SUPPLEMENT TO "AN EMPIRICAL EQUILIBRIUM MODEL OF A DECENTRALIZED ASSET MARKET"
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THE CASE $\delta_2 = 0$

WHEN ASSETS ARE NOT DEPRECIATING—that is, $\delta_2 = 0$—we can obtain the analytical solution of the model (Duffie, Gârleanu, and Pedersen (2005, 2007)). Since δ_2 is estimated to be small, this restricted version of the model provides a reasonable approximation to the more general model, thus providing a clean intuition for the identification of the parameters.

Since assets are not depreciating, we can assume that the mass of assets is fixed and equal to A; thus, $x = 0$ and $T \to \infty$. Moreover, without loss of generality, we can set $\delta_1 = 0$ and $\delta_0 = 1$.

The steady-state evolution of the masses μ_i determines the equilibrium allocation. Up to terms in $o(\varepsilon)$, in equilibrium, these masses evolve from time t to time $t + \varepsilon$ according to

$$\mu_{lo}(t + \varepsilon) = \lambda \varepsilon \mu_{ho}(t) + (1 - \gamma_s \varepsilon - \gamma_{sd} \varepsilon) \mu_{lo}(t),$$

$$\mu_{ho}(t + \varepsilon) = \gamma_b \varepsilon \mu_{hn}(t) + \gamma_{bd} \varepsilon \mu_{hn}(t) + (1 - \lambda \varepsilon) \mu_{ho}(t),$$

$$\mu_{hn}(t + \varepsilon) = \varepsilon \mu + (1 - \lambda \varepsilon - \gamma_b \varepsilon - \gamma_{bd} \varepsilon) \mu_{hn}(t),$$

$$\mu_{ln}(t + \varepsilon) = \lambda \varepsilon \mu_{hn}(t) + \gamma_s \varepsilon \mu_{lo}(t) + \gamma_{sd} \varepsilon \mu_{lo}(t),$$

$$\mu_{do}(t + \varepsilon) = \alpha_{db} \varepsilon \mu_{dn}(t) + (1 - \alpha_{ds} \varepsilon) \mu_{do}(t),$$

$$\mu_{dn}(t + \varepsilon) = \alpha_{ds} \varepsilon \mu_{do}(t) + (1 - \alpha_{db} \varepsilon) \mu_{dn}(t).$$

The intuition for these equations is similar to the intuition reported in Section 5.2.5. For example, the first equation states that the mass of low-valuation agents with an asset results from the flows of three sets of agents: (1) the inflow of high-valuation owners whose valuation just dropped—the term $\lambda \varepsilon \mu_{ho}(t)$; (2) the outflow of low-valuation owners who found a dealer—the term $\gamma_{sd} \varepsilon \mu_{lo}(t)$; and (3) the mass of low-valuation owners who have not found a buyer—the term $(1 - \gamma_s \varepsilon) \mu_{lo}(t)$.

Steady state implies that the total mass of agents with high valuation $\mu_{hn} + \mu_{ho}$ is equal to $\frac{\mu}{\lambda}$; that the total mass of owners $\mu_{ho} + \mu_{lo} + \mu_{do}$ is equal to the mass of assets A; and that the total mass of dealers μ_d is equal to $\mu_{do} + \mu_{dn}$. Hence, $\mu_{hn} + A - \mu_{lo} - \mu_{do} = \frac{\mu}{\lambda}$. In turn, since $A < \frac{\mu}{\lambda}$, this implies that $\mu_{hn} > \mu_{lo} + \mu_{do}$—that is, sellers are the "short" side of the market. Moreover, the masses of assets sold and purchased by dealers must be equal to the mass of assets purchased and sold to dealers: $\alpha_{ds} \mu_{do} = \gamma_{bd} \mu_{hn}$ and $\alpha_{db} \mu_{dn} = \gamma_{sd} \mu_{lo}$. Furthermore, steady state requires $\alpha_{ds} \mu_{do} = \gamma_{bd} \mu_{hn} = \alpha_{db} \mu_{dn} = \gamma_{sd} \mu_{lo}$ since dealers' aggregate inventories do not change over time.

Rearranging and taking the limit for $\varepsilon \to 0$, the masses of agents are

$$\mu_{lo} = \frac{\lambda}{\gamma_s + \gamma_{sd}} \mu_{ho},$$

$$\mu_{ho} = \frac{\gamma_b \mu_{hn} + \gamma_{bd} \mu_{hn}}{\lambda},$$

$$\mu_{hn} = \frac{\mu}{\lambda + \gamma_b + \gamma_{bd}},$$

$$\mu_{ln} = \lambda \mu_{hn} + \gamma_s \mu_{lo} + \gamma_{sd} \mu_{lo} = \mu,$$

$$\mu_{do} = \frac{\alpha_{db}}{\alpha_{ds}} \mu_{dn} = \frac{\gamma_{sd}}{\alpha_{ds}} \mu_{lo}.$$

We can now calculate the moments—that is, equations (18)–(21)—that we seek to match to their empirical analogs in the quantitative analysis. Specifically, the fraction of aircraft for sale is equal to

$$\frac{\mu_{lo} + \mu_{do}}{A} = \frac{\lambda(\alpha_{ds} + \gamma_{sd})}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.$$

The fraction of aircraft for sale by dealers (or dealers' inventories) is equal to

$$\frac{\mu_{do}}{A} = \frac{\lambda \gamma_{sd}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.$$

The fraction of retail-to-retail transactions to total aircraft is equal to

$$\frac{\gamma_s \mu_{lo}}{A} = \frac{\lambda \gamma_s \alpha_{ds}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.$$

The fraction of dealer-to-retail transactions to total aircraft is equal to

$$\frac{\alpha_{ds} \mu_{do}}{A} = \frac{\lambda \gamma_{sd} \alpha_{ds}}{\lambda \alpha_{ds} + \lambda \gamma_{sd} + \gamma_s \alpha_{ds} + \alpha_{ds} \gamma_{sd}}.$$

The equilibrium prices are determined by the solution to the following system of equations:

$$\rho U_{ho} = z_h + \lambda(S_{lo} - U_{ho}),$$

$$\rho S_{lo} = z_l - c_s + \gamma_s(p + V_{ln} - S_{lo}) + \gamma_{sd}(p_B + V_{ln} - S_{lo}),$$

$$\rho S_{hn} = -c_s + \lambda(V_{ln} - S_{hn}) + \gamma_b(U_{ho} - p - S_{hn})$$
$$\qquad + \gamma_{bd}(U_{ho} - p_A - S_{hn}),$$

$$\rho V_{ln} = 0,$$

$$\rho J_{do} = -k + \alpha_{ds}(p_A + J_{dn} - J_{do}),$$

$$\rho J_{dn} = -k + \alpha_{db}(J_{do} - p_B - J_{dn}),$$

$$p = (1 - \theta_s)(S_{lo} - V_{ln}) + \theta_s(U_{ho} - S_{hn}),$$

$$p_A = (1 - \theta_d)(J_{do} - J_{dn}) + \theta_d(U_{ho} - S_{hn}),$$

$$p_B = (1 - \theta_d)(J_{do} - J_{dn}) + \theta_d(S_{lo} - V_{ln}).$$

The first four equations are the agents' value functions; the fifth and the sixth equations are the dealers' value functions; and the last three equations are the negotiated prices.

REFERENCES

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