SUPPLEMENT TO "BOUNDS ON ELASTICITIES WITH OPTIMIZATION FRICTIONS: A SYNTHESIS OF MICRO AND MACRO EVIDENCE ON LABOR SUPPLY"
(*Econometrica*, Vol. 80, No. 3, May 2012, 969–1018)

By Raj Chetty

APPENDIX A: Theoretical Derivations

A.1. *Bounds on Intensive Margin Elasticities With Income Effects and Stochastic Prices*

This section establishes two results. First, the bounds in Proposition 1 apply to the Hicksian elasticity when the quasilinearity assumption in (2) is relaxed. Second, allowing for stochastic prices p_t does not affect the bounds. To simplify notation, I ignore heterogeneity across agents and assume all agents have a flow utility function $v(x_t, y_t)$. Heterogeneity does not affect the result under the assumption that the structural elasticity does not vary locally across agents, as discussed below.

Let $\mathbb{E}_t$ denote the conditional expectation operator over prices given information available in period t and let $p = (p_1, \ldots, p_T)$ denote the realized price vector. To account for stochastic prices, I redefine the nominal model so that the agent maximizes expected lifetime utility

$$(23) \qquad \mathbb{E}_t \sum_{s=t}^{T} v(x_s, y_s)$$

subject to the dynamic budget constraint $Z_{t+1} = Z_t - p_t x_t - y_t$ and the terminal condition $Z_{T+1} = 0$.

Let $V_t(p, Z_t) = \sum_{s=t}^{T} v(x_s(p), y_s(p))$ denote the utility the agent attains from periods t to T with a realized price vector of p and wealth Z_t. Following Helms (1985), I define the agent's expenditure function with stochastic prices as the minimum wealth required to attain expected utility above a given threshold U. The agent's partial expenditure function (on all other goods) conditional on consuming $\widetilde{x}_t$ units of good x_t in period t is

$$\widetilde{e}(\widetilde{x}_t, U) = \min_{Z} Z - p_t \widetilde{x}_t \quad \text{such that}$$

$$\mathbb{E}_t V_t(p, Z) \geq U \quad \text{and} \quad x_t = \widetilde{x}_t$$

and hence the total expenditure function can be written as

$$(24) \qquad E(p_t, U) = \min_{x_t} p_t x_t + \widetilde{e}(x_t).$$

Let the expenditure-minimizing choice of x_t be denoted by $x_t^{c,*}(p_t, U_t)$, the structural Hicksian demand function under the nominal model in (23). Let $x_t^c(p_t, U_t)$ denote the observed Hicksian demand function with frictions. Let $\varepsilon(p_t) = -\frac{\partial x^{c,*}}{\partial p_t} \frac{p_t}{x^c}$ denote the structural Hicksian price elasticity of demand at price p_t. When utility is not quasilinear, identifying $\varepsilon(p_t)$ requires variation in prices *within* period t because price changes across periods conflate the Frisch and Hicksian elasticities (MaCurdy (1981)). Consider an experiment in which some agents face a price of p_A and others face a price of p_B in period t, and let

$$\widehat{\varepsilon}(p_A, p_B) = -\frac{\log x_B^c(p_B) - \log x_A^c(p_A)}{\log(p_B) - \log(p_A)}$$

denote the observed elasticity from this experiment. Our objective is to identify $\varepsilon(p_t)$ from estimates of $\widehat{\varepsilon}$ in an environment with frictions.

In this setting, the δ class of models is defined by the condition

$$(25) \qquad [p_t x_t^c + \widetilde{e}(x_t^c)] - [p_t x_t^{c,*} + \widetilde{e}(x_t^{c,*})] \le \delta p_t x_t^{c,*}.$$

I first establish an analog of Lemma 1 to characterize the choice set with frictions.

LEMMA A1: *For small δ, the set of observed Hicksian demands is approximately*

$$(26) \qquad X_t^c(p_t, \delta) = \left\{ x_t^c : |\log x_t^c - \log x_t^{c,*}| \le [2\varepsilon(p_t)\delta]^{1/2} \right\}.$$

PROOF: The first-order condition for (24) is

$$(27) \qquad \widetilde{e}_x(x_t^{c,*}) = -p_t.$$

Using a quadratic approximation to the partial expenditure function, we can exploit this first-order condition to obtain

$$[p_t x_t^c + \widetilde{e}(x_t^c)] - [p_t x_t^{c,*} + \widetilde{e}(x_t^{c,*})]$$
$$\simeq \frac{1}{2}(x_t^{c,*})^2(\log x_t^c - \log x_t^{c,*})^2 \widetilde{e}_{xx}(x_t^{c,*})$$

and, hence, we can rewrite (25) as

$$(28) \qquad |\log x_t^c - \log x_t^{c,*}| \le \left[2\delta \frac{p_t}{x_t^{c,*}} \frac{1}{\widetilde{e}_{xx}(x_t^{c,*})} \right]^{1/2}.$$

Differentiating (27) with respect to p_t implies $1/\widetilde{e}_{xx}(x_t^{c,*}) = -\frac{\partial x^{c,*}}{\partial p_t}$ and substituting this equation into (28) completes the proof.   *Q.E.D.*

Next, I establish the analog of Proposition 1. When utility is not quasilinear, the structural elasticity $\varepsilon(p_t)$ varies with the price p_t. Let $\varepsilon(p_A)$ and $\varepsilon(p_B)$ denote the structural point elasticities at the initial and final prices, and let $\varepsilon(p_A, p_B) = -\frac{\log x_B^{c,*}(p_B) - \log x_A^{c,*}(p_A)}{\log(p_B) - \log(p_A)}$ denote the structural arc elasticity between the two prices. Then the upper bound on $\varepsilon(p_A, p_B)$ is characterized by an equation analogous to (12):

$$\widehat{\varepsilon}(p_A, p_B) = -\frac{\log x_B^c(p_B) - \log x_A^c(p_A)}{\log(p_B) - \log(p_A)}$$

$$= \varepsilon(p_A, p_B) - \frac{2(2\varepsilon(p_B)\delta)^{1/2}}{\Delta \log p}.$$

Solving this equation requires a parametric assumption about utility to relate the two point elasticities at p_A and p_B to the arc elasticity. I make the following local isoelasticity assumption, which is analogous to Assumption 2 in the extensive margin case.

ASSUMPTION 2′: The structural Hicksian elasticity is constant between p_A and p_B: $\varepsilon(p_t) = -\frac{\partial x^{c,*}}{\partial p_t}\frac{p_t}{x^{c,*}} = \varepsilon(p_A, p_B)$ for $p_t \in [p_A, p_B]$.

Under Assumption 2′, the upper and lower bounds on the structural arc elasticity $\varepsilon(p_A, p_B)$ are characterized by the same equations as (12) and (13):

$$\widehat{\varepsilon} = \varepsilon \pm 2\frac{(2\varepsilon\delta)^{1/2}}{\Delta \log p}.$$

PROPOSITION A1: *Under Assumption 2′, for small δ, the range of structural Hicksian elasticities $\varepsilon(p_A, p_B)$ consistent with an observed Hicksian elasticity $\widehat{\varepsilon}(p_A, p_B)$ is approximately $(\varepsilon_L, \varepsilon_U)$, where*

$$\varepsilon_L = \widehat{\varepsilon} + \frac{4\delta}{(\Delta \log p)^2}(1 - \rho) \quad and \quad \varepsilon_U = \widehat{\varepsilon} + \frac{4\delta}{(\Delta \log p)^2}(1 + \rho)$$

with

$$\rho = \left(1 + \frac{1}{2}\frac{\widehat{\varepsilon}(p_t)}{\delta}(\Delta \log p)^2\right)^{1/2}.$$

The proof is identical to the proof of Proposition 1.

In a model with heterogeneous utilities $v_i(x_s, y_s)$, Proposition A1 requires a stronger isoelasticity assumption, namely that the structural elasticity $\varepsilon(p_t)$ does not vary across agents between p_A and p_B. It also requires an assump-

tion analogous to Assumption 1, that is, that tastes are orthogonal to the price change used for identification.

A.2. *Bounds on Extensive Margin Elasticities*

With quasilinear utility, the agent's flow utility in period t is $v_{i,t}(x, y) = y + b_{i,t}x$. Recognizing that the consumption path of y does not affect lifetime utility, the flow utility cost of choosing x suboptimally in period t is

$$u_{i,t}(x^*(p_t)) - u_{i,t}(x) = (x_{i,t}^* - x)(b_{i,t} - p_t).$$

I define a δ class of models around the nominal model by a condition analogous to (7):

$$(29) \qquad (x_{i,t}^* - x)(b_{i,t} - p_t) \le \delta_i p_t \quad \text{and} \quad \frac{1}{N} \sum_i \delta_{i,t} \le \delta \quad \text{and}$$

$$F(b_{i,t}|\delta_{i,t}) = F(b_{i,t}).$$

The last condition in (29)—that the taste distribution cannot vary across agents with different frictions—is needed to ensure that the choice set has the same width for the marginal agents at each level of p.[32] This condition was not necessary in the intensive margin case because there the marginal agent did not vary with p.

PROOF OF LEMMA 2: Equation (29) implies that agent i's observed demand for x is

$$x_{i,t} = \begin{cases} 1, & \text{if } b_{i,t} - p_t > \delta_{i,t} p_t, \\ \{0, 1\}, & \text{if } |b_{i,t} - p_t| \le \delta_{i,t} p_t, \\ 0, & \text{if } b_{i,t} - p_t < -\delta_{i,t} p_t. \end{cases}$$

Let $\theta_{\delta_{i,t}}(p_t)$ denote the observed participation rate for agents who have frictions $\delta_{i,t}$ and let $\theta_t = \mathbb{E}\theta_{\delta_{i,t}}(p_t)$ denote the observed participation rate in the aggregate economy. Under the condition that $F(b_{i,t}|\delta_{i,t}) = F(b_{i,t})$, it follows that $\theta_{\delta_{i,t}}(p_t)$ lies in the set

$$\begin{aligned} &\left[1 - F((1 + \delta_{i,t})p_t), 1 - F((1 - \delta_{i,t})p_t)\right] \\ &= \left[\theta_t^* + F(p_t) - F((1 + \delta_{i,t})p_t), \theta_t^* + F(p_t) - F((1 - \delta_{i,t})p_t)\right] \\ &\simeq \left[\theta_t^* - f(p_t)p_t\delta_{i,t}, \theta_t^* + f(p_t)p_t\delta_{i,t}\right], \end{aligned}$$

---

[32]To see why this condition is needed, suppose agents with $b_{i,t}$ close to p_t have very large $\delta_{i,t}$ while those away from the margin have $\delta_{i,t} = 0$. This would result in a wide choice set for the participation rate at p_t even if $\mathbb{E}\delta_{i,t} < \delta$.

where the last line uses a first-order Taylor expansion of $F(p_t)$ around p_t. Under Assumptions 1' and 2', $\eta = -\frac{d\log[1-F(p_t)]}{d\log p_t} = \frac{f(p_t)}{\theta^*(p_t)}p_t$. Hence

$$\theta_{\delta_{i,t}}(p_t) \in [\theta_t^* \cdot (1 - \eta\delta_{i,t}), \theta_t^* \cdot (1 + \eta\delta_{i,t})]$$
$$\Rightarrow \quad \mathbb{E}\theta_{\delta_{i,t}}(p_t) \in [\theta_t^* \cdot (1 - \eta\mathbb{E}\delta_{i,t}), \theta_t^* \cdot (1 + \eta\mathbb{E}\delta_{i,t})]$$
$$\Rightarrow \quad \theta_t(p_t)/\theta_t^*(p_t) \in [1 - \eta\delta, 1 + \eta\delta].$$

The approximation $\log(1 + \eta\delta) \simeq \eta\delta$ for small δ yields $|\log\theta_t - \log\theta_t^*| \leq \eta\delta$. $\qquad$ *Q.E.D.*

PROOF OF PROPOSITION 2: Given a structural elasticity η, the maximal observed response to a price change of $\Delta\log p$ is $\Delta\log\theta = \eta\Delta\log p + 2\delta\eta$ and the minimal observed response is $\Delta\log\theta = \eta\Delta\log p - 2\delta\eta$. Therefore, the observed elasticity $\widehat{\eta} = \frac{\Delta\log\theta}{\Delta\log p}$ must satisfy

$$(30) \qquad (1 - \rho_\eta)\eta \leq \widehat{\eta} \leq (1 + \rho_\eta)\eta,$$

where $\rho_\eta = \frac{2\delta}{\Delta\log p}$. If $\rho_\eta \geq 1$, η is unbounded above for a given value of $\widehat{\eta}$ because both inequalities in (30) are satisfied for arbitrarily large η. If $\frac{2\delta}{\Delta\log p} < 1$, then the upper and lower bounds on η are obtained when (30) holds with equality. Solving these equations yields (16). $\qquad$ *Q.E.D.*

PROOF OF COROLLARY 2: Suppose $\widehat{\eta} = 0$. Then $\rho_\eta < 1 \Rightarrow \eta_U = 0$. Hence a positive structural elasticity ($\eta > 0$) can only generate a 0 observed elasticity if $\rho_\eta = \frac{2\delta}{\Delta\log p} \geq 1 \Leftrightarrow \Delta u_{\text{ext},\%} = \Delta\log p \leq 2\delta$. $\qquad$ *Q.E.D.*

A.3. *Intuition for 4δ Threshold in Corollary 1*

This section explains why $\Delta u_\%(\varepsilon)$ must be below 4δ so as to observe $\widehat{\varepsilon} = 0$. Let $d = x_A^*(p_A) - \min(X_A(p_A, \delta))$ denote the difference between the mean optimal demand and the lowest mean demand in the initial choice set. Figure 1(a) shows that at the upper bound ε_U, the difference between the optimal demands at the two prices is $x^*(p_A) - x^*(p_B) = 2d$. By definition, the percentage utility cost of choosing $\min(X_A(p_A, \delta))$ instead of $x^*(p_A)$ is δ. Given that the utility cost of deviating by d units is δ, the utility cost of deviating by $2d$ units is 4δ, as illustrated in Figure 1(b).

APPENDIX B: SOURCES AND CALCULATIONS FOR STUDIES IN TABLE I

This appendix describes how the values in columns 3–5 in Table I are calculated. The papers used for the analysis along with comprehensive documentation of the calculations are available at http://obs.rc.fas.harvard.edu/chetty/bounds_opt_meta_analysis.zip.
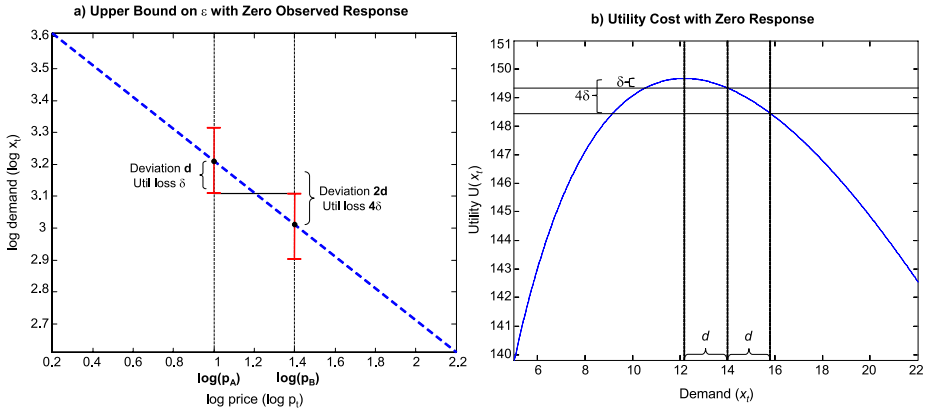
FIGURE 1.—Construction of upper bound when observed elasticity is zero.

I use compensated intensive margin estimates reported in each paper when available and use the Slutsky equation to calculate compensated elasticities in cases where uncompensated elasticities are reported.

The studies do not always directly report the relevant inputs, especially the net-of-tax change $\Delta \log(1 - \tau)$. For studies whose estimates are identified from a single quasi-experiment (e.g., Feldstein (1995)), I define $\Delta \log(1 - \tau)$ as the change in the marginal NTR for the group that the authors' define as the "treated" group. For studies that pool multiple tax or wage changes of different sizes and do not explicitly isolate a treatment group (e.g., Gruber and Saez (2002)), I define $\Delta \log(1 - \tau)$ as twice the standard deviation (SD) of $\Delta \log(1 - \text{MTR})$ in the sample. The logic for this approach is as follows. In a linear regression $Y_i = \alpha + \beta_1 X_i + u_i$, the standard error of $\hat{\beta}_1$ is the square root of $(\text{var}(u)/\text{var}(X))/N$, where N is the sample size. Consider a second regression $Y_i = \alpha + \beta_2 Z_i + u_i$, where $Z_i = 0$ for half the observations (the "control group") and $Z_i = 2 \cdot \text{SD}(X)$ for the remaining observations (the "treatment group"). Setting the size of the single treatment to $2 \cdot \text{SD}(X)$ yields $\text{var}(Z) = \text{var}(X)$. Hence, the standard error of $\hat{\beta}_2$ equals the standard error of $\hat{\beta}_1$. A single tax change of $2 \cdot \text{SD}(\Delta \log(1 - \text{MTR}))$ therefore produces an estimate of $\hat{\varepsilon}$ with the same precision as the original variation in marginal tax rates used for identification.

I calculate the bounds by assuming that agents face a linear budget set whose slope is given by their marginal tax rate (MTR) and apply Proposition A1 using $\Delta \log(1 - \text{MTR})$ in place of $\Delta \log p$. This yields valid bounds on ε for agents who remain in the interior of budget segments in a progressive tax system. However, the bounds cannot be applied to agents who locate at kinks. Given

that most of the studies in Table I estimate elasticities from changes in the behavior of agents away from kinks, this is not a serious limitation.[33]

The remainder of this appendix describes how I calculate $\widehat{\varepsilon}$, the standard error of $\widehat{\varepsilon}$, and $\Delta \log(1 - \text{MTR})$ for each study in Table I.

A. Hours Elasticities

1. MaCurdy (1981). $\widehat{\varepsilon}$ is reported in the text on page 1083; s.e.$(\widehat{\varepsilon})$ is imputed from the t-statistic for δ reported in row 5 of Table 1 as $0.15/0.98$, because the estimate of compensated elasticity is approximately equal to δ; $\Delta \log(1 - \tau)$, the relevant within-person annual wage variation, is not reported in the paper, so I use $2 \times \text{SD} = 2 \times (0.152^2 + 2 \cdot 0.086^2)^{1/2}$ from Table 1, column 4 of Low, Meghir, and Pistaferri (2010), who estimated the standard deviation of changes in log wages. Note that this is likely an overestimate of the size of $\Delta \log(1 - \tau)$, resulting in bounds that are too tight, because MaCurdy used family background characteristics, age, and year dummies as instruments for wage growth and did not use all elements of wage growth for identification.

2, 3. Eissa and Hoynes (1998). $\widehat{\varepsilon}$ is reported for men as an intensive margin "wage elasticity" of 0.07 and an income elasticity of -0.03 in Table 8, column 3. This "wage elasticity" uses the total hours change, which includes the hours change induced by the increased EITC rebate, which raised the *average* net of tax rate by 0.042 for a couple earning \$15,000 with two children (for whom the average net-of-tax rate changed from 107.5% in 1993 to 112.1% in 1994 computed using TAXSIM). This rebate should have changed hours (in log terms) by -0.03×0.042, giving an uncompensated elasticity of 0.069. The compensated elasticity is $\widehat{\varepsilon}_{lc,w}^{\text{men}} = \widehat{\varepsilon}_{l,w} - \frac{wl}{y}\widehat{\varepsilon}_{l,y} = 0.200$, with w, l, and y from Table 3, column 4. A parallel calculation using Table 9 gives $\widehat{\varepsilon}_{lc,w}^{\text{women}} = 0.088$. The s.e.$(\widehat{\varepsilon})$ assumed that w, l, y, and the change in income from the EITC expansion are measured without error. Then using the t-statistics from the coefficients on ln(wage) and virtual inc to impute the standard errors for the elasticities yields $\text{SE}(\widehat{\varepsilon}_{lc,w}^{\text{men}}) = \{\text{SE}(\widehat{\varepsilon}_{l,w})^2 + [\frac{wl}{y}\text{SE}(\widehat{\varepsilon}_{l,y})]^2\}^{1/2} = 0.074$ and $\text{SE}(\widehat{\varepsilon}_{lc,w}^{\text{women}}) = 0.067$. Note that this calculation is limited because the full variance–covariance matrix for the regression coefficients is not reported. $\Delta \log(1 - \tau)$ is defined as $2 \times \text{SD}$ of log net-of-tax-rate in the phase-out EITC rates listed in Table 1 for 1984–1996, because most married couples who receive the EITC are in the phase-out region (Table 2).

---

[33]Recent studies that identify observed elasticities from bunching at kinks (e.g., Saez (2010), Chetty, Friedman, Olsen, and Pistaferri (2011b)) are an exception. I incorporate these studies into the linear-demand framework by exploiting the fact that they also study *movements* in the kinks over time, which create reductions in marginal rates for the subgroup of individuals located between the old and new bracket cutoffs. These studies imply that these individuals do not increase labor supply significantly when their marginal tax rates are lowered. This constitutes an observed elasticity estimate based on choices at interior optima, permitting application of Proposition 1.

4. Blundell, Duncan, and Meghir (1998). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 4, row 1. I interpret this estimate as an intensive margin elasticity because the variation in wages from the grouping estimator does not appear to affect participation, based on the discussion on page 845. $\Delta \log(1 - \tau)$ is defined as $2 \times \text{SD}(\log \widehat{w}_{gt} - \log \widehat{w}_g - \log \widehat{w}_t) = 0.23$, which is reported in Table 9, because the variation arises from group–time interactions in wages.

5. Ziliak and Kniesner (1999). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 1, column 3. $\Delta \log(1 - \tau)$ is the study that effectively uses within-person annual wage variation, because lagged wage growth is included as an instrument. Since within-person annual wage variation is not reported in the paper, I again use $2 \times \text{SD} = 2 \times (0.152^2 + 2 \cdot 0.086^2)^{1/2}$ from Table 1, column 4 of Low, Meghir, and Pistaferri (2010).

B. Taxable Income Elasticities

6. Bianchi, Gudmundsson, and Zoega (2001). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are the average percent change in earnings for men and women weighted by observations (columns 1–4 of Table 6) divided by the percent change in the net-of-tax rate. The standard error is computed from the standard errors reported for the changes in earnings. I interpret this estimate as an intensive margin elasticity because Table 6 conditions on work in 1986, and tax rates were generally lower in 1987 and 1988 than in 1986. I take this to be a compensated elasticity because Bianchi, Gudmundsson, and Zoega argued that income effects are small on page 1565–1566, although this is somewhat tenuous. Note that the elasticity estimates provided by the authors are computed using *average* rather than marginal tax rates, necessitating the use of the computation described above. $\Delta \log(1 - \tau)$ is the log change from a tax rate of 0 in 1987 to 0.3875, which is an average of the flat tax in 1988 and the mean of the top marginal tax rate and bottom marginal tax rate in 1986 reported in Table 1, because the change in earnings estimate compares 1987 to the average earnings in 1986 and 1988.

7. Gruber and Saez (2002). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are averages of the estimates in column 2 of Table 9 for individuals with taxable income between \$10,000 and \$50,000 and those with taxable income between \$50,000 and \$100,000. These estimates are compensated elasticities, as Gruber and Saez note on page 20 that income effects are essentially zero in their sample. $\Delta \log(1 - \tau)$ is defined as $2 \times \text{SD}$ of the change in log net-of-tax-rate and is computed separately for columns 3 and 4 of Table 3 using the means and standard deviations for each year. The two estimates of $\Delta \log(1 - \tau)$ are then averaged in the same way as in the elasticity calculation described above.

8. Saez (2004). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 7B, column 6 for the top 5% to 1% of tax units. Note that Saez used gross income, not taxable income. I interpret his estimate as an intensive margin elasticity because his sample consists of repeated cross sections of workers and because the extensive margin is unlikely to be important for the top 5% to 1% of taxpayers. I interpret this

estimate as a compensated elasticity following the aforementioned evidence from Gruber and Saez (2002) that income effects are small. $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of the log net-of-tax-rate for the top 5% to 1% of tax units listed in column 8 of Table 5.

9. Jacob and Ludwig (2008). For $\widehat{\varepsilon}$, these authors report in Table 3 that head of households' quarterly earnings conditional on working changed by \$228 from a control mean of \$5558. As with Eissa and Hoynes, I calculate how much income would have changed absent the grant worth \$6860 (page 9) so as to compute a compensated wage elasticity. Jacob and Ludwig did not report the effect of unearned income on earnings, so I use an estimate from Imbens, Rubin, and Sacerdote (2001), who reported in Table 4, specification V, column 1, a marginal propensity to earn out of unearned income (MPE) of -0.114 with a standard error of 0.015. In an earlier version, Imbens, Rubin, and Sacerdote (1999) reported earnings and participation elasticities of "around" -0.20 and -0.14, respectively, so I assume an intensive MPE of $\frac{d[wl]}{dY} = -0.114\{1 - (0.14/0.20)\} = -0.034$. On a quarterly basis, the grant should have lowered earnings by $-0.034 \cdot (6860/4) = 58.65$. Dividing the change in earnings absent the grant by the tax change gives an uncompensated elasticity of $\{\log(5558 - 228 + 58.65) - \log(5558)\}/\{\log(1) - \log(1 - 0.3)\} = 0.086$. Finally, the elasticity is $\widehat{\varepsilon} = \widehat{\varepsilon}^u - \frac{d[wl]}{dY} = 0.086 + 0.034 = 0.121$. For s.e.$(\widehat{\varepsilon})$, assuming that the standard error on the intensive MPE is proportional to the error on the total MPE and that the change in income due to the grant is measured without error, then the standard error is 0.031. For $\Delta \log(1 - \tau)$, the MTR changed from 0 to 0.30 for those receiving the housing voucher as described in footnote 29 so that $\log(1) - \log(1 - 0.3) = 0.36$.

10, 11. Gelber (2010). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 3, column 1 for men and column 2 for women. These estimates use earned income since it is less susceptible to manipulation than taxable labor income. These estimates presumably reflect primarily intensive margin responses since the extensive margin is unlikely to be important for the high-income group affected by the change in top bracket tax rates. $\Delta \log(1 - \tau)$ is the percent change in net-of-tax rate from 1989 to 1991 for the highest tax brackets reported in Table 1.

12. Saez (2010). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 2, row 1 of column 6 for wage earners with two or more children. $\Delta \log(1 - \tau)$ is the change in NTR at the first kink in the EITC benefit schedule from 1995 to 2004.

13, 14. Chetty et al. (2011b). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are observed elasticities at middle and top kinks, calculated using equation 6 as $b/K\Delta \log(1 - \tau)$. In this equation, K is the location of the tax bracket cutoff (DKr 164,300 for the middle tax and DKr 267,600 for the top tax). The estimated excess mass at the kink (b) is 1.79 (s.e. 0.05) for married women at the top kink (Figure IIIb) and 0.06 (s.e. 0.03) at the middle kink (Figure VIa). $\Delta \log(1 - \tau)$ is the size of tax changes at the middle and top tax kinks as reported in Figure II.

15. Chetty et al. (2011b). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 2, column 1. $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of the changes in the log net-of-tax rate reported in the last row of Table 1, column 1.

C. Top Income Elasticities

16. Feldstein (1995). $\widehat{\varepsilon}$ is the high minus medium tax rate specification in Table 2. For this and other studies based on TRA86, I follow the literature in interpreting elasticities as compensated elasticities because the reform was revenue neutral. s.e.$(\widehat{\varepsilon})$ was not reported. For a rough estimate, rescaling the standard error cited by Feldstein on page 566 for Auten and Carroll (1994) by the ratio of sample sizes in the two studies yields s.e.$(\widehat{\varepsilon}) = 0.15\sqrt{14{,}425/3735} = 0.295$. $\Delta \log(1 - \tau)$ is reported in Table 2 for the high tax rate group.

17. Auten and Carroll (1999). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 2, column 6. $\Delta \log(1 - \tau)$ was reported by Goolsbee (1999) for the highest income group in Table 3, row C for 1985–1989 because TRA86 "provided tax variation mostly at the top of the income scale, so that their overall estimates are identified primarily by reactions of high income taxpayers" (Gruber and Saez (2002, pp. 24–25)).

18. Goolsbee (1999). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 4, column 1. $\Delta \log(1 - \tau)$ is from Table 3, row C for 1985–1989 based on the quote above.

19. Saez (2004). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 3C, column 3 for the top 1% of tax units. Note that Saez used gross income, not taxable income. I interpret his estimate as an intensive margin elasticity because his sample consists of repeated cross sections of workers and because the extensive margin is unlikely to be important for the top 1% of taxpayers. I interpret this estimate as a compensated elasticity following the aforementioned evidence from Gruber and Saez (2002) that income effects are small. $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of the log net-of-tax-rate for the top 1% of tax units listed in column 3 of Table 5.

20. Kopczuk (2010). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ are from Table 9, second panel, column 1, 2002–2005, with standard error imputed from the reported t-statistic. This is a compensated elasticity following Gruber and Saez (2002, equation (2)). $\Delta \log(1 - \tau)$ is reported on page 17.

D. Macro/Cross-Sectional

21. Prescott (2004). $\widehat{\varepsilon}$ and s.e.$(\widehat{\varepsilon})$ were calculated by regressing log hours per worker on log net-of-tax rates using Organization for Economic Cooperation and Development (OECD) data reported by Prescott in Table 2 on hours per adult, which are converted to hours per worker using labor force participation rates from OECD Stat Extracts.[34] The data on labor force participation

---

[34]Data are for men and women aged 15–64 for 1970–1974 and 1993–1996 so as to match Prescott's data. Data are available from OECD Stat Extracts at http://stats.oecd.org/Index.aspx?DataSetCode=LFS_SEXAGE_I_R.

rates are missing for Canada and the United Kingdom in the 1970's, and these observations are therefore excluded. The elasticity estimate can be interpreted as a compensated labor supply elasticity if government expenditure is viewed as unearned income in the aggregate. $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of the change in log net-of-tax rate for the 12 observations with nonmissing data on hours per employed person.

22. Davis and Henrekson (2005). $\widehat{\varepsilon}$ is computed using log differences in annual hours per employed adult based on the slope coefficient in Table 2.3 (middle panel, Sample C) and the sample means of annual hours per employed person and tax rates in Table 2.1 for the corresponding sample. The elasticity estimate can be interpreted as a compensated labor supply elasticity if government expenditure is viewed as unearned income in the aggregate. s.e.$(\widehat{\varepsilon})$ is calculated from the standard error reported for the slope coefficient in Table 2.3 (middle panel, Sample C). $\Delta \log(1 - \tau)$ is computed as $2 \times$ SD of log 1 minus the sum of tax rates for the 19 countries in Sample C.[35]

23. Blau and Kahn (2007). $\widehat{\varepsilon}$ is computed from intensive margin (with selection correction) elasticities reported in Table 6, defining the income elasticity as the elasticity of women's hours with respect to husband's wages and using the Slutsky equation to compute compensated elasticities in corresponding fashion. Mean values of wl and y are from Tables A2 and A3. I report an unweighted average of the elasticities from Model 1 for each of the three time periods. s.e.$(\widehat{\varepsilon})$ is calculated from the standard error reported for the regression coefficients in Table 7 of NBER Working Paper 11230. I assume that the covariance between the coefficient estimates is zero because the full variance–covariance matrix for the regression coefficients is not reported. $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of log wage rates because the study effectively exploits cross-sectional variation in wage rates for identification; the instruments used in Table 6 correct only for measurement error. The standard deviation of log wages for married women is not reported and is, therefore, taken from Rothstein (2008), who reported a value of 0.50 in column 4 of Table 1 for married women in 1992–1993. This estimate is consistent with other published estimates of the standard deviations of women's log wages in the Current Population Survey (e.g., Blau and Kahn (2000), Card and DiNardo (2002)).

---

[35]Data are for 1995 for all countries except New Zealand and Australia, for which I use 1986 and 1985 values following Davis and Henrekson's data appendix. Austria is excluded because Davis and Henrekson exclude it from Sample C. The variable of interest in the data set is tw, which stands for "tax wedge." See Davis and Henrekson for more details. The mean (0.496 vs. 0.500) and standard deviation (0.14 vs. 0.133) reported for Sample C in Table 2.1 differ slightly from those used in this calculation. The data were accessed from the .zip appendix at http://cep.lse.ac.uk/pubs/number.asp?number=502.

APPENDIX C: Sources and Calculations for Extensive Margin
Studies in Table II

This appendix describes the sources of the values in columns 3–5 of Table II
for each study. For studies 1–7, the elasticity estimates $(\widehat{\eta})$ and standard errors in columns 3 and 4 are taken from Table 1 in Chetty, Guren, Manoli,
and Weber (2011a); details on the sources of these estimates are given in Appendix B of that paper. Studies 8–10 are also from Chetty et al. (2011a); details on these estimates can be found in Appendix C of that paper. I follow
the same methods as in Appendix B to calculate $\Delta \log(1 - \tau)$, defined here
as the change in the net-of-average tax rate. The papers used for the analysis
along with comprehensive documentation of the calculations are available at
http://obs.rc.fas.harvard.edu/chetty/bounds_opt_meta_analysis.zip.

A. Quasi-Experimental Elasticities

1. Eissa and Liebman (1996). $\Delta \log(1 - \tau)$ is from Meyer and Rosenbaum
(2000), who used the same data source and, in Table 2, calculated the financial
gain from working for single mothers in 1990 as \$8458, compared with \$7469
in 1984. I therefore define $\Delta \log(1 - \tau) = \log(8458) - \log(7469)$.

2. Graversen (1998). $\Delta \log(1 - \tau)$ is from Table 3, which reports level
changes in employment rates and participation elasticities, from which I back
out $\Delta \log(1 - \tau) = (\Delta \theta / \overline{\theta}) / \widehat{\eta}$, where $\Delta \theta = -0.031$ is the estimated change in
employment rates for single women, $\overline{\theta} = 0.7$ is the mean employment rate for
single women using an average of the six participation rates in Table 2 weighted
by sample sizes, and $\widehat{\eta} = -0.174$ is the elasticity estimate reported in Table 3.

3. Devereux (2004). $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of the deviations from
the mean log wage change for each region/age/education group in Table A1
for women because the variation used for identification is across region and
time by education/age group. Note that this table conditions on some work,
whereas in the sample used to estimate $\widehat{\eta}$, nonparticipants' wages are imputed
as the average for their group.

4. Meyer and Rosenbaum (2001). $\Delta \log(1 - \tau)$ is from the discussion of study
4 in Chetty et al. (2011a), who defined $\Delta \log(1 - \tau) = 45\%$ after accounting for
taxes and transfers as in Meyer and Rosenbaum (2000, p. 1043).

5. Eissa and Hoynes (2004). $\Delta \log(1 - \tau)$ is from Meyer and Rosenbaum
(2000, p. 1043), who reported a tax change of 45% from 1984 to 1996 for the
group studied by Eissa and Hoynes.

6. Liebman and Saez (2006). $\Delta \log(1 - \tau)$ is defined as $\log(1 - 0.419) -
\log(1 - 0.31)$ based on the change in tax rates reported on pages 10–11 for
OBRA93.

7. Blundell, Bozio, and Laroque (2011). $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of
log net-of-tax rates for participation. A standard deviation of 0.37 was obtained
from personal correspondence with authors.

B. Macro/Cross-Sectional Elasticities

8. Nickell (2003). $\widehat{\eta}$ is computed using the average point estimate of 2% (reported on page 8) and the sample means of employment rates and tax rates from Tables 1 and 2, respectively. s.e.$(\widehat{\eta})$ was not reported because Nickell did not report standard errors for the studies in Table 4 on which his point estimate is based. $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of log net-of-tax rates using values listed in Table 2 because most of the studies in Table 4 used in Nickell's estimate of the effect of taxation on employment used panel or cross-sectional data for OECD countries.

9. Prescott (2004). $\widehat{\eta}$ and s.e.$(\widehat{\eta})$ are calculated by regressing log labor force participation rates from OECD Stat Extracts on log net-of-tax rates using the same sample of countries and years as Prescott.[36] The data on tax rates are taken from Table 2 of Prescott. The data on labor force participation rates are missing for Canada and the United Kingdom in the 1970's and these observations are therefore excluded. $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of the change in log net-of-tax rate for the 12 observations with nonmissing data on labor force participation rates.

10. Davis and Henrekson (2005). $\widehat{\eta}$ is computed using the log difference in employment based on the slope coefficient in Table 2.3 (bottom panel, Sample C) and the sample means of labor force participation and tax rates in Table 1 for the corresponding sample. s.e.$(\widehat{\eta})$ is calculated from the standard error reported for the slope coefficient in Table 2.3 (bottom panel, Sample C). $\Delta \log(1 - \tau)$ is computed as $2 \times$ SD of log 1 minus the sum of tax rates for the 19 countries in Sample C.

11. Blau and Kahn (2007). For $\widehat{\eta}$, I report an unweighted average of the own wage participation elasticities for each of the three time periods in Table 6, Model 1. For s.e.$(\widehat{\eta})$, the standard error is calculated from the standard error reported for own log wage in Table 7 of NBER Working Paper 11230. I assume that the covariance between the coefficient estimates is zero because the full variance–covariance matrix for the parameters in the probit model is not reported. $\Delta \log(1 - \tau)$ is defined as $2 \times$ SD of log wages, calculated as described in study 23 in Appendix B above.

REFERENCES

AUTEN, G., AND R. CARROLL (1994): "Taxpayer Behavior and the 1986 Tax Reform Act," Paper NBER Summer Institute, Cambridge, MA. [10]
BLAU, F. D., AND L. M. KAHN (2000): "Gender Differences in Play," *The Journal of Economic Perspectives*, 14, 75–99. [11]
CARD, D., AND J. E. DINARDO (2002): "Skill-Biased Technological Change and Rising Wage Inequality: Some Problems and Puzzles," *Journal of Labor Economics*, 20, 733–783. [11]

---

[36]Data are for men and women aged 15–64 for 1970–1974 and 1993–1996 so as to match Prescott's data. Data are available from OECD Stat Extracts at http://stats.oecd.org/Index.aspx?DataSetCode=LFS_SEXAGE_I_R.

HELMS, L. J. (1985): "Expected Consumer's Surplus and the Welfare Effects of Price Stabilization," *International Economic Review*, 26, 603–617. [1]

IMBENS, G. W., D. B. RUBIN, AND B. SACERDOTE (1999): "Estimating the Effect of Unearned Income on Labor Supply, Earnings, Savings, and Consumption: Evidence From a Survey of Lottery Players," Working Paper 14836, NBER. [9]

——— (2001): "Estimating the Effect of Unearned Income on Labor Earnings, Savings, and Consumption: Evidence From a Survey of Lottery Players," *American Economic Review*, 91, 778–794. [9]

LOW, H., C. MEGHIR, AND L. PISTAFERRI (2010): "Wage Risk and Employment Risk Over the Life Cycle," *American Economic Review,* 100, 1432–1467. [7,8]

MEYER, B., AND D. ROSENBAUM (2000): "Making Single Mothers Work: Recent Tax and Welfare Policy and Its Effects," *National Tax Journal*, 53, 1027–1059. [12]

ROTHSTEIN, J. (2008): "The Unintended Consequences of Encouraging Work: Tax Incidence and the EITC," Working Paper 165, CEPS. [11]

Dept. of Economics, Harvard University, Cambridge, MA 02142, U.S.A. and NBER; chetty@fas.harvard.edu.