y + fixedcost)/mu;

q*deltauprime*u*k(-1) = alpha*(y + fixedcost)/mu;

k = ((1 - deltau) - (phik/2)*(inv/k(-1) - delta0)^(2))*k(-1)
    + inv;

deltau = delta0 + delta1*(u-1) + (delta2/2)*(u-1)^(2);
```

```
deltauprime = delta1 + delta2*(u-1);

sdf = beta*(a/a(-1))
      *((c^(eta)*(1 - n)^(1 - eta))
      /(c(-1)^(eta)*(1 - n(-1))^(1 - eta)))^((1 - sigma)
      /thetavf)
      *(c(-1)/c)*(vf^(1 - sigma)/expvfsigma(-1))^(1 -
      1/thetavf);

1 = rr*sdf(+1);

1 = r*sdf(+1)*(pie(+1))^(-1);

1 = sdf(+1)*(de(+1) + pe(+1))/pe;

log(r) = rhor*log(r(-1))
         + (1 - rhor)*(log(rss) + rhopie*log(pie/piess)
         + rhoy*log(y/y(-1)));

de = y - w*n - inv - (phip/2)*(pie/piess - 1)^(2)*y
     - leverageratio*(k(-1) - k/rr);

1 = sdf(+1)*(u(+1)*rrk(+1)
    + q(+1)*((1 - deltau(+1)) - (phik/2)*(inv(+1)/k
    - delta0)^(2)
    + phik*(inv(+1)/k - delta0)*(inv(+1)/k)))/q;

1/q = 1 - phik*(inv/k(-1) - delta0);

phip*(pie/piess - 1)*(pie/piess)
    = (1 - thetamu) + thetamu/mu
      + sdf(+1)*phip*(pie(+1)/piess - 1)*(y(+1)/y)
      *(pie(+1)/piess);

expre = (de(+1) + pe(+1))/pe;

expre2 = (de(+1) + pe(+1))^(2)/pe^(2);

varexpre = expre2 - (expre)^(2);

a = (1 - rhoa)*ass + rhoa*a(-1) + vola(-1)*ea;

vola = rhovola*vola(-1) + (1 - rhovola)*volass + volvola*evola;

z = (1 - rhoz)*zss + rhoz*z(-1) + volzss*ez;
```

Since the capital stock is predetermined, we lag the capital stock *K* variables by one period relative to the timing in the model derivation.

APPENDIX C: ESTIMATING VARs ON SIMULATED MODEL DATA

This section provides additional details on our simulation exercises from Sections 5.3 and 6.2 of the main text. After a burn-in sample, we simulate 30 years of data from our theoretical model. We then estimate our baseline empirical VAR, described in Appendix A.1, using the simulated model data. In Section 5.3, the variables in the VAR are the model-implied VXO, output, consumption, investment, hours worked, the price level, and the short-term nominal policy rate. All variables, except the policy interest rate, enter the VAR in log levels. To match our empirical specification, we use the annualized, net nominal interest rate as our measure of monetary policy. After estimating the VAR, we compute the impulse responses to an identified uncertainty shock using a Cholesky decomposition with the VXO ordered first. We then repeat this exercise 10,000 times, which provides us with the probability distribution of the impulse response function. In Section 6.2, we also add in the log of the model-implied markup μ_t after the policy rate.

Since we compute a third-order approximation for this exercise, rather than a full non-linear solution, the simulated variance for the model-implied stock market can occasionally go negative. To reproduce our empirical framework where the VXO is always positive, we truncate simulated values for the VXO at a lower bound of 1 percent, which implies that the log VXO always remains positive. Since the estimated impulse responses reflect the average effect of uncertainty on the economy, we plot the unconditional generalized impulse responses as the true model responses in Figures 5 and 6 of the main text.

APPENDIX D: EXAMINING MODEL FEATURES AND PREDICTIONS

Our baseline model is consistent with both the qualitative comovement and quantitative predictions of an identified uncertainty shock in the data. In the following sections, we provide further details on the key model ingredients and further examine some additional predictions of the model.

D.1. *Countercyclical Markups in a Real Model*

Our mechanism for generating macroeconomic comovement relies on countercyclical markups following an uncertainty shock. Following much of the literature, we implement countercyclical markups by assuming prices adjust slowly to changing economic conditions. However, our main results are unchanged if we instead consider a model with countercyclical markups but without nominal rigidities. To illustrate this idea, we replace our New Keynesian Phillips Curve in Appendix Equation (S.8) with the following equation:

$$\log(\mu_t/\mu) = \varepsilon^{\mu y} \log(Y_t/Y), \tag{S.11}$$

where μ_t is the markup over marginal cost and Y_t is output. $\varepsilon^{\mu y}$ denotes the elasticity of the markup with respect to output. The variables without t-subscripts denote their steady-state values. Following the empirical evidence of Bils, Klenow, and Malin (2014), we calibrate $\varepsilon^{\mu y} = -1.8$. Figure D.1 plots the impulse responses to a demand uncertainty shock in this entirely real model with countercyclical markups. Even without nominal rigidities, countercyclical markups can generate macroeconomic comovment in response to an uncertainty shock.
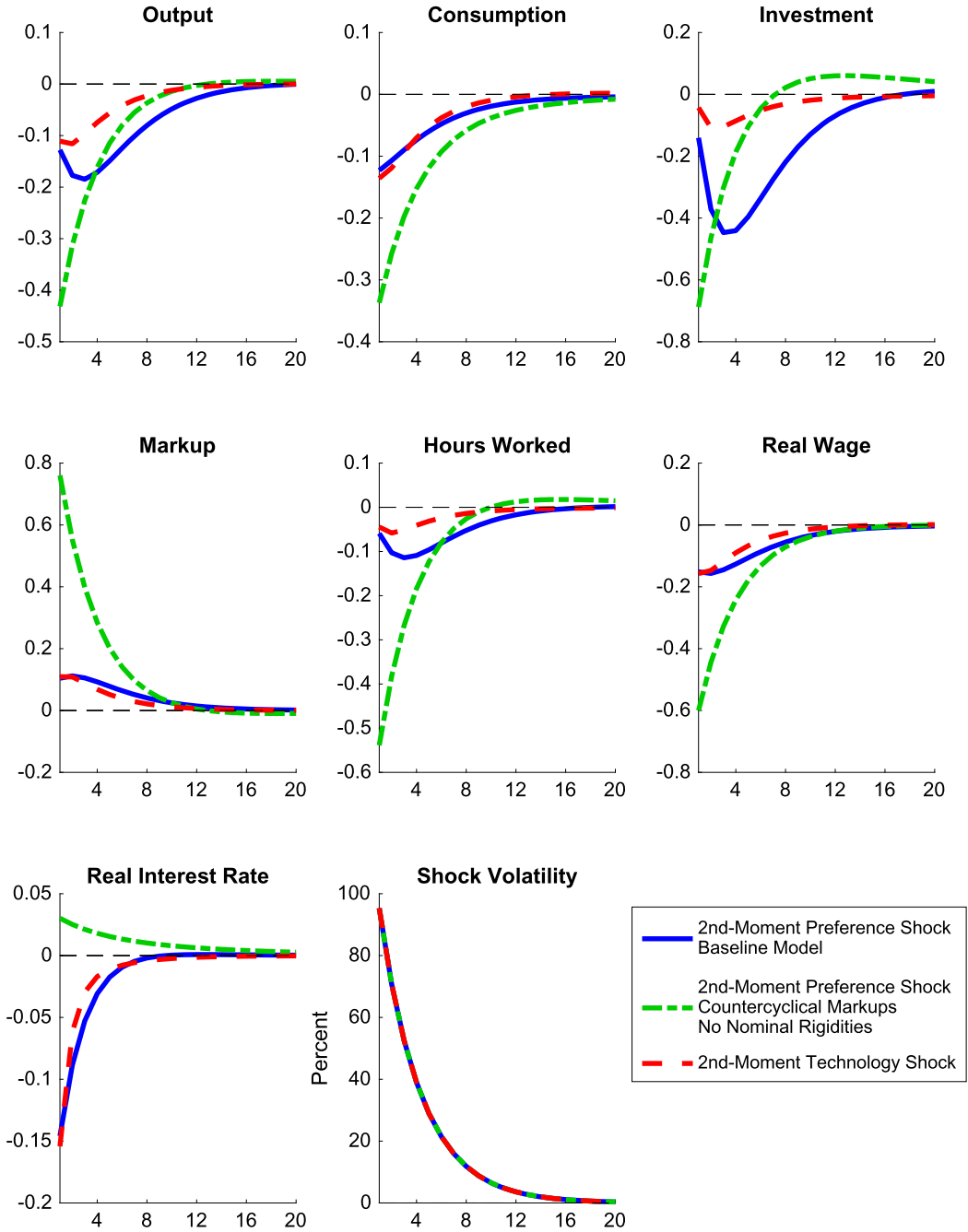
FIGURE D.1.—Countercyclical markups in a real model and technology uncertainty shocks.

D.2. *Uncertainty Shock About Future Technology*

In our baseline model, we consider second-moment shocks to household discount factors. We now extend our model to examine uncertainty shocks with respect to future

technology. We replace our stochastic process for technology with the following two equations:

$$Z_t = (1 - \rho_Z)Z + \rho_Z Z_{t-1} + \sigma^Z_{t-1}\varepsilon^Z_t,$$

$$\sigma^Z_t = (1 - \rho_{\sigma^Z})\sigma^Z + \rho_{\sigma^Z}\sigma^Z_{t-1} + \sigma^{\sigma^Z}\varepsilon^{\sigma^Z}_t.$$

Figure D.1 shows the impulse responses to a technology uncertainty shock.[5] Similarly to a demand uncertainty shock, higher uncertainty about future technology can generate a decline in output, consumption, investment, and hours worked.

However, there are two reasons why second-moment technology shocks are not as good at matching the quantitative aspects of the economy's response to an uncertainty shock. First, technology uncertainty shocks require significantly larger shocks to produce significant fluctuations in output and its components. To produce the outcomes in Figure D.1, we needed a five-fold increase in the volatility of the technology shocks relative to our baseline calibration. This more volatile shock process implies too much unconditional and stochastic volatility in key macroeconomic aggregates. Second, conditional on a given movement in consumption, a technology impulse response generates a much smaller movement in investment than a demand uncertainty shock. When the uncertainty about future technology increases, higher capital provides a hedge against possible negative shocks to future marginal costs. This additional substitution effect, which is not present under a demand uncertainty shock, provides an incentive for a firm to avoid disinvesting in the capital stock when uncertainty about future technology increases. Accordingly, investment falls by only 10 basis points after a technology uncertainty shock but falls by over 50 basis points after a demand uncertainty shock. Since capital and labor are complements in production, the time path of investment implies that equilibrium hours worked also falls by less after a technology uncertainty shock.

D.3. *Model-Implied Unconditional Moments*

Building on the discussion in Section 5.2 of the main text, we now provide further comparison of the unconditional moments generated by our model with their empirical counterparts. Given that uncertainty shocks generate stochastic volatility in key macroeconomic aggregates, a key litmus test for our model will be its ability to match the time-varying volatility in the data.

To evaluate the model's fit, we compare its simulated moments with their data counterparts along three dimensions. First, we assess the model's ability to match the unconditional volatility in the data as measured by the sample standard deviation. Second, we evaluate the amount of stochastic volatility in key macro aggregates both in the data and in the model. Finally, we examine the average real interest rate, equity premium, and implied stock market volatility generated by the model. We examine the model's predictions for output, consumption, investment, hours worked, the real interest rate, equity premium, and the implied stock market volatility. For output and its components, we construct the data as outlined in Appendix A.1. For the real interest rate and equity premium, we calculate the quarterly, annualized ex post returns.[6]

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[5]For this exercise, we calibrate $\sigma^Z = 0.0064$ and $\sigma^{\sigma^Z} = 0.0061$. This calibration implies a 95% increase in the volatility of future technology shocks, which is similar to our baseline model using demand shocks. We also slightly increase the investment adjustment costs ($\phi_K = 10$) to generate a larger decline in investment.

[6]We calculate the real interest rate by subtracting the ex post GDP deflator inflation rate from the effective federal funds rate.

To compare the distance between the model-implied moments and their empirical counterparts, we generate small-sample bootstrapped probability intervals from the model. Our empirical moments come from about a 30-year sample of quarterly data. We want to determine the likelihood that the moments from this given 30-year sample of data could be generated by our baseline model. To compute the probability interval for each moment, we simulate the model economy for 30 years after an initial burn-in period. Then, we compute and save all the desired model-implied moments using this small sample of simulated data. We repeat this exercise 10,000 times, which provides us with a series of small-sample estimates for each moment of interest. Table D.I reports the mean and the 95% probability interval for each model-implied moment as well as their empirical counterparts. If the empirical moment falls outside of this model-implied probability interval, it is highly statistically unlikely that the model is able to generate moments consistent with the data. In Table II of the main text, we report the mean for each model-implied moment.

Our model is generally consistent with the unconditional and stochastic volatility in output, consumption, and investment. For each of these variables, the empirical moment falls within the small-sample probability interval generated by the model. The model does struggle slightly in generating sufficient unconditional and stochastic volatility in hours worked. However, the general fit of the model suggests that we would likely draw similar conclusions about the effects of uncertainty shocks if we instead chose to calibrate our model directly using the stochastic volatility in key macro variables (as opposed to our impulse response matching procedure).

However, the model's predictions for the real interest rate, equity premium, and implied stock market volatility could be improved. On average, the model generates real interest rates that are too high relative to the data. In addition, the average equity premium and VXO in the model are lower than in the data. Finally, the model does not generate enough volatility in the real interest rate, equity premium, or implied stock market volatility.

Higher firm leverage, however, greatly improves the model's predictions for the equity premium and implied stock market volatility. In our baseline model, we calibrated leverage $\nu = 0.9$. This large value is near the upper end of the values examined by Jermann (1998). If we raise leverage to an even higher value, $\nu = 0.985$, the last column of Table D.I shows that the model can generate an equity premium and implied stock market volatility similar to the data. While this calibrated value for leverage is quite high, two important caveats are important to keep in mind. First, adding additional shocks (such as government spending and monetary policy) would allow the model to match the volatility of the equity return with a much smaller amount of leverage.[7] Second, since the Modigliani and Miller (1958) theorem holds in our model, the amount of leverage does not affect firm decisions or firm value. Figure D.2 shows that the responses of the key macro variables to an uncertainty shock are unchanged under this alternative leverage calculation. In our baseline calibration, we calibrate a lower degree of firm leverage to stay as close as possible to the previous literature.

D.4. *Other Shocks and the Persistence of the Uncertainty Shock*

While the amount of leverage does not affect the response to an uncertainty shock, the inclusion of technology shocks does slightly affect the results of our impulse response

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[7]The addition of monetary policy shocks would also generate additional volatility in the real interest rate.

TABLE D.I

EMPIRICAL AND MODEL-IMPLIED UNCONDITIONAL MOMENTS[a]

Moment	Data	Baseline Model	Higher Leverage
Average			
Real Interest Rate	1.7	2.3	2.3
		(2.2, 2.4)	(2.2, 2.4)
Equity Premium	6.3	0.9	4.7
		(−0.6, 2.5)	(−6.9, 15.4)
Implied Stock Market Volatility	20.8	2.8	13.4
		(2.2, 3.3)	(4.8, 21.0)
Unconditional Volatility			
Output	1.1	1.0	1.0
		(0.6, 1.7)	(0.6, 1.7)
Consumption	0.7	0.8	0.8
		(0.4, 1.2)	(0.4, 1.2)
Investment	3.8	4.7	4.7
		(2.5, 7.9)	(2.5, 7.9)
Hours Worked	1.4	0.8	0.8
		(0.4, 1.3)	(0.4, 1.3)
Real Interest Rate	2.4	0.2	0.2
		(0.1, 0.2)	(0.1, 0.2)
Equity Premium	25.7	7.7	61.7
		(6.3, 9.3)	(42.8, 89.5)
Implied Stock Market Volatility	8.2	0.9	6.0
		(0.6, 1.2)	(3.3, 8.5)
Stochastic Volatility			
Output	0.4	0.2	0.2
		(0.1, 0.4)	(0.1, 0.4)
Consumption	0.2	0.2	0.2
		(0.1, 0.3)	(0.1, 0.3)
Investment	1.6	1.2	1.2
		(0.6, 2.2)	(0.6, 2.2)
Hours Worked	0.5	0.2	0.2
		(0.1, 0.4)	(0.1, 0.4)

[a]Unconditional volatility is measured with the sample standard deviation. We measure stochastic volatility using the standard deviation of the time-series estimate for the 5-year rolling standard deviation. The 95% small-sample bootstrapped probability intervals appear in parentheses. The empirical sample period is 1986–2014.

matching estimator. In Figure D.2, we simulate a demand uncertainty shock but double the unconditional volatility of the technology shocks $\sigma^Z = 0.0025$. While more volatile technology shocks do not affect the macroeconomic responses of output and its components, we see that the log of the model-implied VXO rises by less when technology shocks are more volatile. Higher volatility of technology shocks raises the average model-implied VXO but does not change how much the VXO moves in response to a demand uncertainty shock. Thus, our estimator would choose a different size of uncertainty shock under this alternative calibration. Obviously, doubling the volatility of the technology shocks has significant implications for the unconditional moments implied by the model. Thus, it provides an additional rationale for checking the unconditional moments implied by our model against the data.
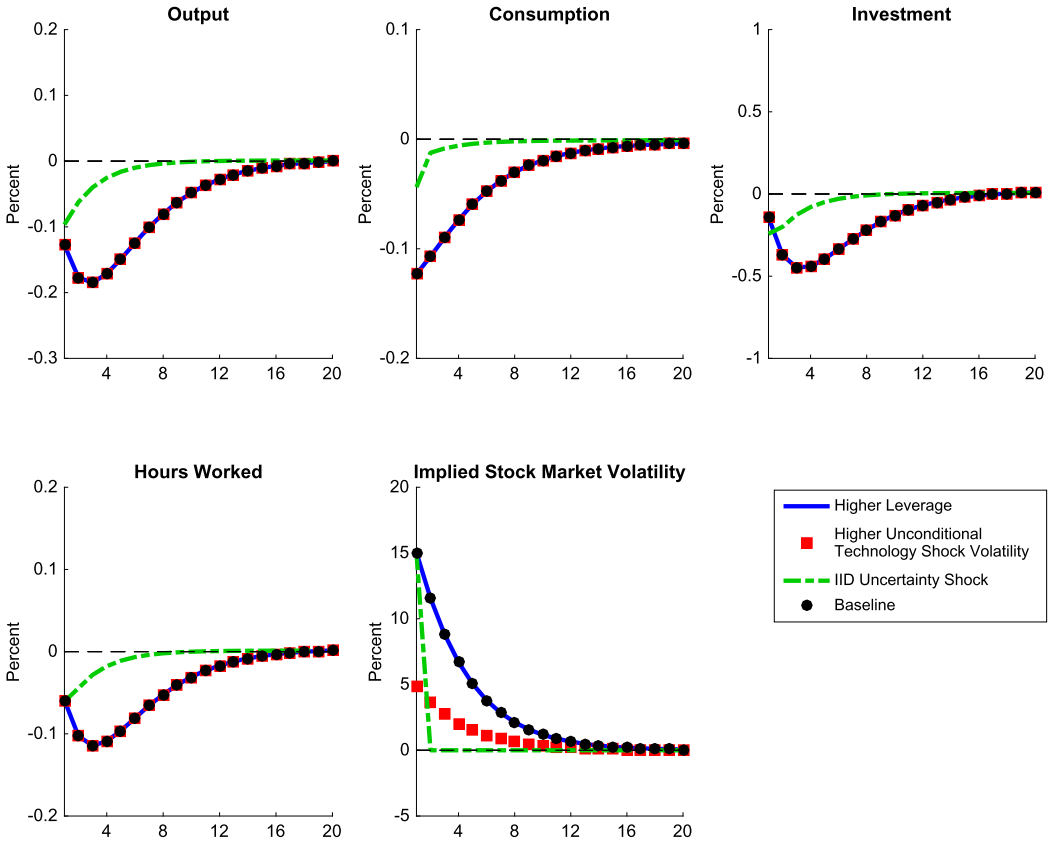
FIGURE D.2.—The effects of leverage, other shocks, and uncertainty shock persistence.

Finally, Figure D.2 also plots the responses to an independent and identically distributed uncertainty shock process by setting $\rho_{\sigma^a} = 0$. Even without exogenous persistence in the uncertainty shock process, output and its components continue to decline after an uncertainty shock. However, like nearly all simple macroeconomic models, the model does not have a great deal of internal propagation. Therefore, the impulse response estimator prefers a somewhat persistent uncertainty shock process to match the quantitative responses.

D.5. *Risk Aversion and Epstein–Zin Preferences*

In loose terms, the precautionary labor supply by households after an uncertainty shock depends on the 'price' of risk multiplied by the 'quantity' of risk. The price of risk is mainly influenced by risk aversion σ and the quantity of risk is determined by the size of the uncertainty shock. We find the model can match the VAR evidence with a calibration of $\sigma = 80$ and an uncertainty shock that increases the volatility of shocks by around 95% relative to its steady-state value.[8] If we divide the risk aversion parameter by about ten

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[8]Since households can adjust their labor margin, Swanson (2012) showed that σ in our model is not comparable to fixed-labor risk aversion estimates.
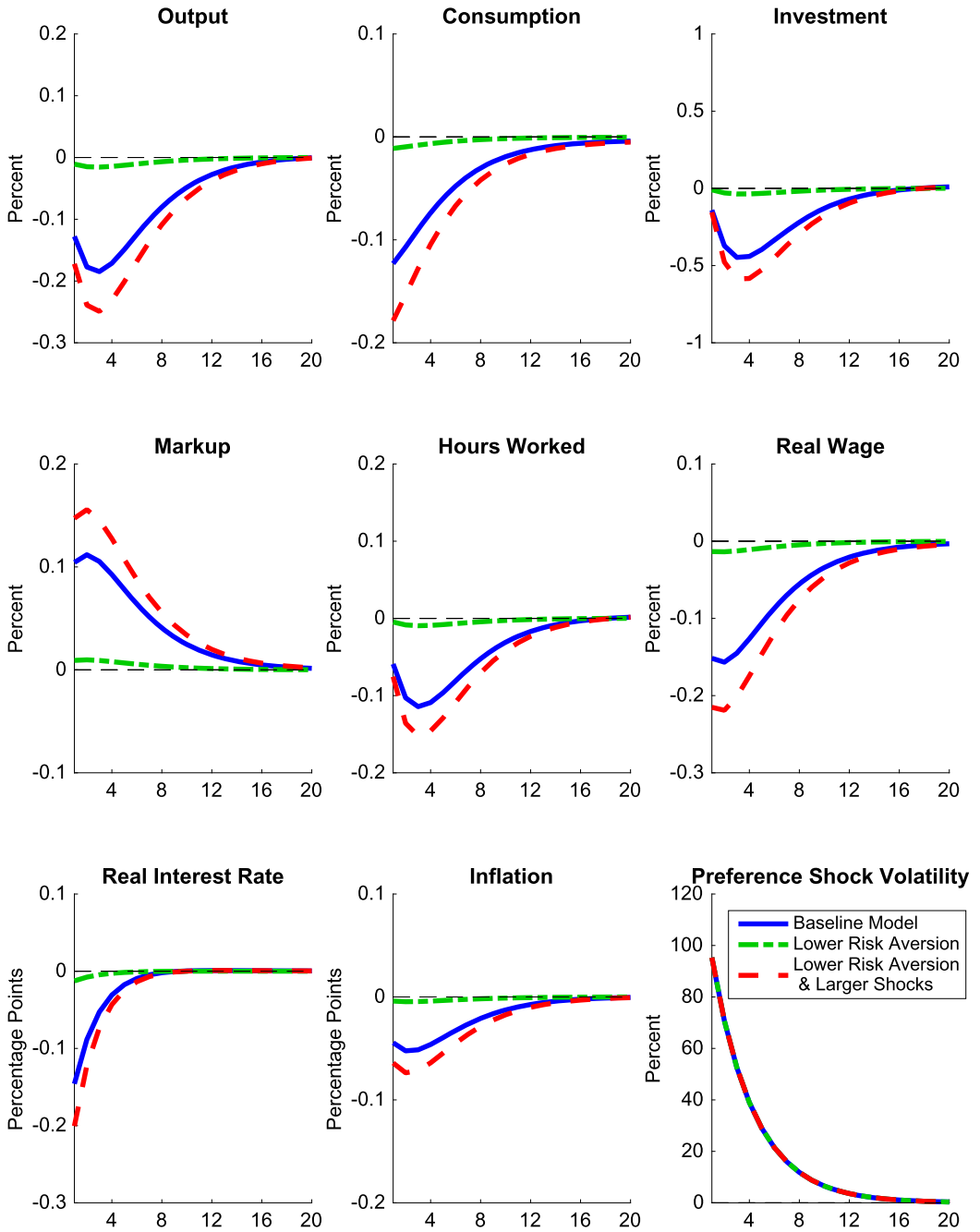
FIGURE D.3.—The role of Epstein–Zin preferences.

($\sigma = 8$), then Figure D.3 shows that the resulting impulse responses are roughly one-tenth as large as the baseline calibration. However, if we set $\sigma = 8$ but dramatically increase the volatility of the demand shock process, the model can generate responses that look

like the baseline model even with substantially less risk-averse households.[9] Thus, the inclusion of Epstein–Zin preferences allow us to match the VAR evidence with smaller movements in the expected volatility of the exogenous shocks.

D.6. *Investment Adjustment Costs and Variable Capital Utilization*

Our baseline model features a very small amount of investment adjustment costs. To illustrate the role of these adjustment frictions, Figure D.4 shows the impulse responses to a demand uncertainty shock under two alternative calibrations. In the first alternative, we turn off the adjustment costs completely $\phi_K = 0$. In the second calibration, we set the costs of adjusting investment and capital utilization to extremely large values, which resembles a model with a fixed capital stock.

Investment adjustment costs help the model generate a significant decline in investment after an uncertainty shock. With zero adjustment costs, Figure D.4 shows that the resulting decline in investment is much smaller than our baseline model. Investment adjustment costs make it more difficult for households to convert their desired savings into physical assets. Thus, small adjustment frictions cause a larger decline in investment after the shock. However, for extremely large adjustment frictions, firms find it too costly to disinvest. Thus investment stays fixed in response to the uncertainty shock but firms greatly decrease their demand for household labor. Figure D.4 also shows that investment adjustment costs are helpful in generating a decline in inflation following an uncertainty shock.

Without the ability to adjust capital or utilization, the fixed capital stock calibration resembles a simple New Keynesian model without capital. Figure D.4 plots the impulse responses for this fixed capital model under both flexible and sticky prices. When prices are fully flexible and the capital stock is fixed, higher uncertainty causes a decline in real interest rates and inflation but output and consumption remain unchanged. When prices are sticky, an increase in uncertainty looks similar to standard time-preference or discount factor shock. If the central bank can always close the gap between the real rate and the natural rate of interest, then higher uncertainty has no effect on output in the economy. In our companion paper, Basu and Bundick (2015), we examined the effects of uncertainty shocks in a simple New Keynesian model without capital and discussed the optimal policy response to higher uncertainty both at and away from the zero lower bound.

Our baseline model also features variable capital utilization. To illustrate the role of this feature, Figure D.5 shows the response to a demand uncertainty shock under our baseline calibration and a calibration with a significantly higher value of δ_2. As discussed by Christiano, Eichenbaum, and Evans (2005), this parameter controls the elasticity of capital utilization with respect to the rental rate. Figure D.5 shows that highly elastic capital utilization helps generate a larger decline in output and investment in our model. Capacity utilization extends the half-life of price stickiness, and hence the period of time over which our results diverge substantially from those of a flexible-price model. Under nominal rigidities, firms set prices according to the expected present value of marginal cost. Variable capacity utilization creates an elastic supply of capital services and reduces the responsiveness of marginal cost to output, just as elastic labor supply does.

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[9]For this exercise, we set $\sigma^a = 0.0105$ and $\sigma^{\sigma^a} = 0.01$ such that, as in the baseline model, an uncertainty shock raises the preference shock volatility by roughly 95%.
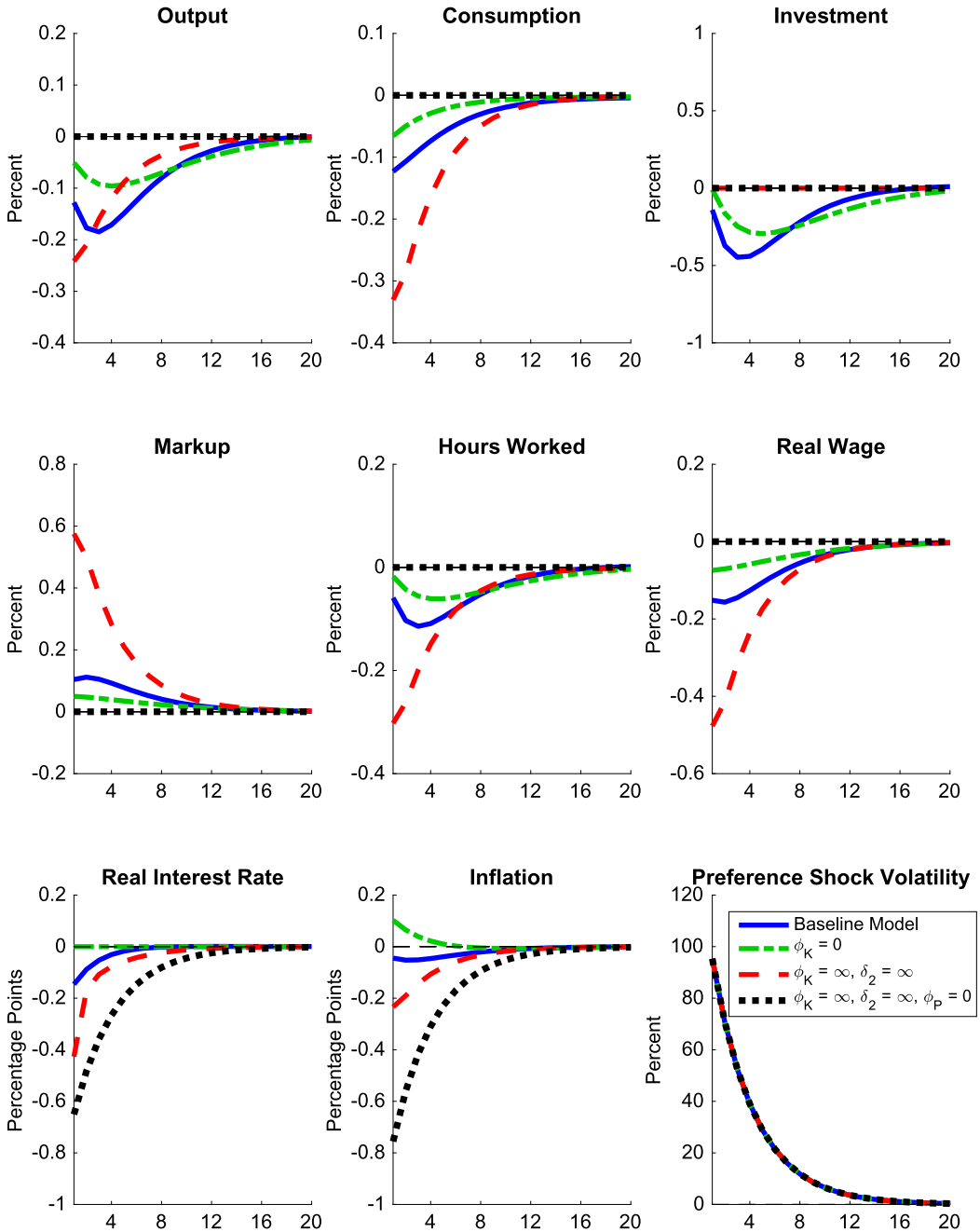
FIGURE D.4.—The role of investment adjustment costs.

D.7. *Features of the Labor Supply Curve*

In our model, increased precautionary savings and precautionary labor supply by households in response to higher uncertainty leads to a contraction in economic activity. In this section, we examine how the exact features of the household's labor supply curve influ-
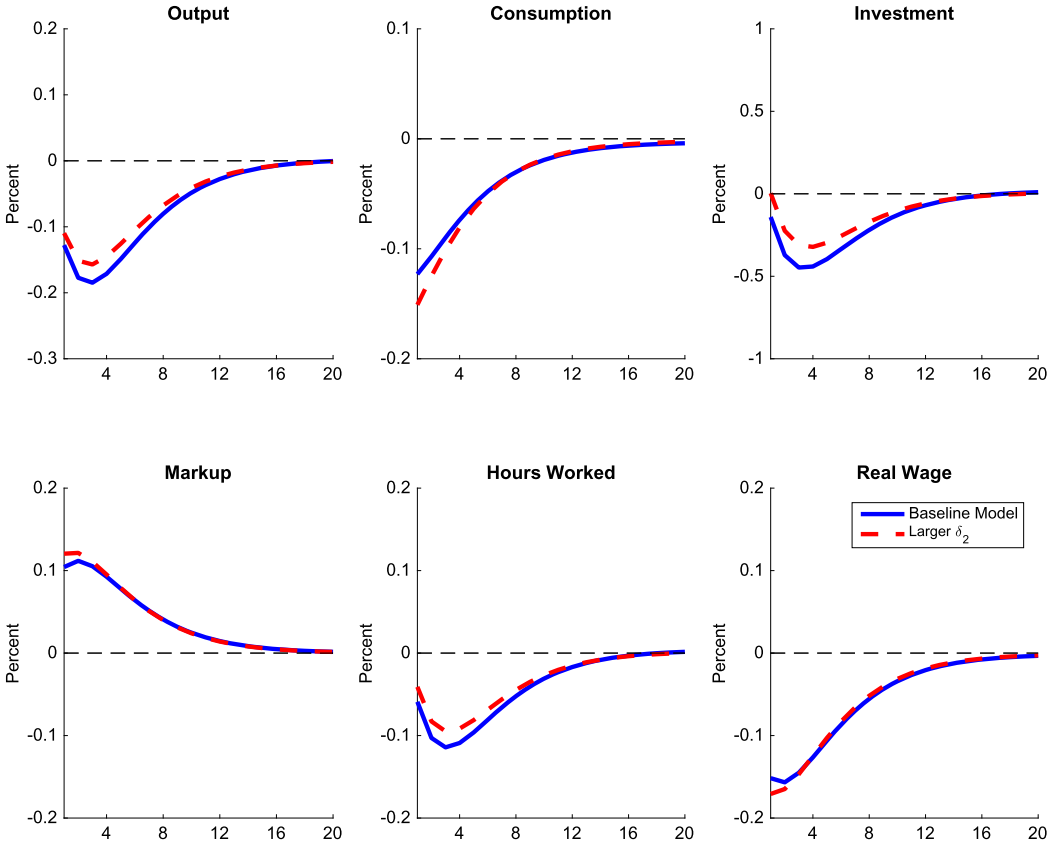
FIGURE D.5.—The role of variable capital utilization.

ence the equilibrium outcomes for the macroeconomy. Figure D.6 plots the impulse responses to a demand uncertainty shock under several different labor supply curves. In our baseline model, we set the Frisch labor supply elasticity equal to 2. If we instead calibrate a labor supply elasticity of one half, Figure D.6 shows that a given uncertainty shock generates smaller movements in output and its components. In terms of the labor supply and labor demand diagrams from Figure 2 of the main text, a smaller labor supply elasticity implies a steeper labor supply curve. For given movement in the marginal utility of wealth λ_t, a steeper labor supply curve implies a much smaller decline in wages and firm marginal costs.

Households in our model desire to increase saving and labor input in response to an uncertainty shock. To decompose the relative effects of the distinct precautionary labor supply and precautionary saving channels, we solve a version of our model that does not feature a wealth effect on labor supply. Taking Appendix Equation (S.5), we divide each side by its steady-state value and then remove the term involving consumption:

$$\frac{W_t^R}{W^R} = \frac{1-N}{1-N_t}, \tag{S.12}$$

where $W_t^R = W_t/P_t$ denotes the real wage. In the spirit of Greenwood, Hercowitz, and Huffman (1988) preferences, this alternative labor supply curve features the same labor
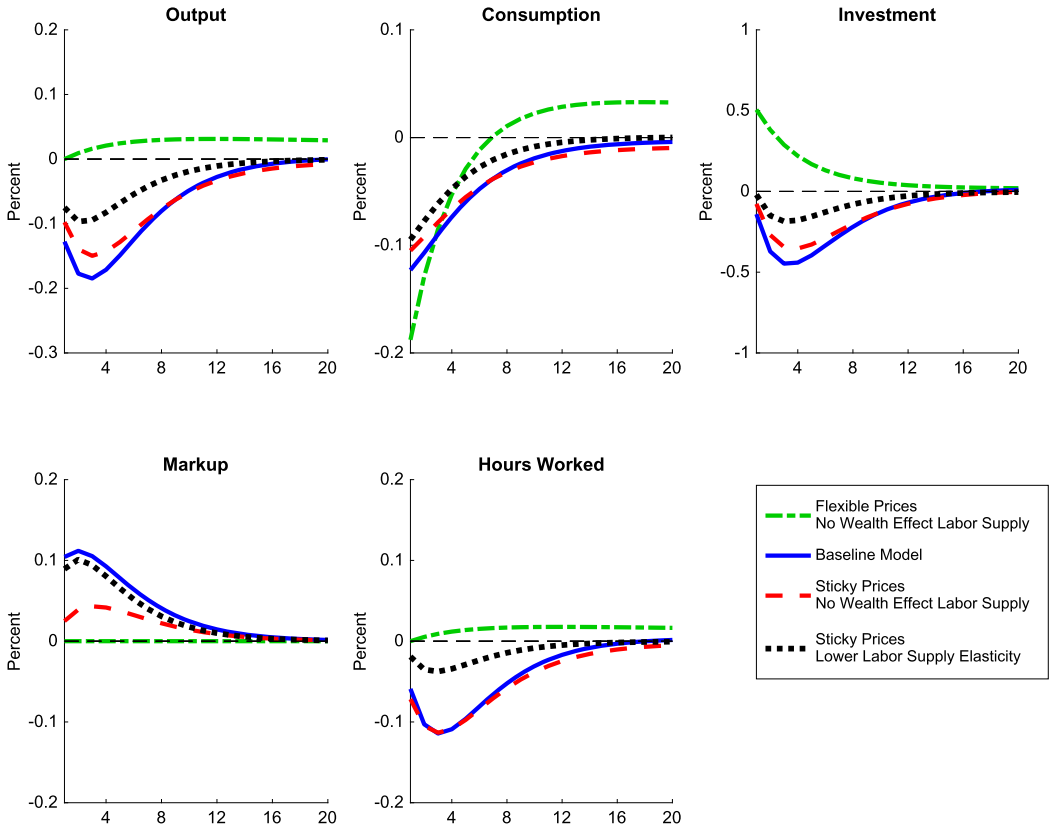
FIGURE D.6.—Alternative labor supply curves.

supply elasticity as our baseline model but does not shift outward in response to an uncertainty shock. In Figure D.6, we plot the impulse responses using this alternative labor supply curve under both flexible and sticky prices. Without a wealth effect on labor supply, hours worked remain unchanged at impact under flexible prices. Since the current level of technology and capital remain fixed, total output remains unchanged at impact as well. However, households still lower their consumption and increase their savings, which leads to higher investment by firms.

Under sticky prices, Figure D.6 shows that the precautionary saving channel alone can cause output and its components to decline after an uncertainty shock. The decreased demand for consumption goods reduces firm marginal costs. Lower marginal costs raise markups, which causes the labor demand curve to shift inward. With an unchanged labor supply curve, an inward shift of the labor demand curve results in a decline in equilibrium hours worked. Compared with our baseline results, these results suggest that about half the movement in markups is due to precautionary saving and about half of the increase is due to precautionary labor supply. Thus, while both mechanisms can qualitatively generate macroeconomic comovement, the combination of the two mechanisms is helpful in matching the quantitative responses to an identified uncertainty shock in the data.[10]

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[10]In related work, Leduc and Liu (2016) showed that labor market search frictions can amplify the negative effects of an uncertainty shock on unemployment.

D.8. *Extension to Sticky Nominal Wages*

In our baseline model, we generate macroeconomic comovment after an uncertainty shock by assuming that output prices are sticky, but household wages are fully flexible. However, various types of evidence suggest that nominal wages are sticky, especially at high frequencies. At the macro level, Christiano, Eichenbaum, and Evans (2005) found that nominal wage stickiness is actually more important than nominal price stickiness for explaining the observed impact of monetary policy shocks. At the micro level, Barattieri, Basu, and Gottschalk (2014) found that the wages of individual workers change less than once a year on average.

In this subsection, we show that our results extend readily to the case where nominal wages are sticky. Rather than writing down an extended model with two nominal frictions, we make our point heuristically using the graphical labor supply–labor demand apparatus from Section 3 of the main text. If households act competitively in the labor market,

$$U_2(C_t, 1 - N_t) = \lambda_t W_t, \tag{S.13}$$

where W is the nominal wage and λ_t is now the utility value of a marginal dollar. Assuming firms have market power, we can reorganize Equations (5) and (6) in the main text as follows:

$$W_t = \frac{P_t}{\mu_t^P} Z_t F_2(K_t, Z_t N_t), \tag{S.14}$$

$$\frac{U_2(C_t, 1 - N_t)}{\lambda_t P_t} = \frac{1}{\mu_t^P} Z_t F_2(K_t, Z_t N_t), \tag{S.15}$$

where μ_t^P is the price-markup over marginal cost.

Now assume a new model, where households also have market power and set wages with a markup over their marginal disutility of work. Equation (3) in the main text and the resulting equilibrium are modified as follows:

$$W_t = \mu_t^W \frac{U_2(C_t, 1 - N_t)}{\lambda_t}, \tag{S.16}$$

$$\frac{U_2(C_t, 1 - N_t)}{\lambda_t P_t} = \frac{1}{\mu_t^W} \frac{1}{\mu_t^P} Z_t F_2(K_t, Z_t N_t). \tag{S.17}$$

Compared with the competitive labor market model, we can replace the labor supply curve in Figure 2 in the main text with $U_2(C_t, 1 - N_t)/\lambda_t P_t$. This quantity has the interpretation of the disutility faced by the household of supplying one more unit of labor, expressed in units of real goods (the real marginal cost of supplying labor). On the vertical axis, we now plot the equilibrium level of the real marginal disutility of work. This alternative 'supply curve' is shifted in exactly the same way by uncertainty as the standard labor supply curve—higher uncertainty raises λ, which shifts the supply curve out. But now the 'demand curve' in the right-hand side of Equation (S.17) is shifted by both price and wage markups—only the product of the two matters.

Take the polar opposite of the case we have analyzed so far: Assume perfect competition in product markets but Rotemberg wage setting by monopolistically competitive households in the labor market. Then, the price markup is always fixed at 1, but the wage markup would jump up in response to an increase in uncertainty (since the marginal cost

of supplying labor falls but the wage is sticky). This alternative assumption would make the qualitative outcome exactly the same as in our previous results. Thus, while introducing nominal wage stickiness would certainly affect quantitative magnitudes, it would not change our qualitative results.

APPENDIX E: SOLVING MODEL WITH A ZERO LOWER BOUND CONSTRAINT

To analyze the impact of uncertainty shocks at the zero lower bound, we solve our model using the policy function iteration method of Coleman (1990) and Davig (2004). This global approximation method allows us to model the occasionally-binding zero lower bound constraint. To make the model computationally feasible using policy function iteration, we simplify our baseline model by removing technology shocks and leverage. We also assume that households receive firm dividends as a lump-sum payment. Finally, we keep the number of grid points reasonable by slightly lowering our risk aversion parameter $\sigma = 15$, increasing the amount of investment adjustment costs $\phi_K = 10$, and slightly reducing the size of the uncertainty shocks $\sigma^{\sigma^a} = 0.0015$.

The policy function algorithm is implemented using the following steps:

1. Discretize the state variables of the model: $\{K_t \times Y_{t-1} \times a_t \times \sigma_t^a\}$.

2. Conjecture initial guesses for the policy functions of the model $N_t = N(K_t, Y_{t-1}, a_t, \sigma_t^a)$, $U_t = U(K_t, Y_{t-1}, a_t, \sigma_t^a)$, $I_t = I(K_t, Y_{t-1}, a_t, \sigma_t^a)$, $\Pi_t = \Pi(K_t, Y_{t-1}, a_t, \sigma_t^a)$, and $\mathbb{E}_t V_{t+1}^{1-\sigma} = EV(K_t, Y_{t-1}, a_t, \sigma_t^a)$.

3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variables a_t and σ_t^a to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space.

4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

We implement the policy function iteration method in FORTRAN using the nonlinear equation solver DNEQNF from the IMSL numerical library. The model is solved using a Linux computing cluster and the model solution is parallelized using Open MP.

To compute the impulse response of an uncertainty shock at the zero lower bound, we generate two time paths for the economy. In the first time path, we simulate an economy hit by a negative first-moment demand shock such that the zero lower bound binds for about six quarters. In the second time path, we simulate the same first-moment demand shock, but also simulate an uncertainty shock. After the uncertainty shock, neither economy is hit with any further shock. We present the (percent) difference between the time paths of variables in the two simulations as the impulse response to the uncertainty shock at the zero lower bound. We choose the size of the uncertainty shock such that, at the stochastic steady state, the simplified model generates roughly the same movements in output as our baseline model from Section 4.

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Dept. of Economics, Boston College, Chestnut Hill, MA 02467, U.S.A. and National Bureau of Economic Research, Cambridge, MA 02138, U.S.A.; susanto.basu@bc.edu; http://www.bc.edu/schools/cas/economics/ faculty-and-staff/faculty-listing/basu-susanto.html; http://www.nber.org/people/susanto_basu

and

Federal Reserve Bank of Kansas City, 1 Memorial Drive, Kansas City, MO 64198, U.S.A.; brent.bundick@kc.frb.org; https://www.kansascityfed.org/people/brentbundick.