Econometrica Supplementary Material

SUPPLEMENT TO "COSTLY SELF-CONTROL AND RANDOM SELF-INDULGENCE"

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OMITTED PROOF

LEMMA 3: *$w_1 C_u w_2$ if and only if there exists $v \in \mathcal{V}$ such that $w_i = v\sqrt{1 - A_i^2} + A_i u$, $i = 1, 2$, with $A_1 \geq A_2$.*

Recall that $\eta(A) = \sqrt{1 - A^2}$.

The proof uses the following lemma.

LEMMA S1: *If $w_1 C_u w_2$ and $w_2 \notin \{-u, u\}$, then there exists $C, c \geq 0$, at least one strictly positive, such that $w_1 = Cu + cw_2$.*[2]

PROOF: Suppose not. Let

$$W = \{w' \mid w' = Cu + cw_2 + e\mathbf{1},\ \text{for some } C, c \geq 0,\ \text{some } e\}.$$

Obviously, W is closed, convex, and nonempty. Since $w_1 \notin W$ by hypothesis, there is a separating hyperplane. So there exists a vector $p \neq 0$, such that $p \cdot w_1 < p \cdot w'$ for all $w' \in W$; that is,

$$p \cdot w_1 < Cp \cdot u + cp \cdot w_2 + ep \cdot \mathbf{1}$$

for all $C, c \geq 0$ and all e.

Since the sign of e is arbitrary, this implies that $\sum_k p_k = 0$. Otherwise, we can take $e \to -\infty$ or $e \to \infty$ to make $ep \cdot \mathbf{1}$ arbitrarily negative and force a contradiction. Similarly, $p \cdot u \geq 0$ and $p \cdot w_2 \geq 0$. To see this, suppose to the contrary that $p \cdot u < 0$. Then we can take C arbitrarily large to generate a contradiction. Obviously, w_2 is analogous. Finally, we must have $p \cdot w_1 < 0$. Otherwise, take $C = c = e = 0$ for all i to get a contradiction.

Hence there exists a vector p, such that $\sum_k p_k = 0$, $p \cdot u \geq 0$, $p \cdot w_2 \geq 0$, and $p \cdot w_1 < 0$. It is not difficult to show that we can rewrite the vector p as a difference between two interior lotteries, α and β, to obtain the conclusion that $u \cdot \alpha \geq u \cdot \beta$ and $w_2 \cdot \alpha \geq w_2 \cdot \beta$, but $w_1 \cdot \alpha < w_1 \cdot \beta$.

Since $w_1 C_u w_2$, it must be true that $u \cdot \alpha = u \cdot \beta$. We can write $w_2 = \eta(A_2)v_2 + A_2 u$. Fix $\varepsilon > 0$ and let

$$\alpha^* = \alpha + \varepsilon[\eta(A_2)u - A_2 v_2].$$

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[2]This is a version of the Harsanyi aggregation theorem. See Weymark (1991).

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It is not hard to show that if ε is sufficiently small, then α^* is a lottery. Note that $u \cdot \alpha^* = u \cdot \alpha + \varepsilon\eta(A_2)$ as $u \cdot u = 1$ and $u \cdot v_2 = 0$. Since $w_2 \notin \{-u, u\}$, we have $A_2 \in (-1, 1)$, so $\eta(A_2) > 0$. Hence $u \cdot \alpha^* > u \cdot \alpha = u \cdot \beta$.

Also,

$$w_2 \cdot \alpha^* = w_2 \cdot \alpha + \varepsilon[\eta(A_2)A_2 - A_2\eta(A_2)] = w_2 \cdot \alpha = w_2 \cdot \beta.$$

For ε sufficiently small, the fact that $w_1 \cdot \alpha < w_1 \cdot \beta$ implies $w_1 \cdot \alpha^* < w_1 \cdot \beta$, contradicting $w_1 C_u w_2$. *Q.E.D.*

PROOF OF LEMMA 3: *If*. First, suppose there exists $v \in \mathcal{V}$ such that $w_i = \eta(A_i)v + A_i u$, $i = 1, 2$, with $A_1 \geq A_2$. If $A_2 = 1$, this requires $A_1 = 1$ also, in which case $w_1 = w_2 = u$ and $w_1 C_u w_2$. If $A_2 = -1$, then it is easy to see that every w satisfies $w C_u w_2$, so w_1 certainly does.

So suppose $A_2 \in (-1, 1)$, implying $\eta(A_2) > 0$. Obviously, if $A_1 = A_2$, then $w_1 = w_2$, so $w_1 C_u w_2$. So without loss of generality, assume $A_1 > A_2$. Then we have

$$\begin{aligned} w_1 = A_1 u + \eta(A_1)v &= \left[A_1 - A_2\frac{\eta(A_1)}{\eta(A_2)}\right]u + A_2\frac{\eta(A_1)}{\eta(A_2)}u + \eta(A_1)v \\ &= \left[A_1 - A_2\frac{\eta(A_1)}{\eta(A_2)}\right]u + \frac{\eta(A_1)}{\eta(A_2)}[A_2 u + \eta(A_2)v] \\ &= \left[A_1 - A_2\frac{\eta(A_1)}{\eta(A_2)}\right]u + \frac{\eta(A_1)}{\eta(A_2)}w_2. \end{aligned}$$

The coefficient on w_2 is nonnegative. Also, $A_1 > A_2$ implies that the coefficient on u is strictly positive. To see this, note that the conclusion is obvious if $A_1 > 0 \geq A_2$ since $\eta(A_1)/\eta(A_2) \geq 0$. If $A_1 > A_2 > 0$, the fact that η is strictly decreasing in A in this range implies

$$A_1 - A_2\frac{\eta(A_1)}{\eta(A_2)} > A_1 - A_2 > 0.$$

If $0 \geq A_1 > A_2$, the fact that η is strictly increasing in A in this range implies exactly the same conclusion. So the coefficient on u is strictly positive. Hence if $u(\alpha) > u(\beta)$ and $w_2(\alpha) \geq w_2(\beta)$, we must have $w_1(\alpha) > w_1(\beta)$. Hence $w_1 C_u w_2$.

Only if. Suppose $w_1 C_u w_2$. If $w_2 = u$, then this requires $w_1 = u$ and the claim follows trivially. If $w_2 = -u$, again the claim follows trivially, since for any $v \in \mathcal{V}$, we have $w_2 = \eta(A_2)v + A_2 u$ with $A_2 = -1$. So suppose $w_2 \notin \{-u, u\}$. Then by Lemma S1, there exists $C, c \geq 0$, at least one strictly positive, such that $w_1 = Cu + cw_2$. Since $w_2 \notin \{-u, u\}$, there is a unique $v \in \mathcal{V}$ and $A_2 \in (-1, 1)$ such that $w_2 = \eta(A_2)v + A_2 u$. Hence $w_1 = c\eta(A_2)v + (C + cA_2)u$. If $c = 0$, then $w_1 = u$, implying that $w_1 = \eta(A_1)v + A_1 u$ with $A_1 = 1 \geq A_2$, so the conclusion follows. If $C = 0$, we must have $c = 1$ implying $w_1 = w_2$, so again the conclusion follows. Hence we can assume that $C > 0$ and $c > 0$. Thus we have $w_1 = \eta(A_1)v + A_1 u$. So we only need to show that $A_1 \geq A_2$.

So suppose $1 > A_2 > A_1$. If $w_1 = -u$, then we cannot have $w_1 C_u w_2$, so $A_1 > -1$. Hence $\eta(A_i) > 0$, $i = 1, 2$. Fix any interior α and $\varepsilon > 0$. Let

$$\beta = \alpha + \varepsilon[\eta(A_2)u - A_2 v].$$

It is easy to show that β is a lottery for all sufficiently small ε. Then $u \cdot \beta = u \cdot \alpha + \varepsilon\eta(A_2)$. Since $\eta(A_2) > 0$, then $u(\beta) > u(\alpha)$. Also, it is easy to see that $w_2 \cdot \beta = w_2 \cdot \alpha$. Finally, $w_1 \cdot \beta = w_1 \cdot \alpha + \varepsilon[\eta(A_2)A_1 - A_2\eta(A_1)]$. Hence $w_1 \cdot \beta < w_1 \cdot \alpha$ iff $A_1/\eta(A_1) < A_2/\eta(A_2)$ which holds as $A_1 < A_2$. Thus there is a pair of lotteries for which w_2 agrees with u and w_1 does not, so we cannot have $w_1 C_u w_2$, a contradiction. *Q.E.D.*

REFERENCE

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