Econometrica Supplementary Material

SUPPLEMENT TO "PLATFORM DESIGN WHEN SELLERS USE PRICING ALGORITHMS"

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JUSTIN P. JOHNSON
Graduate School of Management, Cornell University

ANDREW RHODES
Toulouse School of Economics, University of Toulouse Capitole and CEPR

MATTHIJS WILDENBEEST
Department of Economics, University of Arizona and CEPR

This supplement contains additional results related to platform design. It contains results related to (i) revenue sharing, (ii) endogenous platform fees, and (iii) two different kinds of Dynamic Price Directed Prominence.

1. REVENUE SHARING

IN SECTION 1 OF THE MAIN TEXT, we focused on per-unit fees, but in practice some platforms take a fraction of any revenues generated by sellers. We now show that under certain conditions our main theoretical insights extend to this case. Suppose that, for given $\omega \in [0, 1]$, the platform's payoff in period t is

$$\widehat{\Omega}(p^t) = \omega r\left(\sum_{j\in\mathcal{N}^t} p_j^t D_j(p^t)\right) + (1-\omega)U(p^t), \tag{1}$$

where $\mathcal{N}^t$ is again the set of displayed firms, and where $r < 1$ is the platform's revenue share. As we did with per-unit fees, we take r as fixed so as to focus on platform design. We can write the profit of firm i when it is displayed in period t as

$$\left((1-r)p_i^t - \tilde{c}\right)D_i(p^t) = (1-r)\left(p_i^t - \hat{c}\right)D_i(p^t), \tag{2}$$

where $\hat{c} = \tilde{c}/(1-r) > 0$ is now the effective marginal cost.

Our existing results for how PDP and DPDP affect prices and consumer surplus hold exactly as stated in the main text, once c is replaced by $\hat{c}$. The reason is that, from the perspective of each firm, what matters is the effective marginal cost and the platform's design rule (if any). For this reason, in the remainder of this section we focus on the platform's objective $\widehat{\Omega}$.

The impact of PDP and DPDP on the platform's payoff $\widehat{\Omega}$ depends on parameters. For example, if the platform places full weight on its own fees ($\omega = 1$) and $\hat{c}$ is sufficiently close to zero, then PDP (in a competitive market) and DPDP (with ADV sufficiently large) both reduce the platform's payoff. The reason is that platform design reduces prices down to $\hat{c}$, such that when $\hat{c}$ is small revenue must fall.

Justin P. Johnson: jpj25@cornell.edu
Andrew Rhodes: andrew.rhodes@tse-fr.eu
Matthijs Wildenbeest: wildenbeest@arizona.edu
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Nonetheless, there are also circumstances in which our platform design tools raise the platform's payoff $\widehat{\Omega}$ and perform well relative to more sophisticated rules. To see this, start with the case of PDP in a competitive market. Let $\widehat{\Omega}^{\mathrm{PDP}}$ and $\widehat{\Omega}^{\mathrm{BN}}$ be the platform's payoffs under PDP and Bertrand–Nash, respectively. Similarly, conditional on each seller earning nonnegative profit in each period, let $\widehat{\Omega}^{\mathrm{Max}}$ be the highest achievable platform profit.

REMARK 1: Suppose the market is competitive, revenue sharing is in effect, and either (i) $\hat{c}$ is sufficiently high, or (ii) $a < \hat{c}$ and product differentiation μ is sufficiently low. Then PDP with $k = n - 1$ increases the platform's payoff and, moreover,

$$\frac{\widehat{\Omega}^{\mathrm{PDP}}}{\widehat{\Omega}^{\mathrm{Max}}} \geq 1 - \frac{1}{n} \quad \text{and} \quad \frac{\widehat{\Omega}^{\mathrm{PDP}} - \widehat{\Omega}^{\mathrm{BN}}}{\widehat{\Omega}^{\mathrm{Max}} - \widehat{\Omega}^{\mathrm{BN}}} \geq 1 - \frac{e}{n(e-1)} - \frac{\mu}{\hat{c}}. \tag{3}$$

Remark 1 provides some conditions under which PDP performs well under revenue-sharing.[1]

These conditions ensure that PDP increases industry revenue, and hence also the platform's total commissions; the platform therefore gains from implementing PDP (given that PDP increases consumer surplus as well). Intuitively, under these conditions, market prices without PDP exceed the revenue-maximizing level. Thus, PDP brings prices closer to those that maximize industry revenue and, moreover, this beneficial effect is large enough to outweigh the variety loss.

Under these conditions, PDP also performs well relative to more sophisticated platform rules. In fact, we obtain similar bounds on PDP's performance as with per-unit fees.[2]

However, closely following our analysis of per-unit fees, we also find that PDP can perform poorly under revenue-sharing when the market is cartelized.

REMARK 2: Suppose δ is large enough that full collusion is sustainable (with or without PDP). Then PDP (for any $k = 1, \dots, n-1$) decreases the platform's payoff.

The intuition is straightforward. Proposition 4 showed that under full collusion, PDP reduces both prices and total industry output. Hence, PDP necessarily decreases industry revenue, and thus total commissions as well. Since PDP also decreases consumer surplus under full collusion (again, from Proposition 4), it reduces the platform's payoff.

Finally though, under certain conditions, Dynamic PDP also works well under revenue sharing. A simple example of this is the following.

[1]When the platform puts enough weight $1 - \omega$ on consumer surplus, and sellers compete, PDP also raises the platform's payoff (irrespective of $\hat{c}$ and μ). This is simply because PDP (with $k = n - 1$) always benefits consumers. For the same reason, at high $1 - \omega$ PDP also satisfies performance bounds similar to those from our earlier Proposition 2.

[2]The first performance bounds in equations (4) in the main text and (3) above coincide. This is because prices under PDP, as well as the platform's ideal prices under revenue-sharing, are both equal to effective marginal cost $\hat{c}$ (under the conditions in Remark 1). Thus, intuitively, a comparison of platform payoffs under PDP and the platform's ideal scenario reduces to a comparison of total industry output, just as in the per-unit fee case. The second performance bounds in equations (4) in the main text and (3) above do not perfectly coincide, however. This is because under Bertrand–Nash prices strictly exceed $\hat{c}$, whereas under PDP and the platform's ideal scenario they equal $\hat{c}$—and so the second performance bound does not reduce to a simple comparison of industry outputs. Indeed the resulting bounds differ by $\mu/\hat{c}$, which is related to the percentage mark-up over $\hat{c}$ under Bertrand–Nash.

REMARK 3: Suppose δ is sufficiently high that, absent platform design, firms would fully collude. Suppose also that $n = 2$, and either (i) $\hat{c}$ is sufficiently high, or (ii) $a < \hat{c}$ and product differentiation μ is sufficiently low. Then Dynamic PDP with sufficiently large ADV raises the platform's payoff.

Closely following earlier intuition, the conditions on $\hat{c}$ and μ ensure that the prices charged by a fully collusive cartel exceed those that maximize industry revenue. Therefore, Dynamic PDP, by reducing price closer to the level that maximizes industry revenue, increases total revenue—and hence the platform's commissions—despite the reduction in variety. Since Dynamic PDP is guaranteed to benefit consumers when $n = 2$ (see Proposition 6), it raises the platform's total payoff.

2. ENDOGENOUS PLATFORM FEES

So far, we have studied the impact of platform design when the per-unit fee f or the revenue share r are taken as given. We now show that our main insights are robust when these fees are chosen optimally by the platform. We assume throughout that sellers are treated symmetrically, meaning that given a particular design rule, each seller faces the same per-unit fee f or revenue share r.

Start with per-unit fees and recall the platform's payoff $\Omega(p)$ defined in equation (3) in the main text.

REMARK 4: Suppose the market is competitive and that the platform chooses the per-unit platform fee f to maximize $\Omega(p)$. Suppose also that $k/n > e^{-1}$. Then PDP increases the payoff of both the platform and consumers.

The intuition behind this result is as follows. Fixing the per-unit fee at the optimal level under Bertrand–Nash, Proposition 1 shows that in a competitive market PDP raises the payoffs of both the platform and consumers. Since the platform can then further reoptimize its fee under PDP, it is guaranteed to benefit. Moreover, one can show that even accounting for the platform's reoptimized fee, PDP continues to benefit consumers.

PDP also continues to perform well in competitive markets relative to more sophisticated rules. To see this, let $\Omega^{*\text{PDP}}$ denote the platform's payoff under PDP (with $k = n - 1$) when f is chosen optimally. Similarly, let $\Omega^{*\text{Max}}$ denote the highest achievable platform payoff when f is chosen optimally, subject to the condition that each firm makes nonnegative profit in each period; by the usual argument, $\Omega^{*\text{Max}}$ must involve all n firms being displayed and pricing at $\tilde{c} + f$, where f is optimally chosen by the platform. It is easy to see that

$$\frac{\Omega^{*\text{PDP}}}{\Omega^{*\text{Max}}} \geq 1 - \frac{1}{n}, \tag{4}$$

because fixing the per-unit fee at the optimal level in the platform's ideal scenario, Proposition 2 shows that PDP obtains at least a share $1 - 1/n$ of $\Omega^{*\text{Max}}$. Since the platform can reoptimize its fee under PDP, the share of $\Omega^{*\text{Max}}$ that it achieves can only increase further.[3]

[3]Note that this reoptimization argument is not sufficient to establish the second performance bound in Proposition 2, because our measure of the relative performance of PDP contains three different optimal levels of f.

REMARK 5: Suppose δ is sufficiently high that, absent platform design, firms would fully collude. Suppose the platform chooses the per-unit platform fee f to maximize $\Omega(p)$. Suppose also that ADV is sufficiently large that, with exogenous fees, Dynamic PDP leads to marginal cost pricing, and it also would benefit the platform and consumers (e.g., the condition in Proposition 6 is satisfied). Then Dynamic PDP also benefits the platform and consumers with endogenous fees.

The intuition is the same as for Remark 4. In particular, start with full collusion. Suppose that, fixing the per-unit fee, a shift toward Dynamic PDP increases the platform's payoff. Since the platform can then reoptimize its fee, it is guaranteed to be better off. Moreover, one can show that when reoptimizing, the platform either reduces its per-unit fee or else does not increase it by too much—and so consumers also still gain from Dynamic PDP.

Now consider revenue-sharing and recall the platform's payoff $\widehat{\Omega}(p)$ from equation (1).

REMARK 6: Suppose the market is competitive and that the platform chooses the revenue share r to maximize $\widehat{\Omega}(p)$. Suppose also that either (i) $\tilde{c}$ is sufficiently high, or (ii) $a < \tilde{c}$ and product differentiation μ is sufficiently low. Then PDP (with $k = n - 1$) increases the payoff of the platform.

The intuition behind this result closely follows the above case of per-unit fees. Suppose that $\tilde{c}$ is sufficiently large, or that $a < \tilde{c}$ and μ is sufficiently small, that Remark 1 applies. Note that for any revenue share, $\hat{c} \geq \tilde{c}$. Fixing the optimal revenue share under Bertrand–Nash, PDP therefore increases the platform's payoffs. Since the platform can reoptimize its revenue fee under PDP, its payoff can only further increase. Similarly, and following the same logic as for per-unit fees, one can also show that PDP performs well relative to more sophisticated rules. Specifically,

$$\frac{\widehat{\Omega}^{*\mathrm{PDP}}}{\widehat{\Omega}^{*\mathrm{Max}}} \geq 1 - \frac{1}{n}, \tag{5}$$

where $\widehat{\Omega}^{*\mathrm{PDP}}$ and $\widehat{\Omega}^{*\mathrm{Max}}$ are, respectively, the platform's payoff under PDP and in the platform's ideal scenario when r is endogenous.

One important difference with the per-unit case is that consumers do not always gain from PDP when r is endogenous. However, a sufficient condition for consumers to gain is that the platform places enough weight ($1 - \omega \geq 1/2$) on their surplus.[4] Moreover, numerical computations show that consumers can gain much more widely, including for many parameterizations where the platform places full weight ($\omega = 1$) on its own total commissions.

Finally, and again following earlier intuitions, one can derive similar results on the performance of Dynamic PDP under full collusion. For example, if $\tilde{c}$ is sufficiently large, or $a < \tilde{c}$ and μ is sufficiently small, and the other conditions in Remark 3 hold, then Dynamic PDP benefits the platform even with an endogenous revenue share r due to reoptimization of the revenue share.

[4]We already know that, fixing r at the platform-optimal level under Bertrand–Nash, PDP benefits consumers. When $\omega \leq 1/2$, the platform's optimal revenue share is weakly lower under PDP, which further benefits consumers.

3. A RICHER FORM OF DYNAMIC PDP: RANDOM TURNOVER OF THE ADVANTAGE

So far, we have focused on a simple version of Dynamic PDP where, once a firm is displayed, it continues to be displayed forevermore provided that it neither raises its price nor is substantially undercut by its rivals. Here, we consider a richer form of Dynamic PDP in which the pricing advantage exogenously turns over from time to time.

In particular, consider the following modified version of Dynamic PDP.

DEFINITION 1—Dynamic PDP with Random Turnover of the Advantage: In period $t = 0$, firms set prices and one firm with the lowest price is the only firm shown to consumers, and is given an "advantage" in period 1.

At the start of any period $t > 0$, with probability $\tau \in (0, 1)$ the platform decides to favor the advantaged firm, and with probability $1 - \tau$ decides not to favor it. Firms observe this and then set prices.

Suppose the platform has chosen in period t not to favor the advantaged firm. Then one firm with the lowest price is the only firm shown to consumers in period t, and it is given an advantage in period $t + 1$.

Suppose the platform has chosen in period t to favor the advantaged firm. Suppose it is firm i that has the advantage in period t. Then firm i is the only firm shown to consumers, and also receives the advantage in period $t + 1$, so long as:

(1) firm i has not raised its price in period t compared to its price in period $t - 1$, and
(2) no rival in period t undercuts firm i by strictly more than a fixed value $\mathrm{ADV} > 0$.

If either of these two conditions is violated, then in period t a firm with the lowest price is the only firm shown to consumers, and that firm also receives the advantage in period $t + 1$.

In this modified version of Dynamic PDP, with some probability $1 - \tau \in (0, 1)$ the platform decides at the start of a period to "reset" the advantage and treat all firms on an equal footing—thereby making it easier for a firm that was not displayed in the previous period to nevertheless be shown to consumers in the current period.

We now show that under certain conditions this modified version of Dynamic PDP also leads to marginal cost pricing, even if absent platform design the market would be cartelized.

REMARK 7: Consider Dynamic PDP with an advantage $0 < \mathrm{ADV} \leq p^m(1)$.

(1) There exists a $\widehat{\delta} \geq \widehat{\delta}_1$ such that if $\delta < \widehat{\delta}$, then in any pure-strategy subgame-perfect Nash equilibrium the transaction price equals effective marginal cost in all periods.
(2) Moreover, for any δ, there exist ADV and τ sufficiently large such that the equilibrium transaction price is marginal cost in all periods.

Intuitively, when τ is sufficiently large, firms compete fiercely for the display slot whenever it is up for grabs, because they anticipate being able to occupy it for a relatively long amount of time; this fierce competition then leads to marginal cost pricing.

4. A RICHER FORM OF DYNAMIC PDP: SHOWING $k > 1$ FIRMS

So far, we have assumed that under Dynamic PDP a single firm is displayed each period. We now show that, depending on parameters, Dynamic PDP can be extended such that multiple firms are displayed each period and marginal cost pricing again ensues.

Consider the following modified version of Dynamic PDP.

DEFINITION 2—Dynamic PDP with $k > 1$ Displayed Firms: In period $t = 0$, firms set prices and k of the firms with the lowest prices are shown to consumers, and given an "advantage" in period 1. In any period $t > 0$ in which firm i has an advantage, firm i is shown to consumers and also receives an advantage in period $t + 1$, so long as:

(1) firm i has not raised its price in period t compared to its price in period $t - 1$, and

(2) no rival in period t undercuts firm i by strictly more than a fixed value ADV > 0.

Let $\mathcal{K}^t$ denote the set of firms with the advantage in period t that satisfy both of these conditions. If $|\mathcal{K}^t| < k$, then in period t the $k - |\mathcal{K}^t|$ lowest-priced firms not in $\mathcal{K}^t$ are also displayed to consumers and receive an advantage in period $t + 1$.

The modified version of Dynamic PDP in Definition 2 works in a similar way to the version considered in the main text. In particular, in the initial period the platform displays $k > 1$ of the lowest-priced firms, breaking any ties randomly. In each subsequent period, a firm that was just displayed continues to be shown to consumers provided it has not raised its price and is not undercut by more than a fixed advantage ADV. Any firm that violates these conditions temporarily loses its "display slot"; any vacant display slots are then offered to the lowest-priced firms currently without a slot.

Focusing for simplicity on the case of high ADV, we now show that under certain conditions marginal cost pricing again ensues. To interpret the following result, recall that $p^m(k)$ is the optimal price charged by a k-product monopolist.

REMARK 8: Consider the modified version of Dynamic PDP in Definition 2, and suppose ADV $\geq p^m(k)$. If $k(k-1)\exp(a/\mu) < n - k$, then in any pure-strategy subgame perfect Nash equilibrium the k products displayed to consumers in each period are priced at c.

Remark 8 shows that marginal cost pricing ensues provided the number of displayed firms k is below a threshold which depends on the demand parameters a and μ. To illustrate the intuition as simply as possible, suppose that firms try to implement a fully collusive cartel by charging $p^m(k)$ on each displayed product every period, and rotating demand so that each firm gets the same share of monopoly profits. Intuitively, an increase in k helps cartel stability in two ways. First, a higher k means that the profits from participating in collusion are higher. Second, a higher k means that if a firm cheats on the cartel by undercutting $p^m(k)$ (in the initial period, say) then in future periods it will be displayed alongside more firms that can punish it with a low price.

Finally, if the parameters are such that $k = n - 1$ is achievable, the same performance bounds derived earlier in Proposition 2 also hold for Dynamic PDP.

5. PROOFS OF SUPPLEMENTAL RESULTS

We begin with the following preliminary lemma.

LEMMA 1: *Suppose all n firms are displayed. Industry revenue is maximized with symmetric prices, $p_1 = p_2 = \cdots = p_n = p$. Industry revenue is quasiconcave in p. Moreover, $p < \hat{c}$ if* (i) *$\hat{c}$ is sufficiently high, or* (ii) *$a < \hat{c}$ and product differentiation μ is sufficiently low.*

PROOF OF LEMMA 1: Note that industry revenue is equal to industry profit when firms have zero marginal cost. Therefore, symmetry and quasiconcavity follow from the proof of Lemma 2 in the main text. Condition (i) is obvious because the revenue-maximizing price is independent of $\hat{c}$. Condition (ii) follows from inspection of the first-order condition for profit maximization given in (20) in the main text (setting $c = 0$ and $k = n$). *Q.E.D.*

We now prove the main results from our analysis of revenue sharing.

PROOF OF REMARK 1: Let R^{PDP} and R^{BN} denote industry revenue under, respectively, PDP in a competitive market (with $k = n - 1$) and Bertrand–Nash. Let R^{Max} denote the highest possible industry revenue subject to each firm earning nonnegative profit in each period. Let $\widehat{Q}^{\text{PDP}}$, $\widehat{Q}^{\text{BN}}$, and $\widehat{Q}^{\text{Max}}$ be the associated industry outputs.

We start by showing that PDP increases revenue in a competitive market, under the stated conditions. Using Lemma 1 in the main text, under PDP the $n - 1$ displayed firms charge $\hat{c}$ and, therefore,

$$\widehat{Q}^{\text{PDP}} = \frac{(n-1)\exp\left(\frac{a-\hat{c}}{\mu}\right)}{(n-1)\exp\left(\frac{a-\hat{c}}{\mu}\right)+1} \quad \text{and} \quad R^{\text{PDP}} = \hat{c}\widehat{Q}^{\text{PDP}}.$$

Now consider Bertrand–Nash. We know from Lemma 3 in the main text that $p^*_{\text{BN}} > \hat{c} + \mu$. Meanwhile, Lemma 1 implies that, when all n firms are displayed and charge the same price $p > \hat{c}$, industry revenue is strictly decreasing in p. Hence,

$$R^{\text{BN}} < R^{\text{BN}}_{\dagger} = (\hat{c}+\mu)\widehat{Q}^{\text{BN}}_{\dagger} \quad \text{where } \widehat{Q}^{\text{BN}}_{\dagger} = \frac{n\exp\left(\frac{a-\hat{c}}{\mu}\right)\exp(-1)}{n\exp\left(\frac{a-\hat{c}}{\mu}\right)\exp(-1)+1}.$$

Combining the above and simplifying, we find that

$$R^{\text{PDP}} - R^{\text{BN}} > \frac{\hat{c}\exp\left(\frac{a-\hat{c}}{\mu}\right)}{(n-1)\exp\left(\frac{a-\hat{c}}{\mu}\right)+1} \times \left[n-1-n\exp(-1)\frac{\left(1+\frac{\mu}{\hat{c}}\right)\left[(n-1)\exp\left(\frac{a-\hat{c}}{\mu}\right)+1\right]}{n\exp\left(\frac{a-\hat{c}}{\mu}\right)\exp(-1)+1}\right].$$

Under the stated conditions in (i) and (ii), it is straightforward to show that the square-bracketed term is strictly positive, and hence $R^{\text{PDP}} > R^{\text{BN}}$. Recall from Proposition 1 that consumers also benefit from PDP (with $k = n - 1$), $U^{\text{PDP}} > U^{\text{BN}}$. The platform therefore also gains since its payoff is $\widehat{\Omega} = \omega r R + (1-\omega)U$.

Now consider the performance bounds. Lemma 1 implies that, when all n firms are displayed and make nonnegative profit, industry revenue is maximized when each charges $\hat{c}$. Hence,

$$\widehat{Q}^{\text{Max}} = \frac{n\exp\left(\frac{a-\hat{c}}{\mu}\right)}{n\exp\left(\frac{a-\hat{c}}{\mu}\right)+1} \quad \text{and} \quad R^{\text{Max}} = \hat{c}\widehat{Q}^{\text{Max}}.$$

Using the expressions for $\widehat{Q}^{\text{PDP}}$ and $\widehat{Q}^{\text{Max}}$ and simplifying, we find that

$$R^{\text{PDP}} - \left(1-\frac{1}{n}\right)R^{\text{Max}} = \hat{c}\left[\widehat{Q}^{\text{PDP}} - \left(1-\frac{1}{n}\right)\widehat{Q}^{\text{Max}}\right] \geq 0, \tag{6}$$

where the inequality can be proved in exactly the same way as the one in equation (11) (from the proof of Proposition 2). In addition, Proposition 2 shows that $U^{\text{PDP}}/U^{\text{Max}} > 1-(1/n)$. Therefore, for $\omega < 1$ or $r > 0$ (or both), we have

$$\frac{\widehat{\Omega}^{\text{PDP}}}{\widehat{\Omega}^{\text{Max}}} = \frac{\omega r R^{\text{PDP}} + (1-\omega)U^{\text{PDP}}}{\omega r R^{\text{Max}} + (1-\omega)U^{\text{Max}}} \geq 1 - \frac{1}{n}.$$

Now consider the second performance bound. We will first prove that

$$\left(R^{\text{PDP}} - R^{\text{BN}}\right) - \left[1 - \frac{e}{n(e-1)} - \frac{\mu}{\hat{c}}\right]\left(R^{\text{Max}} - R^{\text{BN}}\right) \geq 0. \tag{7}$$

To show this, note that the left-hand side is decreasing in R^{BN}, and hence it is sufficient to prove that the following is positive:

$$\left(R^{\text{PDP}} - R^{\text{BN}}_{\dagger}\right) - \left[1 - \frac{e}{n(e-1)} - \frac{\mu}{\hat{c}}\right]\left(R^{\text{Max}} - R^{\text{BN}}_{\dagger}\right).$$

Using the definitions of R^{PDP}, $R^{\text{BN}}_{\dagger}$, and R^{Max} and rearranging, this equals

$$\begin{aligned}&\hat{c}\left\{\left(Q^{\text{PDP}} - Q^{\text{BN}}_{\dagger}\right) - \left[1 - \frac{e}{n(e-1)}\right]\left(Q^{\text{Max}} - Q^{\text{BN}}_{\dagger}\right)\right\}\\ &\quad + \mu Q^{\text{Max}} - \left[1 + \frac{e}{n(e-1)} + \frac{\mu}{\hat{c}}\right]\mu Q^{\text{BN}}_{\dagger}.\end{aligned}$$

The curly-bracketed term is positive by the same steps used to sign (15) in the proof of Proposition 2. The remaining terms are in sum positive. To see this, rewrite them as

$$\frac{\mu n Y}{nY+1}\left\{1 - \left[1 + \frac{e}{n(e-1)} + \frac{\mu}{\hat{c}}\right]\exp(-1)\frac{nY+1}{nY\exp(-1)+1}\right\},$$

where $Y \equiv \exp(\frac{a-\hat{c}}{\mu})$, and note that under the conditions stated in (i) and (ii) this is positive. Using (18) in the main text, as well as (7), and assuming either $\omega < 1$ or $r > 0$ (or both), we then have that

$$\begin{aligned}\frac{\widehat{\Omega}^{\text{PDP}} - \widehat{\Omega}^{\text{BN}}}{\widehat{\Omega}^{\text{Max}} - \widehat{\Omega}^{\text{BN}}} &= \frac{\omega r\left(R^{\text{PDP}} - R^{\text{BN}}\right) + (1-\omega)\left(U^{\text{PDP}} - U^{\text{BN}}\right)}{\omega r\left(R^{\text{Max}} - R^{\text{BN}}\right) + (1-\omega)\left(U^{\text{Max}} - U^{\text{BN}}\right)}\\ &\geq 1 - \frac{e}{n(e-1)} - \frac{\mu}{\hat{c}}.\end{aligned}$$

Q.E.D.

We begin with the following preliminary lemma.

LEMMA 2: *Suppose the platform levies a per-unit fee f. Bertrand–Nash and fully collusive prices exhibit pass-through rates less than one*: $\partial p^*_{\text{BN}}/\partial f < 1$ *and* $\partial p^m(n)/\partial f < 1$.

PROOF OF LEMMA 2: Start with the Bertrand–Nash price. Using equation (5) in the main text, p^*_{BN} solves

$$\frac{p^*_{\mathrm{BN}} - \tilde{c} - f}{\mu} = \frac{n\exp\left(\dfrac{a - p^*_{\mathrm{BN}}}{\mu}\right) + 1}{(n-1)\exp\left(\dfrac{a - p^*_{\mathrm{BN}}}{\mu}\right) + 1}.$$

On the way to a contradiction, suppose that $\partial p^*_{\mathrm{BN}}/\partial f \geq 1$. The left-hand side of this equation would increase in f while the right-hand side would strictly decrease in f. But this is impossible. Hence, $\partial p^*_{\mathrm{BN}}/\partial f < 1$.

Next, consider the fully collusive price. Using equation (20) in the main text, $p^m(n)$ solves

$$n\exp\left(\frac{a - p^m(n)}{\mu}\right) + 1 - \left(\frac{p^m(n) - \tilde{c} - f}{\mu}\right) = 0.$$

On the way to a contradiction, suppose that $\partial p^m(n)/\partial f \geq 1$. The left-hand side of this equation would strictly decrease in f, which is impossible. Hence, $\partial p^m(n)/\partial f < 1$. *Q.E.D.*

PROOF OF REMARKS 4 AND 5: The claims that PDP (in a competitive market) and DPDP (with sufficiently large ADV) increase the platform's payoff follow directly from arguments that follow the two remarks. Therefore, here we prove that consumer surplus also increases.

Using equation (3) in the main text and the fact that $U(p) = -\mu\log(1 - Q(p))$, the platform's payoff can be written as

$$\Omega(f) = \omega f Q\big(p(f)\big) - (1-\omega)\mu\log\big[1 - Q\big(p(f)\big)\big], \tag{8}$$

where $p(f)$ denotes the (common) price charged by the displayed firms as a function of f. Differentiating (8) and noting that $Q'(p) = -Q(p)[1 - Q(p)]/\mu$, we obtain

$$\Omega'(f) = Q\big(p(f)\big)\left\{\omega\left[1 - \frac{\big[1 - Q\big(p(f)\big)\big] f p'(f)}{\mu}\right] - (1-\omega)p'(f)\right\}. \tag{9}$$

First, consider the comparison between Bertrand–Nash and PDP (Remark 4). Let f^{BN} and f^{PDP} denote the corresponding optimal fees. To show that consumers are better off under PDP, it is sufficient to prove that $Q^{\mathrm{PDP}} > Q^{\mathrm{BN}}$. Under PDP $p(f) = \tilde{c} + f$, and so the curly-bracketed term in (9) simplifies to

$$\omega\left[1 - \frac{\big[1 - Q\big(p(f)\big)\big] f}{\mu}\right] - (1-\omega). \tag{10}$$

When evaluated at $f = 0$, (10) is negative if and only if $\omega \leq 1/2$. Note also that $[1 - Q(p(f))]f$ is strictly increasing in f. Therefore, when $\omega \leq 1/2$ the optimal fee is $f^{\mathrm{PDP}} = 0$, and when $\omega > 1/2$ the optimal fee f^{PDP} is strictly positive and obtained by setting (9) (equivalently, (10)) to zero.

We now prove that $Q^{\mathrm{PDP}} > Q^{\mathrm{BN}}$.

Start with the case $\omega \leq 1/2$. Fixing $f = f^{\mathrm{BN}}$, we know from Proposition 1 that switching from Bertrand–Nash to PDP strictly increases total output. Reoptimizing the fee under

PDP can only further increase total output; this is because we have just established that $f^{\mathrm{PDP}} = 0$, and hence $f^{\mathrm{PDP}} \leq f^{\mathrm{BN}}$, while total output under PDP is decreasing in the fee.

Now consider the case $\omega > 1/2$. Toward a contradiction, suppose that $Q^{\mathrm{PDP}} \leq Q^{\mathrm{BN}}$. It is immediate from Proposition 1 that, in order for this to be true, we must have $f^{\mathrm{PDP}} > f^{\mathrm{BN}}$. We also know that $p'(f) = 1$ under PDP, and $p'(f) < 1$ under Bertrand–Nash (from Lemma 2). We also established that (9) must be zero under PDP. But then by inspection (9) must be strictly positive under Bertrand–Nash, which is impossible.

We have therefore established that $Q^{\mathrm{PDP}} > Q^{\mathrm{BN}}$, and hence consumers are better off under PDP than Bertrand–Nash.

Second, consider the comparison between full collusion and Dynamic PDP (Remark 5). The proof follows the same steps as above, because under Dynamic PDP $p(f) = \tilde{c} + f$ for ADV sufficiently large. *Q.E.D.*

PROOF OF REMARK 6: The proof follows from arguments in the text and so is omitted. *Q.E.D.*

Finally, we prove the claim that when $\omega \leq 1/2$ and fees are endogenous, PDP (with $k = n - 1$) in a competitive market with revenue sharing is guaranteed to benefit consumers when the conditions in Remark 6 are satisfied.

As a first step, we prove that for $\omega \leq 1/2$ the optimal revenue share under PDP is zero. Let

$$Z = (n-1)\exp\left(\frac{a - \dfrac{\tilde{c}}{1-r}}{\mu}\right),$$

and note that this implies that

$$\frac{\tilde{c}}{1-r} = a - \mu \log\left(\frac{Z}{n-1}\right) \quad \text{and} \quad r = \frac{a - \mu \log\left(\dfrac{Z}{n-1}\right) - \tilde{c}}{a - \mu \log\left(\dfrac{Z}{n-1}\right)}. \tag{11}$$

Recall that under PDP firms Bertrand compete down to effective marginal cost $\tilde{c}/(1-r)$. Hence, the platform's payoff is

$$\begin{aligned} \widehat{\Omega}^{\mathrm{PDP}} &= \omega r \frac{\tilde{c}}{1-r} \frac{Z}{Z+1} + (1-\omega)\mu \log(Z+1) \\ &= \omega\left[a - \mu \log\left(\frac{Z}{n-1}\right) - \tilde{c}\right]\frac{Z}{Z+1} + (1-\omega)\mu \log(Z+1), \end{aligned} \tag{12}$$

where the second equality uses (11) to simplify the first term. The first derivative of (12) with respect to Z is

$$\frac{1}{(Z+1)^2}\left\{\omega\left[a - \mu \log\left(\frac{Z}{n-1}\right) - \tilde{c}\right] + (1-2\omega)(Z+1)\mu\right\}.$$

The first term inside curly-brackets is strictly positive for any $r > 0$ (given equation (11)), while the second term inside curly-brackets is also positive for $\omega \leq 1/2$. Hence, when

$\omega \leq 1/2$ the platform wishes to increase Z as far as possible, which is equivalent to setting $r^{\text{PDP}} = 0$.

It is then straightforward to see that consumers benefit from PDP when $\omega \leq 1/2$ and the platform's commissions are from revenue sharing. Specifically, fixing r at the optimal level under Bertrand–Nash, we know from Proposition 1 that consumers benefit from a move to PDP (with $k = n - 1$). Since we have just proved that $r^{\text{PDP}} = 0$, when the platform re-optimizes under PDP it weakly reduces its revenue share, which further benefits consumers.

PROOF OF REMARK 7: Consider a pure-strategy subgame perfect Nash equilibrium (SPNE), and let $\widehat{p}_t$ denote the price of the product displayed to consumers in period t along the equilibrium path (for a given history of whether the platform favored the advantaged firm or not).

Again, let $\tilde{\pi}(p)$ denote per-period profit of a firm that charges p and is the only firm shown to consumers, and recall that $\tilde{\pi}(p)$ is strictly increasing in p up to $p^m(1)$. Let $\widehat{\delta}$ be the unique solution to

$$\frac{\pi^m(1)}{n} = (1-\delta)\pi^m(1) + \frac{\delta\tau(1-\delta)}{1-\delta\tau}\tilde{\pi}\big(\max\{c, \text{ADV}\}\big). \tag{13}$$

Note for future reference that $\widehat{\delta} = 1 - 1/n$ when $\text{ADV} \leq c$. Note also that when $\text{ADV} > c$ then $1 - 1/n < \widehat{\delta} < (n-1)/(n-\tau)$. To see the latter, note that the right-hand side of (13) is strictly larger than the left-hand side when $\delta = 1 - 1/n$, and because $\tilde{\pi}(\max\{c, \text{ADV}\}) \leq \pi^m(1)$, the right-hand side of (13) is also both strictly decreasing in δ and weakly smaller than the left-hand side when $\delta = (n-1)/(n-\tau)$.[5]

We start with part (1) of the remark. Assume for this part of the proof that $\delta < \widehat{\delta}$.

We first prove that $\widehat{p}_t \leq p^m(1)$ for all t. On the way to a contradiction, suppose this is not true, and let t' denote the first (realized) period in which $\widehat{p}_t > p^m(1)$. Note that a firm with the lowest price in period t' will be displayed and get the advantage going into the following period. (This is immediate if $t' = 0$, or $t' > 0$ and the platform chose at the start of the period not to favor the advantaged firm. If $t' > 0$ and the platform decided at the start of the period to favor the advantaged firm, then the firm that was displayed last period must have raised its price—and so all firms are treated on an even footing.) Note also that in period t', along the equilibrium path all firms are charging strictly more than $p^m(1)$. Hence, any firm could charge $p^m(1)$ in period t' and be displayed for sure, and then (i) if $\text{ADV} < c$, charge c in all future periods, and (ii) if $\text{ADV} \geq c$, charge ADV and keep the slot until the next period in which the platform chooses not to favor the advantaged firm, and then charge c thereafter. Therefore, in period t' the n firms' combined discounted payoff is at least

$$n\left[\pi^m(1) + \sum_{r=1}^{\infty}(\delta\tau)^r\tilde{\pi}\big(\max\{c, \text{ADV}\}\big)\right]$$

[5]In more detail, to see that the right-hand side of (13) is strictly decreasing in δ, rewrite it as

$$(1-\delta)\pi^m(1) + \left[\delta - \frac{\delta(1-\tau)}{1-\delta\tau}\right]\tilde{\pi}\big(\max\{c, \text{ADV}\}\big),$$

and note that $\tilde{\pi}(\max\{c, \text{ADV}\}) \leq \pi^m(1)$.

$$= \frac{n\left[(1-\delta)\pi^m(1) + \left[\delta - \frac{\delta(1-\tau)}{1-\delta\tau}\right]\tilde{\pi}\big(\max\{c, \mathrm{ADV}\}\big)\right]}{1-\delta}$$

$$> \frac{n\left[(1-\widehat{\delta})\pi^m(1) + \left[\widehat{\delta} - \frac{\widehat{\delta}(1-\tau)}{1-\widehat{\delta}\tau}\right]\tilde{\pi}\big(\max\{c, \mathrm{ADV}\}\big)\right]}{1-\delta}$$

$$= \frac{\pi^m(1)}{1-\delta},$$

where the strict inequality uses $\delta < \widehat{\delta}$ and the fact that the square-bracketed term on the right-hand side of the first line is strictly decreasing in δ, and the final equality uses equation (13). But this is a contradiction, because the joint profit in each period cannot exceed $\pi^m(1)$. Hence, $\widehat{p}_t \leq p^m(1)$ for all t.

It also follows that the supremum $\overline{p} \leq p^m(1)$ over equilibrium transaction prices exists. We now prove that $\overline{p} = c$.

On the way to a contradiction, suppose that $\overline{p} \in (c, p^m(1)]$. For any given $\Delta \in (0, \overline{p} - c)$, let t'' be the first (realized) period t in which $\widehat{p}_t \in (\overline{p} - \Delta, \overline{p}]$. Using the same arguments as above, in period t'' along the equilibrium path all firms are charging strictly more than $\overline{p} - \Delta$ and, moreover, a firm with the lowest price will be displayed in period t'' and get the advantage in the following period. Hence, any firm could charge $\overline{p} - \Delta$ in period t'' and be displayed, and then (i) if $\mathrm{ADV} > c$, charge $\min\{\overline{p} - \Delta, \mathrm{ADV}\}$ and keep being displayed until the next period where the platform decides to not favor the advantaged firm, and then charge c thereafter, and (ii) if $\mathrm{ADV} \leq c$, charge c from period $t'' + 1$ onwards. Because this is true for all firms, in period t'' the n firms' combined discounted payoff is at least

$$n\left[\tilde{\pi}(\overline{p} - \Delta) + \sum_{r=1}^{\infty} (\delta\tau)^r \tilde{\pi}\big(\min\{\overline{p} - \Delta, \max\{c, \mathrm{ADV}\}\}\big)\right]$$

$$= \frac{n}{1-\delta}\left[(1-\delta)\tilde{\pi}(\overline{p} - \Delta) + \left[\delta - \frac{\delta(1-\tau)}{1-\delta\tau}\right]\tilde{\pi}\big(\min\{\overline{p} - \Delta, \max\{c, \mathrm{ADV}\}\}\big)\right]. \tag{14}$$

Because $\delta < \widehat{\delta}$, when evaluated at $\Delta = 0$ this is strictly more than $\tilde{\pi}(\overline{p})/(1-\delta)$. To show this, there are three cases to consider. First, if $\mathrm{ADV} \leq c$, then $\widehat{\delta} = 1 - (1/n)$ while (14) simplifies to $n\tilde{\pi}(\overline{p})$ and the claim follows from $\delta < \widehat{\delta}$. Second, if $c < \overline{p} \leq \mathrm{ADV}$, then (14) simplifies to

$$\frac{n\tilde{\pi}(\overline{p})}{1-\delta}\left[\frac{1-\delta}{1-\delta\tau}\right] > \frac{n\tilde{\pi}(\overline{p})}{1-\delta}\left[\frac{1-\widehat{\delta}}{1-\widehat{\delta}\tau}\right] \geq \frac{\tilde{\pi}(\overline{p})}{1-\delta},$$

where the inequalities use the fact that $(1-\delta)/(1-\delta\tau)$ is strictly decreasing in δ and also $\delta < \widehat{\delta} \leq (n-1)/(n-\tau)$. Third, consider $c < \mathrm{ADV} < \overline{p}$. Since the square-bracketed term in (14) is strictly decreasing in δ, and because $\delta < \widehat{\delta}$, (14) is strictly larger than

$$\frac{n}{1-\delta}\left[(1-\widehat{\delta})\tilde{\pi}(\overline{p}-\Delta)+\left[\widehat{\delta}-\frac{\widehat{\delta}(1-\tau)}{1-\widehat{\delta}\tau}\right]\tilde{\pi}\big(\min\{\overline{p}-\Delta,\max\{c,\text{ADV}\}\}\big)\right]$$
$$=\frac{n}{1-\delta}\left[(1-\widehat{\delta})\tilde{\pi}(\overline{p})+\pi^m(1)\left[\frac{1}{n}-(1-\widehat{\delta})\right]\right]\geq\frac{\tilde{\pi}(\overline{p})}{1-\delta},$$

where the equality uses (13), and the inequality uses $\widehat{\delta} > 1-1/n$ and $\pi^m(1) \geq \tilde{\pi}(\overline{p})$.

Hence, we have shown that (14) evaluated at $\Delta = 0$ strictly exceeds $\tilde{\pi}(\overline{p})/(1-\delta)$. By continuity, this implies that, for $\widehat{p}_t \leq \overline{p}$ sufficiently close to $\overline{p}$, firms' combined payoffs strictly exceed $\tilde{\pi}(\overline{p})/(1-\delta)$, a contradiction. We have therefore established that $\overline{p} \leq c$. But since $\overline{p} < c$ is impossible, it must be that $\overline{p} = c$.

We now construct a SPNE where firms charge c in each period. Consider the following strategy:

(1) In period 0, or any period $t > 0$ in which the platform has decided to not favor the advantaged firm, all firms charge c.

(2) Consider a period $t+1$ in which the platform has decided to favor the advantaged firm:
   (a) Suppose that the firm that was displayed in period t charged weakly less than c in period t. Then in period $t+1$ all firms charge c.
   (b) Suppose that the firm that was displayed in period t charged strictly more than c in period t. Then in period $t+1$ that firm charges the minimum of $\min\{c+\text{ADV}, p^m(1)\}$ and its price from period t, while all other firms charge c.

Using the one-shot deviation principle, it is straightforward to check that this strategy forms a SPNE. Moreover, along the equilibrium path the product that is displayed to consumers has price c in every period.

Finally, we prove part (2) of the remark. Note that the right-hand side of (13) is strictly decreasing in δ and strictly increasing in τ and ADV (conditional on ADV $> c$). Hence, $\widehat{\delta}$ is strictly increasing in τ and ADV when ADV $> c$. Moreover, note that when ADV $= p^m(1)$ equation (13) implies that $\widehat{\delta} = (n-1)/(n-\tau)$, and so $\widehat{\delta} \to 1$ as $\tau \to 1$. Hence, for any $\delta < 1$ we can find ADV and τ sufficiently large that $\delta < \widehat{\delta}$. *Q.E.D.*

Before proving Remark 8, we need the following preliminary result.

LEMMA 3: *Suppose that $k(k-1)\exp(a/\mu) < n-k$. Then if a firm charges some price $p > c$, its profit is strictly higher* (i) *when it is displayed for sure alongside $k-1$ rivals that charge zero, compared to* (ii) *when it is displayed with probability k/n, and the other displayed firms have a weakly lower price.*

PROOF OF LEMMA 3: The firm's profit under the first scenario is

$$(p-c)\frac{\exp\left(\dfrac{a-p}{\mu}\right)}{\exp\left(\dfrac{a-p}{\mu}\right)+(k-1)\exp\left(\dfrac{a}{\mu}\right)+1}. \tag{15}$$

The firm's profit under the second scenario is highest when the $k-1$ other firms charge p. Therefore, profit in the second scenario is bounded from above by

$$\frac{k}{n}(p-c)\frac{\exp\left(\dfrac{a-p}{\mu}\right)}{k\exp\left(\dfrac{a-p}{\mu}\right)+1}. \tag{16}$$

After some algebra, (15) strictly exceeds (16) provided that

$$k(k-1)\exp\left(\frac{a}{\mu}\right) < n-k+k(n-1)\exp\left(\frac{a-p}{\mu}\right),$$

and a sufficient condition for this to hold is that $k(k-1)\exp(a/\mu) < n-k$. *Q.E.D.*

We are now ready to prove the main result.

PROOF OF REMARK 8: Consider a pure-strategy subgame perfect Nash equilibrium (SPNE), and let $\widehat{p}_t$ denote the price of the highest-priced product that is displayed to consumers in period t along the equilibrium path.

Let $\tilde{\pi}(p, \mathbf{0}; 1)$ denote per-period profit of a firm that charges p and is displayed for sure alongside $k-1$ firms that charge 0. Let $\tilde{\pi}(p, \mathbf{p}; k/n)$ denote per-period profit of a firm that charges p and is displayed with probability k/n alongside $k-1$ firms that also charge p.

We first prove that $\widehat{p}_t \leq p^m(k)$ for all t. On the way to a contradiction, suppose this is not true, and let t' denote the first period in which $\widehat{p}_t > p^m(k)$. Note that either $t'=0$, or $t'>0$ and at least one firm that was displayed last period has raised its price. Note also that in period t', along the equilibrium path at least $n-k+1$ firms are charging strictly more than $p^m(k)$. Hence, by the definition of (the modified version of) Dynamic PDP, any firm could charge $p^m(k)$ in period t' and be displayed for sure, and also charge $p^m(k)$ in all future periods and be displayed forevermore. Therefore, in period t' the n firms' combined discounted payoff is at least $n\tilde{\pi}(p^m(k), \mathbf{0}; 1)/(1-\delta)$, because the worst outcome for a displayed firm is that all other displayed firms charge the lowest possible price of zero. However, by Lemma 3 this strictly exceeds $n\tilde{\pi}(p^m(k), \mathbf{p^m(k)}; k/n)/(1-\delta)$. But this is impossible because the latter is the discounted profit of a k-product monopolist. Hence, $\widehat{p}_t \leq p^m(k)$ for all t.

It also follows that the supremum $\overline{p} \leq p^m(k)$ over equilibrium transaction prices exists. We now prove that $\overline{p}=c$.

On the way to a contradiction, suppose that $\overline{p} \in (c, p^m(k)]$. For any given $\Delta \in (0, \overline{p}-c)$, let t'' be the first period t in which $\widehat{p}_t \in (\overline{p}-\Delta, \overline{p}]$. Note that either $t''=0$, or $t''>0$ and at least one firm that was displayed in the previous period has raised its price. Hence, by the definition of (the modified version of) Dynamic PDP any firm could charge $\overline{p}-\Delta$ in period t'' and be displayed for sure, and also charge $\overline{p}-\Delta$ in all future periods and be displayed forevermore. Because this is true for all firms, in period t'' the n firms' combined discounted payoff is at least $n \times \tilde{\pi}(\overline{p}-\Delta, \mathbf{0}; 1)/(1-\delta)$, because the worst outcome for a displayed firm is that all other displayed firms charge the lowest possible price of zero. However, by Lemma 3 this strictly exceeds $n\tilde{\pi}(\overline{p}-\Delta, \overline{\mathbf{p}}-\boldsymbol{\Delta}; k/n)/(1-\delta)$, and so by continuity for $\widehat{p}_t$ sufficiently close to $\overline{p}$ it also strictly exceeds $n\tilde{\pi}(\overline{p}, \overline{\mathbf{p}}; k/n)/(1-\delta)$. However, this is a contradiction—because by adapting the proof of Lemma 2 in the main text, it is straightforward to show that a k-product monopolist, which is constrained to charge less than $\overline{p} \leq p^m(k)$, charges exactly $\overline{p}$ on each product and earns discounted profit $n\tilde{\pi}(\overline{p}, \overline{\mathbf{p}}; k/n)/(1-\delta)$.

We have therefore established that $\overline{p} \leq c$. But since $\overline{p} < c$ is impossible, it must be that $\overline{p} = c$.

We now construct a SPNE where firms charge c in each period, and so along the equilibrium path the k displayed products are priced at c in each period. Consider the following strategy:

(1) In period 0, all firms charge c.
(2) Suppose that in period t firm i either was not displayed, or was displayed at a price weakly below c. Then in period $t+1$ firm i charges c.
(3) Suppose that in period t firm i was displayed at a price $p_{i,t} > c$. Then in period $t+1$, firm i charges $\min\{p_{i,t}, p_{i,t}^{\mathrm{BR}}\}$ where $p_{i,t}^{\mathrm{BR}}$ is firm i's (static) best response to the prices of the $k-1$ other firms being displayed in period $t+1$.

Using the one-shot deviation principle, it is straightforward to check that this strategy forms a SPNE. We omit a full proof and focus on part (3). Note that if such a firm i were to charge weakly less than c in period $t+1$ then it would be displayed but would earn weakly negative discounted profit. If instead such a firm i were to charge strictly more than $p_{i,t}$, then it would not be displayed and would earn zero discounted profit. Instead if it charges $\min\{p_{i,t}, p_{i,t}^{\mathrm{BR}}\}$, it maximizes its profit in this period conditional on being displayed, and it is displayed because $p_{i,t}^{\mathrm{BR}} < p^m(1) < p^m(k)$ and so firm i is not undercut by more than $\mathrm{ADV} \geq p^m(k)$ (and moreover this choice of price ensures that this firm's continuation payoffs equal the discounted sum of period t's profits). Instead charging strictly less than $\min\{p_{i,t}, p_{i,t}^{\mathrm{BR}}\}$ both lowers the firm's profits in period t while also weakly decreasing the prices (and hence profits) of all firms in future periods according the posited strategies. (Note that if firm i was the only firm to be displayed in period t at a price strictly above c, then $\min\{p_{i,t}, p_{i,t}^{\mathrm{BR}}\}$ is clearly well-defined. Note if instead multiple firms were displayed in period t at a price strictly above c, then we can view these firms as choosing a price on $[c, \min\{p_{i,t}, p^m(1)\}]$; since each firm's action set is compact and its profit continuous in all prices and quasiconcave in its own price, by the Fan–Glicksberg theorem this pricing game has at least one Nash equilibrium.) *Q.E.D.*

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