Supplement to "Estimating Candidate Valence"

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10 Overview

In the Supplemental Appendix, we provide proofs that we omit from the main text as well as details regarding the model, estimation, data construction and applications to other environments. In Section 10.1, we describe the model of open-seat elections. In Section 10.2, we show that at the estimated parameter values, the challenger's value function, v_C, is not too decreasing in q_C: the condition that ensures that the challenger's entry decision follows a cutoff strategy. In Section 10.3, we provide a derivation of $G_{q_C,p_C}(\cdot|\mathbf{s})$ and $p(\mathbf{s}, q_C, p_C)$ as a function of $\bar{q}_C(\mathbf{s}, p_C)$ for general N. In Section 10.4, we prove Proposition 1 (Injectivity) and Proposition 2 (Sufficient statistic). In Section 10.5, we discuss how we forward-simulate the continuation value. We provide details of the estimation procedure in Section 10.6 and data construction in Section 10.7. In Section 10.8, we show how our approach can be extended to environments in which q_I is time-varying, and one in which there are few uncontested elections. In Section 10.9, we present histograms of our valence measures after accounting for sampling error by applying Bayes shrinkage. In Section 10.10, we describe our bootstrap procedure. In Section 10.11, we discuss our model fit for actions in uncontested elections. In Section 10.12, we discuss the estimation of the policy functions we use for counterfactual simulation.

10.1 Description of the Model of Open-Seat Elections

In an open-seat election, challengers from both parties decide whether or not to enter. The value function of candidate i running against candidate j in the general election is as follows:

$$v_O(X, q_i, q_j, p_i, p_j) = \max_{w_i' \geq 0, d_i \geq 0} B \cdot \Pr(vote_i > 0.5) - C_O(w_i' + d_i, q_i)$$
$$+ H_O(d_i) + \delta \Pr(vote_i > 0.5) \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V_I(\mathbf{s}')],$$

where

$$vote_i = \beta_O \ln d_i - \beta_O \ln d_j + \beta_P (p_i - p^*)^2 - \beta_P (p_j - p^*)^2$$
$$+ D_i \times (\beta_d + \beta_{dn} dn) + \beta_{ue}(ue \times D_i \times D_P) + \text{Election cycle FE} + q_i - q_j + \varepsilon,$$

We assume that ε follows a Normal distribution $N(0.5, \sigma_\varepsilon^2)$. The problem of open-seat candidates is similar to that of challengers that run against incumbents. We allow for the coefficient of campaign spending to be open-seat specific, which we denote by β_O. The functions $C_O(\cdot)$ and $H_O(\cdot)$ are the cost of fund-raising and the benefit of spending in open-seat elections.[1]

Open-seat candidates make simultaneous entry decisions by comparing their expected returns from entry and the cost of entry, κ_O. The ex-ante value function can be expressed as follows:

$$V_O(X, q_i, p_i) = \max \left\{ p_O(X, q_i, p_i) \int v_O(X, q_i, q_j, p_i, p_j) dG_{q,p}(q_j, p_j | X) - \kappa_O, 0 \right\}, (i \neq j),$$

where $p_O(X, q_i, p_i)$ denotes the ex-ante probability that challenger i is selected as a party nominee, defined analogously to Expression (6). Because candidates do not know the valence of the candidate from the opponent party when deciding whether or not to enter, we take expectations with respect to the distribution of valence and policy position of the opponent in the general election, which we denote by $G_{q,p}$.

10.2 Simulating Derivatives of v_C with respect to q_C

In our identification and estimation, we rely on the property that a potential challenger's entry decision is characterized by a cutoff strategy with threshold $\bar{q}_C(\mathbf{s}, p_C)$. A sufficient condition for this property to hold is that $p(\mathbf{s}, q_C, p_C) v_C(\mathbf{s}, q_C, p_C)$ is increasing in q_C. Because we assume that $\pi(q_{C,m}, p_{C,m}, \mathbf{q}_{C,-m}, \mathbf{p}_{C,-m})$ is increasing in $q_{C,m}$, $p(\mathbf{s}, q_C, p_C)$ is increasing in q_C by assumption (see Expression (6)). Hence, $p(\mathbf{s}, q_C, p_C) v_C(\mathbf{s}, q_C, p_C)$ is

[1] In our empirical specification, we assume that $C_O(\cdot) = C_C(\cdot)$ and $H_O(\cdot) = H_C(\cdot)$.

increasing in q_C as long as $v_C(\mathbf{s}, q_C, p_C)$ is not too decreasing in q_C. In this section, we simulate the derivative $\frac{\partial v_C}{\partial q_C}$ for each challenger (at the realized state) at the estimated parameter values to show that the condition generally holds.[2]

Recall that v_C is given as follows.

$$
\begin{aligned}
v_C(\mathbf{s}, q_C, p_C) = \max_{d_C \geq 0, w'_C \geq 0} & B \cdot \Pr(vote_I < 0.5) - C_C(w'_C + d_C, q_C) \\
& + H_C(d_C) + \delta \Pr(vote_I < 0.5) \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[(1 - \lambda(\mathbf{s}'))V_I(\mathbf{s}')].
\end{aligned} \tag{5}
$$

To numerically evaluate $\frac{\partial v_C}{\partial q_C}$ at a given point $\{\mathrm{s}, q_C\}$ in the data, we need to evaluate v_C at state $\{\mathrm{s}, q'_C\}$ such that q'_C is located sufficiently close to q_C. To do so, we compute, at $\{\mathrm{s}, q'_C\}$, the optimal choice of incumbent spending, d_I, challenger spending, d_C, fundraising, fr_C, and savings, w'_C by evaluating the policy functions we estimate for the exercise in Section 7 at $\{\mathrm{s}, q'_C\}$.[3]

Once we compute the optimal actions at $\{\mathrm{s}, q'_C\}$, we evaluate the value of v_C at $\{\mathrm{s}, q'_C\}$ by substituting the optimal actions into Expression (5). To evaluate the continuation payoffs, we use the same polynomial approximation of $\mathbb{E}_{\mathbf{s}'|\mathbf{s}}[(1 - \lambda(\mathrm{s}'))V_I(\mathrm{s}')]$ that we use during the estimation procedure. In Figure 14, we report the histogram of $\frac{\partial v_C}{\partial q_C}$, evaluated at $\{\mathrm{s}, q_C\}$ for each contested election in the data. We find that $\frac{\partial v_C}{\partial q_C} > 0$ for 89.4% of our sample. Moreover, even when $\frac{\partial v_C}{\partial q_C}$ is negative, the magnitude is relatively small, which implies that $v_C(\mathrm{s}, q_C)$ is not too decreasing in q_C. Among challengers for whom $\frac{\partial v_C}{\partial q_C}$ is negative, the mean value of $\frac{\partial v_C}{\partial q_C}$ is -0.02, whereas the mean value of $\frac{\partial v_C}{\partial q_C}$ is 0.18 for those who are positive.

10.3 Derivation of $G_{q_C,p_C}(\cdot|\mathrm{s})$ and $p(\mathrm{s}, q_C, p_C)$ as a Function of $\bar{q}_C(\mathrm{s}, p_C)$

In this section, we show that $G_{q_C,p_C}(\cdot|\mathrm{s})$ and $p(\mathrm{s}, q_C, p_C)$ can be expressed as functions of $\bar{q}_C(\mathrm{s}, p_C)$ and primitives for general N.

Consider first $G_{q_C,p_C}(t_q, t_p|\mathrm{s})$, the probability that the general election challenger has valence less than t_q and policy position less than t_p. Suppose that there are $N = \{1, \cdots, N\}$ potential entrants. The conditional probability that the primary winner has valence less than

[2] We cannot directly test for $\frac{\partial p(\mathbf{s}, q_C, p_C) v_C(\mathbf{s}, q_C, p_C)}{\partial q_C} > 0$, because $p(\mathrm{s}, q_C, p_C)$ is not estimated. The fraction of challengers for whom $\frac{\partial v_C}{\partial q_C} < 0$ is a lower bound for the fraction for whom $\frac{\partial p(\mathbf{s}, q_C, p_C) v_C(\mathbf{s}, q_C, p_C)}{\partial q_C} < 0$.

[3] We discuss the estimation of these policy functions in Appendix 10.12.

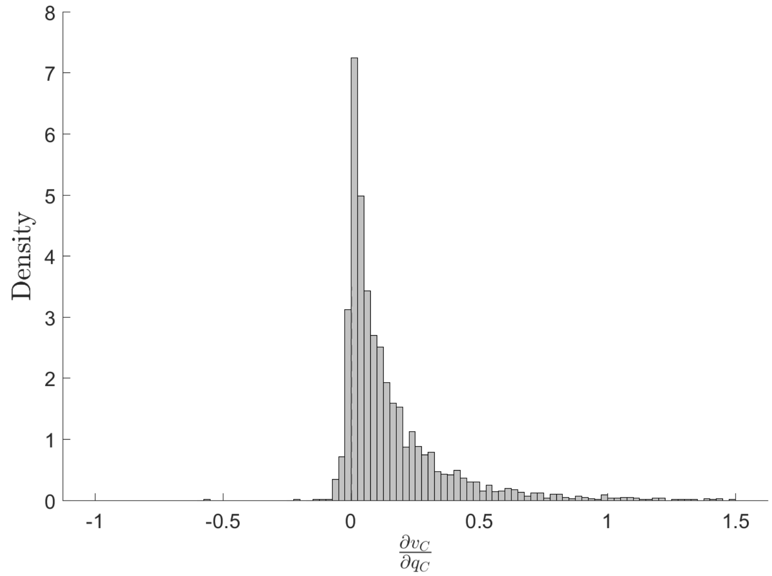


Figure 14: The derivative of v_C with respect to q_C

t_q and policy less than t_p is as follows:

$$G_{q_C,p_C}(t_q, t_p|\mathbf{s}, N) = \Pr(\text{Valence of Primary winner} \leq t_q \cap \text{Policy} \leq t_p|\mathbf{s}, N, \text{At least one entrant})$$
$$= \frac{1}{P_e(\mathbf{s})} \times$$
$$\mathbb{E}_{\mathbf{p}_C}\left[\sum_{S \in 2^{\{1,\cdots,N\}}} \left(\Pr(\text{Set of entrants is } S|\mathbf{p}_C, \mathbf{s}, N) \sum_{i \in S} \Pr(i \text{ wins} \cap q_{C,i} \leq t_q \cap p_{c,i} \leq t_p|\mathbf{p}_C, \mathbf{s}, S)\right)\right] \tag{16}$$

The joint distribution of challengers' valence and policy positions is given by the probability that a particular subset S among potential challengers enters, and an entrant i among them becomes the party nominee with valence less than t_q and policy less than t_p, summed over all possible combinations of S.

Now fix $S \in 2^{\{1,\cdots,N\}}$ and $i \in S$. $\Pr(\text{Set of entrants is } S|\mathbf{p}_C, \mathbf{s}, N)$ and $\Pr(i \text{ wins} \cap q_{C,i} \leq t_q \cap p_{c,i} \leq t_p|\mathbf{p_C}, \mathbf{s}, S)$ can be expressed as follows:

$$\Pr(\text{Set of entrants is } S|\mathbf{p}_C, \mathbf{s}, N) = \prod_{j \in S}(1 - F_{q_C|p_{C,j}}(\bar{q}(\mathbf{s}, p_{C,j}))) \prod_{k \notin S} F_{q_C|p_{C,k}}(\bar{q}(\mathbf{s}, p_{C,k})), \tag{17}$$

and

$$\Pr(i \text{ wins} \cap q_{C,i} \leq t_q \cap p_{c,i} \leq t_p | \mathbf{s}, \text{Set of entrants is } S, \mathbf{p}_C)$$
$$= \left(\prod_{j \in S} (1 - F_{q_C|p_{C,j}}(\bar{q}(\mathbf{s}, p_{C,j}))) \right)^{-1} \times$$
$$\int_{\bar{q}(\mathbf{s},p_{C,1})}^{\infty} \cdots \int_{\bar{q}(\mathbf{s},p_{C,i})}^{t_q} \cdots \int_{\bar{q}(\mathbf{s},p_{C,|S|})}^{\infty} \pi(q_{C,i}, p_{C,i}, \mathbf{q_{C,-i}}, \mathbf{p_{C,-i}}) dF_{\mathbf{q}_C|\mathbf{p}_C} 1\{p_{C,i} \leq t_p \cap \bar{q}(\mathbf{s}, p_{C,i}) \leq t_q\}. \tag{18}$$

Note that $P_e(\mathbf{s})$ can be expressed as a function of $\{\bar{q}(\mathbf{s}, p_{C,k})\}$ and primitives of the model as we discussed in Expression (7) in Section 2. Moreover, from Expressions (17) and (18), we've shown that each term inside the sum in Expression (16) is also expressed as a function of $\{\bar{q}(\mathbf{s}, p_{C,k})\}$ and primitives of the model.

We now turn to $p(\mathbf{s}, q_C, p_C)$, the probability that an entrant with valence q_C and policy position p_C wins the primary. Consider candidate i with valence $q_{C,i}$ and policy position $p_{C,i}$. Let $S_{i|N} \subset 2^{\{1,\cdots,N\}}$ be the collection of sets that includes i. The probability that i wins the Primary conditional on entry is given as follows:

$$p(\mathbf{s}, q_{C,i}, p_{C,i}|N)$$
$$= \mathbb{E}_{\mathbf{p}_{C,-i}} \left[\sum_{S \in S_{i|N}} \Pr(\text{Set of entrants is } S | \mathbf{s}, \mathbf{p}_{C,-i}, N) \Pr(i \text{ wins} | \mathbf{s}, q_{C,i}, p_{C,i}, \mathbf{p}_{C,-i}, S) \right]. \tag{19}$$

Now fix a set of entrants $S \in S_{i|N}$ that includes i and a vector of policy positions $\{p_{C,i}, \mathbf{p}_{C,-i}\}$. The term inside the bracket is given by the following expression.

$$\begin{aligned} &\Pr(\text{Set of entrants is } S | \mathbf{s}, \mathbf{p}_{C,-i}, N) \Pr(i \text{ wins} | \mathbf{s}, q_{C,i}, p_{C,i}, \mathbf{p}_{C,-i}, S) \\ &= \prod_{j \notin S} F_{q_C|p_{C,j}}(\bar{q}(\mathbf{s}, p_{C,j})) \times \\ &\int_{\bar{q}(\mathbf{s},p_{C,1})}^{\infty} \cdots \int_{\bar{q}(\mathbf{s},p_{C,i-1})}^{\infty} \int_{\bar{q}(\mathbf{s},p_{C,i+1})}^{\infty} \cdots \int_{\bar{q}(\mathbf{s},p_{C,|S|})}^{\infty} \pi(q_{C,i}, p_{C,i}, \mathbf{q}_{C,-i}, \mathbf{p}_{C,-i}) dF_{\mathbf{q}_{C,-i}|\mathbf{p}_{C,-i}}. \end{aligned} \tag{20}$$

From Expression (20), it is clear that the term inside the bracket in Expression (19) can be expressed as functions of $\bar{q}(\mathbf{s}, p_C)$ and model primitives.

10.4 Proof of Proposition 1 and 2

In this Section, we give a proof of Proposition 1 (Injectivity) and Proposition 2 (Sufficient statistic).

Proposition 1 (Injectivity): *Assume that the marginal cost of raising money, $\frac{\partial}{\partial x}\tilde{C}_I(x, q_I)$, is strictly decreasing with respect to q_I. Then, the policy functions of an uncontested incumbent, $\{d_I(\mathbf{s})$,$w'_I(\mathbf{s})\}$, are one-to-one from q_I to (d_I, w'_I), holding other state variables fixed.*

Proof. Consider the problem of an uncontested incumbent. The first-order condition for d_I implies

$$\underbrace{\frac{\partial}{\partial d_I}\tilde{H}_I(d_I)}_{\text{MB of spending}} - \underbrace{\frac{\partial}{\partial d_I}\tilde{C}_I(w'_I + d_I - w_I, q_I)}_{\text{MC of fund-raising}} = 0.$$

Now suppose to the contrary that the mapping from q_I to (d_I, w'_I) is not one-to-one, so that q_I and $\tilde{q}_I$ $(q_I > \tilde{q}_I)$ both map to (d_I, w'_I). Then,

$$\frac{\partial}{\partial d_I}\tilde{H}_I(d_I) = \frac{\partial}{\partial d_I}\tilde{C}_I(w'_I + d_I - w_I, q_I)$$

and

$$\frac{\partial}{\partial d_I}\tilde{H}(d_I) = \frac{\partial}{\partial d_I}\tilde{C}_I(w'_I + d_I - w_I, \tilde{q}_I)$$

However, given that $\frac{\partial}{\partial d_I}\tilde{C}_I(\cdot, \cdot)$ is strictly decreasing in the second argument,

$$\frac{\partial}{\partial d_I}\tilde{C}_I(w'_I + d_I - w_I, q_I) < \frac{\partial}{\partial d_I}\tilde{C}_I(w'_I + d_I - w_I, \tilde{q}_I),$$

which is a contradiction. ■

Proposition 1 allows us to invert the policy function of uncontested incumbents to express the unobserved incumbent valence, q_I, as a function of the state and incumbent's actions in uncontested periods, $\bar{\mathbf{s}}_U$ $(q_I(\bar{\mathbf{s}}_U) : \bar{\mathbf{s}}_U \mapsto q_I)$. In our empirical analysis, we make use of the following lemma that allows us to simplify the function $q_I(\bar{\mathbf{s}}_U)$.

Lemma 1 *Assume that $\tilde{C}_I(y; q_I) = c(q_I)(\ln y)^2$, where $c(\cdot)$ is a decreasing function, and $\tilde{H}_I(y) = \gamma_U\sqrt{\ln y}$, as specified in our estimation. Then, the inverse mapping from (d_I, w'_I)*

to q_I simplifies to

$$q_I = c^{-1}\left(\frac{\gamma_U}{4}\frac{w_I' + d_I - w_I}{(\ln d_I)^{1/2} d_I \ln(w_I' + d_I - w_I)}\right).$$

Proof. Suppose that $\tilde{C}_I(y; q_I) = c(q_I)(\ln y)^2$, and $\tilde{H}_I(y) = \gamma_U \sqrt{\ln y}$. Substituting these expressions into the first-order condition, we obtain

$$\begin{aligned} & \frac{\gamma_U}{2}(\ln d_I)^{-1/2}(d_I)^{-1} - 2c(q_I)(\ln fr_I)(fr_I)^{-1} = 0 \\ \iff & c(q_I) = \frac{\gamma_U}{4}(\ln d_I)^{-1/2}(d_I)^{-1}(\ln fr_I)^{-1} fr_I \\ \iff & q_I = c^{-1}\left(\frac{\gamma_U}{4}\frac{fr_I}{(\ln d_I)^{1/2} d_I \ln fr_I}\right), \end{aligned} \tag{21}$$

where fr_I denotes the amount raised ($fr_I = w_I' + d_I - w_I$). We use the fact that $c(\cdot)$ is monotone to obtain the last line of the expression. ■

The fact that we can control for q_I just by conditioning on a one-dimensional object, $z_U \equiv \frac{fr_I}{(\ln d_I)^{1/2} d_I \ln fr_I}$, simplifies our estimation immensely. It would be very difficult to implement our procedure if we had to condition on the full vector of actions and state variables, $\bar{\mathrm{s}}_U$.

We next prove Proposition 2.

Proposition 2 (Sufficient statistic): *Let $m(\mathrm{s}) = \{P_e(\mathrm{s}), F_{p_C}(\cdot|\mathrm{s}, \chi = 1)\}$. Then, $m(\mathrm{s})$ is a sufficient statistic for $G_{q_C}(\cdot|\mathrm{s})$.*

Proof. We say that $h = h(\mathrm{s})$ is a sufficient statistic for $f(\mathrm{s})$ if $h(\mathrm{s}') = h(\mathrm{s}'')$ implies $f(\mathrm{s}') = f(\mathrm{s}'')$. We can see that the function $\bar{q}_C(\mathrm{s}, \cdot) : p_C \mapsto \bar{q}_C(\mathrm{s}, q_C)$ is a sufficient statistic for $G_{q_C}(\cdot|\mathrm{s}) = G_{q_C,p_C}(\cdot, \infty|\mathrm{s})$ from expressions (16) through (18). Now, consider the density of entrant's policy, $f_{p_C}(\cdot|\mathrm{s}, \chi = 1)$. The density can be written as follows:

$$f_{p_C}(p_C|\mathrm{s}, \chi = 1) = \frac{f_{p_C}(p_C) \times (1 - F_{q_C|p_C}(\bar{q}_C(\mathrm{s}, p_C)))}{P_e(\mathrm{s})}$$

This implies that, if we have $f_{p_C}(p_C|\mathrm{s}, \chi = 1) = f_{p_C}(p_C|\mathrm{s}', \chi = 1)$, $\forall p_C$ and $P_e(\mathrm{s}) = P_e(\mathrm{s}')$ for $\mathrm{s} \neq \mathrm{s}'$, then $\bar{q}_C(\mathrm{s}, p_C) = \bar{q}_C(\mathrm{s}', p_C), \forall p_C$. This means that $P_e(\mathrm{s})$ and $f_{p_C}(\cdot|\mathrm{s}, \chi = 1)$ are also sufficient statistics for $G_{q_C}(\cdot|\mathrm{s})$. ■

In our empirical application, we use the sufficient statistics property to control for the sample selection of q_C.

10.5 Forward-Simulation of the Continuation Value

In this section, we discuss how we forward-simulate the continuation value. As we discuss in Section 3.2, our idea is based on Hotz, Miller, Sanders, and Smith (1994) and Bajari, Benkard and Levin (2007). These papers propose a method of simulating the value function by first estimating the policy function and then using the policy function to generate sample paths of outcomes and actions, which are averaged to compute the continuation value as a function of the parameters. Because we do not observe q_C in contested periods, we modify the procedure by estimating the distribution of actions and outcomes conditional on observed state variables instead of the actual policy functions.

In what follows, s refers to the set of variables $(q_I(z_U), w_I, ten_I, p_I, pt, X)$. As we explain in Section 5.1, we specify p^* to be a function of pt and we express q_I as a function of z_U, where $z_U \equiv \frac{fr_I}{\sqrt{\ln d_I d_I \ln fr_I}}$. Below, we describe the details of our procedure.

10.5.1 Estimation of the Transition Probability of the States

We first estimate the transition probability of the state variables. We specify an exogenous AR(1) process for $\tilde{X} = \{ue, pt, \log(dn)\}$, as $\tilde{X}_{t+1} = \alpha_0 + \alpha_1 \tilde{X}_t + \xi_{t+1}$, $\xi_{t+1} \sim N(0, \Sigma_\xi)$ and estimate the parameters by OLS.[4] For the evolution of D_P, we assume that, (i) $D'_P = D_P$ with probability 0.75 when the President ends his first term in the current period; (ii) $D'_P = D_P$ with probability 0.5 when the incumbent President is ending his second term; and (iii) $D'_P = D_P$ with probability one if the election is a Midterm election.

10.5.2 Estimation of Incumbent Policy Position and Retirement

Next, we estimate the reduced-form policy function that determines the incumbent's policy position next period, $p_I(\cdot)$, and the policy function for retirement, $\lambda(\cdot)$. We allow the incumbent to choose a policy position and make a retirement decision after seeing the evolution of the exogenous state variables. Hence, we let $p_I(\cdot)$ and $\lambda(\cdot)$ be functions of $q_I(z_U), w'_I$, ten'_I, pt', X', and her previous policy position, p_I.

[4] In our application, we assume that Σ_ξ is a diagonal matrix. We estimate $\alpha_0^{ue} = 0.02$, $\alpha_1^{ue} = 0.70$, $\sigma_\xi^{ue} = 0.01$, $\alpha_0^{pt} = 0.04$, $\alpha_1^{pt} = 0.92$, $\sigma_\xi^{pt} = 0.16$ and $\alpha_0^{dn} = 0.17$, $\alpha_1^{dn} = 0.97$, $\sigma_\xi^{dn} = 0.38$.

Incumbent Policy Position Our measure of policy positions are discretized into 10 bins. Instead of estimating a 10-by-10 transition matrix as a function of the state variables, we take the absolute value of the policy positions of the candidates and estimate the evolution of $|p_I|$, the extremity of the policy positions. Because we discretize the policy positions of the candidates so that they take discrete values that are symmetric around zero, we pool together observations from both parties and estimate one transition matrix for both parties.[5] This allows us to reduce the size of the transition matrix that we need to estimate to a 5-by-5 matrix.

Formally, let the positive values of the five discrete bins be $\bar{p} = \{p_1, p_2, ..., p_5\}$, where $p_1 < p_2 < \cdots < p_5$. We specify the probability that an incumbent chooses policy p_k next period as follows:

$$\Pr(p'_I = p_k) = \frac{\exp(X_{P_I}\beta_k)}{\sum_k \exp(X_{P_I}\beta_k)}, k \in \{1, 2, 3, 4, 5\}.$$

For X_{P_I}, we include dummies $1\{p_I = p_j\}\ \forall j$, w'_I, z_U, ten'_I, $D_I \times D'_p \times 1\{\text{Midterm}'\}$, $D_I \times D'_p \times ue'$, $D_I \times pt'$, $D_I \times dn'$, $1\{\text{Midterm}'\}$, $1\{\text{First Term}'\}$ and $1\{\text{First Term}'\} \times 1\{\text{Midterm}'\}$. Note that z_U is a sufficient statistic for q_I. We allow the transition probability, $\Pr(p'_I = p_k | p_I = p_j)$ to depend on j by including dummies $1\{p_I = p_j\}$ in X_{P_I}. In addition, we allow for k-specific coefficients on the state variables. We normalize $\beta_5 = 0$.

Table 8 presents the results. All variables (except for the j-specific intercept terms) are standardized by subtracting the mean and dividing by the standard deviation. Elements without numbers are not estimated due to lack of observations.[6] For the coefficient estimates of the j-specific dummies, we find that the diagonals are the highest, except for $j = 5$, where $k = 4$ has the highest estimated coefficient. The estimates imply that choosing the same policy position as the previous period is the most likely outcome, except for candidates with the most extreme policy position who have a high likelihood of moving to a more central position.

Retirement Probability We assume that $\lambda(\cdot)$ takes the following form:

$$\lambda = \frac{\exp(X_\lambda \beta_\lambda)}{1 + \exp(X_\lambda \beta_\lambda)},$$

[5]We discuss our measure of policy positions in more detail in 10.7.

[6]For example, when $p_I = p_5$, no one moves to $p'_I = p_1$ in the following period in the data.

	$k=1$		$k=2$		$k=3$		$k=4$	
$1\{j=1\}$	17.26	(0.30)	15.31	(0.32)	-	-	-	-
$1\{j=2\}$	16.52	(0.53)	18.02	(0.52)	16.20	(0.53)	-	-
$1\{j=3\}$	-	-	17.52	(0.34)	18.25	(0.33)	16.37	(0.38)
$1\{j=4\}$	-	-	-	-	2.25	(0.36)	3.18	(0.35)
$1\{j=5\}$	-	-	-	-	-	-	-0.59	(0.49)
w_I'	-0.69	(0.33)	-0.68	(0.33)	-0.59	(0.31)	-0.48	(0.31)
z_U	1.26	(0.58)	1.22	(0.58)	1.08	(0.57)	1.03	(0.55)
ten_I'	0.74	(0.39)	0.70	(0.39)	0.70	(0.38)	0.65	(0.37)
$D_I \times D_p' \times 1\{\text{Midterm}'\}$	1.57	(0.37)	1.18	(0.35)	0.94	(0.34)	0.73	(0.32)
$D_I \times D_p' \times ue'$	-1.30	(0.45)	-1.09	(0.44)	-0.90	(0.44)	-0.66	(0.41)
$D_I \times pt'$	-1.08	(0.45)	-0.64	(0.44)	-0.57	(0.44)	-0.41	(0.41)
$D_I \times dn'$	0.39	(0.97)	0.54	(0.96)	0.51	(0.96)	-0.40	(0.95)
$1\{\text{Midterm}'\}$	-0.51	(0.42)	-0.63	(0.41)	-0.78	(0.40)	-1.21	(0.37)
$1\{\text{First Term}'\}$	-0.11	(0.37)	-0.17	(0.36)	-0.50	(0.34)	-0.51	(0.32)
$1\{\text{First Term}'\} \times 1\{\text{Midterm}'\}$	0.68	(0.46)	0.58	(0.45)	0.78	(0.44)	0.85	(0.41)

Note: Standard errors are reported in parentheses. All variables (except for the j- specific intercepts) are standardized by subtracting the mean and dividing by the standard deviation.

Table 8: Estimates of $p_I(\cdot)$

where X_λ includes an intercept, w_I', z_U, ten_I', $D_I \times D_p' \times 1\{\text{Midterm}'\}$, $D_I \times D_p' \times ue'$, $D_I \times pt'$, $D_I \times dn'$, p_I, $1\{\text{Midterm}'\}$, $1\{\text{First Term}'\}$ and $1\{\text{First Term}'\} \times 1\{\text{Midterm}'\}$.

Table 9 presents the parameter estimates. Our estimate of the intercept is a large negative value, reflecting the fact that most incumbents do not retire. Consistent with our prior, we find that retirements are negatively associated with war chest and positively associated with tenure. We also find that most of the other variables are statistically not significant and small in magnitude.

10.5.3 Estimation of the Distribution of Actions and Electoral Outcomes Conditional on Observed State Variables

The third set of objects we estimate is the projection of the actions on observed state variables. Because the estimation results in this subsection include many parameters that do not offer much meaningful interpretation on their own, we do not include the results here. The parameter estimates are available upon request.

	$\lambda(\cdot)$	
Intercept	-1.56	(0.53)
w'_I	-0.09	(0.04)
z_U	3.09	(2.93)
ten'_I	0.09	(0.02)
$D_I \times D'_p \times 1\{\text{Midterm}'\}$	-0.03	(0.15)
$D_I \times D'_p \times ue'$	0.47	(1.71)
$D_I \times pt'$	0.50	(0.25)
$D_I \times dn'/10000$	-0.27	(0.55)
p_I	0.18	(0.18)
$1\{\text{Midterm}'\}$	-0.06	(0.20)
$1\{\text{First Term}'\}$	-0.54	(0.22)
$1\{\text{First Term}'\} \times 1\{\text{Midterm}'\}$	0.17	(0.32)

Table 9: Estimates of $\lambda(\cdot)$

Distribution of d_I and fr_I Conditional on s in Contested Periods Recall that the equilibrium spending and amount raised by the incumbent in contested periods, d_I and fr_I, are functions of (s, q_C). The projection of the policy function on s is just the conditional distribution of d_I and fr_I integrated over q_C, $F_{d_I}(\cdot|\mathrm{s})$ and $F_{fr_I}(\cdot|\mathrm{s})$, respectively. We use a (first-order) Hermite series approximation to estimate the conditional distribution, by nonparametric maximum likelihood (Gallant and Nychka 1987). To account for the fact that some of the state variables are discrete, we estimate separate distributions for all combinations of $\{1\{\text{Midterm}\}, D_I \times D_P, 1\{ten_I \leq 5\}\}$.[7]

Distribution of w'_I Conditional on s and $vote_I > 0.5$ in Contested Periods We estimate the distribution of incumbent savings in contested periods in the same way as spending and fund-raising. However, in order to simulate the value function, we need the distribution of savings *conditional on winning*. Hence, we estimate $F_{w'_I}(\cdot|\mathrm{s}, 1\{vote_I > 0.5\})$, where $1\{vote_I > 0.5\}$ corresponds to the event that the vote share of the incumbent is above 50 percent.

[7]Because each of these variables are binary, we estimate 6 nonparametric densities per action. This specification is as flexible as we can allow for given our sample size.

Policy Functions in Uncontested Periods In uncontested periods, we can estimate the policy function directly because the state variables are all observed. We approximate the amount of spending and savings in uncontested periods by least squares. The regresors include a constant, w_I, ten_I, p_I, pt, X and B-spline of z_U. We also include quadratic terms as well as interactions of these variables.

Probability of Winning, $P_{win}(\mathrm{s})$, in Contested Periods We estimate the probability that the incumbent wins in contested periods given s, $P_{win}(\mathrm{s})$, using a Probit. The regressors include $\ln w_I$, $(\ln w_I)^2$, ten_I, $D_I \times D_P \times 1\{\text{Midterm}\}$, $D_I \times D_P \times ue$, D_I, $D_I \times dn$, p_I, p_I^2, $p_I \times pt$, $1\{\text{Midterm}\}$, $1\{\text{First Term}\}$,$1\{\text{Midterm}\} \times 1\{\text{First Term}\}$, z_U and z_U^2.

10.5.4 Computation of the Continuation Value

Once we obtain estimates of the distribution of actions and outcomes conditional on observed states, it is possible to simulate the continuation value for each profile of parameters. The key to our approach is that the incumbent's utility does not depend directly on q_C, which is unobserved. Instead, it depends on q_C only indirectly through the incumbent's own equilibrium actions such as d_I, fr_I, etc, and whether or not the incumbent wins. We compute the continuation value, $\mathbb{E}[V_I(\mathrm{s}')]$ starting from a given state s as follows:

1. Randomly draw pt' and X', given pt and X using the estimated transition matrix. Multiply the saving w_I' by 1.1 to account for interest accumulation, and add 1 to tenure. Given $\{q_I(z_U), 1.1w_I', ten_I', p_I, pt', X'\}$, simulate the incumbent's retirement decision. Draw a random variable U_{RET} from a uniform $\mathrm{U}(0,1)$. If U_{RET} is less than $\lambda(\{q_I(z_U),\ 1.1w_I',\ ten_I',\ p_I,\ pt',\ X'\})$, then the incumbent retires and we terminate the process. If the incumbent runs for reelection, draw p_I' according to $p_I(\{q_I(z_U),$ $1.1w_I',\ ten_I',\ p_I,\ pt',\ X'\})$. We now have a new state vector $\mathrm{s}' = \{q_I(z_U),\ 1.1w_I',$ $ten_I', p_I', pt', X'\}$ for the subsequent election.

2. Draw a random variable U_{ENT} from a uniform $\mathrm{U}(0,1)$. If U_{ENT} is less than the probability of entry, i.e., $U_{ENT} \leq P_e(\mathrm{s}')$, then there is entry (We estimate P_e as a sufficient statistic. See the next section for details). If $U_{ENT} > P_e(\mathrm{s}')$, then there is no entry.

3. Depending on whether or not there is challenger entry in the previous step, draw d_I and fr_I using the conditional distributions $F_{d_I}(\cdot|\mathrm{s}')$ and $F_{fr_I}(\cdot|\mathrm{s}')$, (if there is entry)

or the estimated policy functions (if there is no entry). In case of entry, further draw a random variable U_{WIN} from a uniform $\mathrm{U}(0,1)$.

4. The period utility function is computed as $\tilde{u}_I = B - \tilde{C}_I(fr_I, q_I) + \tilde{H}_I(d_I)$ in the case of no entry. If there is entry, the period utility is either $u_I = B - C_I(fr_I, q_I)$ $+ H_I(d_I)$ or $u_I = -C_I(fr_I, q_I) + H_I(d_I)$, depending on whether U_{WIN} is smaller or larger than $P_{win}(\mathbf{s}')$. A draw of U_{WIN} smaller than $P_{win}(\mathbf{s}')$ is interpreted as a victory for the incumbent, and a larger value is a loss for the incumbent.

5. Terminate the process if the incumbent loses to the entrant. Otherwise, draw w_I' from $F_{w_I'}(\cdot|\mathbf{s}', \{v_I > 0.5\})$. This determines the amount of savings.

6. Go back to step 1 and repeat until termination. Take the discounted sum of u_I.

7. Repeat steps 1 through 6 and take the average.

Note that for the computation of the continuation value, knowledge of the marginal distributions of the actions is sufficient. We do not need to estimate the joint distribution. This follows from the additive separability of u_I and it greatly simplifies the computation.

10.5.5 Computation of the Derivatives of Continuation Value and Challengers' Continuation Value

In evaluating the right-hand side of expression (13), we need to compute the derivative of the value function with respect to w_I. To do so, we approximate the value function with polynomials of the state variables and use its derivative with respect to w_I.[8] We also use this polynomial to evaluate challengers' continuation payoffs. This is possible because the challenger becomes the incumbent conditional on winning.

10.6 Details on the Estimation

We now discuss the details of the estimation procedure that we omit from the main text. The estimation proceeds according to the following steps.

[8]The alternative approach is to use numerical differentiation, but we found the numerical derivative to be less stable, depending heavily on the step size. This may be because the value function is obtained by simulation, and the simulation errors make the function not very smooth.

Estimation of $P_e(\mathbf{s})$ and $F_{p_C}(\cdot|\mathbf{s},\chi=1)$ We estimate $P_e(\mathbf{s})$ with a Probit. We specify $P_e(\mathbf{s})$ as a function of a party-specific constant, $\ln w_I$, $(\ln w_I)^2$, ten, $D_I \times D_P \times 1\{\text{Midterm}\}$, $D_I \times D_P \times ue$, p_I^2, $D_I \times pt$, $D_I \times dn$, $1\{\text{Midterm}\}$, $1\{\text{First Term}\}$ and $1\{\text{Midterm}\} \times 1\{\text{First Term}\}$. We also include B-spline bases of z_U. We take 7 knots, corresponding to $(1/8,...,7/8)$ quantiles of z_U.

For estimation of $F_{p_C}(p_C|\mathbf{s},\chi=1)$, we take the absolute value of p_C and estimate the distribution of $|p_C|$ conditional on $\mathbf{s}$ and challenger entry, pooling all candidates across parties.

Specifically, we assume that $|p_C|$ follows a discretized Normal distribution with mass points at five bins.[9] We specify the mean and the standard deviation as linear functions of a party-specific constant and state variables, i.e., $|p_C| \sim DN(X_p\gamma_p^\mu, (X_p\gamma_p^\sigma)^2)$.

The likelihood function takes the following form:

$$L(|p_C||\mathbf{s},\chi=1) = \prod_{p_k \in \{0.179, 0.552, 0.738, 0.920, 1.242\}} \Delta\Phi\left(p_k\right)^{1\{|p_C|=p_k\}},$$

$$\text{where } \Delta\Phi(p_k) = \Phi\left(\frac{p_k + \Delta_k^+ - X_p\gamma_p^\mu}{|X_p\gamma_p^\sigma|}\right) - \Phi\left(\frac{p_k - \Delta_k^- - X_p\gamma_p^\mu}{|X_p\gamma_p^\sigma|}\right).$$

The term Δ_k^+ is chosen such that $p_k + \Delta_k^+$ is one decile higher than p_k. Similarly, Δ_k^- is defined such that $p_k - \Delta_k^-$ is one decile below p_k.

As regressors we include a party-specific constant, $\ln w_I$, $(\ln w_I)^2$, ten, $D_I \times D_P \times 1\{\text{Midterm}\}$, $D_I \times D_P \times ue$, $|p_I|$, p_I^2, $D_I \times pt$, $D_I \times dn$, $1\{\text{Midterm}\}$, $1\{\text{First Term}\}$, $1\{\text{Midterm}\} \times 1\{\text{First Term}\}$ and B-spline bases of z_U.[10] Columns 2 and 3 in Table 3 present the estimates of γ_p^μ and γ_p^σ.

Estimation of the Vote Share Equation We estimate the vote share equation of the following form (from Section 3.1):

$$\begin{aligned} vote_I = &\beta_I \mathbb{E}[\ln d_I|\mathbf{s}] + \beta_C \mathbb{E}[\ln d_C|\mathbf{s}] + \beta_P(p_I^2 - \mathbb{E}[p_C^2|\mathbf{s}]) - 2\beta_P\beta_{ID}(p_I - \mathbb{E}[p_C|\mathbf{s}])X \\ &+ \beta_{ten} ten_I + \beta_X X + q_I(z_U) - g(m(\mathbf{s})) + \epsilon, \end{aligned}$$

[9] The points are $\{0.179, 0.552, 0.738, 0.920, 1.242\}$. These points correspond to the 10th, 30th, $\cdots$, 90th-percentile of the nondiscretized distribution. We discuss the way we discretize the data in detail in Appendix 10.7.

[10] In other words, we use the same set of variables that we used to estimate $P_e(\mathbf{s})$, with $|p_I|$ added as an extra regressor.

We approximate $g(m(\mathbf{s}))$ with a third-order polynomial of $P_e(\mathbf{s})$ and a linear function of the probability mass function of $F_{p_C}(p_C|\mathbf{s}, \chi = 1)$, i.e., $\{\Delta\Phi(p_k)\}_k$. We also approximate $q_I(z_U)$ with a second-order polynomial in z_U. We then project the residual of the vote share equation on a set of basis functions consisting of pre-determined variables. The set of pre-determined variables includes (i) the set of variables in the vote share equation, excluding those that are endogenous, i.e., $\ln d_I$, $\ln d_C$, p_C and p_C^2, as well as (ii) other pre-determined variables, $\ln w_I$, $(\ln w_I)^2$, $p_I^2 \times pt$, $p_I^2 \times z_U$, $p_I^2 \times w_I$, $p_I^2 \times ten_I$, ue, ue^2, $ue^2 \times D_I \times D_P$, $D_I \times D_P$, ten_I^2, $\ln(ten_I)$, $g(m(\mathbf{s}))$ and B-spline bases of z_U. We then minimize the squared sum.

Estimation of Components of Candidates' Payoffs and σ_ε^2 We estimate the components of the candidates' payoff function and σ_ε^2 using moments constructed from the first-order conditions and orthogonality conditions implied by the model. For each parameter value, we first simulate the continuation value of the incumbents, $\mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V_I(\mathbf{s}')]$, and compute its derivative, $\frac{\partial}{\partial w_I'}\mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V_I(\mathbf{s}')]$, as we describe in Appendix 10.5. We then invert the incumbent's first-order condition with respect to savings (expression (13)) to back out the value of K, and use expression (14) to obtain the value of q_C. Finally, we substitute out K and q_C in expressions (12) and the two first-order conditions of the challengers. The three first-order conditions are then only functions of observed actions, observed states (note that q_I has been estimated in the previous step), and model parameters. We then stack these first-order conditions, the first-order conditions of incumbents in uncontested periods, and two extra moment conditions. One of the extra moment conditions is that the residual of the vote share equation, ε, is mean zero.[11] The other extra moment restriction is that the variance of the residual, ε, is equal to σ_ε^2. The former constrains the location of average challenger valence in the data. The latter provides a direct restriction on σ_ε^2. Although σ_ε^2 can be identified solely from the first-order conditions, using the residual of the vote share equation as a moment provides a more stable estimate of the parameter. We use the identity matrix to equally weight the first-order conditions.[12]

For some of the observations, we encounter trouble inverting $\Phi(\cdot)$ in expression (13),

[11] Note that once we obtain values of q_C, we can also recover the value of ε at each election.

[12] In order to make sure that our estimation procedure is well-behaved, we also impose penalties for observations in which the continuation payoff, or its derivative with respect to savings, are negative. The penalty is proportional to the squared sum of the negative values. We also impose a penalty when $\frac{\partial C_I}{\partial w_I'}/\delta\frac{\partial \mathbb{E}_{X'|X}[V_I(\mathbf{s}')]}{\partial w_I'}$ exceeds one. The first-order condition (Expression (13)) implies that this term is equal to $\Phi(K)$, the relationship we use to invert for K. If the term is larger than 1, we cannot invert $\Phi(\cdot)$.

because the argument inside Φ^{-1} exceeds 1. This corresponds to cases in which the implied winning probability of the incumbent exceeds 1. When this is the case, we replace the value of $\Phi^{-1}(\cdot)$ with $1 - 10^{-6}$. At the estimated parameters, the argument inside Φ^{-1} is larger than 1 in 553 elections out of 1,040 total elections. We acknowledge that this is not ideal. However, note that even at the true parameter values, the argument of Φ^{-1} can exceed 1 when the other parameters (such as the distribution of outcomes and actions) are estimated with noise. Given that there are many elections in which the incumbent is almost sure to win, even small estimation errors can make the term inside Φ^{-1} exceed 1 for a large fraction of elections in the sample.

Estimation of Parameters in Open-seat Elections We estimate β_O and q_O using the first-order conditions for spending as moments (the open-seat counterpart of Expression (12) in Section 3.2).[13] Because we do not estimate the vote share equation for open-seat elections directly, the coefficient of spending, β_O, is also identified from the first-order conditions.[14] The value functions are computed using the polynomial approximation of the incumbents' value function obtained above.

Note that by restricting the estimation sample to candidates whose valence measure is known, we are selecting the sample partly based on the realization of ε, the error term in the vote share equation.[15] However, given that the candidates choose actions so as to satisfy the first-order conditions before ε is realized, the selection on ε does not bias our estimates.

Estimation of Candidate Valence in All Elections Once all of the model parameters are estimated, we recover q_I, q_C and q_O for all of the candidates in our sample. We run a GMM similar to the one we use to estimate payoff parameters, but now the unknowns are the valence measures of the candidates (for whom the valence term is not recovered from the control function). We use as moment conditions the first-order conditions of both candidates from each contested and open-seat elections as well as the first-order conditions from each uncontested election.

When we recover the valence terms using the first-order conditions, there is some degree of freedom as to which first-order conditions we use. This is because there are more

[13] We do not use the first-order conditions for savings, because the vast majority of open-seat candidates do not save money.

[14] Consider an analog of Expression (12) in an open-seat election. Note that β_O appears inside K as well as in a term outside of K that multiplies $\phi(K)$ in the first-order condition.

[15] If the valence measure of one of the candidates is known, it means that the candidate won the open-seat election. Hence, those candidates must have received a favorable value of ε in the election.

equations than unknowns per election, and because we restrict the valence measure of each candidate to be constant across elections. In practice, we minimize the sum of squared differences of all of the first-order conditions with the constraint that the valence measure of a given candidate is invariant across elections. We also impose the constraint that the valence measures estimated in this stage satisfy the vote share equation estimated in the previous stage. The latter condition can be interpreted as a particular weighting scheme for the first-order conditions.

10.7 Data Construction

We construct the sample we use for estimation as follows: We first drop all House elections in Louisiana.[16] We also drop elections in Texas in 1996 which were deemed unconstitutional by the Supreme Court.[17] We also drop special elections held outside of the regular election cycle, elections that occur right after special elections, instances in which two incumbents run against each other, and elections in which a major scandal broke out.[18] Some observations were also dropped because of missing data.[19] We also drop elections in which either candidate spends and saves less than $\$5,000$, or raises less than $\$5,000$ because both of the first-order conditions of the candidate may not hold with equality. Lastly, we drop elections in which the incumbent saves more than $1.2 million since very large savings are invariably for running for higher offices. We are left with a base sample of 3,065 contested elections, 787 uncontested elections and 445 open-seat elections.

Creation of Partisanship Measure One of the variables that we include in the vote share equation is the Republican partisanship measure of the district, pt. In order to construct this variable, we follow Levendusky et. al. (2008) and regress the log difference in the district-level vote shares of the Republicans and the Democrats in the Presidential election between 1952 and 2008 on the following variables; the fraction of those 65 or above, the

[16]Louisiana has a run-off election unlike any other U.S. state.

[17]The Congressional Elections that were affected by the Supreme Court ruling are TX03, TX05, TX06, TX07, TX08, TX09, TX18, TX22, TX24, TX25, TX26, TX29 and TX30 in 1996.

[18]Elections with two incumbents usually occur due to redistricting. Elections that were dropped because of a scandal are CA17 (1990), MA04 (1990), MN06 (1992), NY15 (1992) and NY15 (2000). These events were identified by going through the biography of candidates in the CQ press Congressional Collection.

[19]Some of the entries in the FEC data set are clearly incorrect. Some candidates are listed as having run in a wrong State, for example. Most of these missing data are easily identifiable because the vote shares do not add up to one or there are multiple candidates from the same party. Where the accuracy of the data is suspect, Open Secrets (http://www.opensecrets.org/) was used as a cross-check in order to correct the mistakes.

fraction of blue-collar workers, the fraction of foreign-born residents, the median income, the population density, and the unemployment rate. We include these regressors in logs. In addition, we include the fraction of Blacks and Hispanics (in logs) and their interactions with an indicator variable for Southern states. We also include an indicator variable for whether or not the candidate is local, and year and state fixed effects. The partisanship measure is obtained as the fitted value of the regression for the concurrent or the most recent presidential election.

Creation of a Measure of Candidate Policy Positions We take the measure constructed by Bonica (2023) as our measure of candidate policy positions. His measure is constructed using information on the source of campaign contributions received by each candidate. The measures are normalized so that negative values correspond to liberal policy positions and positive values correspond to conservative policy positions. We then discretize the measures into 10 bins around zero based on quantiles of the data. In doing so, we impose the restriction that the policy position of a Republican candidate must always be positive and that of a Democrat candidate must always be negative.[20] We assign negative observations to the positive bin closest to zero if the candidate is a Republican, and vice versa, if the observation is a Democrat. Specifically, we implement the discretization as follows:

1. Multiply the policy positions of the Democrats by -1.
2. Pool the policy positions from step 1 and the policy positions of the Republicans. Compute the 20%, 40%, 60% and 80% quantiles of the distribution.
3. For all candidates whose position falls within the first quintile, assign their policy position to the 10 percentile of the distribution. For those in the second quintile, assign their policy position to the 30 percentile of the distribution, and so on. We end up with 5 discretized points.
4. Multiply the Democratic candidates' policy positions with -1.

After the discretization, the set of policy positions for Democratic candidates is $\{$-0.179, -0.552, -0.738, -0.920, -1.242$\}$, and that for Republicans is $\{0.179, 0.552, 0.738, 0.920, 1.242\}$.

[20] In the original (non-discretized) data, about 11.5% of Democratic candidates have a policy position that is positive, while 5.9% of Republican candidates have a policy position that is negative.

10.8 Extensions

Time-Varying q_I It is possible to extend our approach to settings in which q_I varies over time. Suppose, for instance, that (i) $q_{I,t}$ evolves as a random walk as $q_{I,t} = q_{I,t-1} + \xi_t$; (ii) ξ_t is revealed after the challengers make their entry decisions but before the candidates decide how much to spend, raise and save. This would be the case if the challengers make entry decisions based on what they know from the previous election and learn ξ_t only after entry. Under this timing assumption, P_e and $F_{p_C}(\cdot|\mathrm{s}, \chi = 1)$ are functions of $q_{I,t-1}$, $w_{I,t}$, $ten_{I,t}$ and X_t.

Consider estimating the vote share equation using the subset of the sample in which the incumbent is contested in period t and the incumbent is uncontested in period $t-1$. Using $\bar{\mathrm{s}}_U$ from one period before to substitute out $q_{I,t}$, the vote share equation can be expressed as follows:

$$\begin{aligned} vote_{I,t} &= \beta_I \ln d_{I,t} + \beta_C \ln d_{C,t} + \beta_P (p_{I,t} - p_t^*)^2 - \beta_P (p_{C,t} - p_t^*)^2 + \beta_{ten} ten_{I,t} \\ &\quad + \beta_X X + q_I(\bar{\mathrm{s}}_{U,t-1}) - g(P_e, F_{p_C}) + \xi_t + (q_{C,t} - g(P_e, F_{p_C})) + \varepsilon_t. \end{aligned}$$

The econometric error term is $\xi_t + (q_{C,t} - g(P_e, F_{p_C})) + \varepsilon_t$, where $\xi_t = (q_{I,t} - q_I(\bar{\mathrm{s}}_{U,t-1}))$. Given that the expectation of the error term conditional on $\mathrm{s}_t \equiv \{\bar{\mathrm{s}}_{U,t-1}, w_{I,t}, ten_{I,t}, p_{I,t}, p_t^*, X_t\}$ is 0, we can proceed as in the main text by regressing $\mathbb{E}[vote_{I,t}|\mathrm{s}_t]$ on regressors projected on s_t.[21] In our estimation, we assume time-invariant q_I due to data limitations.

Extensions to Settings without Uncontested Races We give a sketch of how our approach can be modified to settings with few uncontested elections, such as Senate races. Our original approach does not directly apply, because we cannot construct a control function based on actions in uncontested periods.

For this application, we assume that the researcher has access to auxiliary data such as polling data that directly identify the expected vote share, $\mathbb{E}_\varepsilon[vote_I]$ up to the error term in the vote share equation, ε_t. An implication of this assumption is that we identify the exact realization of ε for each election as the difference between the realized vote and the expected vote share. Moreover, since Expression (14) implies that $K = \frac{1}{\sigma_\varepsilon}\mathbb{E}_\varepsilon(vote_I)$, the variable K is identified in every election.

[21] Note that $\ln d_{I,t}$, $\ln d_{C,t}$, etc. are correlated with ξ_t, but $\mathbb{E}[\ln d_{I,t}|\mathrm{s}_t]$, $\mathbb{E}[\ln d_{C,t}|\mathrm{s}_t]$, etc. are not.

We first show that, under the assumption that the continuation value V_I is increasing in own quality, the policy functions are invertible with respect to q_I and K. To see this, suppose, counterfactually, that incumbents with q_I and q_I' ($q_I > q_I'$) spend and save the exact same amount conditional on $(w_I, ten_I, p_I, p^*, X, K)$. Now consider the first-order condition (12) which equates the marginal cost of fund-raising to the marginal benefit of spending. The marginal cost of fund-raising is higher for q_I' than for q_I given our assumption of $C_I(\cdot)$. On the other hand, the marginal benefit of spending must be higher for q_I than for q_I' under the assumption that the continuation value is higher for q_I (note that K is fixed). This implies that the first-order condition cannot hold with equality at the same levels of spending and savings for both q_I and q_I'. It is easy to see that q_I can be inverted from the policy function. As long as the policy functions are invertible, we can use actions of the incumbent, states and K from any past contested elections to replace out q_I.

In order to control for q_C, we focus on elections in which a challenger defeats an incumbent. As the challenger who defeats an incumbent becomes an incumbent, we observe that candidate's actions in the next election as an incumbent. This implies that we can use the actions, states and K from future contested elections to replace out q_C. We can then identify the vote share equation and the values of q_I and q_C for a subset of the candidates. Once the vote share equation has been identified, it is straightforward to use the first-order conditions to identify the marginal cost of raising money and the marginal benefit of spending. Once these primitives are identified, the first-order conditions can be used to recover the valence measure of all candidates.

10.9 Distribution of Candidate Valence after Applying Bayes Shrinkage

Figure 15 plots the histogram of candidate valence after applying Bayes shrinkage with a Normal prior. Specifically, for each candidate i's valence term q_i, we apply the following formula:

$$\tilde{q}_i = B_i q_i + (1 - B_i)\overline{q},$$
$$\text{where } B_i = \frac{\sigma_{q_i}^2}{\sigma_{q_i}^2 + \sigma_q^2}.$$

We denote by σ_{q_i} the standard error of our estimate of q_i (within-candidate, obtained by bootstrap). We denote by $\overline{q}$ and σ_q the mean and standard deviation of the estimated q_i's across candidates. We compute $\overline{q}$ and σ_q separately for incumbents, challengers, and open-seat candidates.

Even with shrinkage, our discussion in Section 5.3 of the main text remains qualitatively the same. We find that incumbents have higher valence than challengers that run against them, by about 3.6 percentage points. The difference between incumbents and open-seat challengers is 4.3 percentage points. We also find that the distribution of incumbent valence has lower dispersion than that for challengers and for open-seat challengers. We find that the upper tail of the distribution of the open-seat challengers resembles that of the incumbents, but there is also a substantial mass of low valence open-seat challengers.

10.10 Bootstrapping Standard Errors

The standard errors in Table 3, the first column of 4, Table 5 and the standard error of the distribution of valence are obtained based on 500 bootstrap estimates. For each iteration of the bootstrap, we randomly draw congressional districts with replacement.[22] For each bootstrap sample, we estimate the vote share equation and candidate utility terms following the procedure described in Section 3. Once all bootstrap estimates for the vote share equation and utility terms are obtained, we next bootstrap the standard error of the distribution of valence using all candidates in the data. In each bootstrap iteration, we use the full data, but evaluate the first-order conditions of those candidates at the bootstrap parameter values.[23]

10.11 Model Fit for Actions in Uncontested Elections

Figure 16 reports the histograms of the predicted (white bars) and the realized (gray bars) candidate actions in uncontested elections. Panel (A) of the figure corresponds to spending and Panel (B) corresponds to savings. We underestimate spending for the majority of candidates and, because unspent money is saved, overestimate savings. This is partly due to our small estimate of $H(\cdot)$ we present in Figure 3B. Although $H(\cdot)$ should in principle capture candidates' spending incentives in sure-to-win elections such as uncontested elections, estimating $H(\cdot)$ from the first-order conditions also requires the levels of $H(\cdot)$ to

[22]The bootstrap is hence a block bootstrap rather than a bootstrap at the observation level. Because of this, the sample size may vary across bootstrap iterations.

[23]We also impose a lower- and upper-bound for possible value of valence at -0.2 and 0.1, respectively. We find that bootstrapped valence terms obtained without the bound tend to include a small subset of extremely large values.

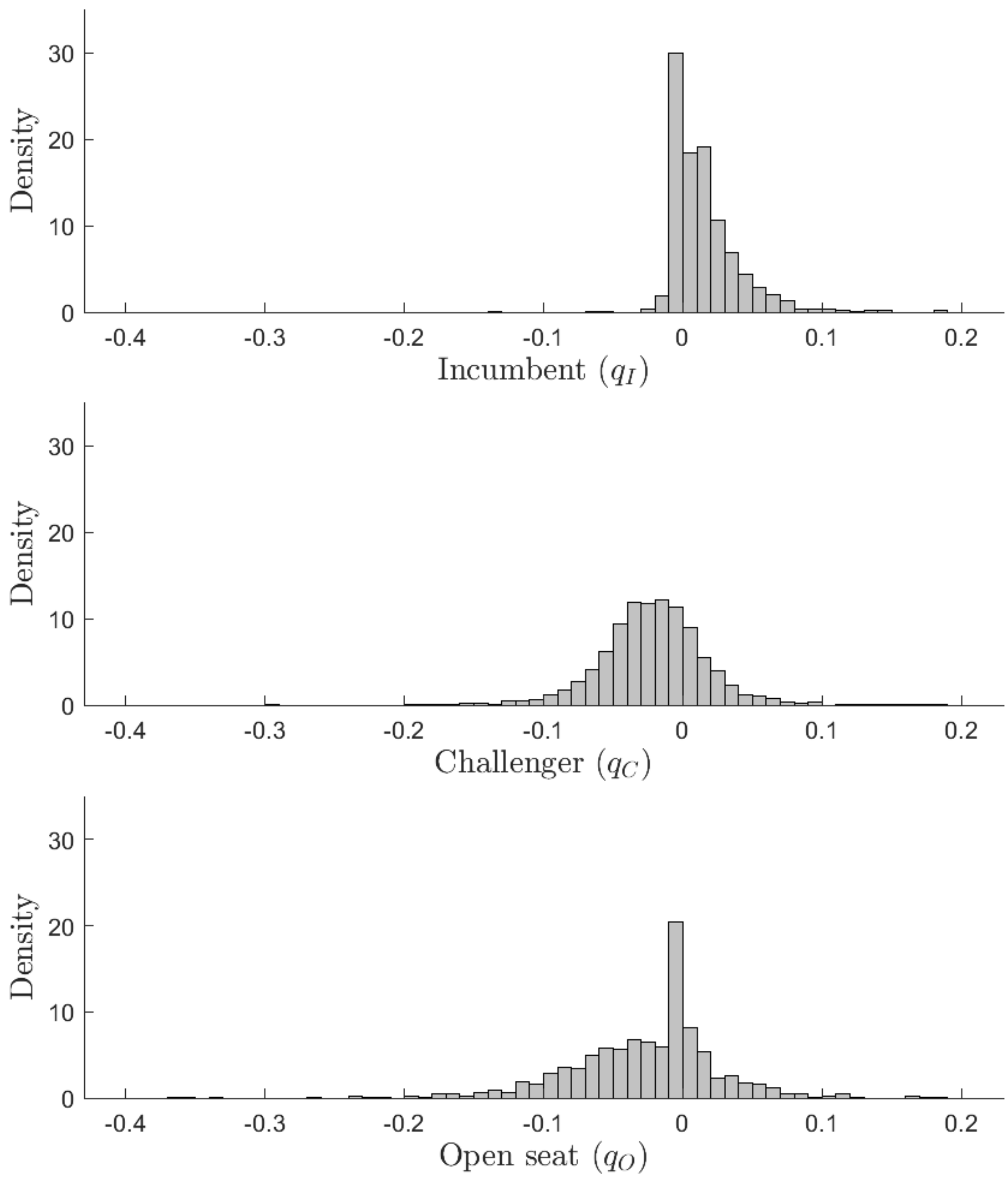
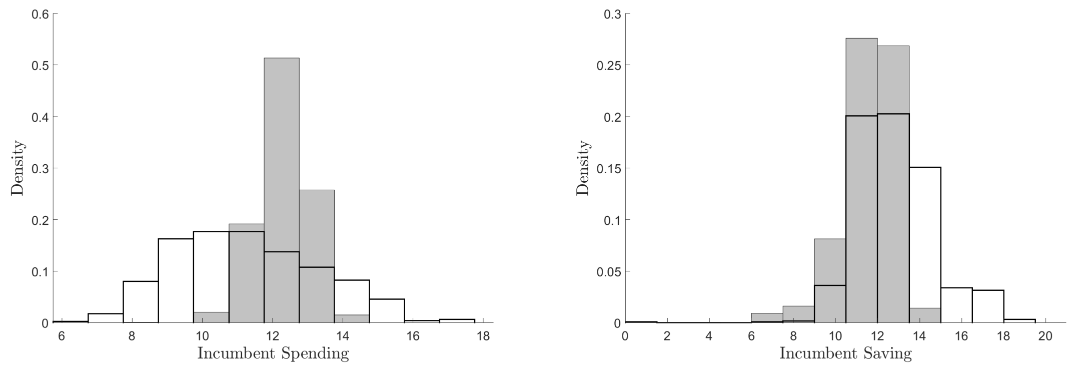


Figure 15: Distribution of Candidate Valence (Shrinkage Applied)

be low.[24] As a result, our estimate of $H(\cdot)$ is lower than what would fully rationalize the spending incentives in uncontested elections.



(A) Spending (B) Saving

Figure 16: Model Fit - Actions in Uncontested Elections

Note: Histogram of observed actions (gray bars) and predicted actions (white bars). Predicted actions are obtained by solving the estimated first-order conditions.

10.12 Policy Estimation for Counterfactual Evaluation

Evaluating the impact of a shift in challenger valence (Section 7) and simulating the derivative of v_C with respect to q_C (Appendix 10.2) both require us to predict candidate actions at realizations of q_C not observed in the data. We approximate the policy functions of the candidates regarding spending, saving and fund-raising as a flexible function of $\{\mathbf{s}, q_C, p_C\}$ and evaluate these functions at various values of q_C to compute the optimal actions.[25] Specifically, using the set of contested elections, we regress d_I, d_C, fr_C and w'_C on a set of state variables and their interactions. We include as regressors a constant, q_C, $\exp(q_C)$, q_I, $\exp(q_I)$, p_I, $p_I \times pt$, p_I^2, p_C, $p_C \times pt$, p_C^2, pt, ten_I, ten_I^2, $1\{\text{Midterm}\} \times D_I \times D_P$, as well as the complete set of interaction terms between $\{ue \times D_I \times D_P,\, dn \times D_I\}$ and $\{1\{\text{Midterm}\}, 1\{\text{First Term}\}\}$.

[24]This is because $H(\cdot)$ also shows up in the continuation payoff. A large value of $H(\cdot)$ inflates the continuation values and results in overestimation of candidates' spending incentives in contested elections.

[25]Note that we cannot solve the dynamic game to compute the policy function because we do not estimate some of the model primitives regarding the Primary, e.g., $\pi(\cdot), \kappa, F_N(\cdot|\mathbf{s})$, etc. See e.g., Barwick and Pathak (2015) for a similar approach. They use a polynomial to approximate the vale function of a dynamic model.

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