SUPPLEMENT TO "NEARLY EFFICIENT LIKELIHOOD RATIO TESTS OF THE UNIT ROOT HYPOTHESIS"
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A.1. MONTE CARLO SIMULATIONS

TO ASSESS THE FINITE SAMPLE PROPERTIES of the likelihood ratio test, we conduct a small Monte Carlo experiment. For specificity, and because the presence of a negative moving average component is known to be problematic in unit root testing, we consider a data generating process (DGP) of the form

$$(A1) \qquad y_t = \rho y_{t-1} + u_t, \quad u_t = \varepsilon_t + \theta \varepsilon_{t-1},$$

where $y_0 = 0$, $\varepsilon_t \sim$ i.i.d. $\mathcal{N}(0, 1)$, and $\theta \in \{0.00, -0.25, -0.50, -0.75\}$. For each of 10^6 replications, we simulate the model with sample sizes $T \in \{100, 250\}$. The parameter ρ is set so that $\rho = 1 + c/T$, with $c \in \{0, -5, \ldots, -30\}$ in the constant mean case and $c \in \{0, -7.5, \ldots, -45\}$ in the linear trend case.

In the simulations, the likelihood ratio test $\widehat{\mathrm{LR}}_T^d$ is compared with the DF-GLS test of Elliott, Rothenberg, and Stock (1996) and with the modified point optimal MP-GLS test of Ng and Perron (2001). All three tests use the lag length chosen by the Modified Akaike Information Criterion of Perron and Qu (2007) applied to the DF-GLS regression. The results of the simulations are presented in Tables A.I–A.IV. Tables A.I and A.II report rejection rates and size-corrected power, respectively, for the constant mean case. The corresponding results for the linear trend case are reported in Tables A.III and A.IV.

In both the constant mean and linear trend cases, the relative performance of the three tests can be summarized as follows. The MP-GLS test has excellent size properties, but it is very conservative and inferior to the other tests in terms of (raw and size-adjusted) power. The DF-GLS and likelihood ratio tests are both somewhat oversized when $\theta = -0.75$, but exhibit only very slight size distortions for $|\theta| \leq 0.5$. In the latter range, the likelihood ratio test is superior in terms of power, and in some cases the difference is quite substantial, for example, for $T = 250$ and $\theta = -0.50$. On the basis of these results, our tentative conclusion is that when size control in the presence of a large negative moving average component is the only concern, the MP-GLS test remains attractive. On the other hand, the likelihood ratio test is attractive when at most a moderate negative moving average component is suspected, in which case it offers substantial power gains over both the MP-GLS and DF-GLS tests without sacrificing size control.

TABLE A.I

SIMULATED REJECTION FREQUENCIES OF UNIT ROOT TESTS, CONSTANT MEAN CASE[a]

θ	ρ	$\widehat{\mathrm{LR}}_T^d$	DF-GLS	MP-GLS	ρ	$\widehat{\mathrm{LR}}_T^d$	DF-GLS	MP-GLS
		$T = 100$				$T = 250$		
0.00	1.00	0.0433	0.0538	0.0389	1.00	0.0521	0.0500	0.0424
	0.95	0.2136	0.2976	0.2295	0.98	0.2563	0.2941	0.2576
	0.90	0.5006	0.6044	0.5151	0.96	0.6138	0.6638	0.6138
	0.85	0.6967	0.7604	0.6970	0.94	0.8368	0.8618	0.8317
	0.80	0.7736	0.8049	0.7577	0.92	0.9110	0.9213	0.9035
	0.75	0.8004	0.8138	0.7708	0.90	0.9359	0.9381	0.9240
	0.70	0.8118	0.8102	0.7647	0.88	0.9471	0.9438	0.9290
-0.25	1.00	0.0515	0.0658	0.0497	1.00	0.0564	0.0561	0.0482
	0.95	0.2366	0.3068	0.2463	0.98	0.2814	0.3080	0.2748
	0.90	0.4750	0.5431	0.4629	0.96	0.6066	0.6316	0.5857
	0.85	0.6351	0.6605	0.5816	0.94	0.8041	0.8046	0.7667
	0.80	0.7127	0.6996	0.6219	0.92	0.8842	0.8659	0.8354
	0.75	0.7493	0.7083	0.6274	0.90	0.9181	0.8875	0.8568
	0.70	0.7703	0.7072	0.6212	0.88	0.9355	0.8932	0.8609
-0.50	1.00	0.0645	0.0744	0.0500	1.00	0.0658	0.0605	0.0486
	0.95	0.2612	0.2901	0.2004	0.98	0.3104	0.3037	0.2491
	0.90	0.4638	0.4571	0.3325	0.96	0.6079	0.5629	0.4805
	0.85	0.5991	0.5369	0.4012	0.94	0.7808	0.6962	0.6108
	0.80	0.6763	0.5730	0.4324	0.92	0.8615	0.7487	0.6613
	0.75	0.7250	0.5929	0.4506	0.90	0.9029	0.7663	0.6774
	0.70	0.7603	0.6093	0.4682	0.88	0.9269	0.7709	0.6760
-0.75	1.00	0.1118	0.0984	0.0457	1.00	0.0954	0.0737	0.0298
	0.95	0.3297	0.2661	0.1325	0.98	0.3695	0.2625	0.1145
	0.90	0.5231	0.3899	0.2275	0.96	0.6195	0.4063	0.1937
	0.85	0.6704	0.4952	0.3334	0.94	0.7829	0.4815	0.2465
	0.80	0.7735	0.5935	0.4491	0.92	0.8737	0.5216	0.2831
	0.75	0.8447	0.6874	0.5719	0.90	0.9253	0.5449	0.3112
	0.70	0.8941	0.7746	0.6895	0.88	0.9556	0.5634	0.3365

[a]The table presents simulated rejection frequencies for the likelihood ratio test $\widehat{\mathrm{LR}}_T^d$, DF-GLS test of ERS, and MP-GLS test of Ng and Perron (2001) using 10^6 replications of the model (A1) allowing only for a constant mean. The entries for $\rho = 1.00$ are the simulated sizes of the tests. All three tests use the MAIC of Perron and Qu (2007) to select lag length.

TABLE A.II

SIZE-CORRECTED REJECTION FREQUENCIES OF UNIT ROOT TESTS, CONSTANT MEAN CASE[a]

		$T = 100$				$T = 250$		
θ	ρ	$\widehat{\text{LR}}_T^d$	DF-GLS	MP-GLS	ρ	$\widehat{\text{LR}}_T^d$	DF-GLS	MP-GLS
0.00	1.00	0.0500	0.0500	0.0500	1.00	0.0500	0.0500	0.0500
	0.95	0.2380	0.2812	0.2812	0.98	0.2488	0.2942	0.2942
	0.90	0.5357	0.5843	0.5830	0.96	0.6026	0.6639	0.6634
	0.85	0.7230	0.7481	0.7441	0.94	0.8301	0.8618	0.8602
	0.80	0.7918	0.7963	0.7899	0.92	0.9074	0.9213	0.9196
	0.75	0.8153	0.8065	0.7948	0.90	0.9333	0.9381	0.9351
	0.70	0.8255	0.8039	0.7859	0.88	0.9452	0.9438	0.9375
-0.25	1.00	0.0515	0.0658	0.0497	1.00	0.0564	0.0561	0.0482
	0.95	0.2298	0.2478	0.2475	0.98	0.2568	0.2821	0.2827
	0.90	0.4651	0.4725	0.4642	0.96	0.5725	0.5987	0.5962
	0.85	0.6276	0.6042	0.5827	0.94	0.7795	0.7820	0.7745
	0.80	0.7069	0.6550	0.6229	0.92	0.8692	0.8513	0.8408
	0.75	0.7444	0.6706	0.6281	0.90	0.9073	0.8757	0.8612
	0.70	0.7656	0.6728	0.6220	0.88	0.9269	0.8827	0.8644
-0.50	1.00	0.0645	0.0744	0.0500	1.00	0.0658	0.0605	0.0486
	0.95	0.2109	0.2080	0.2005	0.98	0.2507	0.2579	0.2547
	0.90	0.3975	0.3639	0.3326	0.96	0.5288	0.5064	0.4877
	0.85	0.5393	0.4562	0.4013	0.94	0.7184	0.6507	0.6178
	0.80	0.6260	0.5035	0.4324	0.92	0.8178	0.7111	0.6664
	0.75	0.6804	0.5330	0.4506	0.90	0.8692	0.7358	0.6820
	0.70	0.7191	0.5569	0.4682	0.88	0.9000	0.7426	0.6803
-0.75	1.00	0.1118	0.0984	0.0457	1.00	0.0954	0.0737	0.0298
	0.95	0.1702	0.1556	0.1425	0.98	0.2137	0.1942	0.1735
	0.90	0.3299	0.2730	0.2389	0.96	0.4256	0.3261	0.2658
	0.85	0.4924	0.3911	0.3443	0.94	0.6070	0.4118	0.3175
	0.80	0.6289	0.5078	0.4579	0.92	0.7410	0.4608	0.3471
	0.75	0.7360	0.6233	0.5789	0.90	0.8311	0.4915	0.3702
	0.70	0.8197	0.7300	0.6944	0.88	0.8902	0.5150	0.3906

[a]The table presents size-corrected rejection frequencies for the likelihood ratio test $\widehat{\text{LR}}_T^d$, DF-GLS test of ERS, and MP-GLS test of Ng and Perron (2001) using 10^6 replications of the model (A1) allowing only for a constant mean. All three tests use the MAIC of Perron and Qu (2007) to select lag length.

TABLE A.III

SIMULATED REJECTION FREQUENCIES OF UNIT ROOT TESTS, LINEAR TREND CASE[a]

θ	ρ	$\widehat{\text{LR}}_T^d$	DF-GLS	MP-GLS	ρ	$\widehat{\text{LR}}_T^d$	DF-GLS	MP-GLS
		$T = 100$				$T = 250$		
0.00	1.000	0.0328	0.0249	0.0172	1.00	0.0350	0.0272	0.0254
	0.925	0.1248	0.0995	0.0721	0.97	0.1372	0.1114	0.1034
	0.850	0.3869	0.3311	0.2667	0.94	0.4409	0.3846	0.3665
	0.775	0.6208	0.5722	0.5135	0.91	0.7155	0.6686	0.6553
	0.700	0.7100	0.6820	0.6497	0.88	0.8238	0.8021	0.8012
	0.625	0.7403	0.7178	0.6982	0.85	0.8564	0.8447	0.8487
	0.550	0.7556	0.7294	0.7102	0.82	0.8692	0.8583	0.8623
−0.25	1.000	0.0446	0.0370	0.0313	1.00	0.0412	0.0336	0.0334
	0.925	0.1405	0.1220	0.1067	0.97	0.1470	0.1254	0.1254
	0.850	0.3484	0.3146	0.2847	0.94	0.4146	0.3701	0.3686
	0.775	0.5292	0.4838	0.4443	0.91	0.6503	0.6045	0.5987
	0.700	0.6302	0.5775	0.5339	0.88	0.7642	0.7251	0.7188
	0.625	0.6843	0.6263	0.5743	0.85	0.8122	0.7753	0.7650
	0.550	0.7158	0.6510	0.5925	0.82	0.8351	0.7963	0.7788
−0.50	1.000	0.0632	0.0499	0.0376	1.00	0.0480	0.0378	0.0316
	0.925	0.1687	0.1401	0.1069	0.97	0.1575	0.1275	0.1068
	0.850	0.3612	0.3037	0.2393	0.94	0.3933	0.3307	0.2802
	0.775	0.5196	0.4407	0.3549	0.91	0.5934	0.5079	0.4337
	0.700	0.6259	0.5332	0.4417	0.88	0.7051	0.6092	0.5239
	0.625	0.6960	0.6045	0.5133	0.85	0.7662	0.6595	0.5658
	0.550	0.7508	0.6660	0.5818	0.82	0.8052	0.6856	0.5835
−0.75	1.000	0.1436	0.1088	0.0733	1.00	0.0733	0.0477	0.0176
	0.925	0.3025	0.2387	0.1694	0.97	0.1950	0.1288	0.0503
	0.850	0.5435	0.4482	0.3542	0.94	0.4076	0.2695	0.1189
	0.775	0.7356	0.6480	0.5627	0.91	0.5941	0.3913	0.1987
	0.700	0.8626	0.8048	0.7468	0.88	0.7285	0.4830	0.2758
	0.625	0.9350	0.9054	0.8738	0.85	0.8165	0.5539	0.3512
	0.550	0.9715	0.9585	0.9437	0.82	0.8773	0.6175	0.4301

[a]The table presents simulated rejection frequencies for the likelihood ratio test $\widehat{\text{LR}}_T^d$, DF-GLS test of ERS, and MP-GLS test of Ng and Perron (2001) using 10^6 replications of the model (A1) allowing for a linear trend. The entries for $\rho = 1.00$ are the simulated sizes of the tests. All three tests use the MAIC of Perron and Qu (2007) to select lag length.

TABLE A.IV

SIZE-CORRECTED REJECTION FREQUENCIES OF UNIT ROOT TESTS, LINEAR TREND CASE[a]

θ	ρ	$\widehat{\text{LR}}_T^d$	DF-GLS	MP-GLS	ρ	$\widehat{\text{LR}}_T^d$	DF-GLS	MP-GLS
			$T = 100$				$T = 250$	
0.00	1.000	0.0500	0.0500	0.0500	1.00	0.0500	0.0500	0.0500
	0.925	0.1787	0.1809	0.1810	0.97	0.1854	0.1874	0.1875
	0.850	0.4809	0.4785	0.4821	0.94	0.5288	0.5271	0.5293
	0.775	0.6881	0.6849	0.6935	0.91	0.7788	0.7777	0.7837
	0.700	0.7543	0.7543	0.7640	0.88	0.8599	0.8637	0.8711
	0.625	0.7780	0.7732	0.7766	0.85	0.8831	0.8879	0.8936
	0.550	0.7882	0.7762	0.7722	0.82	0.8938	0.8957	0.8983
-0.25	1.000	0.0446	0.0370	0.0313	1.00	0.0412	0.0336	0.0334
	0.925	0.1541	0.1587	0.1600	0.97	0.1747	0.1769	0.1770
	0.850	0.3715	0.3741	0.3728	0.94	0.4646	0.4629	0.4620
	0.775	0.5509	0.5384	0.5266	0.91	0.6926	0.6854	0.6819
	0.700	0.6472	0.6210	0.5961	0.88	0.7930	0.7812	0.7747
	0.625	0.6977	0.6595	0.6227	0.85	0.8345	0.8189	0.8069
	0.550	0.7264	0.6798	0.6319	0.82	0.8550	0.8332	0.8143
-0.50	1.000	0.0632	0.0499	0.0376	1.00	0.0480	0.0378	0.0316
	0.925	0.1368	0.1402	0.1352	0.97	0.1631	0.1612	0.1584
	0.850	0.3137	0.3040	0.2804	0.94	0.4025	0.3879	0.3688
	0.775	0.4761	0.4409	0.3931	0.91	0.6021	0.5644	0.5216
	0.700	0.5898	0.5335	0.4716	0.88	0.7120	0.6549	0.5960
	0.625	0.6680	0.6046	0.5368	0.85	0.7718	0.6982	0.6261
	0.550	0.7284	0.6661	0.5986	0.82	0.8104	0.7194	0.6362
-0.75	1.000	0.1436	0.1088	0.0733	1.00	0.0733	0.0477	0.0176
	0.925	0.1306	0.1297	0.1266	0.97	0.1388	0.1339	0.1227
	0.850	0.3279	0.3148	0.2989	0.94	0.3192	0.2770	0.2288
	0.775	0.5723	0.5436	0.5161	0.91	0.5016	0.3991	0.3151
	0.700	0.7740	0.7437	0.7165	0.88	0.6418	0.4905	0.3834
	0.625	0.8974	0.8765	0.8571	0.85	0.7431	0.5600	0.4440
	0.550	0.9576	0.9470	0.9349	0.82	0.8119	0.6225	0.5074

[a]The table presents size-corrected rejection frequencies for the likelihood ratio test $\widehat{\text{LR}}_T^d$, DF-GLS test of ERS, and MP-GLS test of Ng and Perron (2001) using 10^6 replications of the model (A1) allowing for a linear trend. All three tests use the MAIC of Perron and Qu (2007) to select lag length.

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