Econometrica Supplementary Material

SUPPLEMENT TO "INFERRING INEQUALITY WITH HOME PRODUCTION"

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APPENDIX A: PROOFS

IN THIS APPENDIX, WE DERIVE THE EQUILIBRIUM allocations presented in Table I in the main text and prove the observational equivalence theorem. We proceed in four steps. First, in anticipation of the no-trade result, we solve the planner problems. Second, we postulate equilibrium allocations and prices using the solutions to the planner problems. Third, we establish that the postulated equilibrium allocations and prices indeed constitute an equilibrium as defined in Section 2 in the main text. Finally, we show how to invert the equilibrium allocations and identify the sources of heterogeneity leading to these allocations.

A.1. *Preliminaries*

In what follows, we define the following state vectors. The sources of heterogeneity differentiating households within each island ℓ is given by the vector ζ^j:

$$\zeta_t^j = (\kappa_t^j, \upsilon_t^\varepsilon) \in Z_t^j. \tag{A.1}$$

Households can trade bonds within each island contingent on the vector s^j:

$$s_t^j = (B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon). \tag{A.2}$$

We define a household ι by a sequence of all dimensions of heterogeneity:

$$\iota = \{\theta_K^j, D_K^j, B^j, \alpha^j, \kappa^j, \upsilon^\varepsilon\}. \tag{A.3}$$

Finally, the history of all sources of heterogeneity up to period t is given by the vector

$$\sigma_t^j = (\theta_{K,t}^j, D_{K,t}^j, B_t^j, \alpha_t^j, \kappa_t^j, \upsilon_t^\varepsilon, \dots, \theta_{K,j}^j, D_{K,j}^j, B_j^j, \alpha_j^j, \kappa_j^j, \upsilon_j^\varepsilon). \tag{A.4}$$

We denote conditional probabilities by $f^{t,j}(\cdot|\cdot)$. For example, the probability that we observe σ_t^j conditional on σ_{t-1}^j is $f^{t,j}(\sigma_t^j|\sigma_{t-1}^j)$ and the probability that we observe s_t^j conditional on s_{t-1}^j is $f^{t,j}(s_t^j|s_{t-1}^j)$.

We use υ to denote innovations to processes and use Φ_υ to denote the distribution of the innovation. We allow the distributions of innovations to vary over time, $\{\Phi_{\upsilon_t^\alpha}, \Phi_{\upsilon_t^B}, \Phi_{\upsilon_t^\kappa}, \Phi_{\upsilon_t^\varepsilon}, \Phi_{\theta_{K,t}}^j, \Phi_{D_{K,t}}^j\}$, and the initial distributions to vary by cohorts j,

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$\Phi_j^j(\theta_{K,j}^j, D_{K,j}^j, B_j^j, \alpha_j^j, \kappa_j^j)$. We assume that both $\theta_{K,t}^j$ and $D_{K,t}^j$ are orthogonal to the innovations $\{\upsilon_t^B, \upsilon_t^\alpha, \upsilon_t^\kappa, \upsilon_t^\varepsilon\}$ and that all innovations are drawn independently from each other.

A.2. *Planner Problems*

In every period t and in every island ℓ, the planner solves a static problem which consists of finding the allocations maximizing average utility for households on the island subject to an aggregate resource constraint. We omit j, t and ℓ from the notation for clarity.

A.2.1. *No Home Production,* $\omega_K = 0$

The planner chooses an allocation $\{c_M, h_M\}$ to maximize

$$\int_Z \left[\frac{c_M^{1-\gamma} - 1}{1-\gamma} - \frac{\big(\exp(B) h_M\big)^{1+\frac{1}{\eta}}}{1 + \dfrac{1}{\eta}} \right] \mathrm{d}\Phi_\zeta(\zeta) \tag{A.5}$$

subject to an island resource constraint for market goods

$$\int_Z c_M \,\mathrm{d}\Phi_\zeta(\zeta) = \int_Z \tilde{z}_M h_M \,\mathrm{d}\Phi_\zeta(\zeta). \tag{A.6}$$

Denoting by $\mu(\alpha, B)$ the multiplier on the island resource constraint, the solution is characterized by the first-order conditions (for every household ι)

$$[c_M] : c_M^{-\gamma} = \mu(\alpha, B), \tag{A.7}$$

$$[h_M] : \exp(B)^{1+\frac{1}{\eta}} h_M^{\frac{1}{\eta}} = \tilde{z}_M \mu(\alpha, B). \tag{A.8}$$

Equation (A.7) implies that market consumption is equal for every household ι on the island and, thus, there is full consumption insurance. Combining equations (A.6)–(A.8), we solve for market consumption and market hours for every ι:

$$c_M = \left[\frac{\displaystyle\int_Z \tilde{z}_M^{1+\eta} \,\mathrm{d}\Phi_\zeta(\zeta)}{\exp\left(\eta\left(1 + \dfrac{1}{\eta}\right)B\right)} \right]^{\frac{\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}}, \tag{A.9}$$

$$h_M = \tilde{z}_M^{\eta} \frac{\left[\displaystyle\int_Z \tilde{z}_M^{1+\eta} \,\mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{\gamma}{\frac{1}{\eta}+\gamma}}}{\exp\left(\left(1 + \dfrac{1}{\eta}\right)B\right)^{\frac{1}{\frac{1}{\eta}+\gamma}}}. \tag{A.10}$$

A.2.2. *Home Production,* $\omega_K > 0$

The planner chooses $\{c_M, h_M, h_K\}$ to maximize

$$\int_Z \left[\log c - \frac{\left(\exp(B)h_M + \sum \exp(D_K)h_K\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right] \mathrm{d}\Phi_\zeta(\zeta), \tag{A.11}$$

where consumption is given by $c = (c_M^{\frac{\phi-1}{\phi}} + \sum(\theta_K h_K)^{\frac{\phi-1}{\phi}})^{\frac{\phi}{\phi-1}}$ subject to the island market resource constraint (A.6).

Denoting by $\mu(\alpha, B, D_K, \theta_K)$ the multiplier on the island resource constraint, the solution to this problem is characterized by the first-order conditions (for every household ι)

$$[c_M] : \left(c^{\frac{\phi-1}{\phi}}\right)^{-1} c_M^{-\frac{1}{\phi}} = \mu(\alpha, B, D_K, \theta_K), \tag{A.12}$$

$$[h_M] : \left(\exp(B)h_M + \sum \exp(D_K)h_K\right)^{\frac{1}{\eta}} = \tilde{z}_M \frac{\mu(\alpha, B, D_K, \theta_K)}{\exp(B)}, \tag{A.13}$$

$$[h_K] : \left(\exp(B)h_M + \sum \exp(D_K)h_K\right)^{\frac{1}{\eta}} = \theta_K^{\frac{\phi-1}{\phi}} \left(c^{\frac{\phi-1}{\phi}}\right)^{-1} \frac{h_K^{-\frac{1}{\phi}}}{\exp(D_K)}. \tag{A.14}$$

Combining equations (A.12)–(A.14), we solve for the ratio of home hours to consumption:

$$\frac{c_M}{h_K} = \left(\frac{\exp(D_K)}{\exp(B)/\tilde{z}_M}\right)^{\phi} \theta_K^{1-\phi}. \tag{A.15}$$

Substituting these ratios into equations (A.12)–(A.14), we derive

$$c_M = \frac{1}{\mu(\alpha, B, D_K, \theta_K)} \frac{1}{1 + \sum \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}, \tag{A.16}$$

$$h_K = \frac{1}{\mu(\alpha, B, D_K, \theta_K)} \frac{\theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi}}{1 + \sum \theta_K^{\phi-1} \left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}. \tag{A.17}$$

These expressions yield solutions for $\{c_M, h_M, h_K\}$ given a multiplier $\mu(\alpha, B, D_K, \theta_K)$. The multiplier is equal to the inverse of the market value of total consumption:

$$c_M + \tilde{z}_M \sum \frac{\exp(D_K)}{\exp(B)} h_K = \frac{1}{\mu(\alpha, B, D_K, \theta_K)}. \tag{A.18}$$

The equality follows from equations (A.16) and (A.17).

Substituting equation (A.13) into equation (A.6), we obtain the solution for $\mu(\alpha, B, D_K, \theta_K)$:

$$\mu(\alpha, B, D_K, \theta_K) = \frac{\exp(B)}{\left(\int_Z \tilde{z}_M^{1+\eta}\, \mathrm{d}\Phi_\zeta(\zeta)\right)^{\frac{1}{1+\eta}}}. \tag{A.19}$$

The denominator is an expectation independent of ζ. Therefore, μ is independent of ζ. We also note that $\mu(\alpha, B, D_K, \theta_K)$ in the model with home production equals $\mu(\alpha, B)$ in the model without home production under $\gamma = 1$. Given this solution for $\mu(\alpha, B, D_K, \theta_K)$, we obtain the solutions

$$c_M = \frac{\left[\int_Z \tilde{z}_M^{1+\eta}\, \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B)} \frac{1}{1 + \sum \theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}, \tag{A.20}$$

$$h_K = \frac{\left[\int_Z \tilde{z}_M^{1+\eta}\, \mathrm{d}\Phi_\zeta(\zeta)\right]^{\frac{1}{1+\eta}}}{\exp(B)} \frac{\theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi}}{1 + \sum \theta_K^{\phi-1}\left(\frac{\exp(B)/\tilde{z}_M}{\exp(D_K)}\right)^{\phi-1}}, \tag{A.21}$$

$$h_M = \tilde{z}_M^{\eta} \frac{\left[\int_Z \tilde{z}_M^{1+\eta}\, \mathrm{d}\Phi_\zeta(\zeta)\right]^{-\frac{1}{1+\frac{1}{\eta}}}}{\exp(B)} - \sum \frac{\exp(D_K)}{\exp(B)} h_K.$$

A.3. *Postulating Equilibrium*

We postulate an equilibrium in four steps.

Step 1. We postulate that the equilibrium features no trade across islands, $x(\zeta_{t+1}^j; \iota) = 0\ \forall \iota, \zeta_{t+1}^j$.

Step 2. We postulate that the solutions $\{c_{M,t}, h_{M,t}\}$ for the model without home production and $\{c_{M,t}, h_{M,t}, h_{K,t}\}$ for the model with home production from the planner problems in Section A.2 constitute components of the equilibrium for each model.

Step 3. We use the sequential budget constraints to postulate equilibrium holdings for the state-contingent bonds $b^\ell(s_t^j; \iota)$ which are traded within islands. For the models without home production, these are given by

$$b^\ell\left(s_t^j; \iota\right) = \mathbb{E}\left[\sum_{n=0}^{\infty} (\beta\delta)^n \frac{\mu_{t+n}\left(\alpha_{t+n}^j, B_{t+n}^j\right)}{\mu_t\left(\alpha_t^j, B_t^j\right)} (c_{M,t+n} - \tilde{y}_{t+n})\right], \tag{A.22}$$

where $\tilde{y} = \tilde{z}_M h_M = (1 - \tau_0) z_M^{1-\tau_1} h_M$ is after-tax labor income.

For the model with home production, state-contingent bonds $b^\ell(s_t^j; \iota)$ are given by the same expression but using the marginal utility $\mu(\alpha, B, D_K, \theta_K)$ instead of $\mu(\alpha, B)$. As shown above, the two marginal utilities are characterized by the same equation (A.19) under $\gamma = 1$.

Step 4. We use the intertemporal marginal rates of substitution implied by the planner solutions to postulate asset prices for $b^\ell(s^j_{t+1};\iota)$ and $x(\zeta^j_{t+1};\iota)$. For the model without home production, we obtain

$$\begin{aligned}
q^\ell_b\big(s^j_{t+1}\big) = {} & \beta\delta\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon^B_{t+1}\right) \\
& \times\exp\left(-(1-\tau_1)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon^\alpha_{t+1}\right) \\
& \times\left[\frac{\int\exp\big(A\upsilon^\kappa_{t+1}\big)\,\mathrm{d}\Phi_{\upsilon^\kappa_{t+1}}\big(\upsilon^\kappa_{t+1}\big)\int\exp\big(A\upsilon^\varepsilon_{t+1}\big)\,\mathrm{d}\Phi_{\upsilon^\varepsilon_{t+1}}\big(\upsilon^\varepsilon_{t+1}\big)}{\int\exp\big(A\upsilon^\varepsilon_t\big)\,\mathrm{d}\Phi_{\upsilon^\varepsilon_t}\big(\upsilon^\varepsilon_t\big)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} \\
& \times f^{t+1,j}\big(s^j_{t+1}|s^j_t\big),
\end{aligned} \tag{A.23}$$

$$\begin{aligned}
q_x(Z_{t+1}) = {} & \beta\delta\int\exp\left(\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon^B_{t+1}\right)\mathrm{d}\Phi_{\upsilon^B_{t+1}}\big(\upsilon^B_{t+1}\big) \\
& \times\int\exp\left(-(1-\tau_1)\gamma\frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma}\upsilon^\alpha_{t+1}\right)\mathrm{d}\Phi_{\upsilon^\alpha_{t+1}}\big(\upsilon^\alpha_{t+1}\big) \\
& \times\left[\frac{\int\exp\big(A\upsilon^\kappa_{t+1}\big)\,\mathrm{d}\Phi_{\upsilon^\kappa_{t+1}}\big(\upsilon^\kappa_{t+1}\big)\int\exp\big(A\upsilon^\varepsilon_{t+1}\big)\,\mathrm{d}\Phi_{\upsilon^\varepsilon_{t+1}}\big(\upsilon^\varepsilon_{t+1}\big)}{\int\exp\big(A\upsilon^\varepsilon_t\big)\,\mathrm{d}\Phi_{\upsilon^\varepsilon_t}\big(\upsilon^\varepsilon_t\big)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} \\
& \times\mathbb{P}\big(\big(\upsilon^\kappa_{t+1},\upsilon^\varepsilon_{t+1}\big)\in Z_{t+1}\big),
\end{aligned} \tag{A.24}$$

where $A\equiv(1+\eta)(1-\tau_1)$. For the model with home production, we obtain the same expressions under $\gamma=1$.

A.4. *Verifying the Equilibrium Allocations and Prices*

We verify that the equilibrium postulated in Appendix A.3 constitutes an equilibrium by showing that the postulated allocations solve the households' problem and that all markets clear.

A.4.1. *Household Problem*

The problem for a household ι born in period j is described in the main text. We denote the Lagrange multiplier on the household's budget constraint by $\tilde{\mu}_t$. We drop ι from the notation for simplicity.

No Home Production, $\omega_K = 0$. The optimality conditions are

$$(\beta\delta)^{t-j} c_{M,t}^{-\gamma} f^{t,j}\left(\sigma_t^j | \sigma_j\right) = \tilde{\mu}_t, \tag{A.25}$$

$$(\beta\delta)^{t-j} \exp(B_t)^{1+\frac{1}{\eta}} (h_{M,t})^{\frac{1}{\eta}} f^{t,j}\left(\sigma_t^j | \sigma_j\right) = \tilde{z}_{M,t}^j \tilde{\mu}_t, \tag{A.26}$$

$$q_b^\ell\left(s_{t+1}^j\right) = \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t}, \tag{A.27}$$

$$q_x(Z_{t+1}) = \int \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \, \mathrm{d}v_{t+1}^B \, \mathrm{d}v_{t+1}^\alpha. \tag{A.28}$$

Comparing the planner solutions to the household solutions, we verify that they coincide for market consumption and hours when the multipliers are related by

$$\tilde{\mu}_t = (\beta\delta)^{t-j} f^{t,j}\left(\sigma_t^j | \sigma_j\right) \mu\left(\alpha_t^j, B_t^j\right). \tag{A.29}$$

Then the Euler equations become

$$q_b^\ell\left(s_{t+1}^j\right) = \beta\delta \frac{\mu\left(\alpha_{t+1}^j, B_{t+1}^j\right)}{\mu\left(\alpha_t^j, B_t^j\right)} f^{t+1,j}\left(\sigma_{t+1}^j | \sigma_t^j\right), \tag{A.30}$$

$$q_x(Z_{t+1}) = \beta\delta \int \frac{\mu\left(\alpha_{t+1}^j, B_{t+1}^j\right)}{\mu\left(\alpha_t^j, B_t^j\right)} f^{t+1,j}\left(\sigma_{t+1}^j | \sigma_t^j\right) \mathrm{d}v_{t+1}^B \, \mathrm{d}v_{t+1}^\alpha. \tag{A.31}$$

Home Production, $\omega_K > 0$. Total hours, taking into account the respective disutility, are $\tilde{h} = \exp(B)(h_M) + \sum \exp(D_K)(h_K)$. Using again the correspondence between the planner and the household first-order conditions to relate the multipliers $\tilde{\mu}_t$ and $\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)$, we write the optimality conditions as

$$\frac{\tilde{z}_{M,t}}{\exp(B_t)} \left(c^{\frac{\phi-1}{\phi}}\right)^{-1} c_{M,t}^{-\frac{1}{\phi}} = \tilde{h}_t^{\frac{1}{\eta}}, \tag{A.32}$$

$$\frac{\theta_{K,t}^{\frac{\phi-1}{\phi}}}{\exp(D_{K,t})} \left(c^{\frac{\phi-1}{\phi}}\right)^{-1} h_{K,t}^{-\frac{1}{\phi}} = \tilde{h}_t^{\frac{1}{\eta}}, \tag{A.33}$$

$$\begin{aligned} q_b^\ell\left(s_{t+1}^j\right) = \beta\delta \int & \frac{\mu\left(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j\right)}{\mu\left(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j\right)} \\ & \times f^{t+1,j}\left(\sigma_{t+1}^j | \sigma_t^j\right) \mathrm{d}\theta_{K,t+1}^j \, \mathrm{d}D_{K,t+1}^j, \end{aligned} \tag{A.34}$$

$$\begin{aligned} q_x(Z_{t+1}) = \beta\delta \int & \frac{\mu\left(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j\right)}{\mu\left(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j\right)} \\ & \times f^{t+1,j}\left(\sigma_{t+1}^j | \sigma_t^j\right) \mathrm{d}v_{t+1}^B \, \mathrm{d}v_{t+1}^\alpha \, \mathrm{d}\theta_{K,t+1}^j \, \mathrm{d}D_{K,t+1}^j. \end{aligned} \tag{A.35}$$

A.4.2. *Euler Equations*

We next verify that the Euler equations are satisfied at the postulated allocations and prices.

No Home Production, $\omega_K = 0$. Using the marginal utility of market consumption of the planner problem $\mu(\alpha_t^j, B_t^j)$, we write the Euler equation for the state-contingent bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as

$$
\begin{aligned}
q_b^\ell(s_{t+1}^j) &= \beta\delta \frac{\mu(\alpha_{t+1}^j, B_{t+1}^j)}{\mu(\alpha_t^j, B_t^j)} f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j) \\
&= \beta\delta \frac{\exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} B_{t+1}^j\right)\left[\int (\tilde{z}_{M,t+1}^j)^{1+\eta}\, \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} B_t^j\right)\left[\int (\tilde{z}_{M,t}^j)^{1+\eta}\, \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}} \\
&\quad \times f^{t+1,j}(\sigma_{t+1}^j|\sigma_t^j),
\end{aligned}
\tag{A.36}
$$

where the second line follows from equations (A.7) and (A.9). Using that B_t^j follows a random walk process with innovation υ_t^B, we rewrite $q_b^\ell(s_{t+1}^j)$ as

$$
\begin{aligned}
q_b^\ell(s_{t+1}^j) &= \beta\delta \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} \upsilon_{t+1}^B\right) \\
&\quad \times \frac{\left[\int (\tilde{z}_{M,t+1}^j)^{1+\eta}\, \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}}{\left[\int (\tilde{z}_{M,t}^j)^{1+\eta}\, \mathrm{d}\Phi_{\zeta_t^j}(\zeta_t^j)\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}} f^{t+1,j}(s_{t+1}^j|s_t^j).
\end{aligned}
\tag{A.37}
$$

To simplify the fraction in $q_b^\ell(s_{t+1}^j)$, we use that

$$
\tilde{z}_{M,t+1}^j = (1-\tau_0)\exp\big((1-\tau_1)(\alpha_t^j + \upsilon_{t+1}^\alpha + \kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon)\big).
$$

The expectation over the random variables in the numerator is given by

$$
\begin{aligned}
&\int \exp\big(A(\kappa_t^j + \upsilon_{t+1}^\kappa + \upsilon_{t+1}^\varepsilon)\big)\, \mathrm{d}\Phi_{\zeta_{t+1}^j}(\zeta_{t+1}^j) \\
&\quad = \int \exp(A\kappa_t^j)\, \mathrm{d}\Phi_{\kappa_t^j}(\kappa_t^j) \int \exp(A\upsilon_{t+1}^\kappa)\, \mathrm{d}\Phi_{\upsilon_{t+1}^\kappa}(\upsilon_{t+1}^\kappa)
\end{aligned}
$$

$$\times \int \exp\left(A v^{\varepsilon}_{t+1}\right) \mathrm{d}\Phi_{v^{\varepsilon}_{t+1}}\left(v^{\varepsilon}_{t+1}\right), \tag{A.38}$$

where the final equality follows from the assumption that the innovations are drawn independently. Similarly, the expectation over the random variables in the denominator equals

$$\int \exp\left(A\kappa^{j}_{t}\right) \mathrm{d}\Phi_{\kappa^{j},t}\left(\kappa^{j}_{t}\right) \int \exp\left(A v^{\varepsilon}_{t}\right) \mathrm{d}\Phi_{v^{\varepsilon}_{t}}\left(v^{\varepsilon}_{t}\right). \tag{A.39}$$

As a result, the price $q^{\ell}_{b}(s^{j}_{t+1})$ is

$$\begin{aligned} q^{\ell}_{b}\left(s^{j}_{t+1}\right) = \beta\delta \exp&\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} v^{B}_{t+1}\right) \\ &\times \exp\left(-(1-\tau_1)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} v^{\alpha}_{t+1}\right) \\ &\times \left[\frac{\int \exp\left(A v^{\kappa}_{t+1}\right) \mathrm{d}\Phi_{v^{\kappa}_{t+1}}\left(v^{\kappa}_{t+1}\right) \int \exp\left(A v^{\varepsilon}_{t+1}\right) \mathrm{d}\Phi_{v^{\varepsilon}_{t+1}}\left(v^{\varepsilon}_{t+1}\right)}{\int \exp\left(A v^{\varepsilon}_{t}\right) \mathrm{d}\Phi_{v^{\varepsilon}_{t}}\left(v^{\varepsilon}_{t}\right)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}} \\ &\times f^{t+1,j}\left(s^{j}_{t+1}|s^{j}_{t}\right), \end{aligned} \tag{A.40}$$

where $f^{t+1,j}(s^{j}_{t+1}|s^{j}_{t}) = f(v^{B}_{t+1})f(v^{\alpha}_{t+1})f(v^{\kappa}_{t+1})f(v^{\varepsilon}_{t+1})$. This confirms our guess in equation (A.23). The key observation is that the distributions for next-period innovations are independent of the current period state and, therefore, the term in square brackets is independent of the state vector which differentiates islands ℓ. As a result, all islands ℓ have the same state-contingent bond prices: $q^{\ell}_{b}(s^{j}_{t+1}) = Q_{b}(v^{B}_{t+1}, v^{\alpha}_{t+1})$.

We next calculate the state-contingent bond price for a set of states $\mathcal{V}_{t+1} \subseteq \mathbb{V}_{t+1}$:

$$\begin{aligned} q^{\ell}_{b}(\mathcal{V}_{t+1}) = \beta\delta \int_{\mathcal{V}^{B}} \exp&\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} v^{B}_{t+1}\right) \mathrm{d}\Phi_{v^{B}_{t+1}}\left(v^{B}_{t+1}\right) \\ &\times \int_{\mathcal{V}^{\alpha}} \exp\left(-(1-\tau_1)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} v^{\alpha}_{t+1}\right) \mathrm{d}\Phi_{v^{\alpha}_{t+1}}\left(v^{\alpha}_{t+1}\right) \\ &\times \left[\frac{\int \exp\left(A v^{\kappa}_{t+1}\right) \mathrm{d}\Phi_{v^{\kappa}_{t+1}}\left(v^{\kappa}_{t+1}\right) \int \exp\left(A v^{\varepsilon}_{t+1}\right) \mathrm{d}\Phi_{v^{\varepsilon}_{t+1}}\left(v^{\varepsilon}_{t+1}\right)}{\int \exp\left(A v^{\varepsilon}_{t}\right) \mathrm{d}\Phi_{v^{\varepsilon}_{t}}\left(v^{\varepsilon}_{t}\right)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}. \end{aligned} \tag{A.41}$$

Similarly, all islands face the same price $q_b^\ell(\mathcal{V}_{t+1}) = Q_b(\mathcal{V}_{t+1})$.

Finally, we calculate the price for a claim which does not depend on the realization of $(v_{t+1}^B, v_{t+1}^\alpha)$:

$$q_b^\ell(\mathbb{V}_{t+1}) = \beta\delta \int_{\mathbb{V}^B} \exp\left(\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} v_{t+1}^B\right) \mathrm{d}\Phi_{v_{t+1}^B}\left(v_{t+1}^B\right)$$
$$\times \int_{\mathbb{V}^\alpha} \exp\left(-(1-\tau_1)\gamma \frac{\frac{1}{\eta}+1}{\frac{1}{\eta}+\gamma} v_{t+1}^\alpha\right) \mathrm{d}\Phi_{v_{t+1}^\alpha}\left(v_{t+1}^\alpha\right)$$
$$\times \left[\frac{\int \exp\left(Av_{t+1}^\kappa\right) \mathrm{d}\Phi_{v_{t+1}^\kappa}\left(v_{t+1}^\kappa\right) \int \exp\left(Av_{t+1}^\varepsilon\right) \mathrm{d}\Phi_{v_{t+1}^\varepsilon}\left(v_{t+1}^\varepsilon\right)}{\int \exp\left(Av_t^\varepsilon\right) \mathrm{d}\Phi_{v_t^\varepsilon}\left(v_t^\varepsilon\right)}\right]^{-\frac{\frac{\gamma}{\eta}}{\frac{1}{\eta}+\gamma}}. \tag{A.42}$$

All islands face the same price $q_b^\ell(\mathbb{V}_{t+1}) = Q_b(\mathbb{V}_{t+1})$.

By no arbitrage, the prices of bonds x and b, which are contingent on the same set of states, must be equalized. Therefore, the price of a claim traded across islands for some set Z_{t+1} is equalized across islands at the no-trade equilibrium and is given by

$$q_x(Z_{t+1}) = \mathbb{P}\left(\left(v_{t+1}^\kappa, v_{t+1}^\varepsilon\right) \in Z_{t+1}\right) Q_b(\mathbb{V}_{t+1}), \tag{A.43}$$

where $\mathbb{P}((v_{t+1}^\kappa, v_{t+1}^\varepsilon) \in Z_{t+1})$ is the probability of $(v_{t+1}^\kappa, v_{t+1}^\varepsilon)$ being a member of Z_{t+1}. The expression for $q_x(Z_{t+1})$ confirms our guess in equation (A.24)

Home Production, $\omega_K > 0$. For the model with home production, we use the solution for the marginal utility of market consumption in the planner problem $\mu(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j)$ to write the Euler equation for the state-contingent bonds $b^\ell(s_{t+1}^j)$ at the postulated equilibrium as

$$q_b^\ell\left(s_{t+1}^j\right) = \beta\delta \int \frac{\mu\left(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j\right)}{\mu\left(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j\right)} f^{t+1,j}\left(\sigma_{t+1}^j | \sigma_t^j\right) \mathrm{d}\theta_{K,t+1}^j \, \mathrm{d}D_{K,t+1}^j$$
$$= \beta\delta \int \frac{\exp\left(B_{t+1}^j\right)\left[\int \left(\tilde{z}_{M,t+1}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_{t+1}^j}\left(\zeta_{t+1}^j\right)\right]^{-\frac{1}{1+\eta}}}{\exp\left(B_t^j\right)\left[\int \left(\tilde{z}_{M,t}^j\right)^{1+\eta} \mathrm{d}\Phi_{\zeta_t^j}\left(\zeta_t^j\right)\right]^{-\frac{1}{1+\eta}}}$$
$$\times f^{t+1,j}\left(\sigma_{t+1}^j | \sigma_t^j\right) \mathrm{d}\theta_{K,t+1}^j \, \mathrm{d}D_{K,t+1}^j, \tag{A.44}$$

where the second equality follows from equation (A.19). Using equations (A.38) and (A.39), and the fact that $\theta_{K,t+1}^j$ and $D_{K,t+1}^j$ are orthogonal to the innovations, the price

$q_b^\ell(s_{t+1}^j)$ simplifies to

$$q_b^\ell\big(s_{t+1}^j\big) = \beta\delta \exp\big(v_{t+1}^B - (1-\tau_1)v_{t+1}^\alpha\big) \times \left[\frac{\int \exp\big(Av_{t+1}^\kappa\big)\,\mathrm{d}\Phi_{v_{t+1}^\kappa}\big(v_{t+1}^\kappa\big) \int \exp\big(Av_{t+1}^\varepsilon\big)\,\mathrm{d}\Phi_{v_{t+1}^\varepsilon}\big(v_{t+1}^\varepsilon\big)}{\int \exp\big(Av_t^\varepsilon\big)\,\mathrm{d}\Phi_{v_t^\varepsilon}\big(v_t^\varepsilon\big)}\right]^{-\frac{1}{1+\eta}} \times f^{t+1,j}\big(s_{t+1}^j | s_t^j\big). \tag{A.45}$$

The price $q_b^\ell(s_{t+1}^j)$ is identical to equation (A.40) for the model without home production under $\gamma = 1$. The remainder of the argument is identical to the argument for the model without home production.

A.4.3. *Household's Budget Constraint*

We now verify our guess for the state-contingent bond positions $b_t^\ell(s_t^j)$ and confirm that the household budget constraint holds at the postulated equilibrium allocations. The proof to this claim is identical for both models. We define the deficit term by $d_t \equiv c_{M,t} - \tilde{y}_t$. Using the expression for the price $q_b^\ell(s_{t+1}^j)$ in equation (A.30), the budget constraint at the no-trade equilibrium is given by

$$b_t^\ell\big(s_t^j\big) = d_t + \beta\delta \int \frac{\mu\big(\alpha_{t+1}^j, B_{t+1}^j, D_{K,t+1}^j, \theta_{K,t+1}^j\big)}{\mu\big(\alpha_t^j, B_t^j, D_{K,t}^j, \theta_{K,t}^j\big)} b_{t+1}^\ell\big(s_{t+1}^j\big) \times f^{t+1}\big(\sigma_{t+1}^j | \sigma_t^j\big)\,\mathrm{d}s_{t+1}^j\,\mathrm{d}\theta_{K,t+1}^j\,\mathrm{d}D_{K,t+1}^j.$$

By substituting forward using equation (A.30), we confirm the guess for $b_t^\ell(s_t^j)$ in equation (A.22) and show that the household budget constraint holds at the postulated equilibrium allocations.

A.4.4. *Goods Market Clearing*

Aggregating the resource constraints in every island, we obtain that the allocations solving the planner problems satisfy the aggregate goods market clearing condition

$$\int_\iota c_{M,t}\,\mathrm{d}\Phi(\iota) + G_t = \int_\iota z_{M,t} h_{M,t}\,\mathrm{d}\Phi(\iota). \tag{A.46}$$

A.4.5. *Asset Market Clearing*

We now confirm that asset markets clear. The asset market clearing conditions $\int_\iota x(\zeta_t^j;\iota)\,\mathrm{d}\Phi(\iota) = 0$ hold trivially in a no-trade equilibrium with $x(\zeta_t^j;\iota) = 0$. Next, we confirm that asset markets within each island ℓ also clear, that is, $\int_{\iota\in\ell} b^\ell(s_t^j;\iota)\,\mathrm{d}\Phi(\iota) = 0$ $\forall \ell, s_t^j$.

Omitting the household index ι for simplicity, we substitute the postulated state-contingent bond holdings in equation (A.22) into the asset market clearing conditions:

$$\begin{aligned}
&\int b^\ell\big(s_t^j\big)\,\mathrm{d}\Phi(\iota)\\
&\quad=\int \mathbb{E}\left[\sum_{n=0}^{\infty}(\beta\delta)^n\frac{\mu\big(\alpha_{t+n}^j,B_{t+n}^j,D_{K,t+n}^j,\theta_{K,t+n}^j\big)}{\mu\big(\alpha_t^j,B_t^j,D_{K,t}^j,\theta_{K,t}^j\big)}d_{t+n}\right]\mathrm{d}\Phi(\iota)\\
&\quad=\sum_{n=0}^{\infty}(\beta\delta)^n\int\frac{\mu\big(\alpha_{t+n}^j,B_{t+n}^j,D_{K,t+n}^j,\theta_{K,t+n}^j\big)}{\mu\big(\alpha_t^j,B_t^j,D_{K,t}^j,\theta_{K,t}^j\big)}d_{t+n}f\big(\sigma_{t+n}^j|\sigma_{t-1}^j\big)\,\mathrm{d}\sigma_{t+n}^j\,\mathrm{d}\Phi(\iota).
\end{aligned}$$

For simplicity, we omit conditioning on σ_{t-1}^j and write the density function as $f(\sigma_{t+n}^j|\sigma_{t-1}^j)=f(\{\upsilon_{t+n}^B\})f(\{\upsilon_{t+n}^\alpha\})f(\{\upsilon_{t+n}^\kappa\})f(\{\upsilon_{t+n}^\varepsilon\})f(\{\theta_{K,t+n}\})f(\{D_{K,t+n}\})$. Further, the expression for the growth in marginal utility is identical between the two models and equals

$$\mathcal{Q}\big(\upsilon_{t+n}^B,\upsilon_{t+n}^\alpha\big)\equiv\frac{\mu\big(\alpha_{t+n}^j,B_{t+n}^j,D_{K,t+n}^j,\theta_{K,t+n}^j\big)}{\mu\big(\alpha_t^j,B_t^j,D_{K,t}^j,\theta_{K,t}^j\big)}=\frac{\mu\big(\alpha_{t+n}^j,B_{t+n}^j\big)}{\mu\big(\alpha_t^j,B_t^j\big)}.$$

Hence, we write aggregate state-contingent bond holdings $\int b^\ell(s_t^j)\,\mathrm{d}\Phi(\iota)$ as

$$\begin{aligned}
&\sum_{n=0}^{\infty}(\beta\delta)^n\int\int\mathcal{Q}\big(\upsilon_{t+n}^B,\upsilon_{t+n}^\alpha\big)d_{t+n}f\big(\big\{\upsilon_{t+n}^B\big\}\big)f\big(\big\{\upsilon_{t+n}^\alpha\big\}\big)f\big(\big\{\upsilon_{t+n}^\kappa\big\}\big)f\big(\big\{\upsilon_{t+n}^\varepsilon\big\}\big)f\big(\{\theta_{K,t+n}\}\big)\cdots\\
&\qquad\cdots f\big(\{D_{K,t+n}\}\big)\,\mathrm{d}\big\{\upsilon_{t+n}^B\big\}\,\mathrm{d}\big\{\upsilon_{t+n}^\alpha\big\}\,\mathrm{d}\big\{\upsilon_{t+n}^\kappa\big\}\,\mathrm{d}\big\{\upsilon_{t+n}^\varepsilon\big\}\,\mathrm{d}\big\{\theta_{K,t+n}^j\big\}\,\mathrm{d}\big\{D_{K,t+n}^j\big\}\,\mathrm{d}\Phi(\iota)\\
&\quad=\sum_{n=0}^{\infty}(\beta\delta)^n\int\int d_{t+n}f\big(\big\{\upsilon_{t+n}^\kappa\big\}\big)f\big(\big\{\upsilon_{t+n}^\varepsilon\big\}\big)\,\mathrm{d}\big\{\upsilon_{t+n}^\kappa\big\}\,\mathrm{d}\big\{\upsilon_{t+n}^\varepsilon\big\}\,\mathrm{d}\Phi(\iota)\\
&\qquad\times\mathcal{Q}\big(\upsilon_{t+n}^B,\upsilon_{t+n}^\alpha\big)f\big(\big\{\upsilon_{t+n}^B\big\}\big)f\big(\big\{\upsilon_{t+n}^\alpha\big\}\big)f\big(\big\{\theta_{K,t+n}^j\big\}\big)\\
&\qquad\times f\big(\big\{D_{K,t+n}^j\big\}\big)\,\mathrm{d}\big\{\upsilon_{t+n}^B\big\}\,\mathrm{d}\big\{\upsilon_{t+n}^\alpha\big\}\,\mathrm{d}\big\{\theta_{K,t+n}^j\big\}\,\mathrm{d}\big\{D_{K,t+n}^j\big\}.
\end{aligned}$$

Recalling that the deficit terms equal $d_t=c_{M,t}-\tilde{y}_t$, the state-contingent bond market clearing condition holds because the first term is zero by the island-level resource constraint.

A.5. *Observational Equivalence Theorem*

We derive the identified sources of heterogeneity presented in Table II. We invert the equilibrium allocations in Table I and solve for the sources of heterogeneity leading to these allocations. The identification is unique up to constants because $\mathcal{C}_s$ appearing in the equations of Table II depends on the εs.

A.5.1. *No Home Production,* $\omega_K=0$

Given cross-sectional data $\{c_{M,t},h_{M,t},z_{M,t}\}_\iota$ and parameters $\gamma,\eta,\tau_0,\tau_1$, we show that there exists a unique $\{\alpha_t,\varepsilon_t,B_t\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι. We divide the solution for c_M with the solution for h_M to obtain

$$\frac{c_{M,t}}{h_{M,t}} = (1-\tau_0) z_{M,t}^{-\eta(1-\tau_1)} \exp\big((1-\tau_1)(1+\eta)\alpha_t\big) \times \int_{\zeta_t} \exp\big((1-\tau_1)(1+\eta)\varepsilon_t\big)\, \mathrm{d}\Phi_{\zeta_t^j}\big(\zeta_t^j\big). \tag{A.47}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, ε_t is also uniquely determined. Finally, we can use the solution for $c_{M,t}$ or $h_{M,t}$ in Table I to solve for B_t.

A.5.2. *Home Production,* $\omega_K > 0$

Given cross-sectional data $\{c_{M,t}, h_{M,t}, z_{M,t}, h_{N,t}, h_{P,t}\}_\iota$ and parameters $\phi, \gamma, \eta, \tau_0, \tau_1$, we show that there exists a unique $\{\alpha_t, \varepsilon_t, B_t, \theta_{N,t}, D_{P,t}\}_\iota$ such that the equilibrium allocations generated by the model are equal to the data for every household ι.

Dividing the solution for h_N with the solution for c_M, we obtain θ_N from the equation

$$\frac{h_{N,t}}{c_{M,t}} = \theta_{N,t}^{\phi-1} \tilde{z}_{M,t}^{-\phi}. \tag{A.48}$$

Next, we divide the solutions for h_P with the solution for h_N and we solve for the ratio of disutilities $\exp(D_P)/\exp(B)$:

$$\frac{h_{P,t}}{h_{N,t}} = \left(\frac{\theta_{P,t}}{\theta_{N,t}}\right)^{\phi-1} \left(\frac{\exp(B_t)}{\exp(D_{P,t})}\right)^{\phi}. \tag{A.49}$$

Next, we divide the solution for h_T with the solution for c_M and use equation (A.48) to obtain

$$\frac{h_{M,t} + h_{N,t} + \dfrac{\exp(D_{P,t})}{\exp(B_t)} h_{P,t}}{c_{M,t}} = \frac{z_{M,t}^{\eta(1-\tau_1)}}{1-\tau_0} \frac{\exp\big(-(1+\eta)(1-\tau_1)\alpha_t\big)}{\displaystyle\int_{Z_t} \exp\big((1+\eta)(1-\tau_1)\varepsilon_t\big)\, \mathrm{d}\Phi_{\zeta^j,t}\big(\zeta_t^j\big)} \times \left[1 + \left(\frac{\theta_{N,t}}{\tilde{z}_{M,t}}\right)^{\phi-1} + \left(\frac{\exp(B_t)/\tilde{z}_{M,t}}{\exp(D_{P,t})/\theta_{P,t}}\right)^{\phi-1}\right]. \tag{A.50}$$

Since the left-hand side is a positive constant and the right-hand is increasing in α_t, the value of α_t is determined uniquely for every household ι from this equation. Since $\log z_{M,t} = \alpha_t + \varepsilon_t$, the ε_t is also uniquely determined. Next, we can identify B using the first-order conditions with respect to market consumption and equations (A.18), (A.48)

TABLE A.I

ATUS (RAW) VERSUS CEX (IMPUTED) SAMPLES

	ATUS Married Individuals			CEX Married Households		
Age	All	25–44	45–65	All	25–44	45–65
Mean h_M	42.1	41.9	42.2	66.1	66.8	65.5
Mean h_N	12.5	14.6	10.5	21.3	25.4	17.3
Mean h_P	10.6	10.7	10.5	16.7	16.4	17.0
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	−0.15	−0.14	−0.14
$\text{corr}(z_M, h_N)$	0.01	0.04	−0.01	0.10	0.16	0.12
$\text{corr}(z_M, h_P)$	−0.08	−0.06	−0.09	0.02	0.00	0.03
$\text{corr}(h_M, h_N)$	−0.44	−0.46	−0.42	−0.25	−0.36	−0.23
$\text{corr}(h_M, h_P)$	−0.45	−0.44	−0.46	−0.42	−0.42	−0.41
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.15	0.20	0.17

and (A.49) to obtain

$$\exp\bigl((1+\eta)B_t\bigr) = \frac{\left(\dfrac{\bar{c}_{M,t}}{\tilde{z}_{M,t}} + h_{N,t} + \left(\dfrac{\bar{c}_{M,t}}{\bar{h}_{P,t}}\right)^{\frac{1}{\phi}} \theta_{P,t}{}^{\frac{\phi-1}{\phi}} \dfrac{h_{P,t}}{\tilde{z}_{M,t}}\right)^{-\eta}}{\bar{h}_{M,t} + h_{N,t} + \left(\dfrac{\bar{c}_{M,t}}{\bar{h}_{P,t}}\right)^{\frac{1}{\phi}} \theta_{P,t}{}^{\frac{\phi-1}{\phi}} \dfrac{h_{P,t}}{\tilde{z}_{M,t}}}. \tag{A.51}$$

Finally, once we know B, we can solve for D_P from equation (A.49).

APPENDIX B: ADDITIONAL RESULTS

In this appendix, we present summary statistics from various data sets and additional results and sensitivity analyses.

- Table A.I shows summary statistics of wages and hours for married individuals in the ATUS and for married households in the CEX in which we have imputed home hours. The ATUS sample excludes respondents during weekends and, so, market hours are noticeably higher.
- Tables A.II and A.III show summary statistics of wages and hours for married individuals in the ATUS by sex and education.

TABLE A.II

CORRELATIONS IN ATUS MARRIED BY SEX

	ATUS All			ATUS Men			ATUS Women		
Age	All	25–44	45–65	All	25–44	45–65	All	25–44	45–65
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	0.02	0.00	0.04	0.04	0.02	0.06
$\text{corr}(z_M, h_N)$	0.01	0.04	−0.01	0.03	0.07	0.01	0.03	0.05	0.01
$\text{corr}(z_M, h_P)$	−0.08	−0.06	−0.09	−0.02	0.00	−0.04	−0.08	−0.08	−0.09
$\text{corr}(h_M, h_N)$	−0.44	−0.46	−0.42	−0.40	−0.41	−0.39	−0.44	−0.47	−0.43
$\text{corr}(h_M, h_P)$	−0.45	−0.44	−0.46	−0.39	−0.38	−0.41	−0.46	−0.44	−0.47
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.06	0.09	0.05	0.07	0.10	0.07

TABLE A.III

CORRELATIONS IN ATUS MARRIED BY EDUCATION

	ATUS All			ATUS Less than College			ATUS College or More		
Age	All	25–44	45–65	All	25–44	45–65	All	25–44	45–65
$\text{corr}(z_M, h_M)$	0.06	0.03	0.08	0.05	0.03	0.06	0.05	0.02	0.07
$\text{corr}(z_M, h_N)$	0.01	0.04	−0.01	−0.01	0.01	−0.01	−0.02	0.02	−0.05
$\text{corr}(z_M, h_P)$	−0.08	−0.06	−0.09	−0.05	−0.03	−0.07	−0.07	−0.06	−0.09
$\text{corr}(h_M, h_N)$	−0.44	−0.46	−0.42	−0.42	−0.44	−0.41	−0.47	−0.50	−0.45
$\text{corr}(h_M, h_P)$	−0.45	−0.44	−0.46	−0.45	−0.43	−0.46	−0.45	−0.45	−0.45
$\text{corr}(h_N, h_P)$	0.10	0.14	0.08	0.08	0.12	0.06	0.14	0.17	0.14

TABLE A.IV

CEX/ATUS (1995–2016) VERSUS PSID (1975–2014) MOMENTS

	CEX/ATUS			PSID		
Age	All	25–44	45–65	All	25–44	45–65
Mean h_M	66.1	66.8	65.5	67.8	65.3	70.3
Mean $h_N + h_P$	38.0	41.8	34.2	25.9	27.1	24.7
$\text{corr}(z_M, h_M)$	−0.15	−0.14	−0.14	−0.15	−0.15	−0.14
$\text{corr}(z_M, h_N + h_P)$	0.09	0.12	0.10	0.00	0.02	−0.02
$\text{corr}(z_M, c_M^{\text{food}})$	0.22	0.21	0.22	0.28	0.29	0.27
$\text{corr}(h_M, h_N + h_P)$	−0.42	−0.49	−0.42	−0.24	−0.28	−0.20
$\text{corr}(h_M, c_M^{\text{food}})$	0.10	0.09	0.12	0.06	0.06	0.07
$\text{corr}(h_N + h_P, c_M^{\text{food}})$	−0.03	−0.01	−0.02	0.01	0.03	−0.01

TABLE A.V

CEX/ATUS (1995–2016) VERSUS PSID (2004–2014) MOMENTS

	CEX/ATUS			PSID		
Age	All	25–44	45–65	All	25–44	45–65
Mean h_M	66.1	66.8	65.5	64.8	67.6	62.0
Mean $h_N + h_P$	38.0	41.8	34.2	24.3	24.1	24.6
$\text{corr}(z_M, h_M)$	−0.15	−0.14	−0.14	−0.09	−0.15	−0.06
$\text{corr}(z_M, h_N + h_P)$	0.09	0.12	0.10	−0.01	0.03	−0.03
$\text{corr}(z_M, c_M^{\text{nd}})$	0.25	0.24	0.25	0.26	0.29	0.25
$\text{corr}(h_M, h_N + h_P)$	−0.42	−0.49	−0.42	−0.23	−0.27	−0.20
$\text{corr}(h_M, c_M^{\text{nd}})$	0.14	0.16	0.13	0.20	0.21	0.20
$\text{corr}(h_N + h_P, c_M^{\text{nd}})$	−0.05	−0.04	−0.03	−0.03	−0.03	−0.03

- Tables A.IV and A.V present summary statistics of wages, hours, and expenditures in the CEX and PSID samples.
- Table A.VI presents the correlation matrix of observables and sources of heterogeneity in the two models.
- Figure A.1 presents distributions of the sources of heterogeneity in the two models.

TABLE A.VI

WITHIN-AGE CORRELATIONS

	$\log z_M$	$\log c_M$	$\log h_M$	$\log h_N$	$\log h_P$	α	ε	B	D_P	$\log \theta_N$
$\omega_K = 0$										
$\log z_M$	1.00	0.29	−0.07	–	–	0.70	0.42	0.42	–	–
$\log c_M$		1.00	0.13	–	–	0.69	−0.50	−0.55	–	–
$\log h_M$			1.00	–	–	−0.46	0.50	−0.71	–	–
$\log h_N$				–	–	–	–	–	–	–
$\log h_P$					–	–	–	–	–	–
α						1.00	−0.35	0.23	–	–
ε							1.00	0.26	–	–
B								1.00	–	–
D_P									–	–
$\log \theta_N$										–
$\omega_K > 0$										
$\log z_M$	1.00	0.29	−0.07	0.07	−0.02	0.82	0.42	0.45	−0.58	0.69
$\log c_M$		1.00	0.13	0.00	−0.06	0.66	−0.54	−0.43	−0.02	−0.15
$\log h_M$			1.00	−0.17	−0.30	−0.32	0.38	−0.48	0.06	−0.20
$\log h_N$				1.00	0.18	0.13	−0.08	−0.29	−0.36	0.66
$\log h_P$					1.00	0.08	−0.15	−0.03	−0.67	0.12
α						1.00	−0.18	0.23	−0.41	0.46
ε							1.00	0.40	−0.34	0.46
B								1.00	−0.05	0.31
D_P									1.00	−0.66
$\log \theta_N$										1.00

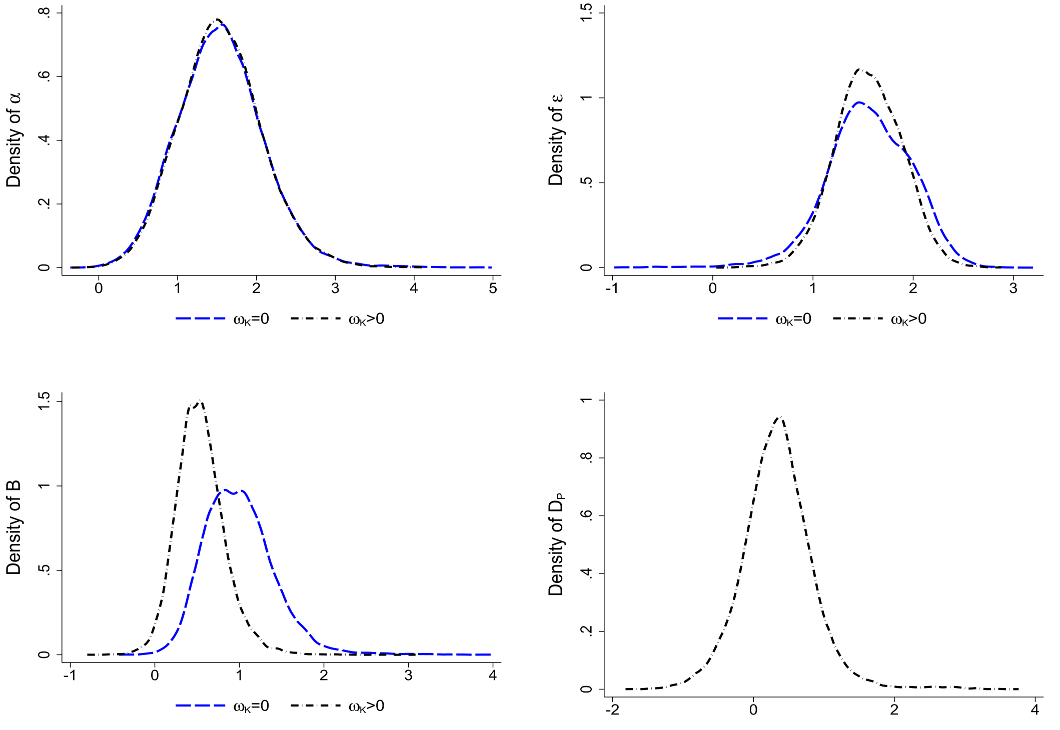

FIGURE A.1.—Distributions of sources of heterogeneity.

TABLE A.VII

WITHIN-AGE HETEROGENEITY AND LIFETIME CONSUMPTION EQUIVALENCE

No Within-Age Dispersion in ...	$\omega_K = 0$ Model	$\omega_K > 0$ model
z_M, θ_N, B, D_P	0.07	0.14
z_M, θ_N	0.07	0.16
θ_N, D_P	–	0.11
θ_N	–	0.12

- Table A.VII presents the welfare effects of eliminating heterogeneity within age groups.
- Table A.VIII compares the four inequality metrics in six versions of the home production model.
  - (i) One sector model with heterogeneity only in home production efficiency θ_N.
  - (ii) Two sector model with heterogeneity in home production efficiency θ_N and disutility of work D_P (the baseline case).
  - (iii) One sector model with heterogeneity only in home disutility of work D_P.
  - (iv) Two sector model with heterogeneity in home production efficiencies θ_N and θ_P.
  - (v) Two sector model with reversal of classification of home hours relative to baseline (efficiency θ_P and disutility D_N).
  - (vi) Two sector model with heterogeneity in home disutilities of work D_N and D_P.

  The first three cases repeat the cases shown in Table VII in the main text. The second panel of Table A.VIII shows the three alternative cases.
- Figures A.2 and A.3 present the life-cycle means and variances of the sources of heterogeneity in the version of the PSID with food expenditures. We obtain these age profiles by regressing each inferred source of heterogeneity on age dummies, year dummies, and an individual fixed effect. Therefore, these age profiles reflect the within-household evolution of the sources of heterogeneity.

TABLE A.VIII

THE ROLE OF HOME EFFICIENCY AND HOME DISUTILITY IN AMPLIFYING INEQUALITY

		Home Production		
Statistics	No Home Production	Efficiency θ_N	Baseline (θ_N, D_P)	Disutility D_P
std(T)	0.78	1.14	0.90	0.76
std(t)	0.55	0.83	0.73	0.65
λ	0.06	0.20	0.12	0.03
τ_1	0.06	0.32	0.24	0.13
		Efficiencies (θ_N, θ_P)	Reversed (θ_P, D_N)	Disutilities (D_N, D_P)
std(T)	0.78	1.13	0.82	0.73
std(t)	0.55	0.83	0.68	0.63
λ	0.06	0.19	0.12	0.02
τ_1	0.06	0.31	0.21	0.09

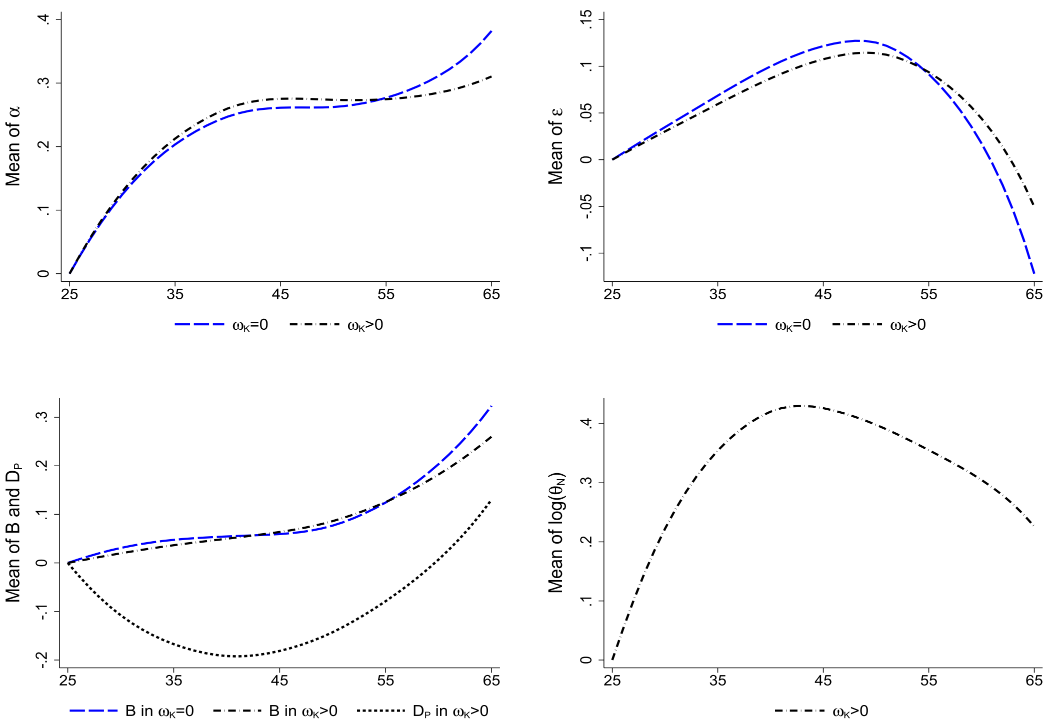

FIGURE A.2.—Means of sources of heterogeneity (PSID food). The plots are the age means of the uninsurable component of market productivity α, the insurable component of market productivity ε, the disutilities of work B and D_P, and the home production efficiency $\log \theta_N$ for the economy with ($\omega_K > 0$, dotted lines) and without home production ($\omega_K = 0$, dashed lines).

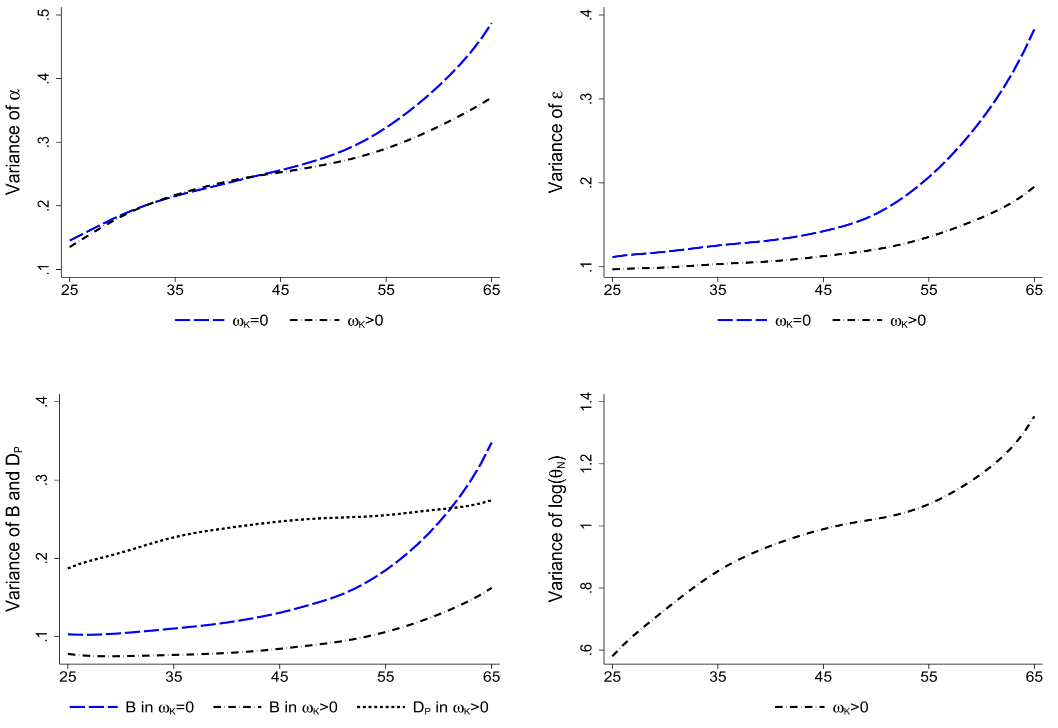

FIGURE A.3.—Variances of sources of heterogeneity (PSID food). The plots are the age variances of the uninsurable component of market productivity α, the insurable component of market productivity ε, the disutilities of work B and D_P, and the home production efficiency $\log\theta_N$ for the economy with ($\omega_K > 0$, dotted lines) and without home production ($\omega_K = 0$, dashed lines).

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