

STRATEGIC ANALYSIS OF AUCTIONS

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THE DIPLOMA FOR the Sveriges Riksbank Prize in Economics Sciences in Memory of Alfred Nobel that I shared in 2020 with Paul Milgrom cites “improvements in auction theory and inventions of new auction formats”.¹ As requested by the Royal Swedish Academy of Sciences, this lecture describes the origin of my work on auctions. It complements the Nobel Lecture by Paul Milgrom (2021) that describes later developments in theory and practice.

1. PROLOG

Undergraduate courses led me to view social sciences as modern forms of moral philosophy, and more incisive. I saw that inattention to actors’ information generated some of the seeming dilemmas. After graduate work with Howard Raiffa on Bayesian analysis of single-person decision problems, I aimed to develop multi-person decision theories. Forays into efficient risk sharing, vote trading in legislatures, and social choice theory led me into economics. I admired the technical accomplishment of establishing sufficient conditions for existence of prices that cleared all markets simultaneously.² But the theory seemed arid because it mostly ignored how markets operate.³ From John Lintner’s MBA course I knew about trading platforms and how dealers or specialists manage markets, the effects of information on traders’ strategies, and from Christenson (1967) about bidding to manage new issues of corporate bonds. From my decision theory viewpoint, I saw information that affected beliefs and thus strategies as primary features. To study imperfectly competitive markets, I relied on formulations and analytical tools of game theory because they allow precise specification of *who knows what when*. So did others in my cohort, and the infusion of game-theoretic models and methods into economics progressed quickly.

Central to this transformation was Leonid Hurwicz’ (1973) view of a market as a mechanism, viz., a procedure that translates agents’ actions into a social outcome, and anticipating that process, each agent’s strategy selects an action that is optimal given his preferences, information, and beliefs about others’ strategies. Instead of a fixed point in prices and quantities, Hurwicz characterized equilibrium as a fixed point in agents’ strategies. I adopted this perspective because it fit my interest in the microstructure of markets, and because it recognized that design of market procedures is important. But it requires a conceptual leap from observable prices and quantities to strategies that are only partially observable via their actions, leaving hidden the motivating information and beliefs.

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¹The Prize in Economic Sciences 2020. [NobelPrize.org](https://www.nobelprize.org). Nobel Media AB 2020.

www.nobelprize.org/prizes/economic-sciences/2020/summary/,

www.nobelprize.org/uploads/2020/09/advanced-economicsciencesprize2020.pdf.

²I made one contribution to this topic in Wilson (1978c).

³My view was reinforced by later critiques by Mirrlees (1971), Akerlof (1970), Spence (1974), and Rothschild and Stiglitz (1976) showing effects of private information even in perfectly competitive (but incomplete) markets.

Further, it is sensitive to predictive criteria, such as Nash equilibrium or one of its refinements, rather than weaker criteria like absence of arbitrage opportunities; and, it takes the market design as an explicit exogenous parameter rather than implicitly specified by market clearing. Empirical study of a mechanism requires a structural model, ample data, and new econometric methods. I was not deterred by these limitations because initially I worked on participants' practical decision problems before studying equilibrium among them.

2. ORIGINS OF MY WORK ON AUCTIONS

The path outlined above doesn't mention auctions, but I studied them often.⁴ Because auctions are tractable examples of market microstructure, with explicit rules determining allocations and prices, analyses of their properties were natural first steps in the broad agenda of detailed elaboration of market organization and performance. Cassidy (1967) describes a wide variety of auctions used in practice, and they are simple examples of trading platforms that are important in economics and amenable to analysis using game theory.⁵ Auctions fit Hayek's (1945) view of markets as information systems, and Hurwicz's (1973) view as mechanisms that translate agents' actions, motivated by their individual preferences and information, into social outcomes. Experimental and empirical studies of auctions can invoke structural models because the rules and participants' objectives are clear.

The following describes three topics that were chief sources of my interest in auctions.

Winner's Curse. William Vickrey's (1961) pioneering analysis assumes each bidder knows privately his value of an item. From a case study of an auction of leases conferring rights to explore for oil in off-shore tracts, I saw that bidders relied on imperfect estimates of an unobserved component of all their realized values, namely the quantity of oil present in the tract and the present value of the costs and prices at which it can be extracted and sold. I published a slight contribution (Wilson (1969)) that analyzed a simple symmetric auction with this feature. The adverse selection affecting the winning bidder explained the later simulation results of Capen, Clapp, and Campbell (1971) that Capen dubbed the 'winner's curse' (Anonymous (2004)). My article attracted the attention of a timber company and then Darius Gaskins at the U.S. Department of the Interior. Throughout the 1970s I worked as a consultant developing models of bidding for off-shore leases, and later advised companies bidding for leases.⁶ I was deeply impressed with the importance of common value components, amplified by extreme imprecision of bidders' estimates, and by decision makers' difficulties in appreciating the implications of adverse

⁴As an MBA student in 1959 I received a failing grade on a written analysis of a case involving competitive bidding because I invoked a mathematical analysis rather than the mandatory 'administrative point of view'.

⁵I encouraged PhD students to study auctions, with success in the dissertations of Armando Ortega-Reichert (1967), Paul Milgrom (1979a), Christopher Avery (1998), David McAdams (2006), and Songzi Du (2012). Other graduates of our PhD program with major contributions to auction theory and practical design include Peter Cramton, Paul Klemperer, Margaret Meyer, and Dennis Yao, and even during their PhD studies an important contribution was published by Peter Cramton, Robert Gibbons, and Paul Klemperer (1987) that is later applied in Cramton, Gall, Sujarittanonta, and Wilson (2013).

⁶It was much later that I published in Wilson (1983) an example of this work. A draft of a book was never completed because I became absorbed in designs of spectrum auctions and wholesale power markets. In recent presentations I resurrected an unpublished model of competing bond dealers' offers to buy or sell to customers in which the predicted bid-ask spread is a multiple, depending only on the number of competing dealers, of the standard deviation of their normally distributed estimates of the bond's future value.

selection for bidding strategies—they were often more interested in ‘bidding to win’ than in improving their valuations.

The theory of such auctions reached maturity in the 1980s in articles by Paul Milgrom and his co-authors that established rigorous foundations for a wide variety of auction formats. Milgrom’s Nobel Lecture (2021) describes some of his many contributions to auction theory, and applications to designs of novel auction formats.

Milgrom and Weber’s (1982) demonstration that ascending auctions diminish bidders’ informational rents via learning from others’ bidding behavior, and thereby increase the seller’s expected revenue and promote efficiency, suggested the (simultaneous) ascending auctions devised in the 1990s for auctions of spectrum licenses. In Wilson (1998) I studied the sequential equilibrium of an ascending auction, showing how a bidder’s initially unbiased estimate is progressively revealed to be extreme as he observes the prices at which other bidders drop out, and thus his own dropout price declines too.

Information in Prices. Auctions with common value components provide apt models for studying the informational content of prices. In Wilson (1977) I studied an auction for a single item when bidders’ estimates are independent and identically distributed conditional on an unobserved value that is the same for all bidders. My argument that the sale price converges almost surely to the common value as the number of bidders increases was awkward because it relied on the support of bidders’ estimates moving monotonically with the common value. A later literature (Milgrom (1979b), Pesendorfer and Swinkels (1997), Reny and Perry (2006)) remedies these deficiencies and shows convergence in probability.⁷

In Wilson (1979) I studied a similar model in which each bidder submits a demand function for shares of a fixed supply, and solved examples.⁸ I derived the Euler condition for a bidder’s optimal strategy, an approach used again in Wilson (2008) for an equilibrium in supply functions suggested as an alternative to the original construction by Klemperer and Meyer (1989). The interesting aspect was that each price-quantity pair on a bidder’s demand function is optimal based on partial inference of others’ estimates from the property that for the market to clear at that price the quantity must be the residual supply net of others’ demands. Hence the clearing price reflects all bidders’ estimates.

Remarkably simple results occur in a double auction with bidders’ valuations linear-quadratic in quantity, all with the same coefficient for the quadratic term so that demand functions are linear in quantity with the same slope. In Wilson (1993a, 1996) I used mechanism design theory to show for independent private values that there exists an *interim* incentive-efficient mechanism whose outcomes are fully *interim* efficient for each profile of traders’ values. For a common value (with bias toward one’s own estimate, like a private value) Du and Zhu (2013) derive an equilibrium of a double auction for which the equilibrium clearing price is precisely the average of all bidders’ estimates. Amazingly, this is actually an *ex post* equilibrium, independent of the probability distributions of traders’ estimates. As in a share auction the key feature is inferring others’ estimates from the market clearing condition, which in this case is simpler because residual-demand prices reflect averages of others’ estimates. This static version, and their extensions to repeated

⁷Even now I work with Paulo Borelli and Srihari Govindan (2021) to extend these results to auctions and double auctions with bidders partitioned into a finite set of asymmetric types with each type observing independent and identically distributed estimates conditional on an unobserved state variable.

⁸Successful bidders were paid the clearing price. In other work I assumed they were paid their bids, as in Hortaçsu and McAdams (2010), but I failed to solve interesting examples.

double auctions, are now workhorses of financial economics (Du and Zhu (2017), Duffie and Zhu (2017)).

Posted Bid-Ask Markets. I saw frenzied oral auctions on the crowded floor of the Chicago Mercantile Exchange that contrasted with the moderate pace of some double auctions with bid and ask prices posted by buyers and sellers. I was also aware of experimental evidence that in such markets a large fraction of transactions are at prices significantly different from Walrasian market-clearing prices. In Wilson (1987b) I studied a dynamic bid-ask market in which each trader has a private value and consistent beliefs about others' values. If a trader were to know the final market-clearing price then he would grab any better offer, but absent foreknowledge his strategy is a dynamic programming problem that implies an induced continuation value after each history. My analysis focused on the feature that at each time the buyer with the currently highest bid and the seller with the lowest ask are endogenously matched in a game of bilateral bargaining. I sought the source of pressure leading either to improve his offer, or for one to accept the other's offer. If traders are impatient for exogenous reasons then the bargaining fits Cramton's model (1992) of mutual signaling in which first one's continuation value is revealed by his delay in improving his offer, and then the other's is revealed by his further delay until offering the price predicted by, say, Rubinstein's (1982) model of bilateral bargaining that is then accepted. I observed that each trader's impatience is endogenous too, namely it is the interest rate (if any) *plus* the hazard rate that another trader on the same side of the market will usurp the opportunity by accepting the other side's current best offer.

The shared aspect of the above three topics is the role of bidders' information, which was always the source of my interest in auctions. I never proved any basic theorems, focusing instead on illustrative examples. I mention two other topics that engaged my interest but for which I failed to obtain strong results.

Incentive Efficiency of Double Auctions. In Wilson (1985a,b, 1987a) I used mechanism design theory to characterize an *interim* incentive-efficient mechanism when buyers and sellers have unit demands and supplies with independent private values. Such a mechanism is like a double auction, but in terms of traders' virtual valuations as in direct revelation games, and thus dependent on their probability distributions. I showed that it can be implemented as an ordinary double auction if there are sufficiently many buyers and sellers and their bid and ask strategies in a symmetric equilibrium are monotone. These sufficiency conditions were disappointingly strong and I retained hope that eventually a general result can be found. If so then it would explain the prevalent use of double auctions with procedural rules that do not depend on the probability distributions of traders' valuations. I was interested more generally in explaining the prevalence over millennia of various standard market procedures, motivated by Myerson's (1981) fundamental result for auctions.

General Equilibrium via Auctions. In Wilson (1978a) I studied a multi-commodity economy with no private information in which one of the traders conducts an auction with other traders as bidders. I showed that there exists an equilibrium whose outcome is a core allocation, like in Böhm-Bawerk (1888), and as bidders are replicated by division into smaller copies the outcomes converge to a Walrasian allocation. I wanted to address economies with dispersed private information but I was stymied by the variety of possible definitions of the core that I discussed in Wilson (1978b). Others' progress on this agenda is reported in Glycypantis and Yannelis (2005).

3. CODA

Progress on auctions encouraged renewed emphasis on strategic analysis of other markets and trading platforms. Because the models and methods derived from game theory, it became the analytical tool in many areas of economic theory. Models of auctions contribute to economic science because they can be tested in experimental and empirical studies using structural models. And, they enable an engineering approach to designing new markets and improving performance of existing ones. Innovative auction designs can be valuable in the private sphere and especially beneficial for government agencies' procurements and allocations of public resources for private uses.

I studied auctions as case studies of imperfectly competitive markets. I was mainly interested in the role of private information, its effects on incentives and strategies, and thus efficiency and revenue, and how these are affected by procedural rules. My early studies, and the rich literature that followed, guided my later work on new designs of auctions for spectrum licenses, and then the elaborate system of auctions used in wholesale electricity markets that combined auction design with my earlier work with Hung-po Chao and Shmuel Oren on pricing and rationing in retail markets (Chao and Wilson (1987), Wilson (1989, 1993b, 2002)).⁹ From practical work I learned the importance of careful attention to institutional and technical features of the industry. Yet an auction design must be robust against variations, and cannot depend on the common knowledge assumed in exploratory analyses using game theory, which can only be indicative of participants' incentives.

My subsequent work diverged into other topics. One strand pursued game-theoretic analyses of topics in industrial organization, including entry deterrence with David Kreps (1982a) and intertemporal monopoly with Faruk Gül and Hugo Sonnenschein (1986). Prolonged cooperation in a perturbed version of the finitely repeated prisoners' dilemma, shown in an article with Kreps, Milgrom and Roberts (1982), fascinated me and even now Srihari Govindan and I pursue a conjecture, derived from computations, that players' bounded recall implies cooperation in the infinitely repeated version. The other strand pursued further implications of rationality, first with David Kreps (1982b) in defining sequential equilibrium, and then with Srihari Govindan (2009, 2012) in identifying behavioral axioms that characterize forward induction and stable equilibria. I was acutely aware of limitations imposed by game theory's assumptions of common belief or knowledge, as I emphasized in Wilson (1987a), and prevalent departures from rationality revealed by experiments, such as framing effects, but alas, I always deferred those topics for later study.

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⁹The innovative design of spectrum auctions is described in the Nobel Lecture by Paul Milgrom (2021). I defer to his exposition because my contributions were minor compared to his invention of procedural rules, especially the 'activity rule' and the closing rule, that improve performance of simultaneous auctions.

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