SUPPLEMENT TO "THE POLITICAL ECONOMY OF ZERO-SUM THINKING"

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The Supplemental Appendix contains all arguments, notation, and proofs for Sections 4 and 5 of Ali, Mihm, and Siga (2024).

1. PROOFS FOR SECTION 4

For a binary collective choice problem $\mathcal{C} = (\mathcal{N}, \Omega, P)$, we let $W_i = \{\omega \in \Omega : V_i^d(\omega) > 0\}$ be the set of states in which voter i is a winner, and $\mathcal{W}(\omega) = \{i \in \mathcal{N} : \omega \in W_i\}$ denote the set of winners in state ω.

1.1. *Proof of Proposition 3*

Suppose $\mathcal{C}$ is binary collective choice problem in which signals are fully-informative. We first show that, for any $\kappa \in \{0, ..., \tau\}$,

$$V^G(\kappa|P_W, v) = \sum_{w=\kappa}^{n} \left(\left(\frac{w-\kappa}{n-\kappa} \right) v_W - \left(\frac{n-w}{n-\kappa} \right) v_L \right) \frac{\binom{w}{\kappa} P_W(w)}{\sum_{w'=\kappa}^{n} \binom{w'}{\kappa} P_W(w')}. \tag{2}$$

For $\kappa \in \{0, ..., \tau\}$, $w \in \{\kappa + 1, ..., n\}$, and $i \in \mathcal{N}$,

$$\begin{aligned} &P(G = \kappa | S_i = s^0, M = \kappa, W = w) \\ &= \left(\frac{w}{n}\right) \frac{\binom{w-1}{\kappa}\binom{n-1-(w-1)}{0}}{\binom{n-1}{\kappa}} + \left(\frac{n-w}{n}\right) \frac{\binom{w}{\kappa}\binom{n-1-w}{0}}{\binom{n-1}{\kappa}} \\ &= \frac{\binom{w}{\kappa}}{\binom{n}{\kappa}} \end{aligned}$$

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and

$$P(G=\kappa|S_i=s^0,M=\kappa,W=\kappa)=\frac{1}{\binom{n}{\kappa}}.$$

Therefore, for any $\kappa\in\{0,...,\tau\}$ and $w\in\{\kappa,...,n\}$, and $i\in\mathcal{N}$,

$$P(W=w|S_i=s^0,G=M=\kappa)=\frac{\binom{w}{\kappa}P_W(w)}{\sum_{w'=\kappa}^{n}\binom{w'}{\kappa}P_W(w')}.$$

Equation (2) then follows by observing that

$$P(W_i|S_i=s^0,G=M=\kappa,W=w)=\frac{w-\kappa}{n-\kappa}.$$

Parts (a) and (b) follow because $V^G(\kappa)$ is increasing in v_W for a fixed v_L and P_W, and decreasing in v_L for a fixed v_W and P_W. Part (c) follows because, for $w'>w$, $P'_W(w')P_W(w)\geq P'_W(w)P_W(w')$ implies that

$$\frac{\binom{w'}{\kappa}P'_W(w')}{\sum_{w''=\kappa}^{n}\binom{w''}{\kappa}P'_W(w'')}\frac{\binom{w}{\kappa}P_W(w)}{\sum_{w''=\kappa}^{n}\binom{w''}{\kappa}P_W(w'')}\geq\frac{\binom{w}{\kappa}P'_W(w)}{\sum_{w''=\kappa}^{n}\binom{w''}{\kappa}P'_W(w'')}\frac{\binom{w'}{\kappa}P_W(w')}{\sum_{w''=\kappa}^{n}\binom{w''}{\kappa}P_W(w'')},$$

and so $P_W\succcurlyeq_{LR}P'_W$ implies $P_W(.|S_i=s^0,G=M=\kappa)\succcurlyeq_{LR}P'_W(.|S_i=s^0,G=M=\kappa)$. Since $P(W_i|S_i=s^0,G=M=\kappa,W=w)$ is increasing in w, $P_W(.|S_i=s^0,G=M=\kappa)\succcurlyeq_{LR}P'_W(.|S_i=s^0,G=M=\kappa)$ then implies that $V^G(\kappa|P_W,v_W,v_L)\geq V^G(\kappa|P'_W,v_W,v_L)$. *Q.E.D.*

1.2. *Proof of Proposition 4*

For the n-voter collective choice problem, let

$$V_n(g,b)=\left(\frac{\lceil qn\rceil-g}{n-g-b}\right)v_w-\left(\frac{n-\lceil qn\rceil-b}{n-g-b}\right)v_l$$

be the expected payoff difference for an uninformed voter who learns that g other voters received good news and b other voters received bad news. Hence, the n-voter collective choice problem has adverse correlation if there exists $g\in\{0,...,\frac{n-1}{2}\}$ such that $V_n(g,0)<0$. In particular, since $V_n(g,0)$ is strictly decreasing in g, there is adverse correlation if and only if $V_n(\frac{n-1}{2},0)<0$, i.e.,

$$\frac{v_\ell}{v_w}>\frac{\lceil qn\rceil-\dfrac{n-1}{2}}{n-\lceil qn\rceil}=\frac{\dfrac{\lceil qn\rceil}{n}-\dfrac{1}{2}+\dfrac{1}{2n}}{1-\dfrac{\lceil qn\rceil}{n}}.$$

The term on the right-hand side is strictly greater than ρ, converging to ρ as $n \to \infty$. Hence, if $v_\ell/v_w \leq \rho$, the n-voter collective choice problem is advantageously correlated for any population size n; if $v_\ell/v_w > \rho$, then there exists $N(q, v_\ell/v_w)$ such that n-voter collective choice problem has adverse correlation for all $n > N(q, v_\ell/v_w)$. *Q.E.D.*

1.3. *Proof of Proposition 5*

Given $\lambda \in (0,1)$ and a strategy-profile σ_n of the n-voter collective choice problem, let $P_{\sigma_n,\lambda}(g, b|piv, S_i = s_0)$ be the probability that an uninformed voter i attaches to there being g informed winners and b informed losers, conditional on that voter being pivotal (as long as pivotality is a non-null event). If pivotality is a non-null event, the expected payoff difference conditional on being pivotal is, therefore,

$$\Pi_0^i(\sigma_n, \lambda) = \sum_{g=0}^{\frac{n-1}{2}} \sum_{b=0}^{n-\lceil qn \rceil} V_n(g, b) P_{\sigma_n,\lambda}(g, b|piv, S_i = s_0),$$

where we can omit the i-superscript if σ_n is symmetric.

Now suppose $v_\ell/v_w > \rho$ and fix some $\varepsilon \in (0,1)$. For the n-voter collective choice problem, denote by σ_n^* the symmetric weakly undominated strategy profile where uninformed voters vote for the inferior policy. Then, an uninformed voter is pivotal if and only if $g = \frac{n-1}{2}$, which is a non-null event, and

$$P_{\sigma_n^*,\lambda}(g, b|piv, S_i = s_0) = \begin{cases} \binom{n-\lceil qn \rceil}{b} \lambda^b (1-\lambda)^{n-\lceil qn \rceil - b} & \text{if } g = \frac{n-1}{2} \\ 0 & \text{otherwise} \end{cases}.$$

Therefore,

$$\Pi_0(\sigma_n^*, \lambda) = \sum_{b=0}^{n-\lceil qn \rceil} V_n\left(\frac{n-1}{2}, b\right) \binom{n-\lceil qn \rceil}{b} \lambda^b (1-\lambda)^{n-\lceil qn \rceil - b},$$

which is strictly increasing in λ since $V_n\left(\frac{n-1}{2}, b\right)$ is strictly increasing in b. Moreover, by the weak law of large numbers and the Portmanteau Theorem,

$$\lim_{n\to\infty} \Pi_0(\sigma_n^*, \lambda) = \left(\frac{q - \frac{1}{2}}{\frac{1}{2} - \lambda(1-q)} \right) v_w - \left(\frac{(1-q)(1-\lambda)}{\frac{1}{2} - \lambda(1-q)} \right) v_l,$$

which is strictly increasing in λ and strictly negative if $\lambda < 1 - (v_\ell/v_w)^{-1}\rho$. Hence, for any $\lambda < 1 - (v_\ell/v_w)^{-1}\rho$, there exists N^* such that for all $n > N^*$, $\Pi_0(\sigma_n^*, \lambda) < 0$; hence, σ_n^* is a strict equilibrium. Moreover, under the strategy-profile σ_n^* only informed winners vote for the optimal policy. The proportion of informed winners is strictly increasing in λ and, by the weak law of large numbers, converges in probability to λq as $n \to \infty$. Since $\lambda^* q < \frac{1}{2}$, it follows that there exists N_ε such that for all $n > N_\varepsilon$, the inferior policy wins with probability greater than $1 - \varepsilon$ when voters follow the strategy-profile σ_n^*. Hence, for all $n > \max\{N^*, N_\varepsilon\}$, there is a strict equilibrium in which the inferior policy wins with probability at least $1 - \varepsilon$.

Conversely, suppose $v_\ell/v_w \leq \rho$ and fix any population size n and $\lambda \in (0,1)$. Since $V_n(g, b)$ is strictly decreasing in g and strictly increasing in b, it follows that for any $g \in \{0, ..., \frac{n-1}{2}\}$

and $b \in \{0, ..., n - \lceil qn \rceil\}$,

$$V_n(g,b) \geq \left(\frac{\frac{\lceil qn \rceil}{n} - \frac{1}{2} + \frac{1}{2n}}{\frac{1}{2} + \frac{1}{2n}} \right) v_w - \left(\frac{1 - \frac{\lceil qn \rceil}{n}}{\frac{1}{2} + \frac{1}{2n}} \right) v_l > \left(\frac{q - \frac{1}{2}}{\frac{1}{2} + \frac{1}{2n}} \right) v_w - \left(\frac{1-q}{\frac{1}{2} + \frac{1}{2n}} \right) v_l > 0.$$

Now, for sake for contradiction, suppose there exists a weakly undominated equilibrium σ^n in which the inferior policy wins with strictly positive probability. Let $\mathcal{N}^* = \{i \in \{1, ..., n\} : \sigma_i^n < 1\}$ be the set of voters who vote for the inferior policy with strictly positive probability when they are uninformed, and let $n^* = |\mathcal{N}^*|$. Since the inferior policy wins with strictly positive probability, $n - \lceil qn \rceil + n^* \geq \frac{n+1}{2}$, while $n - \lceil qn \rceil \leq \frac{n-1}{2}$. Hence, any of the voters in $\mathcal{N}^*$ can be pivotal with strictly positive probability when they are uninformed. However, since $V_n(g,b) > 0$ for all $g \in \{0, ..., \frac{n-1}{2}\}$ and $b \in \{0, ..., n - \lceil qn \rceil\}$, it follows that $\Pi_0^i(\sigma_n, \lambda) > 0$, which contradicts that σ^n is an equilibrium in which $\sigma_i^n < 1$. Hence, in any weakly undominated equilibrium, the optimal policy wins with probability 1. *Q.E.D.*

1.4. *Proof of Proposition 6*

Suppose $\mathcal{C}$ is a binary collective choice problem in which signals convey only aggregate news. For any signal profile $s \in \mathcal{S}^n$, it follows that

$$V_i^d(s) = \sum_{w=0}^{n} V_i^d(s, W = w) P(W = w | S = s) = \sum_{w=0}^{n} V_i^d(W = w) P(W = w | S = s),$$

and therefore, $V_i^d(s) = V_j^d(s)$ for all $i, j \in \mathcal{N}$. Now consider a signal profile s such that $s_i \in \mathcal{G}$ and $s_j \in \mathcal{M}$ for some voters $i \neq j$. Then $V_i^d(s_i) > 0$ and so $V_i^d(s) > 0$ by Assumption 2(b). It follows that $V_j^d(s) > 0$, and so $V_j^d(s_j) > 0$ by Assumption 2(b). Hence, $s_j \in \mathcal{G}$. Analogously, if $s_i \in \mathcal{B}$ and $s_j \in \mathcal{M}$, then $s_j \in \mathcal{B}$.

Let $G(\kappa) = \{s \in \mathcal{M}^n : G(s) \geq \kappa\}$. By Lemma 1 and the preceding argument, for a voter who is uninformed, and learns that $\kappa \in \{1, ..., \tau\}$ other voters received good news,

$$\begin{aligned} P(W_i | S_i = s^0, M = G = \kappa) &= \lambda^\kappa (1-\lambda)^{n-\kappa} \sum_{s \in G(\kappa)} P(W_i|s) P(s | s \in G(\kappa)) \\ &= \lambda^\kappa (1-\lambda)^{n-\kappa} \sum_{s \in \mathcal{G}^n} P(W_i|s) P(s | s \in \mathcal{G}^n) \\ &= \lambda^\kappa (1-\lambda)^{n-\kappa} P(W_i | S \in \mathcal{G}^n) \end{aligned}$$

and, therefore, $V^G(\kappa) > 0$. *Q.E.D.*

1.5. *Proof of Proposition 7*

We say that binary collective choice problems $\mathcal{C} = (\mathcal{N}, \Omega, P)$ and $\mathcal{C}' = (\mathcal{N}, \Omega', P')$ are informationally equivalent if, for any $\mathcal{N}' \subseteq \mathcal{N}$ and $s \in \mathcal{S}^n$, $P(\mathcal{W} = \mathcal{N}', S = s) = P'(\mathcal{W} = \mathcal{N}', S = s)$; that is, the problems differ in terms of the payoff but not the information structure. We show that, when all news is distributional, for any binary collective choice problem, there is an informationally equivalent problem that has adverse correlation.

Suppose $\mathcal{C} = (\mathcal{N}, \Omega, P)$ is a binary collective choice problem in which signals convey only distributional news. Without loss of generality, let $P(W_i|S_i = s^k) \geq P(W_i|S_i = s^{k+1})$ for $k = 1, ..., K-1$ (where it does not matter in the following how ties are broken). Let $\mathcal{G}' = \{k = 1, ..., K : P(W_i|S_i = s^K) \geq P(W_i)\}$ and $\mathcal{B}' = \mathcal{M} - \mathcal{G}'$. Since $P(B \geq 1) > 0$, both $\mathcal{G}'$ and $\mathcal{B}'$ are non-empty, and $P(W_i|S_i = s^1) > P(W_i)$. For a signal profile $s \in \mathcal{S}^n$, $G'(s)$ is the number of voters with a signal in $\mathcal{G}'$.

We first show that, for any voter $h \in \mathcal{N}$,

$$P(W_h) > P(W_h|S_h = s^0, G' = M = 1). \tag{3}$$

Let $w \in \{0, ..., n\}$ and E be any event such that $E \cap W^{-1}(w)$ is non-null. Then,

$$\begin{aligned}\sum_{i=1}^{n} P(W_i|E, w) &= \sum_{i=1}^{n} \sum_{\omega \in \Omega(w) \cap E} P(W_i|E, w, \omega) P(\omega|E, w) \\ &= \sum_{\omega \in \Omega(w) \cap E} P(\omega|E, w) \sum_{i=1}^{n} P(W_i|E, w, \omega) \\ &= w \sum_{\omega \in \Omega(w) \cap E} P(\omega|E, w) = w\end{aligned}$$

Hence, for any voter $i \neq h$ and $w \in \{0, ..., n\}$ with $P(W = w) > 0$, Assumption 1 implies that

$$\begin{aligned}\sum_{j=1}^{n} P(W_j|S_i \in \mathcal{G}', M = 1, w) &= P(W_i|S_i \in \mathcal{G}', M = 1, w) + \sum_{j \neq i} P(W_j|S_i \in \mathcal{G}', M = 1, w) \\ &= P(W_i|S_i \in \mathcal{G}', M = 1, w) + (n-1) P(W_h|S_i \in \mathcal{G}', M = 1, w) \\ &= \sum_{j=1}^{n} P(W_j|w) = P(W_i|w) + \sum_{j \neq i} P(W_j|w) \\ &= P(W_i|w) + (n-1) P(W_h|w).\end{aligned}$$

Since $P(W_i|w) < P(W_i|S_i \in \mathcal{G}', M = 1, w)$, it follows that $P(W_h|w) > P(W_h|S_i \in \mathcal{G}', M = 1, w)$. Moreover, by Assumption 1,

$$\begin{aligned}P(W_h|S_k = s^0, G' = M = 1, w) &= \sum_{j \neq h} P(W_h|S_j \in \mathcal{G}', M = 1, w) P(S_j \in \mathcal{G}'|G' = M = 1, w) \\ &= \frac{1}{n-1} \sum_{j \neq h} P(W_h|S_j \in \mathcal{G}', M = 1, w) \\ &= P(W_h|S_i \in \mathcal{G}', M = 1, w),\end{aligned}$$

and, therefore, $P(W_h|w) < P(W_h|S_k = s^0, G' = M = 1, w)$. Since signals convey only distributional information,

$$P(W_h) = \sum_{w=0}^{n} P(W_h|w) P(w)$$

$$
\begin{aligned}
&< \sum_{w=0}^{n} P(W_h|S_k = s^0, G' = M = 1, w)P(w) \\
&= \sum_{w=0}^{n} P(W_h|S_k = s^0, G' = M = 1, w)P(w|S_h = s^0, G' = M = 1) \\
&= P(W_h|S_h = s^0, G' = M = 1),
\end{aligned}
$$

and, therefore, $P(W_h) > P(W_h|S_k = s^0, G' = M = 1)$.

Now let $k^* = \min\{k = 1, ..., K : s^k \in \mathcal{B}'\}$ and let $P^* = \max\{P(W_i|S_i = s^{k^*}), P(W_h|S_k = s^0, G' = M = 1)\}$. From the preceding argument, $P(W_h) > P^*$ and so there exists $(v'_W, v'_L) >> 0$ such that

$$P(W_h)v'_W - (1 - P(W_h))v'_L > 0 > P^* v'_W - (1 - P^*)v'_L.$$

The binary collective choice problem $\mathcal{C}' = (\mathcal{N}, \Omega, P')$ uniquely defined by letting $P'(\mathcal{W} = \mathcal{N}', S = s) = P(\mathcal{W} = \mathcal{N}', S = s)$ for any $\mathcal{N}' \subseteq \mathcal{N}$ and $s \in \mathcal{S}$ is informationally equivalent to $\mathcal{C}$, and $\mathcal{C}'$ is adversely correlated because the set of good news signals for $\mathcal{C}'$ is exactly $\mathcal{G}'$. As a result, for any binary collective choice problem $\mathcal{C}$ in which signals convey only distributional information there exists an informationally equivalent collective choice problem with adverse correlation. *Q.E.D.*

2. PROOFS FOR SECTION 5

2.1. *Symmetric Equilibria (Section 5.1)*

Preliminaries: Let σ^α be the symmetric strategy-profile defined in the proof of Proposition 1. Recall that, for $g \in \{0, ..., \tau\}$ and $m \in \{g, ..., g + \tau\}$,

$$p_i(\sigma^\alpha|g, m) = \begin{cases} \mathbb{1}[g = \tau] & \text{if } \alpha = 0 \\ \mathbb{1}[m - g = \tau] & \text{if } \alpha = 1 \\ \binom{n-1-m}{\tau-g}\alpha^{\tau-g}(1-\alpha)^{\tau-(m-g)} & \text{if } \alpha \in (0,1) \end{cases},$$

and so

$$\Pi_0(\sigma^1, \lambda) = \sum_{g=0}^{\tau} \binom{n-1}{\tau+g} \lambda^{\tau+g}(1-\lambda)^{\tau-g} Z(g, \tau+g),$$

$$\Pi_0(\sigma^0, \lambda) = \sum_{m=\tau}^{n-1} \binom{n-1}{m} \lambda^m (1-\lambda)^{n-1-m} Z(\tau, m),$$

and, for $\alpha \in (0, 1)$,

$$
\begin{aligned}
\Pi_0(\sigma^\alpha, \lambda) &= \sum_{g=0}^{\tau} \sum_{m=g}^{\tau+g} \mathcal{M}(g, m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau-g}(1-\alpha)^{\tau+g-m} Z(g, m), \\
&= \alpha^\tau(1-\alpha)^\tau(1-\lambda)^{n-1} \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)}\right)^g \sum_{m=g}^{\tau+g} \left(\frac{\lambda}{(1-\alpha)(1-\lambda)}\right)^{m-g} \mathcal{M}(g, m)Z(g, m)
\end{aligned}
$$

where $\mathcal{M}(g, m)$ is shorthand for the multinomial coefficient $\binom{n-1}{\tau-g, m, \tau+g-m}$.

Proof of Theorem 3

(1) Suppose $\mathcal{C}$ is strongly adversely correlated: $\sum_{\kappa=0}^{\tau} \theta^{\kappa}\binom{\tau}{\kappa} Z(\kappa,\kappa) < 0$ for some $\theta \in \mathbb{R}_{++}$, and fix some $\varepsilon \in (0,1)$.

By Case 1 in the proof of Theorem 1, if $Z(\tau,\tau) < 0$, then there exists $\bar{\lambda} \in (0, 1-(1-\varepsilon)^{\frac{1}{n}})$ such that σ^0 is a symmetric equilibrium in which p_* wins with probability exceeding $1-\varepsilon$ for all $\lambda \in (0,\bar{\lambda})$. Therefore, we can focus on the case $Z(\tau,\tau) > 0$. In that case, since

$$\lim_{\lambda\to 0} \sum_{m=\tau}^{n-1} \binom{n-1}{m} \lambda^{m-\tau}(1-\lambda)^{n-1-m} Z(\tau,m) = \binom{n-1}{\tau} Z(\tau,\tau)$$

it follows that $\Pi_0(\sigma^0,\lambda) > 0$ for $\lambda > 0$ sufficiently small.

Let $\bar{\alpha} \equiv 1-(1-\varepsilon)^{\frac{1}{2(\tau+1)}}$. If $\alpha < \bar{\alpha}$ and $\lambda < 1-(1-\varepsilon)^{\frac{1}{2n}}$, then p_* wins in strategy profile σ^{α} with probability exceeding

$$(1-\bar{\alpha})^{\tau+1}(1-\lambda)^n > (1-\varepsilon)^{\frac{\tau+1}{2(\tau+1)}}(1-\varepsilon)^{\frac{n}{2n}} = 1-\varepsilon$$

Since $\Pi_0(\sigma^0,\lambda) > 0$ for λ sufficiently small, it therefore sufficies to show that there exists $\bar{\lambda} \in (0,1)$ such that, for all $\lambda \in (0,\bar{\lambda})$, there is a $\alpha_\lambda \in (0,\bar{\alpha})$ such that $\Pi_0(\sigma^{\alpha_\lambda},\lambda) < 0$.

For any $\lambda \in (0,1)$, let $\alpha_\lambda \equiv \frac{\lambda}{(1-\lambda)\theta+\lambda}$; hence, $\alpha_\lambda \in (0,1)$, is increasing in λ, and converges to 0 as $\lambda \to 0$. Therefore,

$$\lim_{\lambda\to 0} \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha_\lambda)}{\alpha_\lambda(1-\lambda)}\right)^g \sum_{m=g}^{\tau+g} \left(\frac{\lambda}{(1-\alpha_\lambda)(1-\lambda)}\right)^{m-g} \mathcal{M}(g,m)Z(g,m)$$
$$= \sum_{g=0}^{\tau} \theta^g \mathcal{M}(g,g)Z(g,g) = \binom{n-1}{\tau}\mathcal{K}(\theta) < 0.$$

Hence, there is $\bar{\lambda} \in (0,1)$ such that $\Pi_0(\sigma^{\alpha_\lambda},\lambda) < 0$ for all $\lambda \in (0,\bar{\lambda})$.

(2) Suppose $\mathcal{C}$ is weakly advantageously correlated: $\sum_{\kappa=0}^{\tau} \theta^{\kappa}\binom{\tau}{\kappa} Z^G(\kappa) > 0$ for all $\theta \in \mathbb{R}_{++}$.

The correlation structure implies that $Z(\tau,\tau) > 0$, and so there exists $\bar{\theta}$ such that

$$\sum_{\kappa=0}^{\tau} \theta^{\kappa}\binom{\tau}{\kappa} Z(\kappa,\kappa) > Z(0,0)$$

for all $\theta > \bar{\theta}$. Since $\lim_{\theta\to 0}\sum_{\kappa=0}^{\tau} \theta^{\kappa}\binom{\tau}{\kappa} Z(\kappa,\kappa) = Z(0,0)$, it follows that $\mathcal{K}$ attains a minimum on $[0,\bar{\theta}]$, which is strictly positive. As a result, there exists $\delta > 0$ such that $\sum_{\kappa=0}^{\tau} \theta^{\kappa}\binom{\tau}{\kappa}\left(Z^G(\kappa)-\delta\right) > 0$ for all $\theta \in \mathbb{R}_{++}$.

Now fix some $\varepsilon \in (0,1)$. Let $\bar{\alpha} \equiv (1-\varepsilon)^{\frac{1}{2(\tau+1)}} \in (0,1)$. If $\alpha \in [\bar{\alpha},1]$ and $\lambda < 1-(1-\varepsilon)^{\frac{1}{2n}}$, then p^* wins with probability exceeding

$$\alpha^{\tau+1}(1-\lambda)^n > (1-\varepsilon)^{\frac{\tau+1}{2(\tau+1)}}(1-\varepsilon)^{\frac{n}{2n}} = 1-\varepsilon$$

in the strategy profile σ^{α}.

For $g \in \{0,...,\tau\}$, let $\phi(g) \equiv \mathcal{M}(g,g)Z(g,g)$ and, for $\alpha, \lambda \in (0,1)$, let

$$\phi(g,\alpha,\lambda) \equiv \sum_{m=g+1}^{\tau+g} \left(\frac{\lambda}{(1-\alpha)(1-\lambda)}\right)^{m-g} \mathcal{M}(g,m)Z(g,m)$$

so that

$$\begin{aligned}\Pi_0(\sigma^\alpha,\lambda) &= \alpha^\tau(1-\alpha)^\tau(1-\lambda)^n \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)}\right)^g \Big(\phi(g) + \phi(g,\alpha,\lambda)\Big) \\ &\geq \alpha^\tau(1-\alpha)^\tau(1-\lambda)^n \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)}\right)^g \Big(\phi(g) - |\phi(g,\alpha,\lambda)|\Big)\end{aligned}$$

For $g \in \{0,...,\tau\}$ and $\alpha \in (0,\bar{\alpha})$, $|\phi(g,\alpha,\lambda)| \leq |\phi(g,\bar{\alpha},\lambda)|$, and so there exists $\bar{\lambda} \in (0,1)$ such that $|\phi(g,\bar{\alpha},\lambda)| \leq \binom{n-1}{\tau}\delta$ for all $\lambda \in (0,\bar{\lambda})$ and $g \in \{0,...,\tau\}$. Hence, for all $\lambda \in (0,\bar{\lambda})$,

$$\begin{aligned}\Pi_0(\sigma^\alpha,\lambda) &\geq \alpha^\tau(1-\alpha)^\tau(1-\lambda)^n \sum_{g=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)}\right)^g \left(\phi(g) - \binom{n-1}{\tau}\delta\right) \\ &= \alpha^\tau(1-\alpha)^\tau(1-\lambda)^n \binom{n-1}{\tau} \sum_{\kappa=0}^{\tau} \left(\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)}\right)^\kappa \binom{\tau}{\kappa} \left(Z(\kappa,\kappa) - \delta\right) > 0,\end{aligned}$$

since $\frac{\lambda(1-\alpha)}{\alpha(1-\lambda)} \in \mathbb{R}_{++}$. Therefore, for all $\lambda \in (0,\bar{\lambda})$, σ^α is not an equilibrium for any $\alpha \in (0,\bar{\alpha})$. Moreover, if $Z(\tau,\tau) > 0$, there exists $\bar{\lambda}_0 \in (0,1)$ such that $\Pi_0(\sigma^0,\lambda) > 0$ for all $\lambda \in (0,\bar{\lambda}_0)$.

Hence, for $\lambda_\varepsilon = \min\{1-(1-\varepsilon)^{\frac{1}{2n}}, \bar{\lambda}, \bar{\lambda}_0\}$, the preceding arguments show that, for all $\lambda \in (0,\lambda_\varepsilon)$ and any $\alpha \in [0,1]$, either p^* wins with probability exceeding $(1-\varepsilon)$ in the strategy profile σ^α or the strategy profile σ^α is not an equilibrium. *Q.E.D.*

2.2. *Population Uncertainty (Section 5.2)*

Preliminaries: We first adapt our main assumptions from Section 2 of Ali, Mihm, and Siga (2024) to the setting with population uncertainty. As previously, let $\mathcal{M} = \{s^1,...,s^K\}$ be the set of informative signals for any population size. For $\omega \in \Omega^n$, $V(\omega)$ and $S(\omega)$ are the payoff and signal profiles in state ω, and $V_i^d(E,n)$ is the expected payoff difference between the ex-ante optimal and inferior policies for a voter i who conditions on the population size n and the event $E \subseteq \Omega^n$. We continue to define $V_i^d(E,n) \equiv 0$ when E is a null-event and assume that $V_i^d(E,n) \neq 0$ otherwise.

ASSUMPTION 5: *Voters are exchangeable for any population size $n \in \mathcal{Q}$: if $\omega,\omega' \in \Omega^n$ and ω permutes ω', then $P_n(\omega) = P_n(\omega')$.*

Let p_n^* be the optimal policy when voters learn only that the population size is n. By Assumption 5, voters agree on p_n^*. We assume that p_n^* does not depend on n.

ASSUMPTION 6: *For all $n,n' \in \mathcal{N}$, $p_n^* = p_{n'}^*$.*

ASSUMPTION 7: *There is an uninformative signal, and other signals are sufficient:*

(a) **Uninformative signal:** *For* $n \in \mathcal{Q}$, $\omega \in \Omega^n$ *with* $S_i(\omega) = s^0$,

$$P_n(\omega) = P_n(V(\omega), S_{-i}(\omega))(1-\lambda).$$

for some $\lambda \in (0,1)$.

(b) **Informative signals:** *For* $n \in \mathcal{Q}$ *and* $s_i \in \mathcal{M}$, $V_i^d(s_i, n) > 0$ *if and only if* $V_i^d(s', n') > 0$ *for all* $n' \in \mathcal{Q}$, $s' \in \mathcal{S}^{n'}$ *such that* $s_i' = s_i$.

By Assumption 7, we can again classify informative signals as good or bad news. We let $\tau_0 \equiv \tau(n_0)$ and $P_0 \equiv P_{n_0}$.

ASSUMPTION 8: $P_0(B \geq 1) > 0$ *and* $P_0(G \geq \tau_0) > 0$.

We denote the mean population size by μ and the CDF of Q by F. As observed by Myerson (1998), being selected to participate in an election, leads a voter to update their beliefs about the size of the electorate. To perform this updating when $\mathcal{Q}$ may be countably infinite, we follow Myerson (1998) by first assuming $\bar{N} \in \mathcal{Q}$ players are pre-selected, each of whom is equally likely to be recruited as a voter. We then calculate voter i's beliefs about the size of the electorate, conditional on the event R_i that i is a voter, and take the limit as $\bar{N} \to \infty$. Hence,

$$\begin{aligned} Q(N=n|R_i) &\equiv \lim_{\bar{N}\to\infty} Q(N=n|R_i, N \leq \bar{N}) \\ &= \lim_{\bar{N}\to\infty} \frac{Q(R_i|N=n, N\leq \bar{N})Q(N=n|N\leq\bar{N})}{\sum_{n'=n_0}^{\bar{N}} Q(R_i|N=n', N\leq\bar{N})Q(N=n|N\leq\bar{N})} \\ &= \lim_{\bar{N}\to\infty} \frac{\frac{n}{\bar{N}} \frac{\mathbb{1}[n\leq\bar{N}]Q(N=n)}{F(\bar{N})}}{\sum_{n'=n_0}^{\bar{N}} \frac{n'}{\bar{N}}\frac{Q(N=n')}{F(\bar{N})}} = \frac{nQ(N=n)}{\mu} \end{aligned}$$

By Assumption 7(b), a voter who receives an informative signal has a unique undominated action. Given the population uncertainty, we focus on the set of symmetric undominated strategy profiles σ^α, where voters with good signals vote for p^*, voters with bad signals vote for p_*, and voters with the signal s^0 independently vote for p^* with probability α for some $\alpha \in [0,1]$. Adapting our previous notation, let $\Pi(\alpha, \lambda)$ be the expected payoff difference between a vote for p^* and vote for p_* for a voter who receives signal s^0 when $P(S_i \in \mathcal{M}) = \lambda$ and other voters follow the strategy-profile σ^α. A subscript n means "conditional on population size n," with a subscript 0 for the case when $n = n_0$. By Assumption 7(a), signal s^0 is not informative about the population size, payoff-profile or signal-profile of the other voters. Hence, for a voter i and $m \in \{0, ..., n-1\}$,

$$P_n(M=m|S_i=s^0, N=n) = \binom{n-1}{m}\lambda^m(1-\lambda)^{n-1-m}.$$

We formulate the following property of absolutely convergent series for later reference.

LEMMA 2: *Let* $a: \mathbb{N}^2 \to \mathbb{R}$ *such that, for all* t, $\sum_{n=0}^\infty a(n,t)$ *is absolutely convergent and, for all* n, $a(n,t)$ *converges monotonically to* 0. *Then,* $\lim_{t\to 0}\sum_{n=0}^\infty a(n,t) = 0$.

PROOF: For $(n,t) \in \mathbb{N}^2$, let $a^+(n,t) = \mathbb{1}[a(n,t) \geq 0]a(n,t)$ and $a^-(n,t) = \mathbb{1}[a(n,t) < 0]|a(n,t)|$. Since $\sum_{n=0}^{\infty} a(n,t)$ is absolutely convergent for any t,

$$\sum_{n=0}^{\infty} a(n,t) = \sum_{n=0}^{\infty} a^+(n,t) - \sum_{n=0}^{\infty} a^-(n,t)$$

(where both series on the right-hand side converge, hence converge absolutely). We show that $\lim_{t\to 0} \sum_{n=0}^{\infty} a^+(n,t) = 0$, and analogous argument then applies for the series of negative terms.

Let $\varepsilon > 0$. First fix some some t^*. Since $\sum_{n=0}^{\infty} a^+(n,t^*)$ converges absolutely, there exists $\bar{n}$ such that $\sum_{n=\bar{n}+1}^{\infty} a^+(n,t^*) \leq \frac{\varepsilon}{2}$. Now fix $\bar{n}$, since $\lim_{t\to 0} a^+(n,t) = 0$ for all $n \in \{0,...,\bar{n}\}$, there exists $\bar{t} \geq t^*$ such that $\sum_{n=0}^{\bar{n}} a^+(n,t) < \frac{\varepsilon}{2}$ for all $t \geq \bar{t}$. Moreover, since $a(n,t)$ is decreasing in t, $\sum_{n=\bar{n}+1}^{\infty} a^+(n,t) \leq \sum_{n=\bar{n}+1}^{\infty} a^+(n,t^*) = \frac{\varepsilon}{2}$ for all $t \geq t^*$. Hence, $\sum_{n=0}^{\infty} a^+(n,t) \leq \varepsilon$ for all $t \geq \bar{t}$. *Q.E.D.*

Proof of Theorem 4

For notational convenience, let $\mathcal{R}_n(g,m) \equiv \mathcal{M}_n(g,m)Z_n(g,m)\frac{nQ(n)}{\mu}$, where $\mathcal{M}_n(g,m) \equiv \binom{n-1}{m,\tau(n)-g,\tau(n)+g-m}$, and

$$Z_n(g,m) = P(G = g|M = m, S_i = s^0, N = n)V_i^d(S_i = s^0, G = g, M = m, N = n),$$

and let $v^* = \max\{|v^{p^*} - v^{p_*}| : (v^{p^*}, v^{p_*}) \in \mathcal{V}^{p^*} \times \mathcal{V}^{p_*}\}$.

PROOF: First, suppose $\mathcal{K}_*(n_0) < 0$ and fix $\varepsilon \in (0,1)$. We consider two cases.

Case 1: Suppose $V_0^G(\tau_0) < 0$, which implies $\binom{n_0-1}{\tau_0}Z_0(\tau_0,\tau_0)\frac{n_0 Q(n_0)}{\mu} < 0$ by Assumption 8. By Assumption 4, $\lim_{n\to\infty} nQ(n) = 0$, and so

$$\sum_{n=n_0+1}^{\infty} \sum_{m=\tau(n)}^{n-1} \binom{n-1}{m} \lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0}|Z_n(\tau(n),m)|\frac{nQ(n)}{\mu} \leq \frac{v^*}{\lambda^{\tau_0}(1-\lambda)^{\tau_0}},$$

hence, the series is absolutely convergent. Moreover, for $m \geq \tau(n)$, it follows that $\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0}$ is strictly increasing in $\lambda \in (0,1/2)$ and converges to 0 as $\lambda \to 0$. As a result, there exists $\bar{\lambda} \in (0,1)$ such that, for all $\lambda \in (0,\bar{\lambda})$,

$$\sum_{n=n_0+1}^{\infty} \sum_{m=\tau(n)}^{n-1} \mathcal{R}_n(\tau(n),m)\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0} \leq \frac{1}{2}\binom{n_0-1}{\tau_0}|Z_0(\tau_0,\tau_0)|\frac{n_0 Q(n_0)}{\mu}$$

Therefore, for all $\lambda \in (0,\bar{\lambda})$,

$$\begin{aligned}\Pi(0,\lambda) &= \sum_{n=n_0}^{\infty} \sum_{m=\tau(n)}^{n-1} \mathcal{R}_n(\tau(n),m)\lambda^m(1-\lambda)^{n-1-m} \\ &\leq \frac{1}{2}\lambda^{\tau_0}(1-\lambda)^{\tau_0}\binom{n_0-1}{\tau_0}Z_0(\tau_0,\tau_0)\frac{n_0 Q(n_0)}{\mu} < 0.\end{aligned}$$

Let λ'_ε be the unique solution to $\sum_{n=0}^{\infty}(1-\lambda)^n Q(n) = 1-\varepsilon$, and $\lambda_\varepsilon = \min\{\bar{\lambda}, \lambda'_\varepsilon\}$. Then, for all $\lambda \in (0, \lambda_\varepsilon)$, σ^0 is an equilibrium in which p_* wins with probability exceeding $1-\varepsilon$.

Case 2: Suppose that $V_0^G(\tau_0) > 0$ but $\sum_{\kappa=0}^{\tau_0} \theta^\kappa \binom{\tau_0}{\kappa} Z_0(\kappa,\kappa) < 0$ for some $\theta \in \mathbb{R}_{++}$.

For any $\lambda \in (0, \frac{\theta}{1-\theta})$, let $\alpha_\lambda \equiv \frac{\lambda}{\theta(1-\lambda)}$; then, $\alpha_\lambda \in (0,1)$, is strictly increasing in λ, and converges to 0 as $\lambda \to 0$.

Since $\lim_{n\to\infty} nQ(n) = 0$,

$$\sum_{n=n_0+1}^{\infty} \sum_{g=0}^{\tau(n)} \sum_{m=g}^{g+\tau(n)} \mathcal{R}_n(g,m) \lambda^m (1-\lambda)^{n-n_0-m} \alpha_\lambda^{\tau(n)-\tau_0-g} (1-\alpha_\lambda)^{\tau(n)-\tau_0+g-m} \le \frac{v^*}{\alpha_\lambda^{\tau_0}(1-\alpha_\lambda)^{\tau_0}}$$

and so the series on the left-hand side is absolutely convergent for any $\lambda \in (0, \frac{\theta}{1+\theta})$. Moreover,

$$\lambda^m (1-\lambda)^{n-n_0-m} \alpha_\lambda^{\tau(n)-\tau_0-g} (1-\alpha_\lambda)^{\tau(n)-\tau_0+g-m}$$
$$= \theta^{-n+n_0+m} (\lambda(1-\lambda)\theta - \lambda^2)^{\tau(n)-\tau_0} \left(\frac{\lambda}{\theta(1-\lambda)-\lambda}\right)^{m-g},$$

which is strictly increasing in $\lambda \in (0, \frac{\theta}{2(1+\theta)})$ and converges to 0 as $\lambda \to 0$. As a result, there exists $\bar{\lambda} \in (0, \frac{\theta}{2(1+\theta)})$ such that, for all $\lambda \in (0, \bar{\lambda})$,

$$\sum_{n=n_0+1}^{\infty} \sum_{g=0}^{\tau(n)} \sum_{m=g}^{g+\tau(n)} |\mathcal{R}_n(g,m)| \lambda^m (1-\lambda)^{n-n_0-m} \alpha_\lambda^{\tau(n)-\tau_0-g} (1-\alpha_\lambda)^{\tau(n)-\tau_0+g-m}$$
$$\le \frac{1}{4} \binom{n_0-1}{\tau_0} \frac{n_0 Q(n_0)}{\mu} \sum_{\kappa=0}^{\tau_0} \theta^\kappa \binom{\tau_0}{\kappa} |Z_0(\kappa,\kappa)|$$

Moreover,

$$\sum_{g=0}^{\tau_0} \sum_{m=g+1}^{g+\tau_0} |\mathcal{R}_0(g,m)| \lambda^m (1-\lambda)^{-m} \alpha_\lambda^{-g} (1-\alpha_\lambda)^{g-m}$$
$$= \sum_{g=0}^{\tau_0} \sum_{m=g+1}^{g+\tau_0} |\mathcal{R}_0(g,m)| \theta^m \left(\frac{\lambda}{\theta(1-\lambda)-\lambda}\right)^{m-g},$$

which converges to 0 as $\lambda \to 0$. Therefore, there exists $\bar{\lambda}' \in (0, \bar{\lambda})$ such that

$$\sum_{g=0}^{\tau_0} \sum_{m=g+1}^{g+\tau_0} |\mathcal{R}_0(g,m)| \lambda^m (1-\lambda)^{-m} \alpha_\lambda^{-g} (1-\alpha_\lambda)^{g-m}$$
$$\le \frac{1}{4} \binom{n_0-1}{\tau_0} \frac{n_0 Q(n_0)}{\mu} \sum_{\kappa=0}^{\tau_0} \theta^\kappa \binom{\tau_0}{\kappa} |Z_0(\kappa,\kappa)|$$

for all $\lambda \in (0, \bar{\lambda}')$. Therefore, for all $\lambda \in (0, \bar{\lambda}')$,

$$\Pi(\alpha_\lambda, \lambda) = \sum_{n=n_0}^{\infty} \sum_{g=0}^{\tau(n)} \sum_{m=g}^{g+\tau(n)} |\mathcal{R}_n(g,m)| \lambda^m (1-\lambda)^{n-1-m} \alpha_\lambda^{\tau(n)-g} (1-\alpha_\lambda)^{\tau(n)+g-m}$$

$$\leq \frac{1}{2}\alpha_\lambda^{\tau_0}(1-\alpha_\lambda)^{\tau_0}(1-\lambda)^{n_0-1}\binom{n_0-1}{\tau_0}\frac{n_0 Q(n_0)}{\mu}\sum_{\kappa=0}^{\tau_0}\theta^\kappa\binom{\tau_0}{\kappa}Z_0(\kappa,\kappa) < 0.$$

Finally, analogous to the argument in Case 1, $V_0^G(\tau_0) > 0$ implies that there exists $\bar{\lambda}'' \in (0, \bar{\lambda}')$ such that $\Pi(0,\lambda) > 0$ for all $\lambda \in (0, \bar{\lambda}'')$. As a result, for any $\lambda \in (0, \bar{\lambda}'')$ there exists $\alpha'_\lambda \in (0, \alpha_\lambda)$ such that $\Pi(\alpha'_\lambda, \lambda) = 0$; hence an equilibrium.

Now let λ'_ε be the unique solution to $\sum_{n=n_0}^\infty \left(\frac{\theta(1-\lambda)-\lambda}{\theta}\right)^n Q(n) = 1-\varepsilon$ when $\varepsilon \leq \theta^{-1}$ and 1 otherwise, and let $\lambda_\varepsilon = \min\{\bar{\lambda}'', \lambda'_\varepsilon\}$. Then, for all $\lambda \in (0, \lambda_\varepsilon)$, $\sigma^{\alpha'_\lambda}$ is an equilibrium in which p_* wins with probability exceeding $1-\varepsilon$.

Now, suppose $\mathcal{K}_*(n_0) > 0$ and fix $\varepsilon \in (0,1)$. The advantageous correlation condition implies that $Z_0(\tau_0, \tau_0), Z_0(0,0) > 0$, and therefore there exists $\delta > 0$ such that $\sum_{\kappa=0}^{\tau_0}\theta^\kappa\binom{\tau_0}{\kappa}Z_0(\kappa,\kappa) > \delta$ for all $\theta \in \mathbb{R}_{++}$.

Let ν_ε be the unique solution in $(0,1)$ to $\sum_{n=n_0}^\infty \nu^n Q(n) = 1-\varepsilon$, and let $\bar{\lambda} = 1 - \sqrt{\nu_\varepsilon}$ and $\bar{\alpha} = \sqrt{\nu_\varepsilon}$. Then, for any $\alpha \in (\bar{\alpha}, 1]$ and $\lambda \in (0, \bar{\lambda})$, p^* wins with probability exceeding

$$\sum_{n=n_0}^\infty \bar{\alpha}^n(1-\bar{\lambda})^n Q(n) = \sum_{n=n_0}^\infty \nu_\varepsilon^n Q(n) = 1-\varepsilon$$

in the strategy profile σ^α. It therefore suffices to show that there exists $\lambda_\varepsilon \in (0, \bar{\lambda})$ such that, for all $\lambda \in (0, \lambda_\varepsilon)$ and $\alpha \in [0, \bar{\alpha}]$, σ^α is not an equilibrium. We do this by first showing that there exists $\bar{\lambda}_0 \in (0,1)$ such that σ^0 is not an equilibrium for all $\lambda \in (0, \bar{\lambda}_0)$ (step 1), and then showing that there exists $\lambda^* \in (0,1)$ such that, for all $\lambda \in (0, \lambda^*)$, σ^α is not an equilibrium for any $\alpha \in (0, \bar{\alpha})$ (step 2).

Step 1: Since $\lim_{n\to\infty} nQ(n) = 0$,

$$\sum_{n=n_0+1}^\infty \sum_{m=\tau(n)}^{n-1} |\mathcal{R}_n(\tau(n), m)| \binom{n-1}{m}\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0} \leq \frac{v^*}{\lambda^{\tau_0}(1-\lambda)^{\tau_0}}$$

and so the series is absolutely convergent. Moreover, for $m \geq \tau(n) > \tau_0$, $\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0}$ is strictly increasing in $\lambda \in (0, 1/2)$ and converges to 0 as $\lambda \to 0$. As a result, there exists $\lambda_0 \in (0,1)$ such that

$$\sum_{n=n_0+1}^\infty \sum_{m=\tau(n)}^{n-1} |\mathcal{R}_n(\tau(n), m)|\lambda^{m-\tau_0}(1-\lambda)^{n-1-m-\tau_0} \leq \frac{1}{4}\binom{n_0-1}{\tau_0}Z_0(\tau_0,\tau_0)\frac{n_0 Q(n_0)}{\mu}$$

Moreover, since $\lambda^{m-\tau_0}(1-\lambda)^{\tau_0-m}$ is increasing in $\lambda \in (0,1)$ and converges to 0 as $\lambda \to 0$, there exists $\lambda'_0 \in (0,1)$ such that

$$\sum_{m=\tau_0+1}^{n_0-1}\binom{n_0-1}{m}\lambda^{m-\tau_0}(1-\lambda)^{\tau_0-m}|Z_0(\tau_0,m)| \leq \frac{1}{4}\binom{n_0-1}{\tau_0}Z_0(\tau_0,\tau_0)$$

Let $\bar{\lambda}_0 = \min\{\lambda_0, \lambda'_0\}$; then for all $\lambda \in (0, \bar{\lambda}_0)$,

$$\Pi(0,\lambda) = \sum_{n=n_0}\sum_{m=\tau(n)}\mathcal{R}_n(\tau(n), m)\binom{n-1}{m}\lambda^m(1-\lambda)^{n-1-m}$$

$$\geq \frac{1}{2}\binom{n_0-1}{\tau_0}\lambda^{\tau_0}(1-\lambda)^{\tau_0}Z_0(\tau_0,\tau_0)\frac{n_0Q(n_0)}{\mu}>0,$$

and so σ^0 is not an equilibrium.

Step 2: It remains to show that there exists $\lambda^* \in (0,1)$ such that, for all $\lambda \in (0,\lambda^*)$, σ^α is not an equilibrium for any $\alpha \in (0,\bar{\alpha})$. We show this by establishing a contradiction. Suppose that, for any $\lambda^* \in (0,1)$, there exists $\lambda \in (0,\lambda^*)$ and $\alpha_\lambda \in (0,\bar{\alpha})$ such that $\Pi(\alpha_\lambda,\lambda)=0$; hence, there exists a sequence $(\lambda_t,\alpha_t)_{t=1}^\infty$ such that $\lambda_t \to 0$ and, for all $t \geq 1$, $\alpha_t \in (0,\bar{\alpha})$, and $\Pi(\alpha_t,\lambda_t)=0$, where

$$\Pi(\alpha_t,\lambda_t)=\sum_{n=n_0}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g}^{g+\tau(n)}\mathcal{R}_n(g,m)\lambda_t^m(1-\lambda_t)^{n-1-m}\alpha_t^{\tau(n)-g}(1-\alpha_t)^{\tau(n)+g-m}. \tag{4}$$

We consider three collectively exhaustive cases: (i) there is a subsequence such that $\frac{\alpha_t(1-\lambda_t)}{\lambda_t} \to 0$, (ii) there is a subsequence such that $\alpha_t \to 0$ but $\frac{\alpha_t(1-\lambda_t)}{\lambda_t} \geq \gamma$ for some $\gamma > 0$, and (iii) there is a subsequence such that $\alpha_t \geq \gamma$ for some $\gamma > 0$.

Case (i). In this case, there is a subsequence such that $\lambda_t, \alpha_t, \frac{\alpha_t(1-\lambda_t)}{\lambda_t}, \lambda_t(1-\lambda_t)(1-\alpha_t), \frac{\lambda_t}{(1-\lambda_t)(1-\alpha_t)}$ are all decreasing, and converge to 0. From $\Pi(\alpha_t,\lambda_t)=0$, it follows that (for all t, with the subscript suppressed for convenience),

$$\begin{aligned}
&-\binom{n_0-1}{\tau_0}\lambda^{\tau_0}(1-\lambda)^{\tau_0}(1-\alpha)^{\tau_0}Z_0(\tau_0,\tau_0)\frac{n_0Q(n_0)}{\mu}\\
&=\sum_{g=0}^{\tau_0-1}\mathcal{R}_0(g,g)\lambda^g(1-\lambda)^{n_0-1-g}\alpha^{\tau_0-g}(1-\alpha)^{\tau_0}\\
&+\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{g+\tau_0}\mathcal{R}_0(g,m)\lambda^m(1-\lambda)^{n_0-1-m}\alpha^{\tau_0-g}(1-\alpha)^{\tau_0+g-m}\\
&+\sum_{n=n_0+1}^{\infty}\mathcal{R}_n(\tau(n),\tau(n))\lambda^{\tau(n)}(1-\lambda)^{\tau(n)}(1-\alpha)^{\tau(n)}\\
&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)-1}\mathcal{R}_n(g,g)\lambda^g(1-\lambda)^{n-1-g}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)}\\
&+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)+g-m}
\end{aligned}$$

Therefore (dividing both sides by $[\lambda(1-\lambda)(1-\alpha)]^{\tau_0}$),

$$\begin{aligned}
&-\binom{n_0-1}{\tau_0}Z_0(\tau_0,\tau_0)\frac{n_0Q(n_0)}{\mu}\\
&=\sum_{g=0}^{\tau_0-1}\mathcal{R}_0(g,g)\left(\frac{\alpha(1-\lambda)}{\lambda}\right)^{\tau_0-g}
\end{aligned}$$

$$+\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{g+\tau_0}\mathcal{R}_0(g,m)\left(\frac{\alpha(1-\lambda)}{\lambda}\right)^{\tau_0-g}\left(\frac{\lambda}{(1-\lambda)(1-\alpha)}\right)^{m-g}$$

$$+\sum_{n=n_0+1}^{\infty}\mathcal{R}_n(\tau(n),\tau(n))[\lambda(1-\alpha)(1-\lambda)]^{\tau(n)-\tau_0}$$

$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)-1}\mathcal{R}_n(g,g)\left(\frac{\alpha(1-\lambda)}{\lambda}\right)^{\tau(n)-g}[\lambda(1-\alpha)(1-\lambda)]^{\tau(n)-\tau_0}$$

$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\left(\frac{\alpha(1-\lambda)}{\lambda}\right)^{\tau(n)-g}\left(\frac{\lambda}{(1-\lambda)(1-\alpha)}\right)^{m-g}[\lambda(1-\alpha)(1-\lambda)]^{\tau(n)-\tau_0}.$$

By Lemma 2, the left-hand side converges to 0 but the right-hand side is constant and bounded away from 0.

Case (ii). In this case, there is a subsequence such that $\lambda_t, \alpha_t, \frac{\alpha_t}{1-\alpha_t}, \alpha_t(1-\lambda_t)^2(1-\alpha_t)$ are all decreasing and converge to 0. From $\Pi(\alpha_t,\lambda_t)=0$ it follows that (t subscript suppressed)

$$-\sum_{g=0}^{\tau_0}\binom{n_0-1}{g}\binom{n_0-1-g}{\tau_0-g}\lambda^g(1-\lambda)^{n_0-1-g}\alpha^{\tau_0-g}(1-\alpha)^{\tau_0}Z_0(g,g)\frac{n_0Q(n_0)}{\mu}$$

$$=\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{\tau_0}\mathcal{R}_0(g,m)\lambda^m(1-\lambda)^{n_0-1-m}\alpha^{\tau_0-g}(1-\alpha)^{\tau_0+g-m}$$

$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\mathcal{R}_n(g,g)\lambda^g(1-\lambda)^{n-1-g}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)}$$

$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)+g-m}$$

Therefore (dividing both sides by $\alpha^{\tau_0}(1-\alpha)^{\tau_0}(1-\lambda)^{n_0-1}$),

$$-\binom{n_0-1}{\tau_0}\frac{n_0Q(n_0)}{\mu}\sum_{g=0}^{\tau_0}\binom{\tau_0}{g}\left(\frac{\lambda}{\alpha(1-\lambda)}\right)^g Z_0(g,g)$$

$$=\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{\tau_0}\mathcal{R}_0(g,m)\left(\frac{\lambda}{\alpha(1-\lambda)}\right)^m\left(\frac{\alpha}{1-\alpha}\right)^{m-g}$$

$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\mathcal{R}_n(g,g)\left(\frac{\lambda}{\alpha(1-\lambda)}\right)^g[\alpha(1-\alpha)(1-\lambda)^2]^{\tau(n)-\tau_0}$$

$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\left(\frac{\lambda}{\alpha(1-\lambda)}\right)^m\left(\frac{\alpha}{1-\alpha}\right)^{m-g}[\alpha(1-\alpha)(1-\lambda)^2]^{\tau(n)-\tau_0}$$

$$\equiv\tilde{\Pi}(\alpha,\lambda)$$

If there exists a further subsequence such that $\frac{\lambda_t}{\alpha_t(1-\lambda_t)}$ is decreasing, then the left-hand side converges to 0 by Lemma 2 while the right-hand side is constant. Otherwise, there exists a subsequence such that $\frac{\lambda_t}{\alpha_t(1-\lambda_t)}$ converges up to some $\theta^* > 0$. For each t in that subsequence, let $\alpha_t^* = \frac{\lambda_t}{\theta^*(1-\lambda_t)}$. Eventually, $\alpha_t^* \in (0,1)$, and so

$$\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{\tau_0}\mathcal{R}_0(g,m)\,(\theta^*)^m\left(\frac{\alpha^*}{1-\alpha^*}\right)^{m-g}$$
$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\mathcal{R}_n(g,g)\,(\theta^*)^g\,[\alpha^*(1-\alpha^*)(1-\lambda)^2]^{\tau(n)-\tau_0}$$
$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\,(\theta^*)^m\left(\frac{\alpha^*}{1-\alpha^*}\right)^{m-g}[\alpha^*(1-\alpha^*)(1-\lambda)^2]^{\tau(n)-\tau_0}$$

is absolutely convergent. Since, for each t there exists $t' \geq t$ such that $\frac{\alpha_{t'}}{1-\alpha_{t'}} \leq \frac{\alpha_t}{1-\alpha_t}$ and $\alpha_{t'}(1-\alpha_{t'})(1-\lambda_{t'})^2 \leq \alpha_t^*(1-\alpha_t^*)(1-\lambda_t)^2$, it follows that

$$\sum_{g=0}^{\tau_0}\sum_{m=g+1}^{\tau_0}\mathcal{R}_0(g,m)\,(\theta^*)^m\left(\frac{\alpha}{1-\alpha}\right)^{m-g}$$
$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\mathcal{R}_n(g,g)\,(\theta^*)^g\,[\alpha(1-\alpha)(1-\lambda)^2]^{\tau(n)-\tau_0}$$
$$+\sum_{n=n_0+1}^{\infty}\sum_{g=0}^{\tau(n)}\sum_{m=g+1}^{g+\tau(n)}\mathcal{R}_n(g,m)\,(\theta^*)^m\left(\frac{\alpha}{1-\alpha}\right)^{m-g}[\alpha(1-\alpha)(1-\lambda)^2]^{\tau(n)-\tau_0}$$

is eventually absolutely convergent, and then converges 0 by Lemma 2.

Case (iii). In this case, there is a subsequence such that λ_t is decreasing and, since $\alpha \in (0,\bar{\alpha})$, there exists some $\gamma \in (0,1/2)$ such that $\alpha_t \in [\gamma,(1-\gamma)]$ for all t. From $\Pi(\alpha_t,\lambda_t) = 0$ it follows that (t subscript suppressed)

$$-\sum_{n=n_0}^{\infty}\binom{n-1}{\tau(n)}(1-\lambda)^{n-1}\alpha^{\tau(n)}(1-\alpha)^{\tau(n)}Z_n(0,0)\frac{nQ(n)}{\mu}$$
$$=\sum_{n=n_0}^{\infty}\sum_{m=1}^{\tau(n)}\mathcal{R}_n(0,m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau(n)}(1-\alpha)^{\tau(n)-m}$$
$$+\sum_{n=n_0}^{\infty}\sum_{g=1}^{\tau(n)}\sum_{m=g}^{g+\tau(n)}\mathcal{R}_n(g,m)\lambda^m(1-\lambda)^{n-1-m}\alpha^{\tau(n)-g}(1-\alpha)^{\tau(n)+g-m}$$

Since $\alpha \in [\delta, 1-\delta]$ it follow that $\alpha^{\tau(n)}(1-\alpha)^{\tau(n)} \geq \gamma^{\tau(n)}$, and so the left-hand side is greater $\sum_{n=n_0}^{\infty} \binom{n-1}{\tau(n)}(1-\lambda)^{n-1}\delta^{\tau(n)} Z_n(0,0)\frac{nQ(n)}{\mu}$, which converges to

$$\sum_{n=n_0}^{\infty} \binom{n-1}{\tau(n)} \delta^{\tau(n)} Z_n(0,0) Q(n) > 0,$$

while the right-hand side converges to 0 by Lemma 2. *Q.E.D.*

2.3. *The Role of Elites (Section 5.3)*

Preliminaries: For any state ω, we denote by $G_E(\omega)$ the number of elites who receive good news, $M_E(\omega)$ the number of elites who receive informative signals, $G_N(\omega) = G(\omega) - G_E(\omega)$ and $M_N(\omega) = M(\omega) - M_E(\omega)$, with typical realizations of these random variables denoted, respectively, by g_E, m_E, g_N, and m_N.

For $g_E \in \{0, ..., |\mathcal{E}|\}$, $g_N \in \{0, ..., |\mathcal{NE}|\}$, $m_E \in \{g_e, ..., |\mathcal{E}|\}$, and $m_N \in \{g_e, ..., |\mathcal{NE}|\}$,

$$Z_i(g_E, g_N, m_E, m_N) \equiv P(g_E, g_N | S_i = s^0, m_E, m_N) V_i(S_i = s^0, g_E, g_N, m_E, m_N).$$

Proof of Proposition 8

Fix $\varepsilon \in (0,1)$ and let $\sigma^* \in \Sigma^*$ with $\sigma_i^*(s^0) = \mathbb{1}[i \in \mathcal{E}]$. Since $|\mathcal{E}| \leq \tau$, p_* wins for the strategy profile σ^* in the event $\{S = s_0\}$, and therefore wins with probability exceeding $1-\varepsilon$ for all $\lambda \in (0, 1-(1-\varepsilon)^{\frac{1}{n}})$. Hence, it is sufficient to show that σ^* is a strict equilibrium for λ sufficiently small.

If $i \in \mathcal{E}$ receives signal s^0, then

$$\Pi_i(\sigma^*, \lambda) = \lambda^{\tau - |\mathcal{E}|+1} \sum_{g_E=0}^{|\mathcal{E}|-1} \sum_{m_E=g_E}^{|\mathcal{E}|-1} \sum_{m_N=\hat{g}(m_E,m_N)}^{|\mathcal{NE}|} \binom{|\mathcal{E}|-1}{m_E}\binom{|\mathcal{NE}|}{m_N} \lambda^{m_E+m_N-\tau+|\mathcal{E}|-1}(1-\lambda)^{n-1-m_E-m_N} Z_i(g_E, \hat{g}(m_E,m_N), m_E, m_N),$$

where $\hat{g}(m_E, m_N) = \tau - (|\mathcal{E}| - 1 - (m_e - g_e))$. Since

$$\lim_{\lambda\to 0} \Pi_i(\sigma^*, \lambda)\lambda^{-\tau+|\mathcal{E}|-1} = \binom{|\mathcal{NE}|}{\tau - |\mathcal{E}|+1} Z_i(0, \tau - |\mathcal{E}|+1, 0, \tau - |\mathcal{E}|+1),$$

which is strictly positive by elite-adverse correlation, there exists $\bar{\lambda}_E \in (0,1)$ such that $\Pi_i(\sigma^*, \lambda) > 0$ for all elites who receive the signal s^0 for all $\lambda \in (0, \bar{\lambda}_E)$.

If $i \in \mathcal{NE}$ receives signal s^0, then

$$\Pi_i(\sigma^*, \lambda) = \lambda^{\tau - |\mathcal{E}|} \sum_{g_E=0}^{|\mathcal{E}|} \sum_{m_E=g_E}^{|\mathcal{E}|} \sum_{m_N=\hat{g}(m_E,m_N)-1}^{|\mathcal{NE}|-1} \binom{|\mathcal{E}|}{m_E}\binom{|\mathcal{NE}|-1}{m_N} \lambda^{m_E+m_N-\tau+|\mathcal{E}|}(1-\lambda)^{n-1-m_E-m_N} Z_i(g_E, \hat{g}(m_E,m_N)-1, m_E, m_N).$$

Since

$$\lim_{\lambda\to 0} \Pi_i(\sigma^*, \lambda)\lambda^{-\tau+|\mathcal{E}|} = \binom{|\mathcal{NE}|}{\tau - |\mathcal{E}|} Z_i(0, \tau - |\mathcal{E}|, 0, \tau - |\mathcal{E}|),$$

which is strictly negative by elite-adverse correlation, there exists $\bar{\lambda}_{NE} \in (0,1)$ such that $\Pi_i(\sigma^*, \lambda) < 0$ for all non-elites who receive the signal s^0 for all $\lambda \in (0, \bar{\lambda}_{NE})$.

As a result, σ^* is an equilibrium for all $\lambda \in (0, \min\{\bar{\lambda}_E, \bar{\lambda}_{NE}\})$. *Q.E.D.*

Proof of Proposition 9

For some (P_W, v_W, v_L, e), let $i \in \mathcal{NE}$ and $w \in \{\tau+1, ..., n\}$. Then,

$$
\begin{aligned}
&P(G_N = \tau - e | S_i = s^0, M = M_N = \tau - e, W = w) \\
&= \left(\frac{w-e}{n-e}\right) \frac{\binom{w-e-1}{\tau-e}\binom{n-1-e-(w-e-1)}{0}}{\binom{n-1-e}{\tau-e}} \\
&+ \left(\frac{n-w}{n-e}\right) \frac{\binom{w-e}{\tau-e}\binom{n-1-e-(w-e)}{0}}{\binom{n-e-1}{\tau-e}} \\
&= \frac{\binom{w-e}{\tau-e}}{\binom{n-e}{\tau-e}}
\end{aligned}
$$

So, $P(G_N = \tau - e | S_i = s^0, M = M_N = \tau - e, W = \tau) = \frac{\binom{\tau-e}{\tau-e}}{\binom{n-e}{\tau-e}}$. Hence, for any $w \in \{\tau, ..., n\}$,

$$
P(W = w | E_0(e)) = \frac{\binom{w-e}{\tau-e} P_W(w)}{\sum_{w'=\tau}^{n} \binom{w'-e}{\tau-e} P_W(w')}.
$$

where $E_0(e) = \{S_i = s^0, G = M = M_N = \tau - e\}$ for $i \in \mathcal{NE}$. Since

$$
P(W_i | E_0(e), W = w) = \frac{w-e-(\tau-e)}{n-e-(\tau-e)} = \frac{w-\tau}{n-\tau},
$$

it follows that

$$
V_i(E_0(e)) = \sum_{w=\tau}^{n} \left(\left(\frac{w-\tau}{n-\tau}\right) v_W - \left(\frac{n-w}{n-\tau}\right) v_L \right) \frac{\binom{w-e}{\tau-e} P_W(w)}{\sum_{w'=\tau}^{n} \binom{w'-e}{\tau-e} P_W(w')}.
$$

For $0 \leq e < e' \leq \tau$ and $\tau \leq w < w' \leq n$,

$$
\binom{w'-e'}{\tau-e'}\binom{w-e}{\tau-e} < \binom{w-e'}{\tau-e'}\binom{w'-e}{\tau-e}
$$

and so $P_W(.|E_0(e)) \succcurlyeq_{LR} P_W(.|E_0(e'))$. Since $P(W_i|E_0(e), W = w)$ is strictly increasing in w for $i \in \mathcal{NE}$, it follows that $V_i(E_0(e)) \geq V_i(E_0(e'))$. *Q.E.D.*

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