SUPPLEMENT TO "USING ASSET PRICES TO MEASURE THE PERSISTENCE OF THE MARGINAL UTILITY OF WEALTH"

(*Econometrica*, Vol. 73, No. 6, November 2005, 1977–2016)

BY FERNANDO ALVAREZ AND URBAN J. JERMANN

This supplement contains an additional example that illustrates Proposition 4 in the paper. We also describe the programs and data used to derive the empirical results.

KEYWORDS: Pricing kernel, stochastic discount factor, permanent component, unit roots.

1. EXAMPLE THAT ILLUSTRATES PROPOSITION 4

WE PRESENT here an example that illustrates Proposition 4 in the paper. In particular, this example fosters better understanding of the assumption made in Proposition 4 that

$$\lim_{k\to\infty}\left[\frac{(1+v_{t+1,t+k})}{(1+v_{t,t+k})}\right]=1 \quad \text{a.s.}, \tag{1}$$

where $v_{t,t+k} \equiv \operatorname{cov}_t(M^T_{t+k}M^P_{t+k})/E_t(M^T_{t+k})E_t(M^P_{t+k})$. This assumption holds when the transitory and permanent components are uncorrelated, but it requires only a much weaker condition. In particular, the conditional covariance between the transitory and permanent components does need to stabilize for long forecasting horizons. To better understand this condition, we present a simple example in which it is satisfied. In the example, both components of the pricing kernel are lognormal with correlated innovations: the log of the permanent component is a random walk with drift, and the transitory component is covariance stationary. This type of process has often been used in the measurement of the size of the permanent component for linear time series. See, for instance, Watson (1986) and Cochrane (1988).

EXAMPLE 1: *Assume that*

$$\log M^P_{t+1} = -\frac{1}{2}\sigma_P^2 + \log M^P_t + \varepsilon^P_{t+1},$$

$$\log M^T_{t+1} = \log \beta^{t+1} + \sum_{i=0}^{\infty} \alpha_i \varepsilon^T_{t+1-i},$$

where α is a square summable sequence, and ε^P_t and ε^T_t are iid normal with mean zero and covariance σ_{TP}. Direct computation gives

$$\frac{(1+v_{t+1,t+k})}{(1+v_{t,t+k})} = \exp(-\alpha_{k-1}\sigma_{TP});$$

hence, (1) *is satisfied, since* $\lim_{k\to\infty}\alpha_{k-1}=0$ *because α is square summable.*

2. DESCRIPTION OF PROGRAMS

Programs are given in five separate directories.

2.1. *Zero-Coupon Data: Tables* I *and* II*, and Figures* 1–3

To generate the results in Table I, alldatajen.m has to be run first, followed by tablejen.m. The desired holding period is selected in alldatajen.m, and the maturity and the type of spread is selected in tablejen.m. The program alldatajen.m calls data in mccuyh.mat, blissyh.mat, crspnan.m, stor.m and stormon.m. The original versions of the last three files contain commercially available data from CRSP as described in Appendix B of the paper. Sample files that do not contain the actual data are provided for crspnan.m, stor.m, and stormon.m.

To generate the results of Table II, the option is chosen at the beginning in alldatajen.m as well as the available portfolio returns. The programs alldatajen.m and tablejen.m are then run as for the results in Table I. The program alldatajen.m calls growop.m, growopdat.m, and gmmjen.m.

Figure 1 is created by running first alldatajen.m and then figure1.m. Figure 2 is created by running alldatatemp.m and then figure2.m. Figure 3 is created by running alldatatemp.m, then figure2.m, and then figure3.m.

The zero-coupon data are stored in mat-files, that is, binary Matlab files. These data are also given in ASCII files.

2.2. *Table* III*. Size of Permanent Component Based on Aggregate Equity and Coupon Bonds*

The function gmmf1.m produces one row of Table III. Required input options and output are described in the file. Running gmmf1.m requires the data files Datus.m or Datukad.m. This is commercially available data from either Ibbotson Associates or Global Financial Data as described in Appendix B of the paper. Sample files that do not contain the actual data are provided for Datus.m and Datukad.m.

2.3. *Table* V*. The Size of the Permanent Component Due to Inflation*

The first panel of Table V of the paper can be replicated by running table5.m. This program calls the files pricelevel.m, arspec.m, monti1.m, monti2.m, and sizeprice.m. Variables are explained in the program. The original version of the file pricelevel.m contains commercially available data from Ibbotson Associates as described in Appendix B of the paper. A sample file that does not contain the actual data is provided.

2.4. *Table* VI*. Inflation-Indexed Bonds*

Table VI is replicated by running tab6prg.m. This program calls ukinfsto.m, uknf.m, uknzy.m, ukrf.m, and ukrzy.m. The original version of the file ukinf-

sto.m contains commercially available data from Global Financial Data as described in Appendix B of the paper. A sample file that does not contain the actual data is provided.

2.5. *Figure* 4. $1/k$ *Times the Variance of Consumption*

Figure 4 is replicated by running fig4prg.m. This program calls consdat.m and sizepc1.m.

Dept. of Economics, University of Chicago, Chicago, IL 60637, U.S.A.; and NBER; f-alvarez1@uchicago.edu

and

Dept. of Finance, The Wharton School of the University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104-6367, U.S.A.; and NBER; jermann@wharton.upenn.edu; http://finance.wharton.upenn.edu/~jermann.