Econometrica Supplementary Material

SUPPLEMENT TO "A DYNAMIC MODEL FOR BINARY PANEL DATA WITH UNOBSERVED HETEROGENEITY ADMITTING A $\sqrt{n}$-CONSISTENT CONDITIONAL ESTIMATOR"

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S1. SIMULATION STUDY

WE HERE DESCRIBE the Monte Carlo study of the conditional estimator of the model proposed in the aforementioned paper. The study closely follows that of Honoré and Kyriazidou (2000) and, therefore, we first consider a benchmark design and then other designs. Then we make a comparison of the conditional estimator with some other estimators.

S1.1. *Benchmark Design*

Under the benchmark design, samples of different size ($n = 250, 500, 1000$) are generated from the quadratic exponential model (see equation (4) of the paper) with $T = 3$ time occasions, only one covariate, and parameters $\beta_1 = \beta_2 = 1$, $\gamma = 0.5$, and $\phi = 0.5\gamma$. The covariate is generated by drawing each x_{it} ($i = 1, \dots, n$, $t = 0, \dots, T$) from a Normal distribution with mean 0 and variance $\pi^2/3$, whereas α_i is generated as $(x_{i0} + \sum_t x_{it})/(T+1)$ for $i = 1, \dots, n$. Finally, the initial observation y_{i0} is drawn, for $i = 1, \dots, n$, from a Bernoulli distribution with parameter $\exp(\alpha_i + x_{i0}\beta_1)/[1 + \exp(\alpha_i + x_{i0}\beta_1)]$. To study the sensitivity of the results on T and γ, we also consider the case $T = 7$ and different values of γ (0.25, 1, 2).

Under each scenario, defined by a different combination of n, T, and γ, we generated 1000 samples and, for every sample, we computed the conditional estimator $\hat{\boldsymbol{\theta}} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\phi}, \hat{\gamma})'$. The results in terms of *mean bias*, *root mean squared error* (RMSE), *median bias*, and *median absolute error* (MAE) of the estimators $\hat{\beta}_1$ and $\hat{\gamma}$, which are of most interest, are shown in Table I.

As for the conditional estimator $\hat{\beta}_1$, from Table I we see that its mean and median bias are always negligible and tend to increase with γ, to decrease with n, and to decrease very quickly with T. A similar trend is observed for both RMSE and MAE. In particular, they decrease with n at a rate close to $\sqrt{n}$ and much faster with T. This depends on the fact that the number of observations that contribute to the conditional likelihood increases more than proportionally with T, as an increase of T also determines an increase of the actual sample size. Moreover, both RMSE and MAE of this estimator increase with γ. This is mainly due to the fact that when γ is positive, its increase implies a decrease of the actual sample size.

The conditional estimator $\hat{\gamma}$ has a behavior similar to $\hat{\beta}_1$, but its bias is not always negligible for small sample sizes. In particular, both RMSE and MAE

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TABLE I

PERFORMANCE OF THE CONDITIONAL MAXIMUM LIKELIHOOD ESTIMATORS OF β_1 AND γ UNDER THE BENCHMARK SIMULATION DESIGN

			$\hat{\beta}_1$				$\hat{\gamma}$			
T	γ	n	Mean Bias	RMSE	Median Bias	MAE	Mean Bias	RMSE	Median Bias	MAE
3	0.25	250	0.032	0.164	0.020	0.109	0.012	0.387	−0.001	0.262
		500	0.017	0.114	0.007	0.075	−0.004	0.278	−0.005	0.183
		1000	0.007	0.080	0.003	0.053	−0.010	0.182	−0.018	0.121
	0.50	250	0.027	0.169	0.016	0.107	0.012	0.402	0.020	0.253
		500	0.013	0.117	0.008	0.075	0.004	0.301	−0.004	0.214
		1000	0.009	0.083	0.007	0.055	−0.000	0.190	−0.004	0.124
	1.00	250	0.053	0.203	0.039	0.123	0.052	0.471	0.021	0.304
		500	0.022	0.137	0.007	0.089	0.025	0.323	0.031	0.207
		1000	0.010	0.089	0.007	0.057	−0.010	0.220	−0.014	0.144
	2.00	250	0.100	0.314	0.055	0.165	0.168	0.811	0.073	0.448
		500	0.047	0.182	0.033	0.110	0.074	0.481	0.048	0.295
		1000	0.021	0.124	0.010	0.077	0.032	0.317	0.010	0.200
7	0.25	250	0.002	0.065	−0.002	0.043	−0.008	0.142	−0.008	0.093
		500	0.002	0.045	0.000	0.031	0.003	0.101	0.001	0.068
		1000	−0.001	0.032	−0.003	0.022	−0.005	0.074	−0.004	0.050
	0.50	250	0.008	0.068	0.006	0.046	0.013	0.147	0.010	0.095
		500	0.003	0.046	0.002	0.031	−0.000	0.105	−0.001	0.072
		1000	0.002	0.034	0.001	0.022	0.001	0.075	−0.000	0.052
	1.00	250	0.004	0.073	0.002	0.048	0.006	0.174	0.006	0.111
		500	0.002	0.051	−0.001	0.034	0.008	0.120	0.006	0.081
		1000	0.001	0.036	0.000	0.024	0.000	0.081	−0.000	0.056
	2.00	250	0.014	0.103	0.010	0.066	0.010	0.251	−0.001	0.174
		500	0.008	0.070	0.002	0.047	0.006	0.173	−0.000	0.112
		1000	0.002	0.049	0.003	0.033	0.004	0.122	0.004	0.079

increase with γ, decrease with n at a rate close to $\sqrt{n}$, and decrease much faster with T.

For each sample generated as above, we also constructed 90% and 95% confidence intervals for β_1 and γ. The results, in terms of actual coverage level of these intervals, are reported in Table II.

The good performance of the conditional estimator is confirmed for both β_1 and γ by the fact that the actual coverage levels of these confidence intervals are very close to the nominal levels under each scenario.

S1.2. *Other Designs*

Following Honoré and Kyriazidou (2000), we considered other simulation designs, characterized by the following changes to the benchmark design (un-

TABLE II

ACTUAL COVERAGE LEVELS OF THE CONFIDENCE INTERVALS FOR β_1 AND γ BASED ON THE CONDITIONAL MAXIMUM LIKELIHOOD ESTIMATOR UNDER THE BENCHMARK SIMULATION DESIGN

		$T=3$				$T=7$			
		Interval for β_1		Interval for γ		Interval for β_1		Interval for γ	
γ	n	90%	95%	90%	95%	90%	95%	90%	95%
0.25	250	90.5	96.4	89.9	95.3	89.5	95.4	90.1	94.9
	500	90.5	95.3	89.1	93.9	91.4	95.4	90.3	95.4
	1000	89.2	93.9	92.0	95.3	90.3	95.0	89.8	93.9
0.50	250	91.6	96.1	89.1	95.1	90.2	94.9	90.2	94.8
	500	89.7	95.1	88.8	93.8	89.6	95.2	89.5	95.1
	1000	89.4	94.5	90.5	94.6	89.0	94.0	89.4	94.3
1.00	250	90.3	95.0	89.5	95.5	89.9	94.3	87.9	94.1
	500	89.8	94.6	89.4	94.3	90.1	95.0	89.3	95.0
	1000	90.2	95.2	89.5	94.5	90.3	95.4	91.1	95.8
2.00	250	89.8	95.8	90.0	94.6	89.0	94.1	90.3	95.4
	500	90.6	96.3	88.5	93.2	90.7	94.9	89.5	95.6
	1000	88.9	94.5	90.7	95.1	90.1	94.8	90.6	95.3

der which $T=3$, $\beta_1=\beta_2=1$, $\gamma=0.5$):

- $\chi^2(1)$ *regressor*: The only difference with respect to the benchmark design is that each x_{it} $(i=1,\dots,n,\ t=0,\dots,T)$ is generated from a $\chi^2(1)$ distribution with mean 0 and variance $\pi^2/3$.
- *Additional regressors*: Samples are generated as in the benchmark design, but three more covariates are used in the estimation of the parameters. These covariates are generated from the same Normal distribution used to generate x_{it}.
- *Trending regressors*, $T=3$: The only difference with respect to the benchmark design is that the covariate is generated as $x_{it}=\tau(\psi+0.1t+\zeta_{it})$, with τ and ψ suitably chosen and where $\zeta_{i0},\dots,\zeta_{iT}$ follow a Gaussian AR(1) process with autoregressive coefficient equal to 0.5, normalized so as to have variance $\pi^2/3$.
- *Trending regressors*, $T=7$: As in the previous design, but with $T=7$.

The results in terms of mean bias, RMSE, median bias, and MAE of the conditional estimators $\hat{\beta}_1$ and $\hat{\gamma}$ are shown in Table III, whereas the results in terms of actual coverage levels of the confidence intervals are shown in Table IV.

Based on the results in Table III, we conclude that the good performance of the conditional estimators of β_1 and γ are robust to the different models adopted for the covariates. The differences with respect to the benchmark design are small in terms of both bias and efficiency of the estimators. A similar consideration is drawn about the quality of the proposed procedure for constructing confidence intervals for β_1 and γ (see Table IV).

TABLE III

PERFORMANCE OF THE CONDITIONAL MAXIMUM LIKELIHOOD ESTIMATORS OF β_1 AND γ UNDER OTHER SIMULATION DESIGNS

		$\hat{\beta}_1$				$\hat{\gamma}$			
Type of Design	n	Mean Bias	RMSE	Median Bias	MAE	Mean Bias	RMSE	Median Bias	MAE
Regressors $\chi^2(1)$	250	0.033	0.208	0.013	0.133	0.020	0.328	0.028	0.214
	500	0.022	0.137	0.012	0.095	0.007	0.236	0.000	0.157
	1000	0.012	0.096	0.006	0.067	−0.009	0.163	−0.007	0.112
Additional regressors	250	0.073	0.200	0.054	0.120	0.035	0.423	0.032	0.282
	500	0.037	0.125	0.029	0.077	0.019	0.290	0.014	0.188
	1000	0.019	0.083	0.017	0.052	0.006	0.201	0.006	0.132
Trending regressors	250	0.059	0.216	0.047	0.134	0.008	0.418	−0.004	0.274
($T = 3$)	500	0.022	0.149	0.016	0.097	0.012	0.295	0.022	0.201
	1000	0.010	0.096	0.008	0.066	−0.004	0.204	0.001	0.137
Trending regressors	250	0.008	0.080	0.008	0.057	0.000	0.172	−0.004	0.115
($T = 7$)	500	0.003	0.058	−0.000	0.039	0.001	0.122	−0.000	0.077
	1000	0.001	0.039	−0.000	0.027	−0.000	0.080	0.001	0.053

S1.3. *Comparison With Alternative Estimators*

Following Honoré and Kyriazidou (2000), we report some simulation results also for the fixed effects and the infeasible maximum likelihood estimators. The former, denoted by $\hat{\boldsymbol{\theta}}_F = (\hat{\beta}_{F1}, \hat{\beta}_{F2}, \hat{\phi}_F, \hat{\gamma}_F)'$, estimates all the n incidental

TABLE IV

ACTUAL COVERAGE LEVELS OF THE CONFIDENCE INTERVALS FOR β_1 AND γ BASED ON THE CONDITIONAL MAXIMUM LIKELIHOOD ESTIMATOR UNDER OTHER SIMULATION DESIGNS

		Interval for β_1		Interval for γ	
Type of Design	n	90%	95%	90%	95%
Regressors $\chi^2(1)$	250	90.1	95.3	91.6	95.9
	500	92.3	95.8	88.7	94.7
	1000	91.1	95.5	90.6	95.1
Additional regressors	250	90.5	95.4	89.7	95.0
	500	91.0	95.6	89.5	94.2
	1000	90.7	94.8	89.4	94.0
Trending regressors ($T = 3$)	250	91.5	95.6	91.2	95.6
	500	89.0	94.7	90.3	95.5
	1000	91.7	95.6	89.5	95.1
Trending regressors ($T = 7$)	250	89.6	96.3	89.1	95.0
	500	89.1	94.6	88.0	94.2
	1000	90.9	95.9	91.3	95.4

TABLE V

COMPARISON BETWEEN THE CONDITIONAL AND ALTERNATIVE ESTIMATORS OF β_1 AND γ UNDER THE BENCHMARK DESIGN WITH $n = 250$

		Fixed Effects Estimator				Conditional Estimator				Infeasible Estimator			
		$\hat{\beta}_{F1}$		$\hat{\gamma}_F$		$\hat{\beta}_1$		$\hat{\gamma}$		$\hat{\beta}_{I1}$		$\hat{\gamma}_I$	
γ	T	Bias	MAE	Bias	MAE	Bias	MAE	Bias	MAE	Bias	MAE	Bias	MAE
0.25	3	0.515	0.515	−1.911	1.911	0.020	0.109	−0.001	0.262	0.003	0.069	−0.002	0.072
	7	0.227	0.227	−0.592	0.592	−0.002	0.043	−0.008	0.093	−0.002	0.039	0.001	0.039
	15	0.095	0.095	−0.232	0.232	0.001	0.026	−0.005	0.055	0.001	0.025	−0.003	0.024
0.50	3	0.503	0.503	−1.913	1.913	0.016	0.107	0.020	0.253	0.005	0.076	−0.003	0.072
	7	0.244	0.244	−0.565	0.565	0.006	0.046	0.010	0.095	0.004	0.039	0.003	0.038
	15	0.102	0.102	−0.214	0.214	0.002	0.026	0.009	0.056	0.002	0.025	0.004	0.023
1.00	3	0.539	0.539	−2.001	2.001	0.039	0.123	0.021	0.304	0.012	0.081	0.007	0.081
	7	0.250	0.250	−0.560	0.560	0.002	0.048	0.006	0.111	−0.000	0.043	0.001	0.039
	15	0.114	0.114	−0.227	0.227	0.001	0.029	0.003	0.065	0.001	0.027	0.002	0.024
2.00	3	0.546	0.546	−2.158	2.158	0.055	0.165	0.073	0.448	0.008	0.089	0.012	0.119
	7	0.301	0.301	−0.623	0.623	0.010	0.066	−0.001	0.174	0.003	0.051	0.005	0.071
	15	0.149	0.149	−0.318	0.318	0.001	0.037	0.008	0.094	0.002	0.032	0.003	0.040

parameters and, therefore, is inconsistent for fixed T. The latter, denoted by $\hat{\boldsymbol{\theta}}_I = (\hat{\beta}_{I1}, \hat{\beta}_{I2}, \hat{\phi}_I, \hat{\gamma}_I)'$, treats the unobserved heterogeneity effects as realizations of a covariate (supposed to be known) with its own regression parameter. The results in terms of median bias and MAE are shown in Table V for $n = 250$ and different values of γ and T.

Of course we expect the performance of the conditional estimator to be a compromise between those of the other two estimators. In fact, the conditional estimator performs much better than the fixed-effects estimator in terms of both bias and efficiency even when $T = 15$. On the other hand, the infeasible estimator is even more efficient because it does not discard any observation. Nevertheless, the difference between the conditional and the infeasible estimators tends to decrease as T increases in terms of both bias and efficiency. In particular, the two estimators perform very similarly when $T = 15$ and the parameter of interest is β_1.

In the following discussion, we compare the proposed conditional estimator with available estimators of the parameters of the latter model. In particular, if we compare the results of our simulation study (benchmark design described in Section S1.1 above) with those of Honoré and Kyriazidou (2000), it emerges that our estimator, using a larger number of response configurations, performs better than their estimator in terms of both bias and efficiency. Making a comparison with the bias corrected estimator proposed by Carro (2007), we note that the former performs much better than the latter when the parameter of interest is γ, and performs slightly worse when the parameter of interest is β_1.

TABLE VI

RESULTS FOR β_1 AND γ WITH DIFFERENT PANEL LENGTHS t (BENCHMARK DESIGN, $n = 1000$, $\gamma = 0.5$, $T = 7$)

	β_1		γ		Interval for β_1		Interval for γ	
Panel Length	Median Bias	MAE	Median Bias	MAE	90%	95%	90%	95%
7	0.001	0.022	−0.000	0.052	89.0	94.0	89.4	94.3
6	0.004	0.024	−0.003	0.059	89.8	95.0	89.1	94.7
5	0.004	0.031	−0.012	0.067	89.3	95.0	89.0	94.4
4	0.007	0.039	−0.012	0.083	89.2	94.6	89.2	94.1

However, in taking these conclusions, one must be conscious that the results compared here derive from simulation studies performed under different, although very similar, models.

S1.4. *Model Consistency*

As an illustration of model consistency, which is discussed at the end of Section 3.2 of the paper, we finally performed a series of simulations in which samples are drawn under the benchmark design with $T = 7$, but $\boldsymbol{\theta}$ is estimated on the basis of the covariates and response variables observed at the first t occasions ($t = 4, 5, 6$). The results, in terms of median bias, MAE of the estimators $\hat{\beta}_1$ and $\hat{\gamma}$, and coverage level of the corresponding confidence intervals, are reported in Table VI.

Although γ is different from 0, the bias of both estimators considered above is negligible and comparable to that resulting from computing these estimators on the complete data sequence. This is in agreement with our conjecture about model consistency according to which the distribution resulting from marginalizing the proposed model over a subset of response variables may be adequately approximated by a model of the same family.

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