

SUPPLEMENT TO “THE MICRO ECONOMICS OF EFFICIENT GROUP BEHAVIOR: IDENTIFICATION”: A PARAMETRIC EXAMPLE
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The general results of the paper can readily be illustrated on a simple, parametric example.

CONSIDER INDIVIDUAL PREFERENCES of the LES type:

$$U^s(x_s, X) = \sum_{i=1}^n a_i^s \log(x_s^i - c_s^i) + \sum_{j=n+1}^N A_j^s \log(X^j - C^j) \quad (s = 1, 2),$$

where the parameters a_i^s and A_j^s are normalized by the condition $\sum_i a_i^s + \sum_j A_j^s = 1$ for all s , whereas the parameters c_s^i and C^j are unconstrained. Also, let the Pareto weights have the simple, linear form

$$\lambda_s = \lambda_s^0 + \lambda_s^y y + \lambda_s^z z \quad (s = 1, 2).$$

Again, we normalize the weights by imposing that $\lambda_1 + \lambda_2 = 1$. We then define $\lambda^a = \lambda_1^a$ for $a = 0, y, z$, so that

$$(S1) \quad \lambda_2^0 = 1 - \lambda^0, \quad \lambda_2^y = -\lambda^y, \quad \lambda_2^z = -\lambda^z.$$

S1. AGGREGATE DEMAND

The group solves the program

$$\begin{aligned} \max \sum_{s=1}^2 (\lambda_s^0 + \lambda_s^y y + \lambda_s^z z) \\ \times \left(\sum_{i=1}^n a_i^s \log(x_s^i - c_s^i) + \sum_{j=n+1}^N A_j^s \log(X^j - C^j) \right) \end{aligned}$$

under the budget constraint

$$p'(x_1 + \dots + x_s) + P'X = y,$$

where one price has been normalized to 1. Individual demands for private goods are given by

$$p_i x_s^i = p_i c_s^i + a_i^s (\lambda_s^0 + \lambda_s^y y + \lambda_s^z z) \left(y - \sum_{m,t} p_m c_t^m - \sum_k P_k C^k \right),$$

generating the aggregate demand

$$(S2) \quad p_i x^i = p_i c^i + \sum_s a_i^s (\lambda_s^0 + \lambda_s^y y + \lambda_s^z z) Y,$$

$$(S3) \quad P_j X^j = P_j C^j + \sum_s A_j^s (\lambda_s^0 + \lambda_s^y y + \lambda_s^z z) Y,$$

where $c^m = \sum_s c_s^m$ and $Y = y - \sum_m p_m c^m - \sum_k P_k C^k$. The aggregate group demand is thus a direct generalization of the standard LES, with additional quadratic terms in y^2 , and cross terms in yp_m and yP_k , plus terms involving the distribution factor z .

A first remark is that the c_s^m cannot be individually identified from group demand, since the latter only involves their sums c^m . As discussed above, this indeterminacy is, however, welfare irrelevant, because the collective indirect utility of individual s is, up to an additive constant,

$$\begin{aligned} W^s(p, P, y, z) = & \log Y + \log(\lambda_s^0 + \lambda_s^y y + \lambda_s^z z) \\ & - \sum_i a_i^s \log p_i - \sum_j A_j^s \log P_j, \end{aligned}$$

which only depends on the c^m . Second, the form of aggregate demands illustrates that private and public goods have exactly the same structure. We therefore simplify our notations by defining

$$\xi^i = x^i \quad \text{for } i \leq n, \quad \xi^i = X^i \quad \text{for } n < i \leq N,$$

and similarly

$$\begin{aligned} \alpha_i^s = a_i^s \quad \text{for } i \leq n, \quad \alpha_i^s = A_i^s \quad \text{for } n < i \leq N, \\ \gamma^i = c^i \quad \text{for } i \leq n, \quad \gamma^i = C^i \quad \text{for } n < i \leq N, \\ \pi_i = p_i \quad \text{for } i \leq n, \quad \pi_i = P_i \quad \text{for } n < i \leq N, \end{aligned}$$

so that the group demand has the simple form

$$(S4) \quad \pi_i \xi^i = \pi_i \gamma^i + \sum_s \alpha_i^s (\lambda_s^0 + \lambda_s^y y + \lambda_s^z z) Y,$$

leading to collective indirect utilities of the form

$$W^s(\pi, y, z) = \log Y + \sum_{i=1}^N \alpha_i^s \log(\lambda_s^0 + \lambda_s^y y + \lambda_s^z z) - \sum_i \alpha_i^s \log \pi_i.$$

It is clear, on this form, that the distinction between private and public goods can be ignored. This illustrates an important remark: while ex ante knowledge

of the public versus private nature of each good is necessary for the identifiability result to hold in general, for many parametric forms it is actually not needed.

S2. IDENTIFIABILITY

S2.1. *The General Case*

The question, now, is whether the empirical estimation of the form (S4) allows us to recover the relevant parameters—namely, the α_i^s , the γ^i , and the λ^a . We start by plugging the normalized Pareto weights (S1) into (S4), and get

$$(E'_i) \quad \pi_i \xi^i = \pi_i \gamma^i + (\alpha_j^2 + (\alpha_j^1 - \alpha_j^2) \lambda^0 + (\alpha_j^1 - \alpha_j^2) (\lambda^y y + \lambda^z z)) \\ \times \left(y - \sum_m \pi_m \gamma^m \right).$$

The right-hand side of (E'_i) can in principle be econometrically identified; we can thus recover the coefficients of the variables, namely y , y^2 , yz , the π_m , and the products $y\pi_m$ and $z\pi_m$. For any i and any $m \neq i$, the ratio of the coefficient of y to that of π_m gives γ^m ; the γ^m are therefore vastly overidentified. However, the remaining coefficients are identifiable only up to an arbitrary choice of two of them. Indeed, an empirical estimation of the right-hand side of (E'_i) can only recover for each j the respective coefficients of y , y^2 , and yz , that is, the three expressions $K_y^j = \alpha_j^2 + (\alpha_j^1 - \alpha_j^2) \lambda^0$, $K_{yy}^j = (\alpha_j^1 - \alpha_j^2) \lambda^y$, and $K_{yz}^j = (\alpha_j^1 - \alpha_j^2) \lambda^z$. Now, pick up two arbitrary values for λ^0 and λ^y , with $\lambda^y \neq 0$. The last two expressions give $(\alpha_j^1 - \alpha_j^2)$ and λ^z ; the first gives α_j^2 and therefore α_j^1 .

As expected, a continuum of different models generates the same aggregate demand. Moreover, these differences are welfare-relevant in the sense that the individual welfare gains of a given reform (say, a change in prices and incomes) will be evaluated differently by different models; in practice, the collective indirect utilities recovered above are not invariant across the various structural models compatible with a given aggregate demand.

S2.2. *A Unitary Version*

A unitary version of the model obtains when the Pareto weights are constant: $\lambda^y = \lambda^z = 0$. Then $K_{yz}^j = 0$ for all j (since distribution factors cannot matter¹) and $K_{yy}^j = 0$ for all j (demand must be linear in y , since a quadratic term would violate Slutsky). We are left with $K_y^j = \alpha_j^2 + (\alpha_j^1 - \alpha_j^2) \lambda^0$, and it is obviously impossible to identify independently α_j^1 , α_j^2 , and λ^0 ; as expected, the unitary framework is not identifiable.

¹For a discussion of the role of distribution factors in a unitary context, see Browning, Chiapori, and Lechene (2006).

S2.3. Identification Under Exclusion

We now show that in the nonunitary version of the collective framework, an exclusion assumption per member is sufficient to exactly recover all the coefficients. Assume, indeed, that member i does not consume commodity i for $i = 1, 2$; that is, $\alpha_1^1 = \alpha_2^2 = 0$. Then equation (E'_1) gives

$$\alpha_1^2(1 - \lambda^0) = K_y^1, \quad -\alpha_1^2\lambda^y = K_{yy}^1, \quad -\alpha_1^2\lambda^z = K_{yz}^1,$$

while (E'_2) gives

$$\alpha_2^1\lambda^0 = K_y^2, \quad \alpha_2^1\lambda^y = K_{yy}^2, \quad \alpha_2^1\lambda^z = K_{yz}^2.$$

Combining the first two equations of each block and assuming $\lambda^y \neq 0$, we get

$$\frac{1 - \lambda^0}{\lambda^y} = -\frac{K_y^1}{K_{yy}^1} \quad \text{and} \quad \frac{\lambda^0}{\lambda^y} = \frac{K_y^2}{K_{yy}^2}.$$

Therefore, assuming $K_y^2 K_{yy}^1 - K_y^1 K_{yy}^2 \neq 0$,

$$\frac{1 - \lambda^0}{\lambda^0} = -\frac{K_y^1 K_{yy}^2}{K_y^2 K_{yy}^1} \quad \text{and} \quad \lambda^0 = \frac{K_y^2 K_{yy}^1}{K_y^2 K_{yy}^1 - K_y^1 K_{yy}^2}.$$

It follows that

$$\lambda^y = \frac{K_{yy}^2}{K_y^2} \lambda^0 = \frac{K_{yy}^2 K_{yy}^1}{K_y^2 K_{yy}^1 - K_y^1 K_{yy}^2}$$

and all other coefficients can be computed as above. It follows that the collective indirect utility of each member can be exactly recovered, which allows for unambiguous welfare statements.

Finally, it is important to note that this conclusion requires $\lambda^y \neq 0$; in particular, *it does not hold true* in the unitary version, in which $\lambda^y = \lambda^z = 0$. Indeed, the same exclusion restrictions as above only allow us to recover $\alpha_1^2(1 - \lambda^0) = K_y^1$ and $\alpha_2^1\lambda^0 = K_y^2$; this is not sufficient to identify λ^0 , let alone the α_j^i for $j \geq 3$. Also, identifiability is only generic in the sense that it requires $K_y^2 K_{yy}^1 - K_y^1 K_{yy}^2 \neq 0$. Clearly, the set of parameter values violating this condition is of zero measure.

S3. POLICY IMPLICATIONS: A SIMPLE EXAMPLE

Assume, now, that the group is a household with two decision makers (husband and wife), and consider a simple policy reform that changes the price of the first public good by some fixed amount dP_1 , the household being compensated by a lump sum transfer equal to $X_1 dP_1$ —so that the reform is revenue

neutral at the first order for the group as a whole. For instance, one may think of a subsidy ($dP_1 < 0$) for some child expenditures (e.g., schooling), paid for by income taxation; we disregard distortions on labor supply for simplicity. Then the following implications can be stated:

- In a unitary framework, the impact of the reform on the welfare of each member is given by

$$\frac{\partial W^s}{\partial P_1} + X_1 \frac{\partial W^s}{\partial y} = \frac{X^1 - C^1}{y - \sum_m p_m c^m - \sum_k P_k C^k} - \frac{A_1^s}{P_1}.$$

Since the sign of this expression crucially depends on the parameter A_1^s , the impact of the reform cannot be assessed; there exists a continuum of models (all choices of A_1^1 , A_1^2 , and λ^0 giving the same $K_y^1 = A_1^1 + (A_1^1 - A_1^2)\lambda^0$) that are observationally equivalent but have totally divergent implications in terms of individual welfare. In words, whether a particular member benefits or loses depends on the weight of child expenditures in that member's utility, and this cannot be independently recovered for the two members in the unitary setting.

- In a collective setting, the effect is more complex because of the additional changes in Pareto weights; indeed

$$\begin{aligned} \frac{\partial W^s}{\partial P_1} + X_1 \frac{\partial W^s}{\partial y} \\ = \frac{X^1 - C^1}{y - \sum_m p_m c^m - \sum_k P_k C^k} + \frac{X_1 \lambda_s^y}{\lambda_s^0 + \lambda_s^y y + \lambda_s^z z} - \frac{A_1^s}{P_1}. \end{aligned}$$

However, since all parameters are identified from the exclusion restrictions, the impact on each member can be exactly assessed.

- Finally, assume that a fraction z of the income compensation is given to the wife and the rest to the husband. How does z affect welfare and bargaining weights? Again, this question does not make much sense in a unitary framework. In a collective model, however, if z is found to matter at all (i.e., if $K_{yz}^j \neq 0$ for at least one j), then one can exactly recover λ^z , which provides an unambiguous answer. The collective approach thus provide a theoretical background for analyzing "targeted" policies, as argued by Blundell, Chiappori, and Meghir (2005).

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