

# Appendix I

## Experimental Instructions

### Introduction

This is an experiment in decision-making. Research foundations have provided funds for conducting this research. Your payoffs will depend partly only on your decisions and partly on chance. It will not depend on the decisions of the other participants in the experiments. Please pay careful attention to the instructions as a considerable amount of money is at stake.

The entire experiment should be complete within an hour and a half. At the end of the experiment you will be paid privately. At this time, you will receive \$5 as a participation fee (simply for showing up on time). Details of how you will make decisions and receive payments will be provided below.

During the experiment we will speak in terms of experimental tokens instead of dollars. Your payoffs will be calculated in terms of tokens and then translated at the end of the experiment into dollars at the following rate:

$$2 \text{ Tokens} = 1 \text{ Dollar}$$

### A decision problem

In this experiment, you will participate in 50 independent decision problems that share a common form. This section describes in detail the process that will be repeated in all decision problems and the computer program that you will use to make your decisions.

In each decision problem you will be asked to allocate tokens between three accounts, labeled  $x$ ,  $y$  and  $z$ . Each choice will involve choosing a point on a three-dimensional graph representing possible token allocations,  $x / y / z$ . The  $x$  account corresponds to the  $x$ -axis, the  $y$  account corresponds to the  $y$ -axis and the  $z$  account corresponds to the  $z$ -axis in a three-dimensional graph. In each choice, you may choose any combination of  $x / y / z$  that is on the plane that is shaded in gray. Examples of planes that you might face appear in Attachment 1.

Each decision problem will start by having the computer select such a plane randomly from the set of planes that intersect with at least one of the axes ( $x$ ,  $y$  or  $z$ ) at 50 tokens or more but with no intercept exceeding 100 tokens. The planes selected for you in different decision problems are independent of each other and independent of the planes selected for any of the other participants in their decision problems.

For example, as illustrated in Attachment 2, choice  $A$  represents an allocation in which you allocate approximately 20 tokens in the  $x$  account, 21 tokens in the  $y$  account, and 30 tokens in the  $z$  account. Another possible allocation is  $B$ , in which you allocate approximately 40 tokens in the  $x$  account, 17 tokens in the  $y$  account, and 11 tokens in the  $z$  account.

To choose an allocation, use the mouse to move the pointer on the computer screen to the allocation that you desire. On the right hand side of the program dialog window, you will be informed of the exact allocation that the pointer is located. When you are ready to make your decision, left-click to enter your chosen allocation. After that, confirm your decision by clicking on the Submit button. Note that you can choose only  $x / y / z$  combinations that are on the gray plane. To move on to the next round, press the OK button. The computer program dialog window is shown in Attachment 3.

Your payoff at each decision round is determined by the number of tokens in each account. At the end of the round, the computer will randomly select one of the accounts,  $x$ ,  $y$  or  $z$ . For each participant, account  $y$  will be selected with  $1/3$  chance, account  $x$  will be selected with some chance  $p$  and account  $z$  will be selected with some chance  $q$  such that the sum of  $p$  and  $q$  is equal to  $2/3$ . You will not be informed about the values of  $p$  and  $q$ . You will only receive the number of tokens you allocated to the account that was chosen.

Next, you will be asked to make an allocation in another independent decision. This process will be repeated until all 50 rounds are completed. At the end of the last round, you will be informed the experiment has ended.

### **Earnings**

Your earnings in the experiment are determined as follows. At the end of the experiment, the computer will randomly select one decision round from each participant to carry out (that is, 1 out of 50). The round selected depends solely upon chance. For each participant, it is equally likely that any round will be chosen.

The round selected, your choice and your payment will be shown in the large window that appears at the center of the program dialog window. At the end of the experiment, the tokens will be converted into money. Each token will be worth 0.50 Dollars. Your final earnings in the experiment will be your earnings in the round selected plus the \$5 show-up fee. You will receive your payment as you leave the experiment.

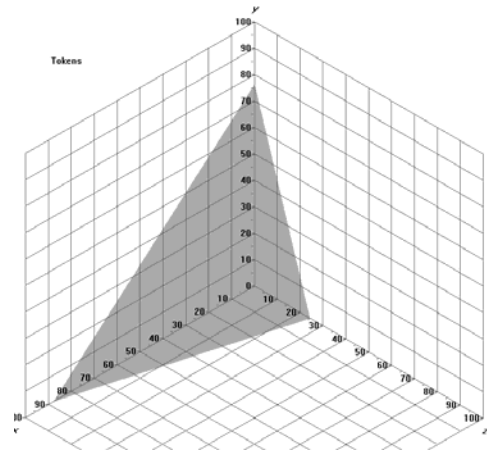
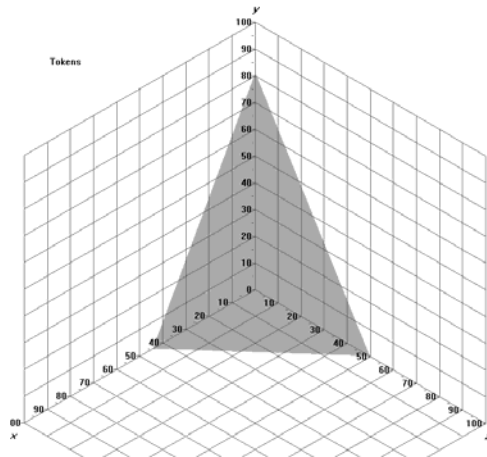
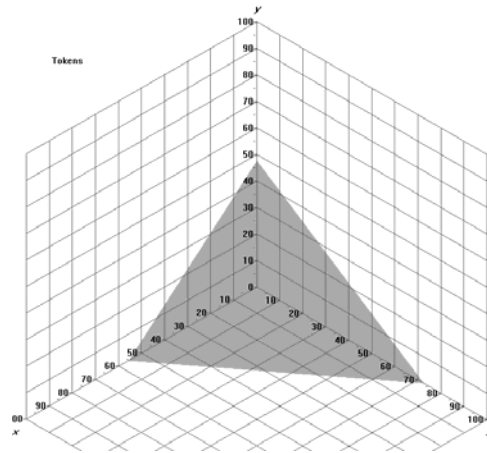
### **Rules**

Your participation in the experiment and any information about your payoffs will be kept strictly confidential. Your payment-receipt and participant form are the only places in which your name and social security number are recorded.

You will never be asked to reveal your identity to anyone during the course of the experiment. Neither the experimenters nor the other participants will be able to link you to any of your decisions. In order to keep your decisions private, please do not reveal your choices to any other participant.

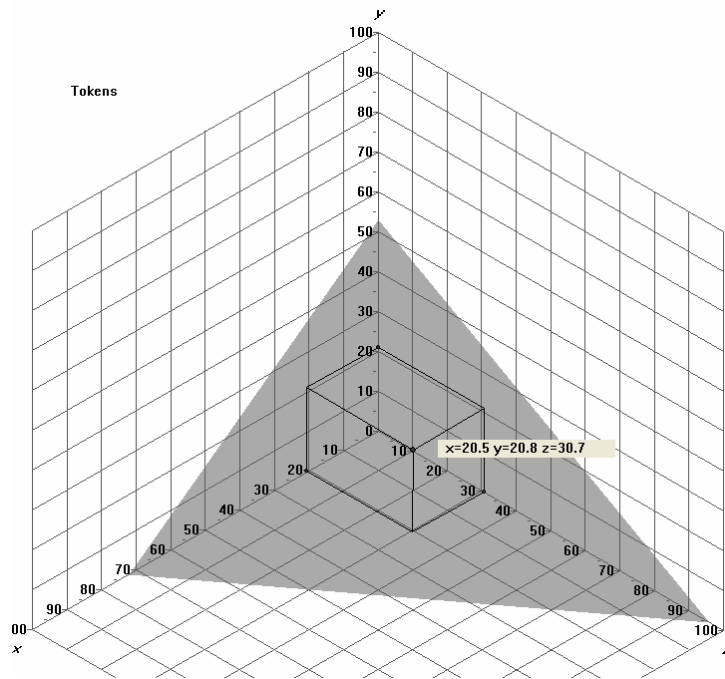
Please do not talk with anyone during the experiment. We ask everyone to remain silent until the end of the last round. If there are no further questions, you are ready to start. An instructor will approach your desk and activate your program.

# Attachment 1

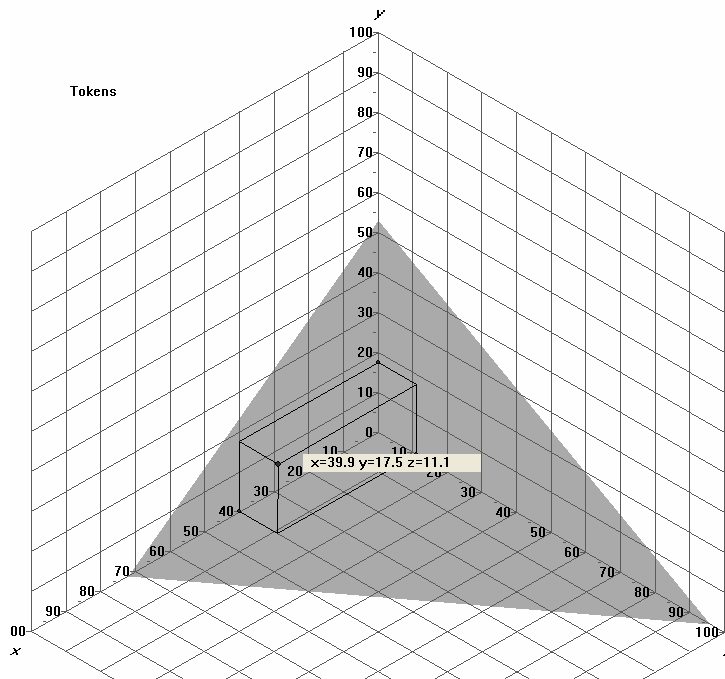


## Attachment 2

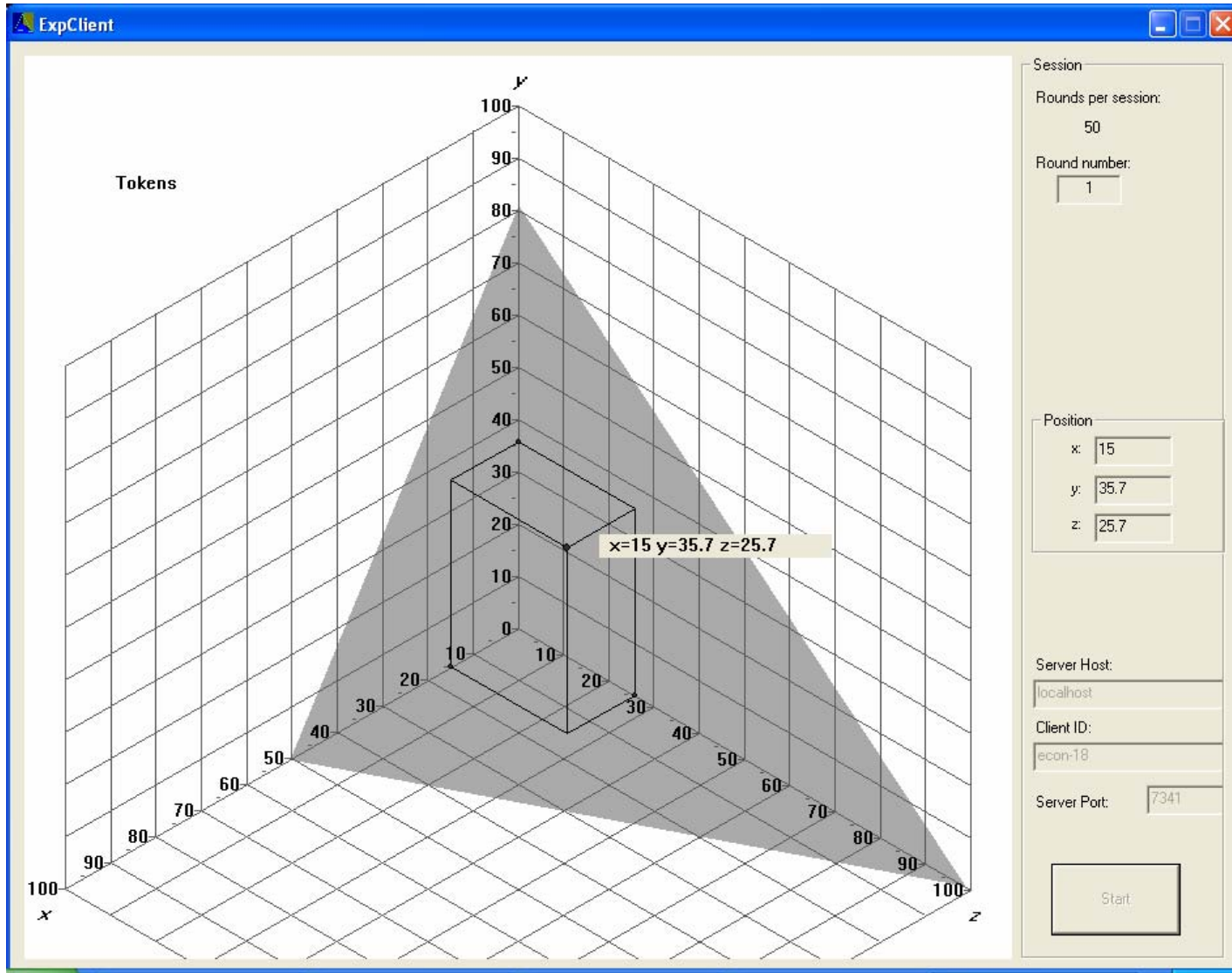
### Choice A



### Choice B



### Attachment 3



## Appendix II

### Appendix IIA

Kernel density estimates of the token share of the cheapest security (the security with the lowest price) depicted in Figure 3 with the data generated by a sample of ambiguity-neutral and ambiguity-averse simulated subjects who make choices from the same set of budget sets the human subjects do. The simulated subjects maximize the kinked specification in (equation 1) using a range of parameter values for ambiguity aversion and risk aversion. Each panel assumes a different ambiguity parameter.

### Appendix IIB

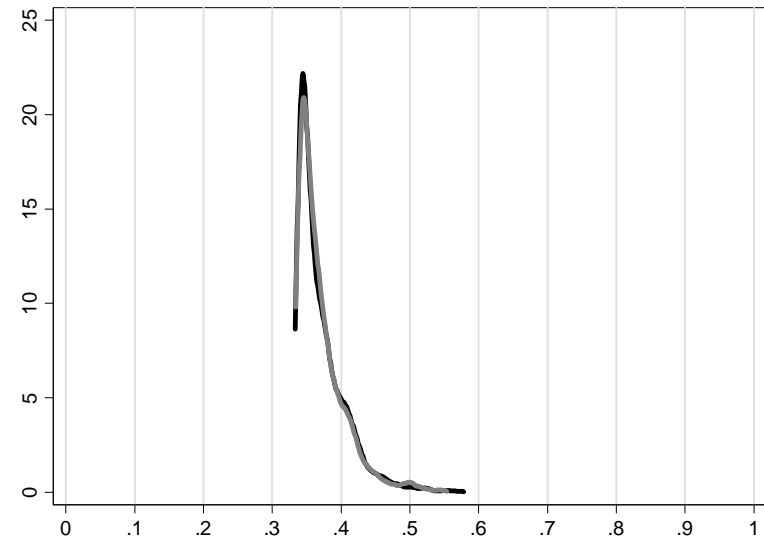
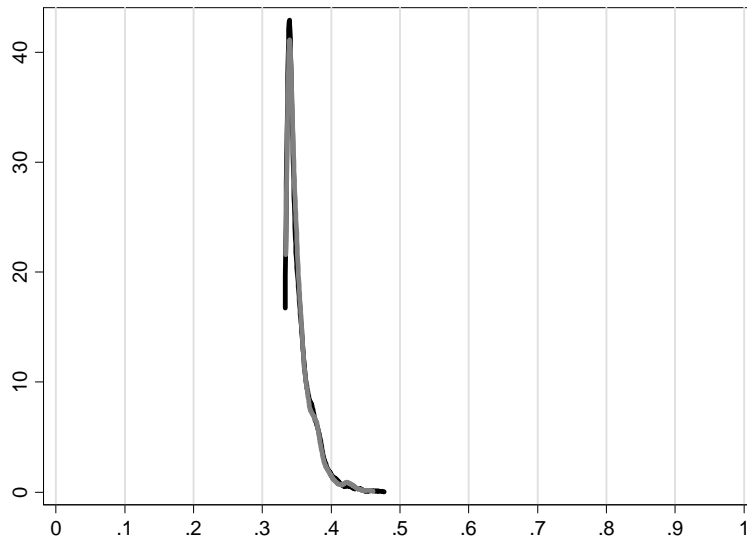
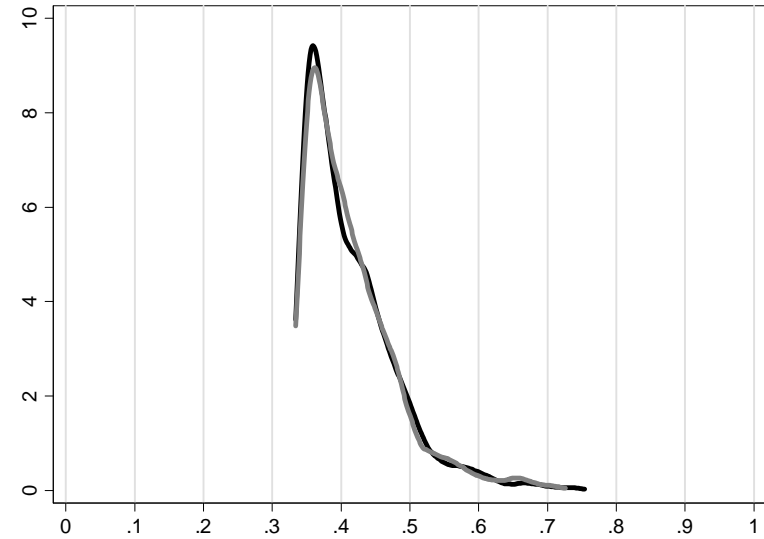
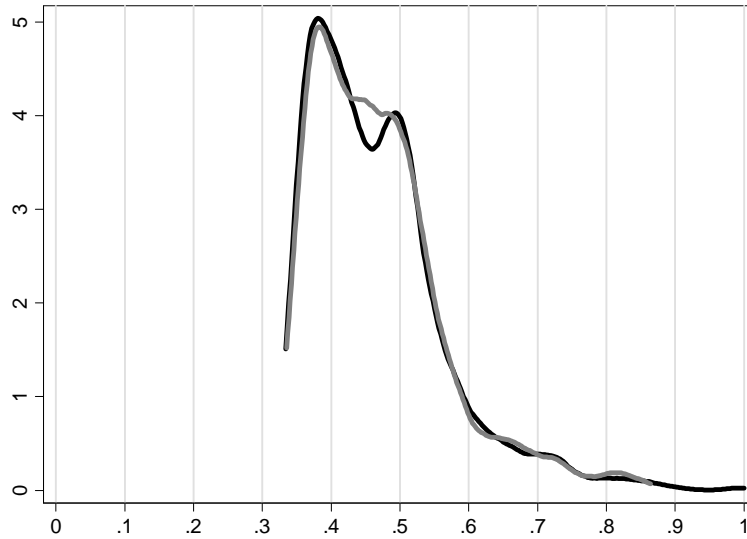
Within-subject comparisons of the average token shares of the cheapest security depicted in Figure 5. The data are generated by the sample of simulated subjects who make choices from the same set of budget sets the human subjects do and maximize the kinked specification (Equation 1). Each panel assumes a different ambiguity parameter.

We distinguish between portfolios where the cheapest security pays off in one of the ambiguous states (vertical axis) and portfolios where the cheapest security pays off in the unambiguous state (horizontal axis). Note that following the  $\alpha$ -MEU model, in the kinked specification (equation 1),  $\alpha$  is a measure of ambiguity aversion. We assume that risk preferences are represented by a von Neumann-Morgenstern utility function with constant absolute risk aversion (CARA) so  $\rho$  is the coefficient of absolute risk aversion.

**A: Kernel density estimates of the distribution of the fraction of tokens allocated to the cheapest security**

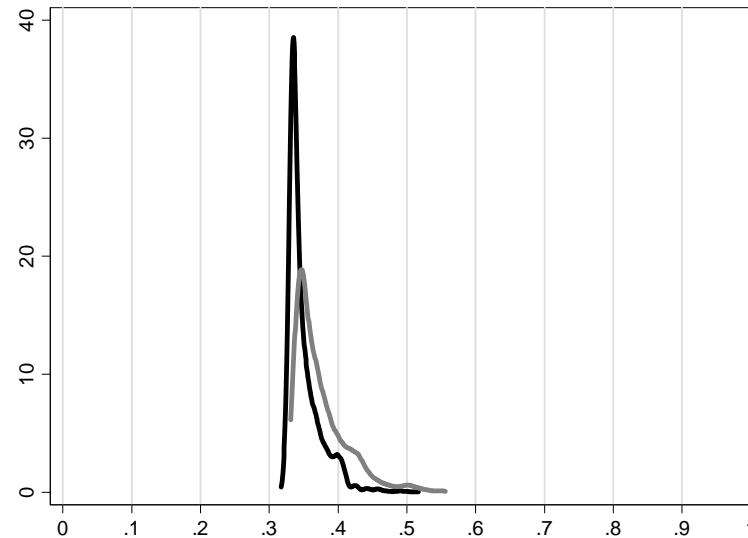
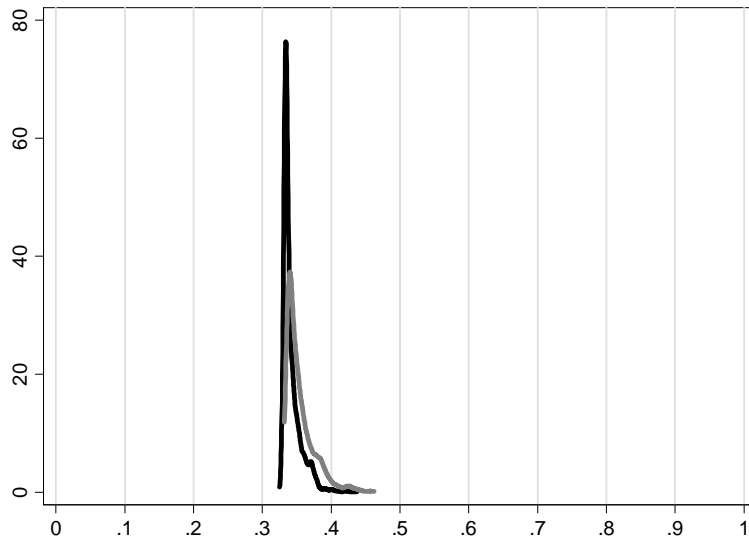
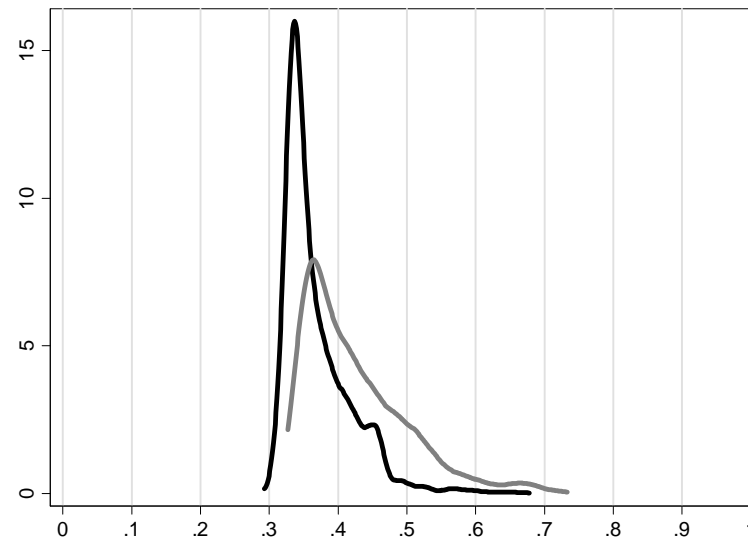
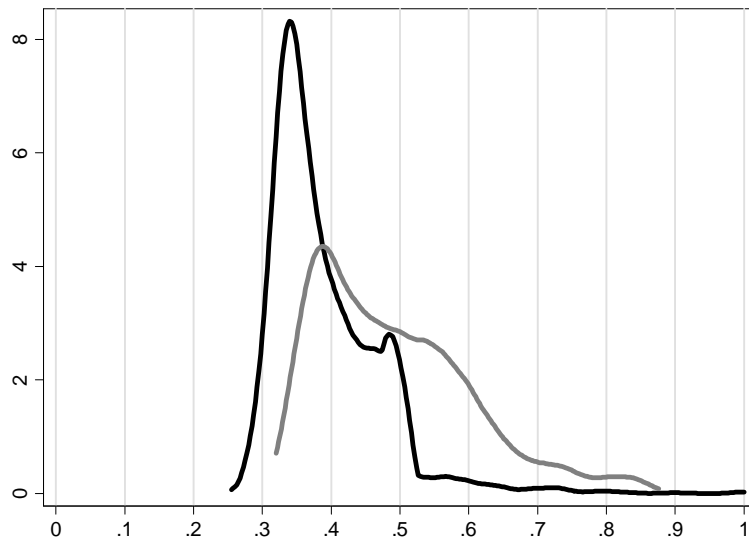
$\alpha = 0.5$

(clockwise from top left:  $\rho = 0.05, 0.1, 0.25, 0.5$ )



**Black:** Portfolios where the cheapest security pays off in one of the *ambiguous* states, 1 or 3. **Gray:** Portfolios where the cheapest security pays off in the *unambiguous* state.

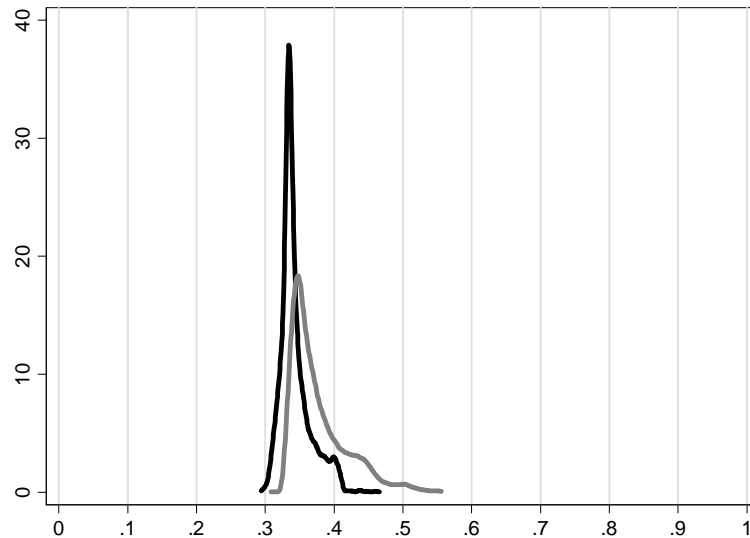
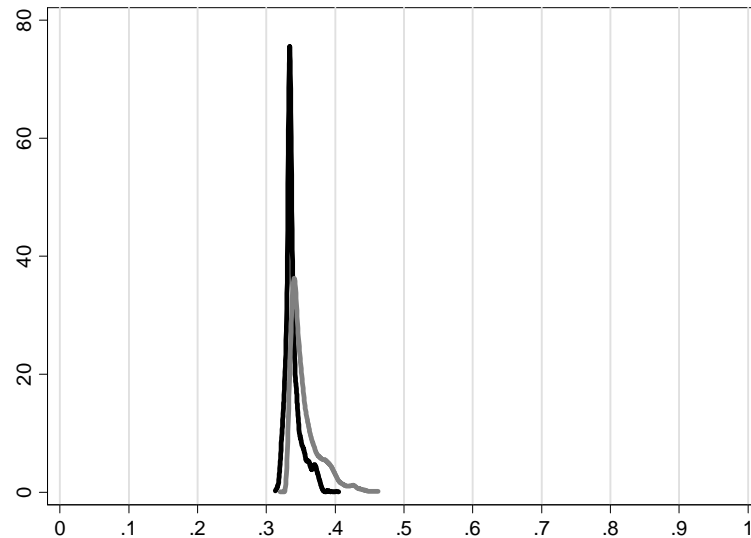
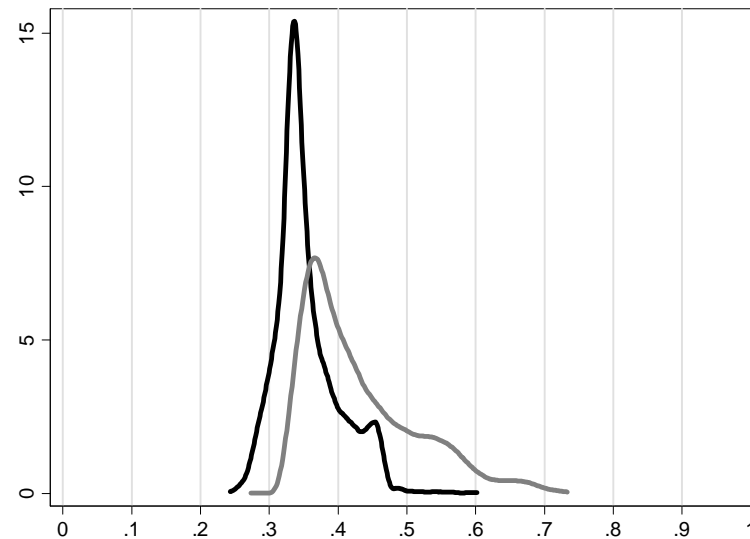
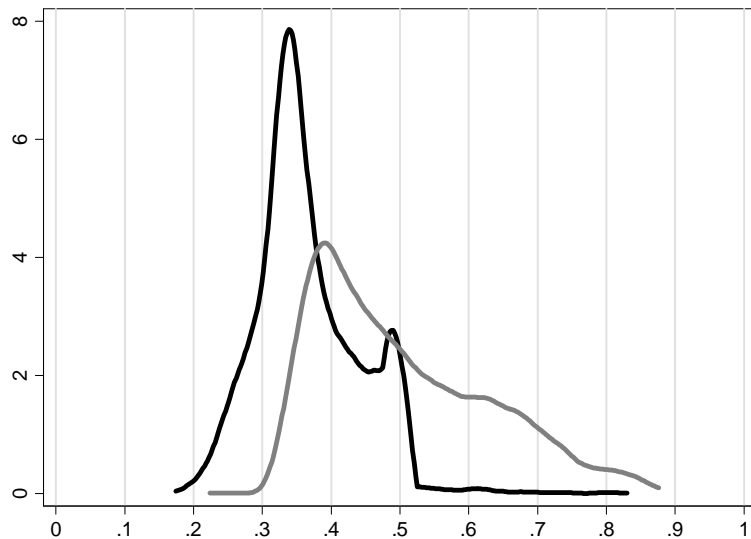
$\alpha = 0.7$   
 (clockwise from top left:  $\rho = 0.05, 0.1, 0.25, 0.5$ )



**Black:** Portfolios where the cheapest security pays off in one of the *ambiguous* states, 1 or 3. **Gray:** Portfolios where the cheapest security pays off in the *unambiguous* state.

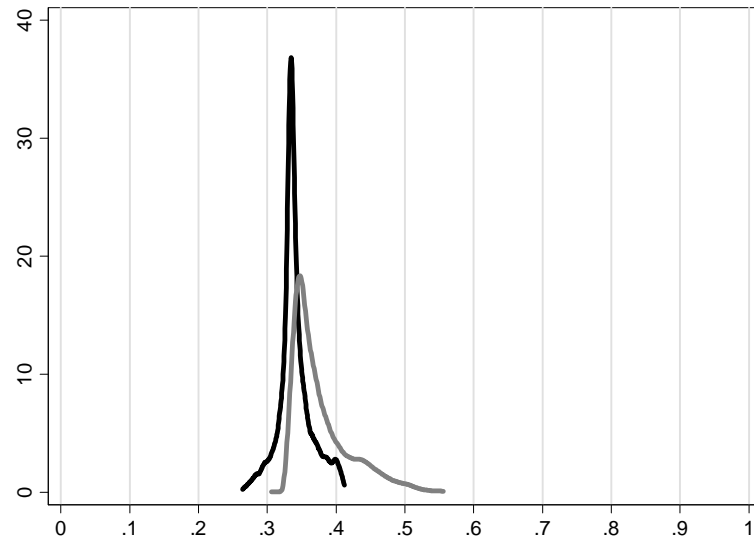
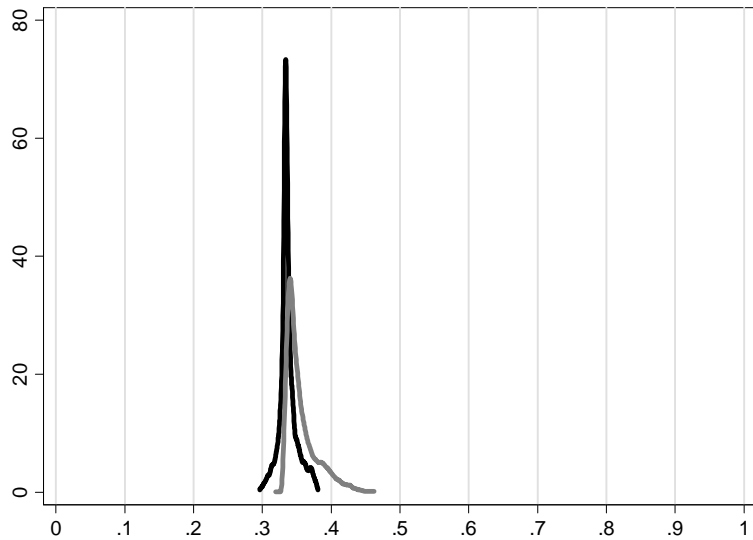
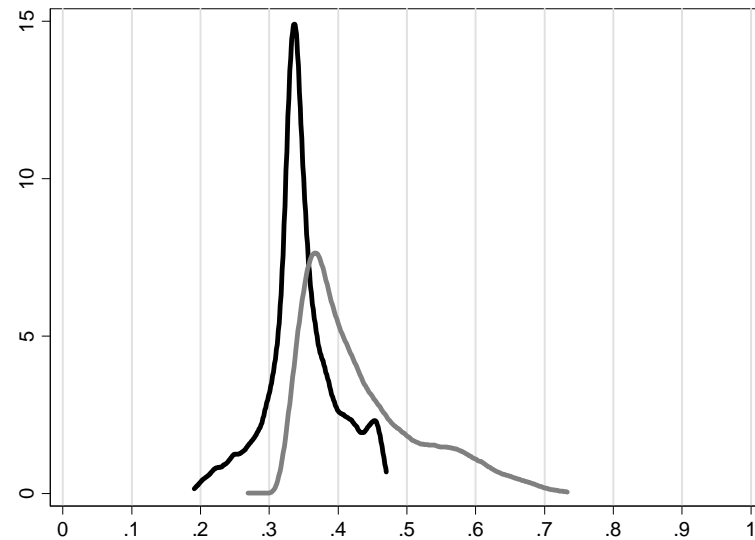
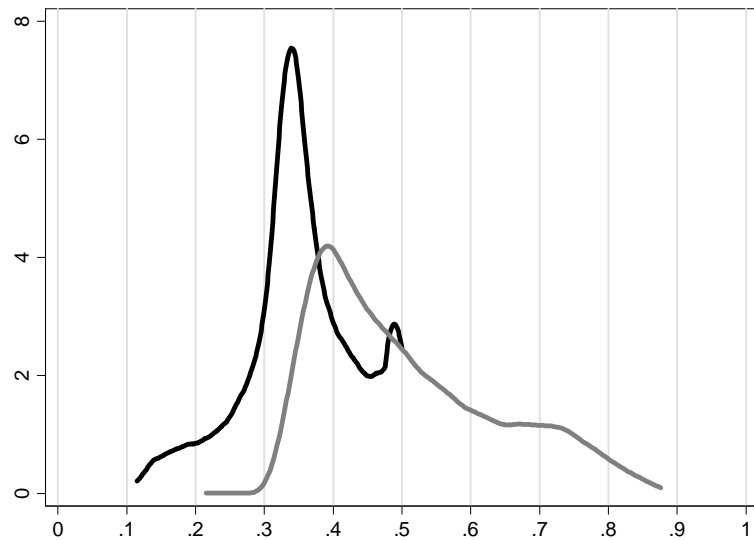


$\alpha = 0.8$   
 (clockwise from top left:  $\rho = 0.05, 0.1, 0.25, 0.5$ )



**Black:** Portfolios where the cheapest security pays off in one of the *ambiguous* states, 1 or 3. **Gray:** Portfolios where the cheapest security pays off in the *unambiguous* state.

$\alpha = 0.9$   
(clockwise from top left:  $\rho = 0.05, 0.1, 0.25, 0.5$ )

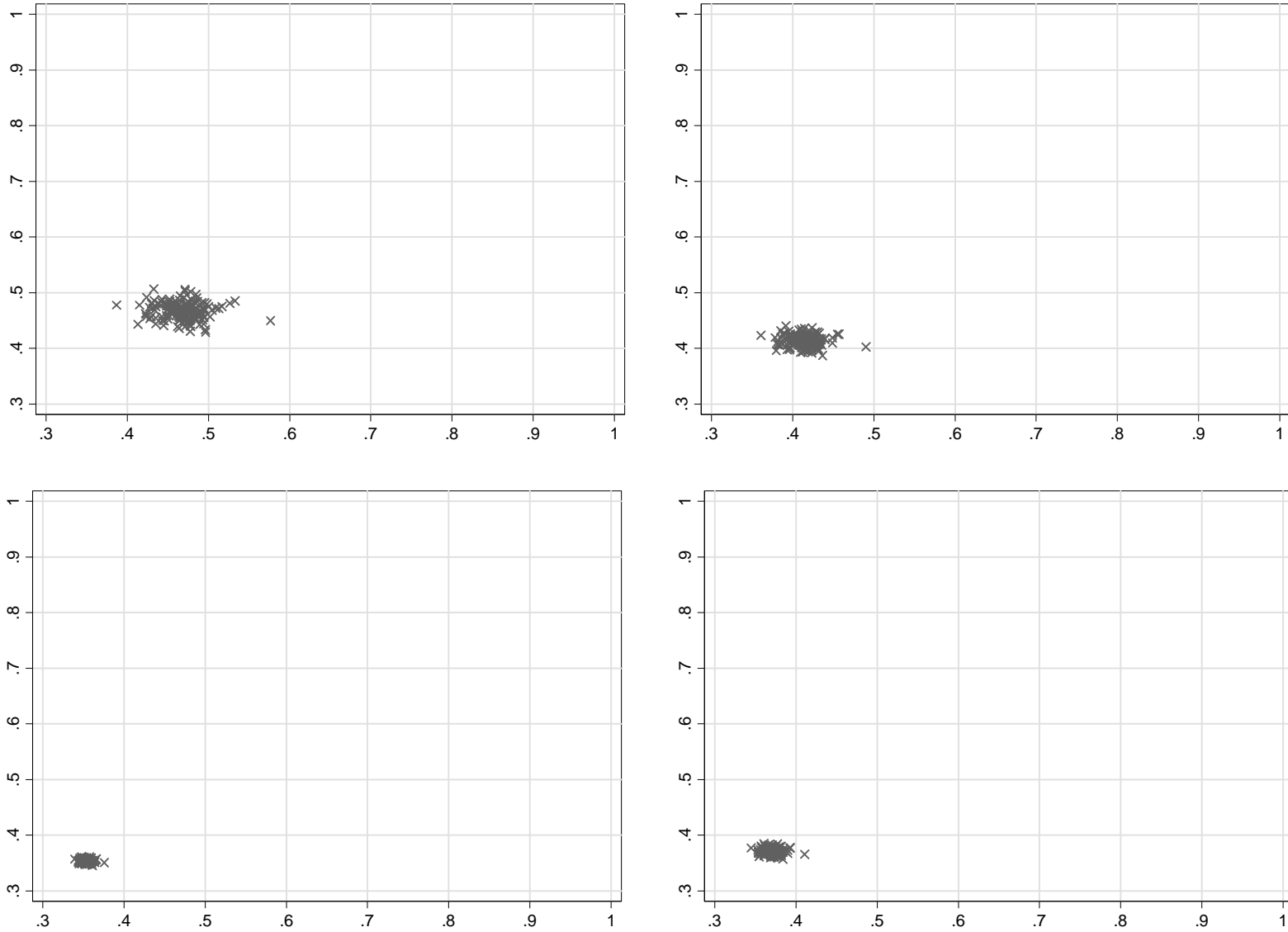


**Black:** Portfolios where the cheapest security pays off in one of the *ambiguous* states, 1 or 3. **Gray:** Portfolios where the cheapest security pays off in the *unambiguous* state.

**B: Scatterplot of the average fraction of tokens allocated to the cheapest security by subject**

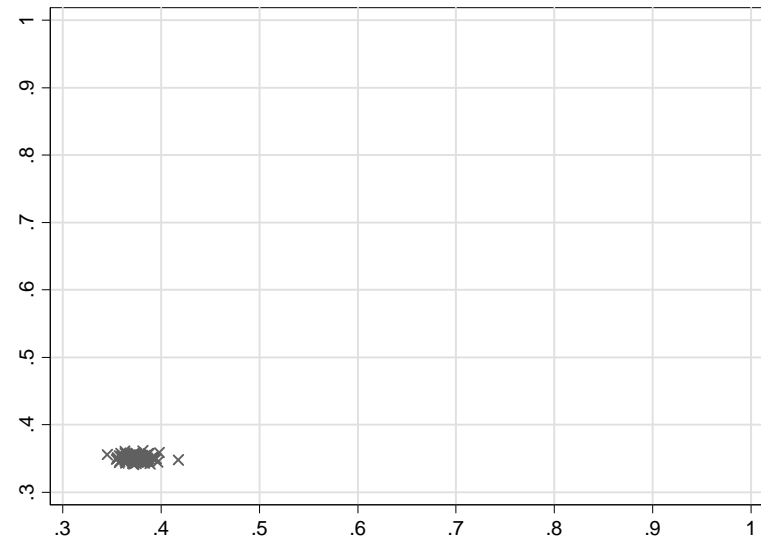
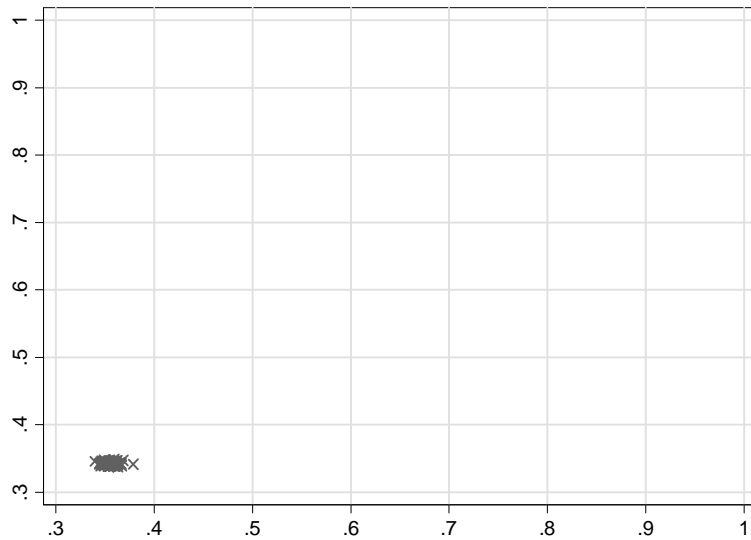
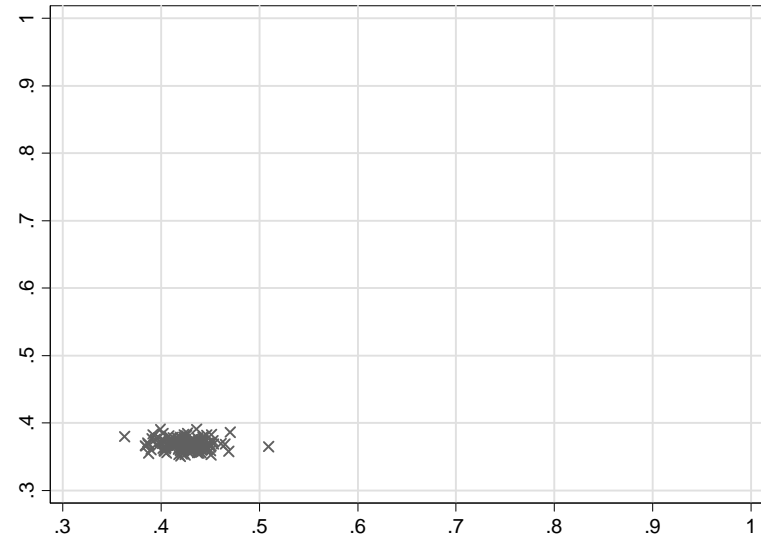
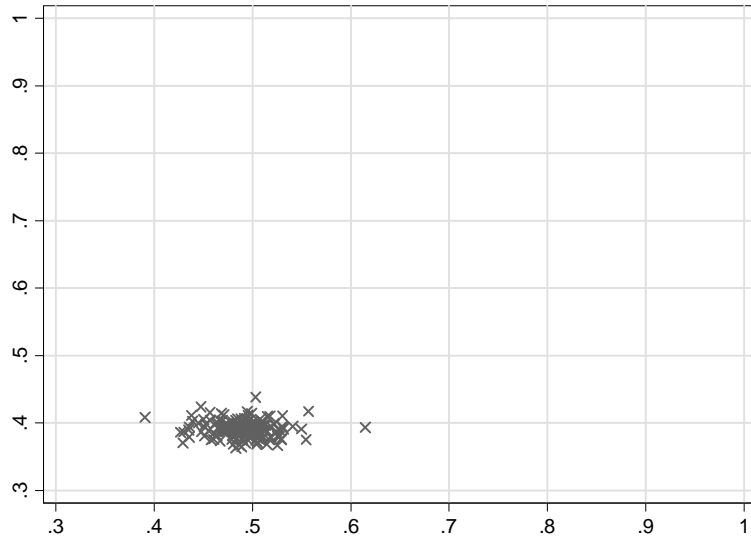
$\alpha = 0.5$

(clockwise from top left:  $\rho = 0.05, 0.1, 0.25, 0.5$ )



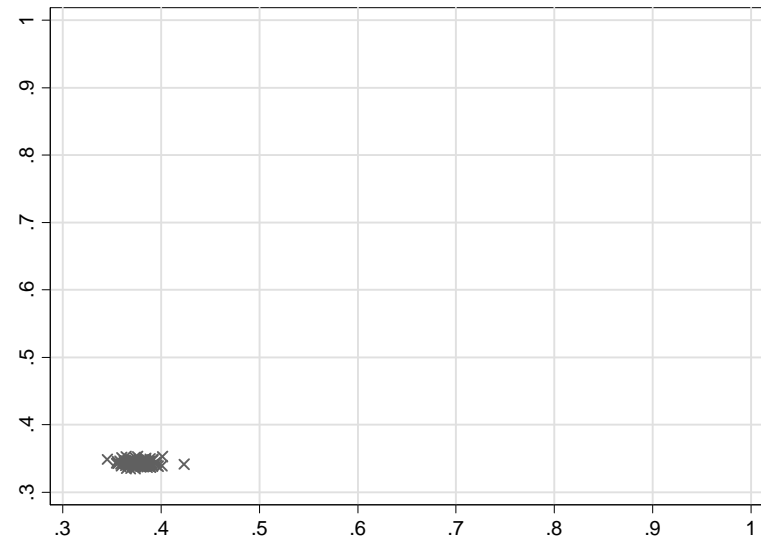
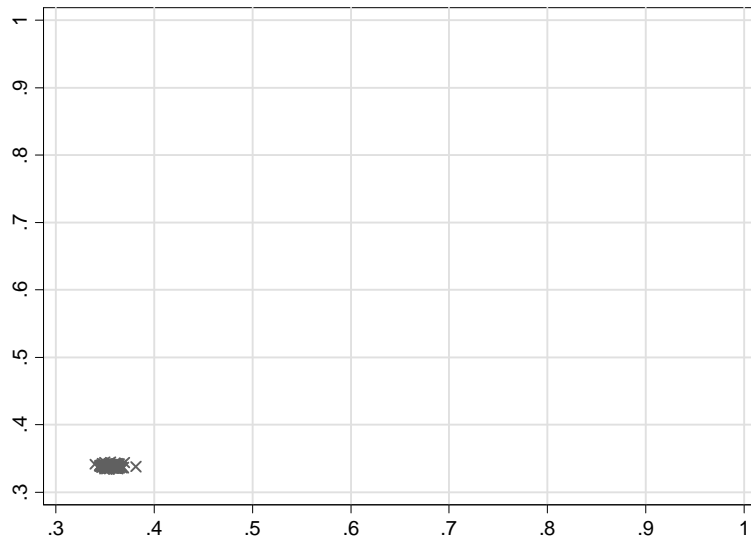
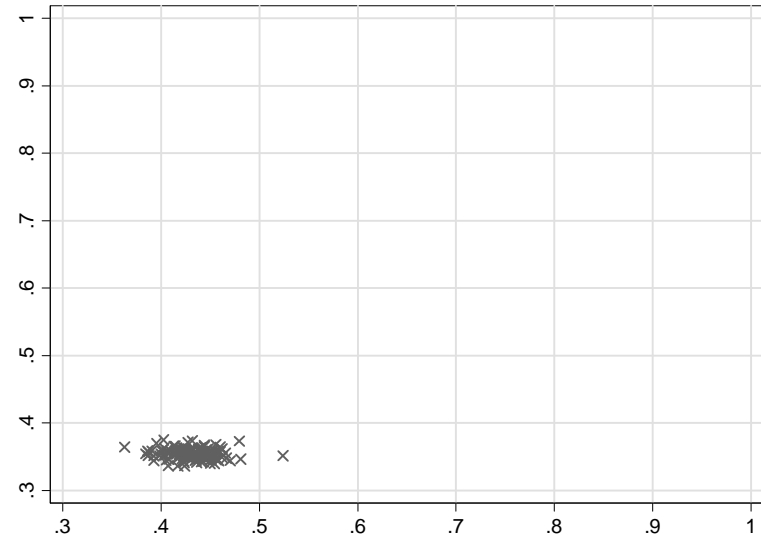
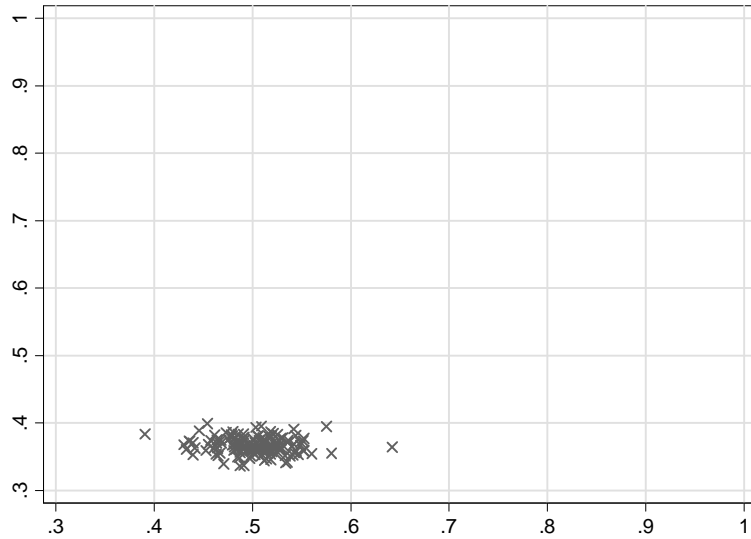
**Vertical axis:** The average fraction of tokens allocated to the cheapest security (the security with the lowest price) when it pays off in an *ambiguous* state. **Horizontal axis:** The average fraction of tokens allocated to the cheapest when it pays off in the *unambiguous* state.

$\alpha = 0.7$   
(clockwise from top left:  $\rho = 0.05, 0.1, 0.25, 0.5$ )



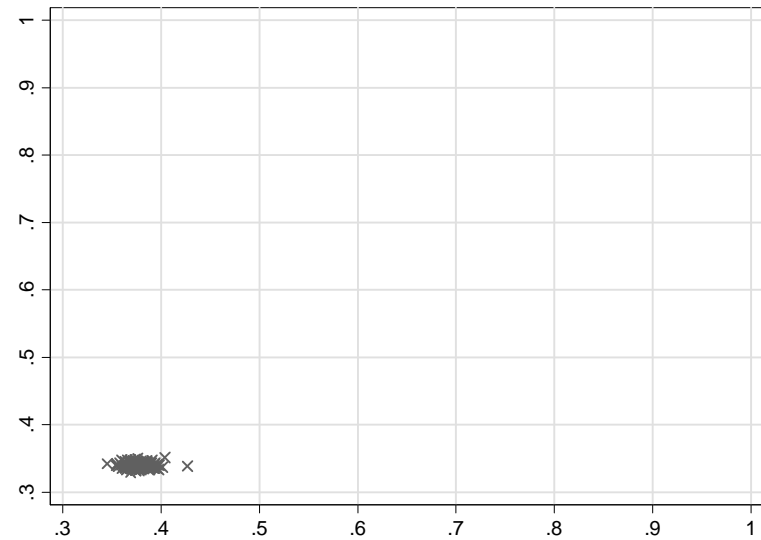
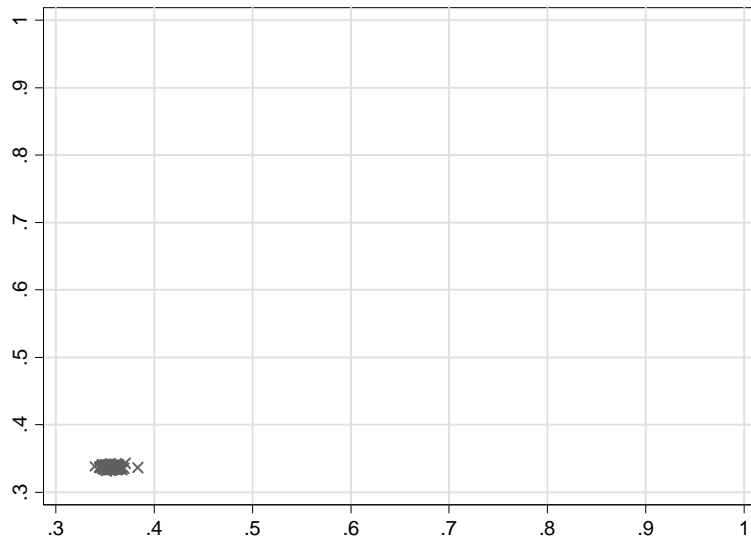
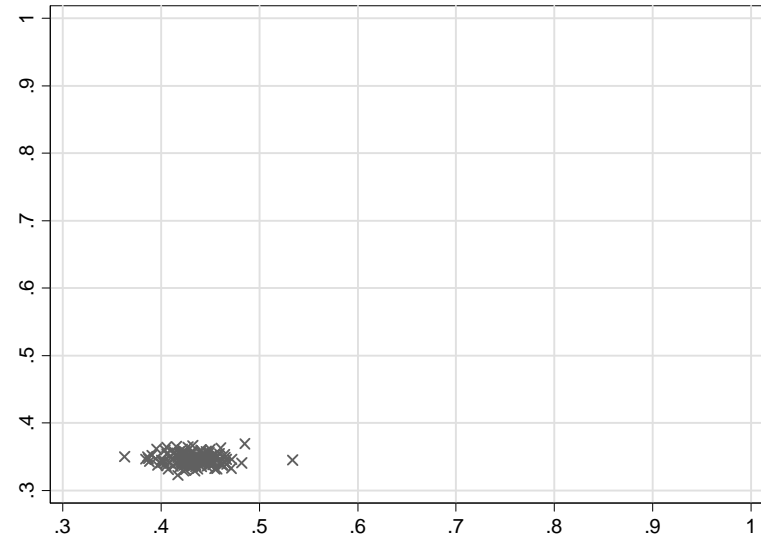
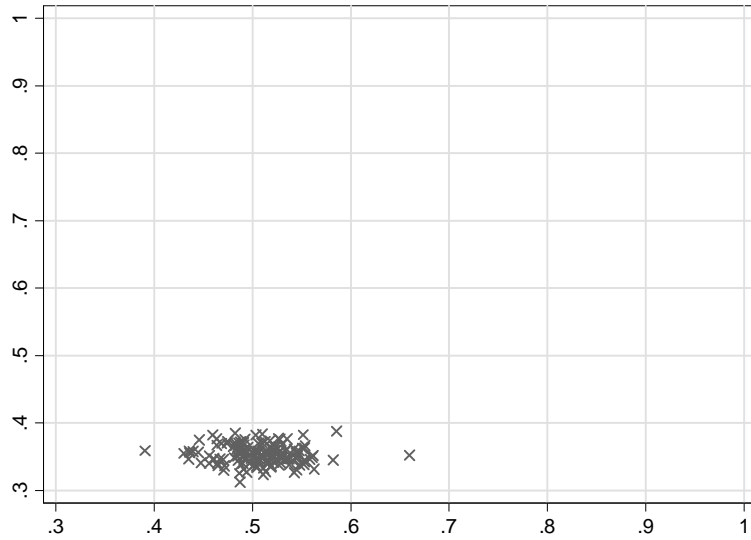
**Vertical axis:** The average fraction of tokens allocated to the cheapest security (the security with the lowest price) when it pays off in an *ambiguous* state. **Horizontal axis:** The average fraction of tokens allocated to the cheapest when it pays off in the *unambiguous* state.

$\alpha = 0.8$   
(clockwise from top left:  $\rho = 0.05, 0.1, 0.25, 0.5$ )



**Vertical axis:** The average fraction of tokens allocated to the cheapest security (the security with the lowest price) when it pays off in an *ambiguous* state. **Horizontal axis:** The average fraction of tokens allocated to the cheapest when it pays off in the *unambiguous* state.

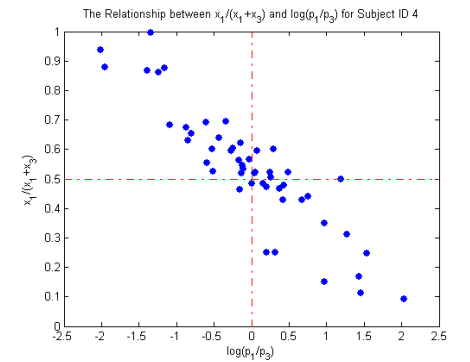
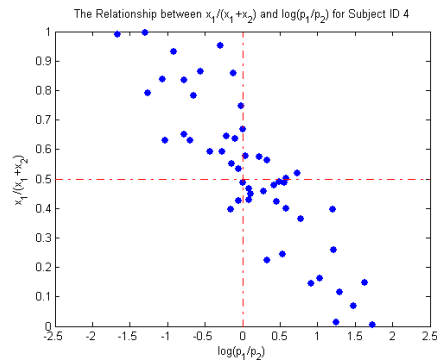
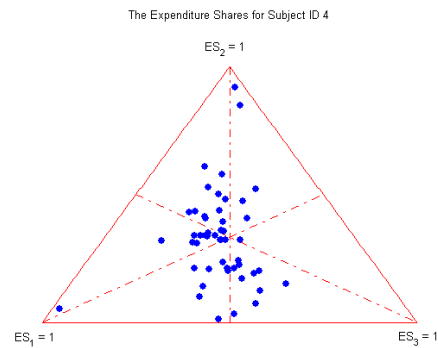
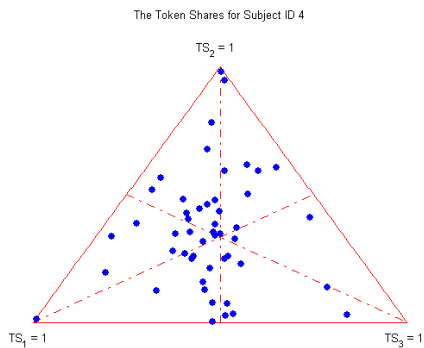
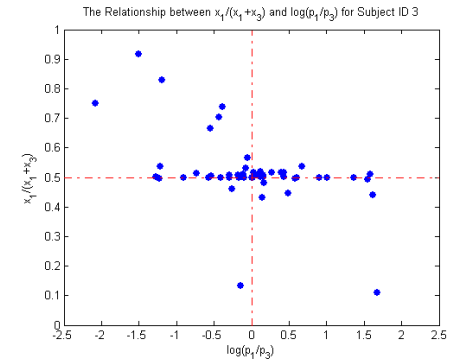
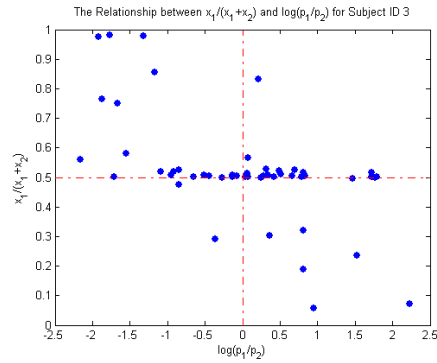
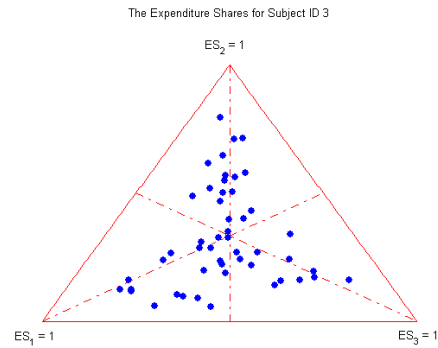
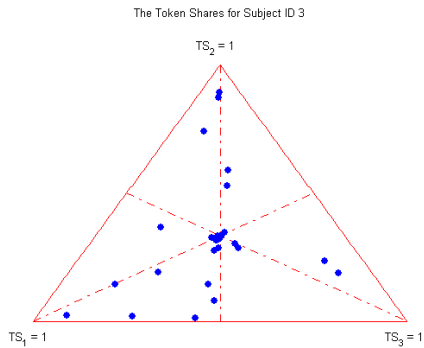
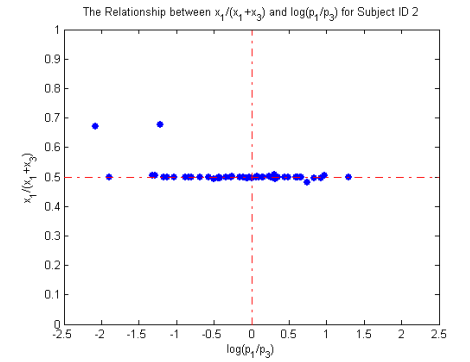
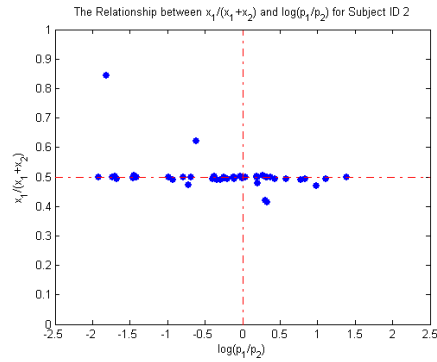
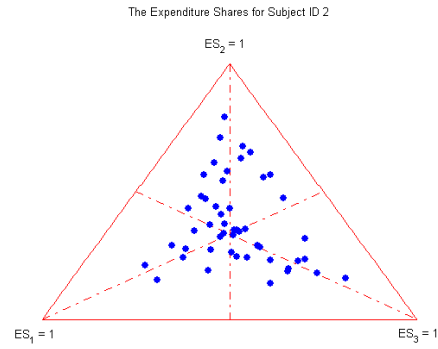
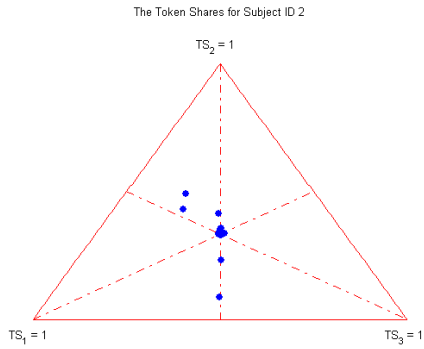
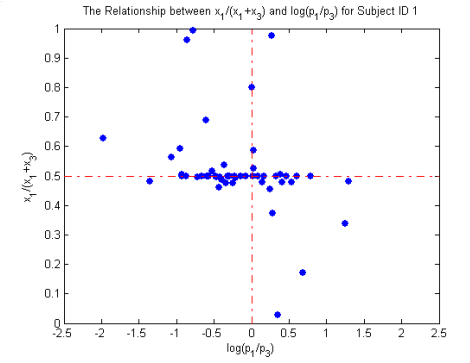
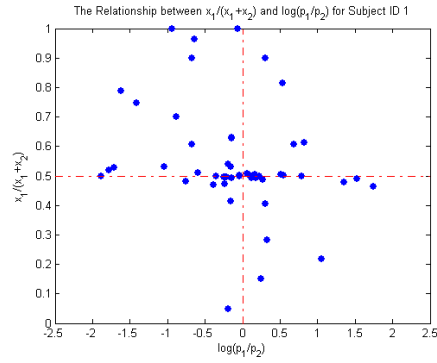
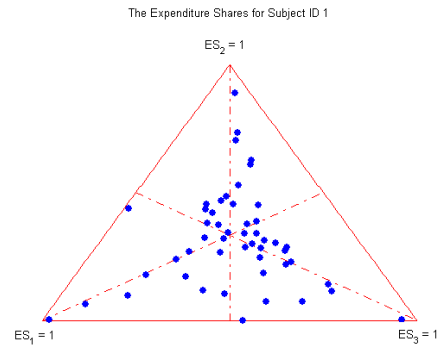
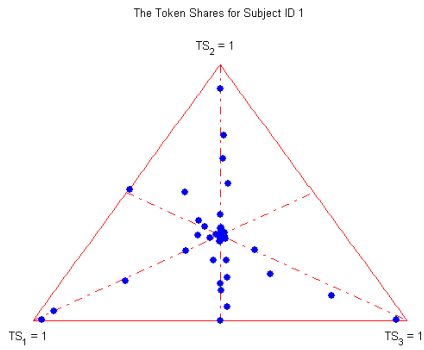
$\alpha = 0.9$   
(clockwise from top left:  $\rho = 0.05, 0.1, 0.25, 0.5$ )



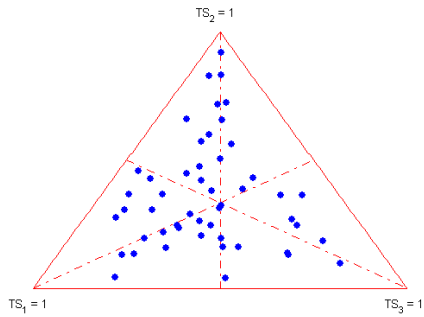
**Vertical axis:** The average fraction of tokens allocated to the cheapest security (the security with the lowest price) when it pays off in an *ambiguous* state. **Horizontal axis:** The average fraction of tokens allocated to the cheapest when it pays off in the *unambiguous* state.

# Appendix III

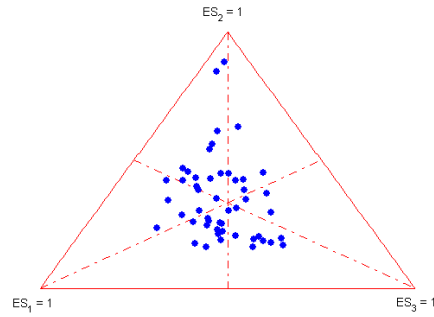
## Individual-level data



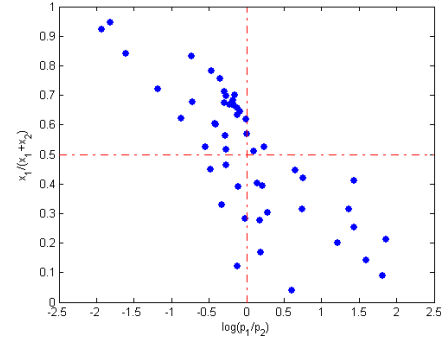
The Token Shares for Subject ID 5



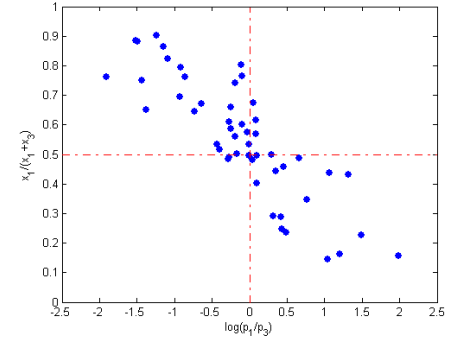
The Expenditure Shares for Subject ID 5



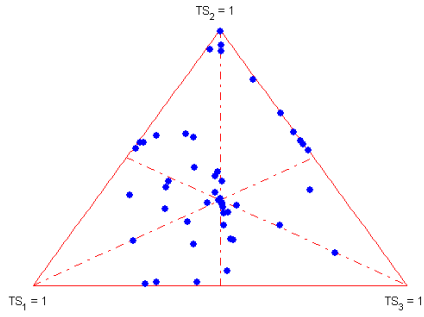
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 5



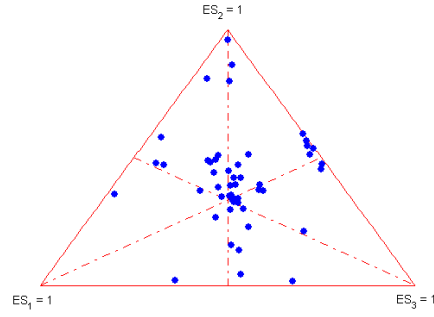
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 5



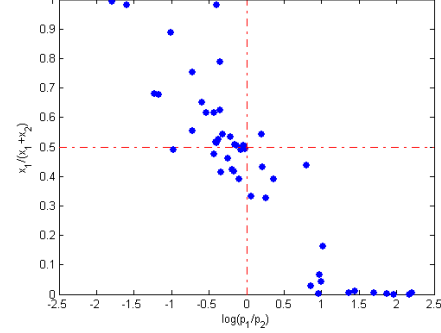
The Token Shares for Subject ID 6



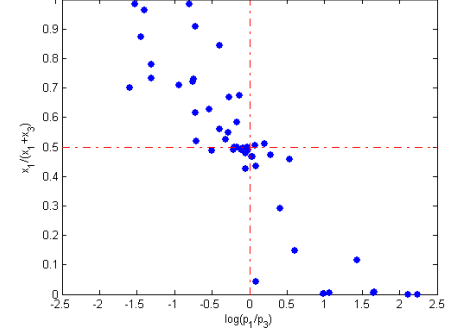
The Expenditure Shares for Subject ID 6



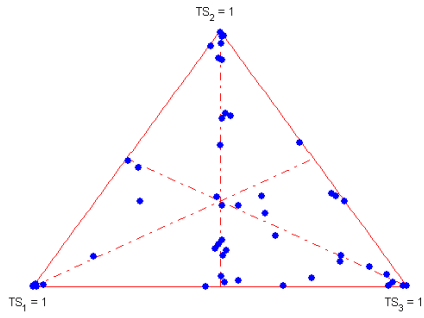
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 6



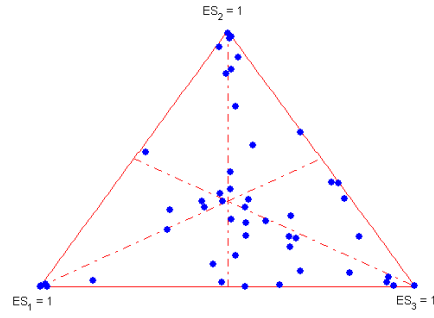
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 6



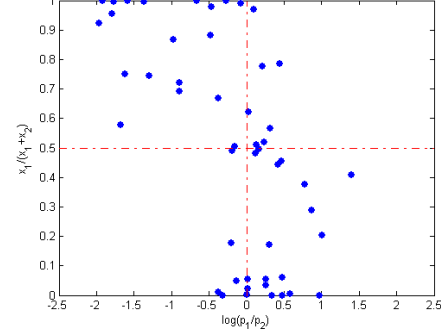
The Token Shares for Subject ID 7



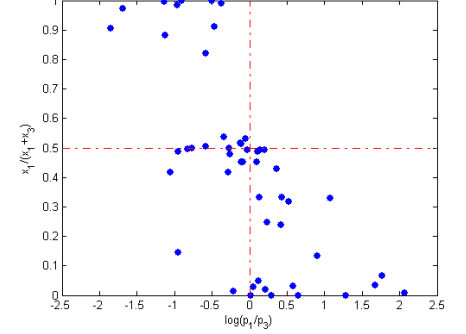
The Expenditure Shares for Subject ID 7



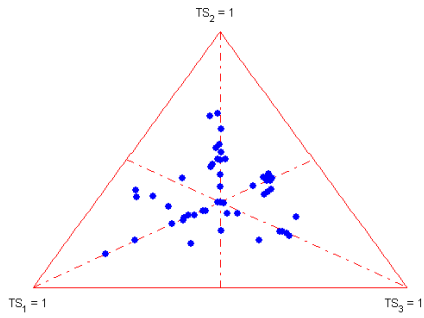
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 7



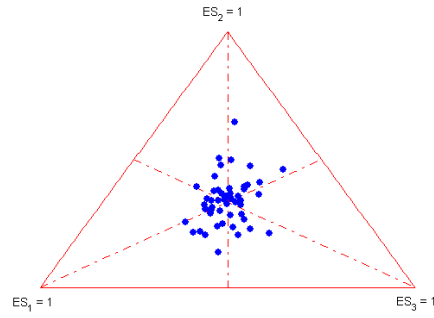
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 7



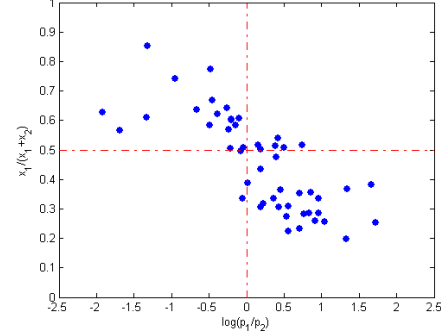
The Token Shares for Subject ID 8



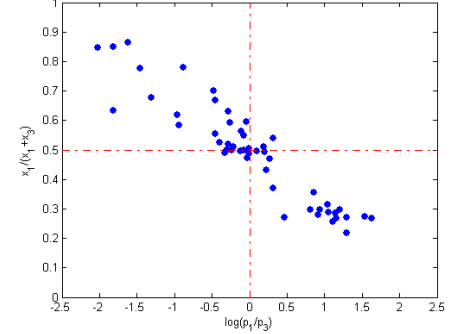
The Expenditure Shares for Subject ID 8



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 8

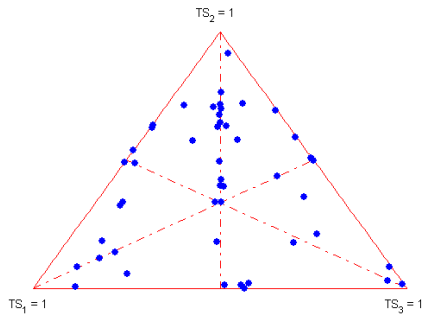


The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 8

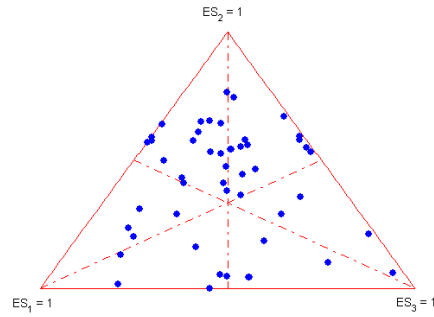




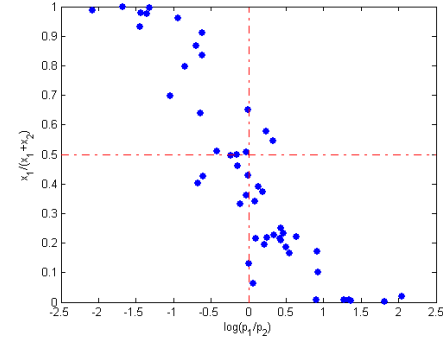
The Token Shares for Subject ID 9



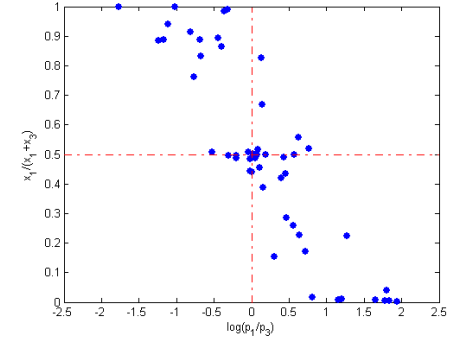
The Expenditure Shares for Subject ID 9



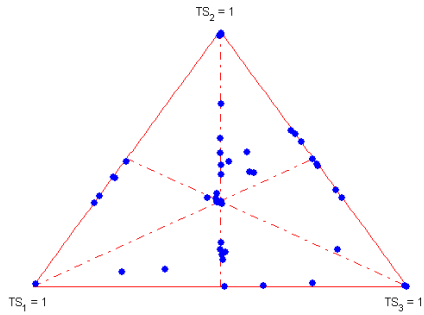
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 9



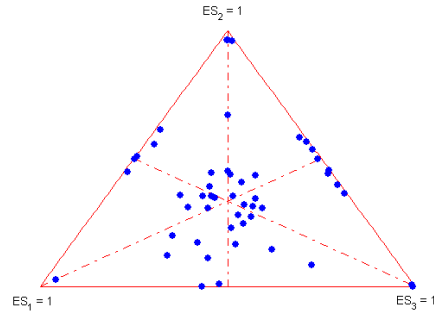
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 9



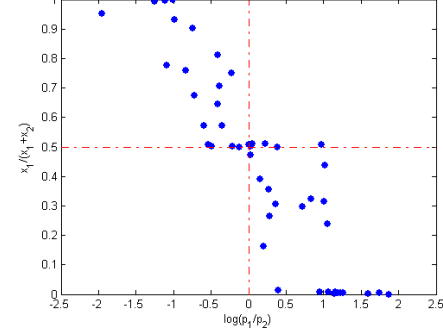
The Token Shares for Subject ID 10



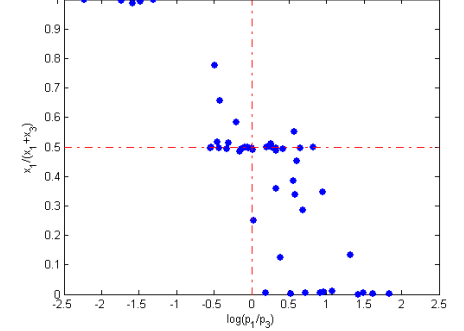
The Expenditure Shares for Subject ID 10



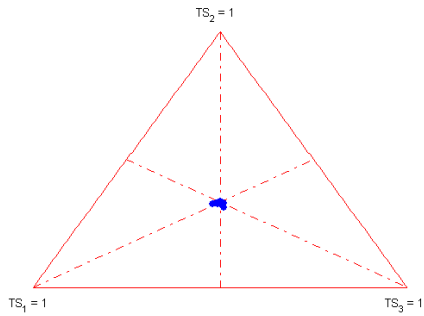
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 10



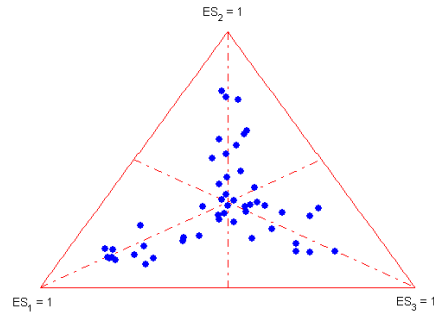
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 10



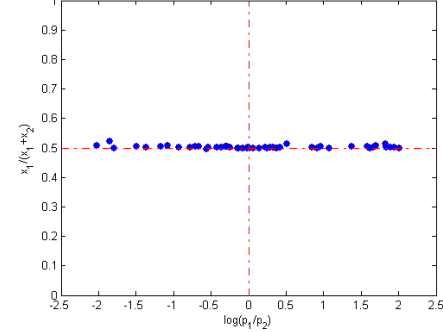
The Token Shares for Subject ID 11



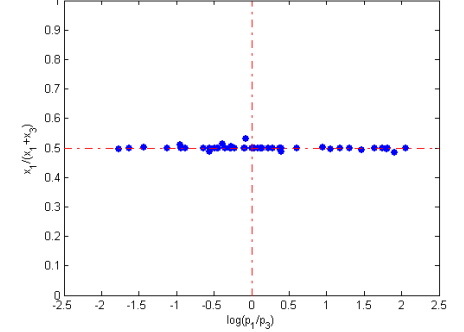
The Expenditure Shares for Subject ID 11



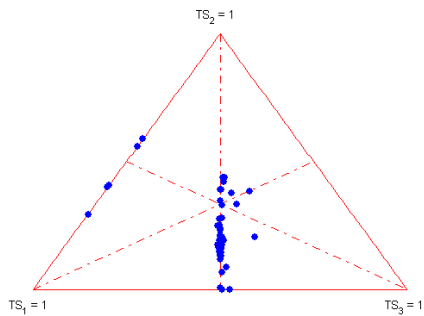
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 11



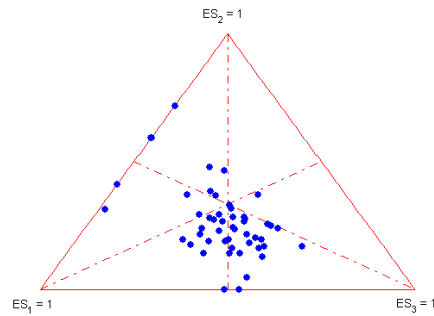
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 11



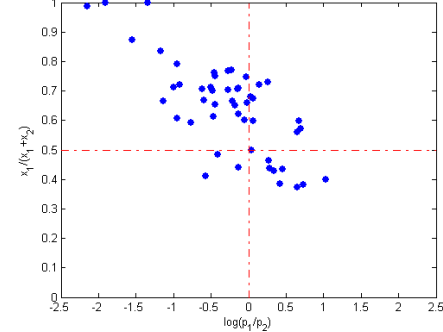
The Token Shares for Subject ID 12



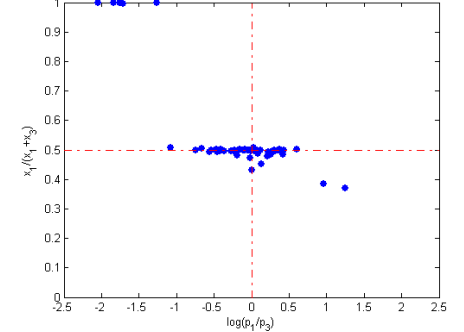
The Expenditure Shares for Subject ID 12



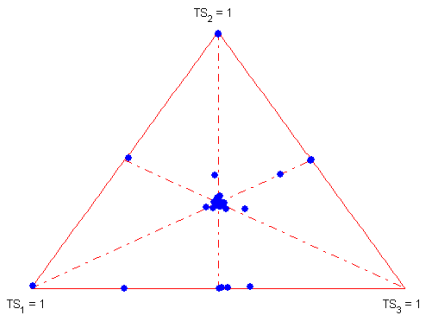
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 12



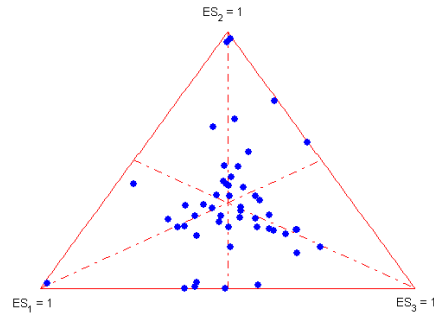
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 12



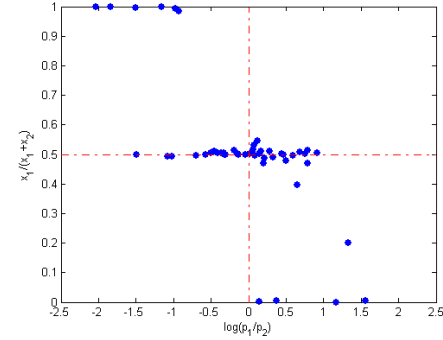
The Token Shares for Subject ID 13



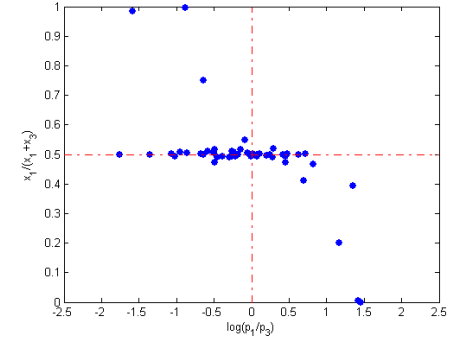
The Expenditure Shares for Subject ID 13



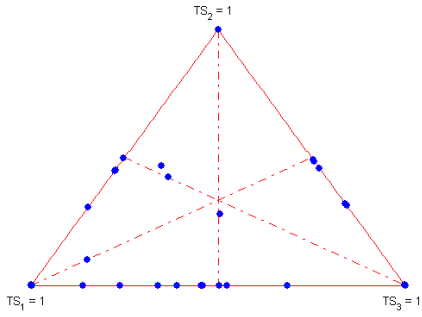
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 13



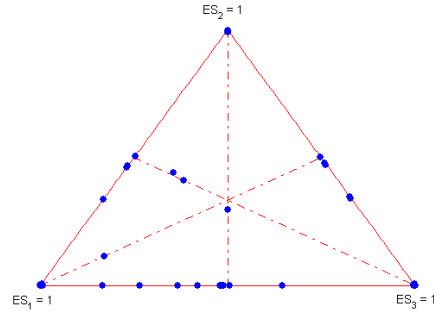
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 13



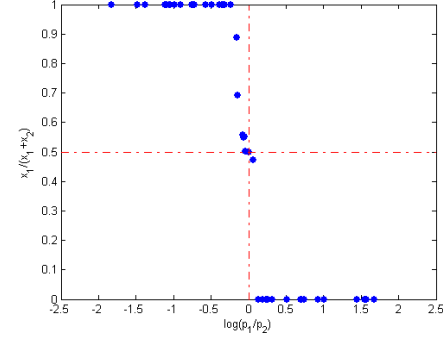
The Token Shares for Subject ID 14



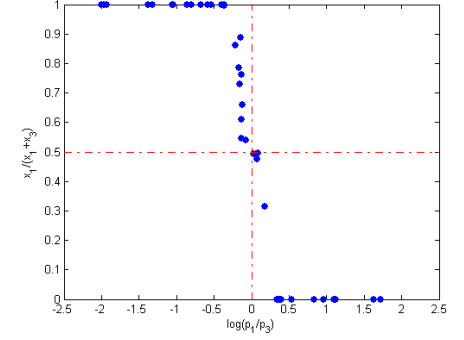
The Expenditure Shares for Subject ID 14



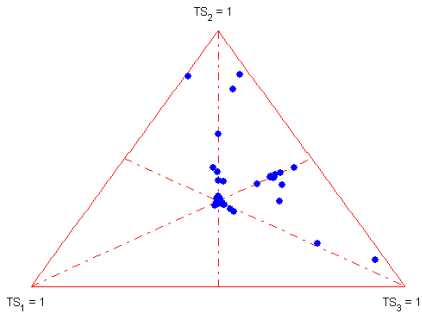
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 14



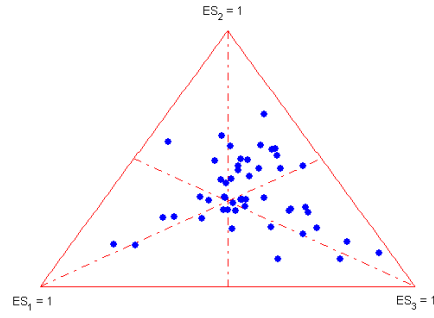
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 14



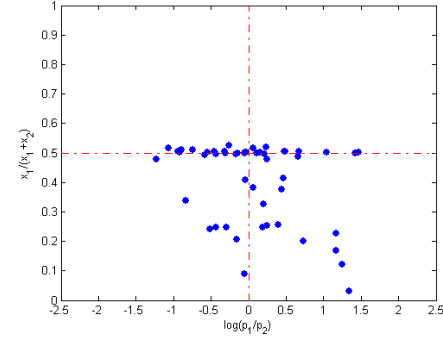
The Token Shares for Subject ID 15



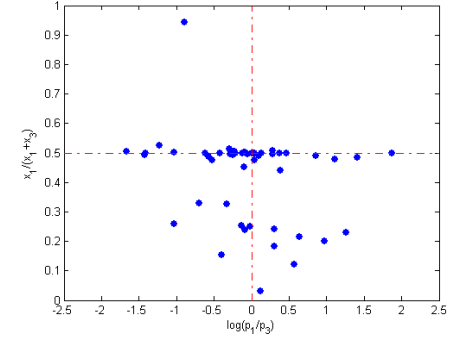
The Expenditure Shares for Subject ID 15



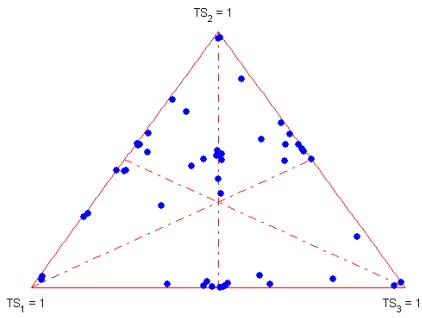
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 15



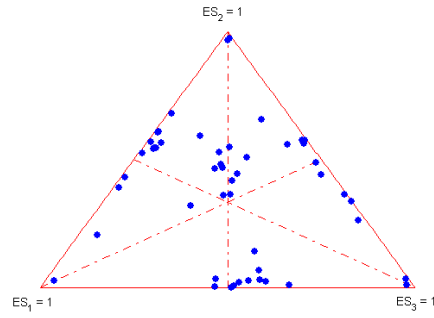
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 15



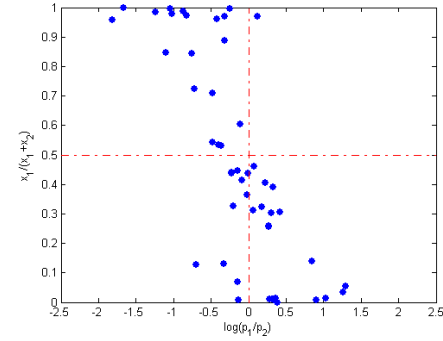
The Token Shares for Subject ID 16



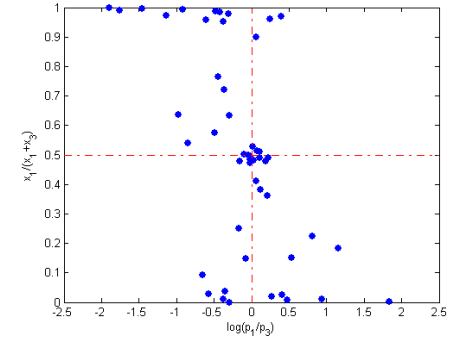
The Expenditure Shares for Subject ID 16



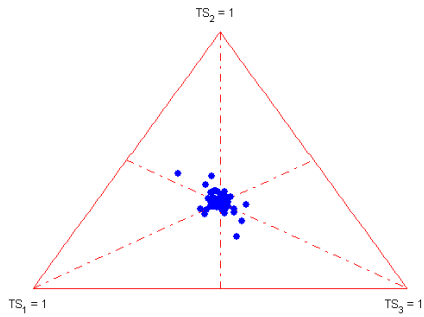
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 16



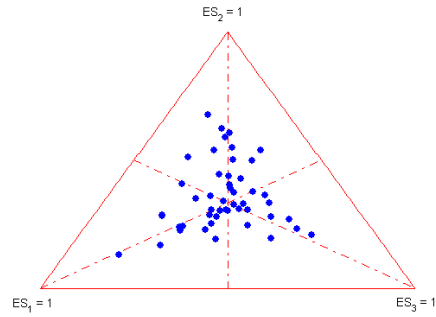
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 16



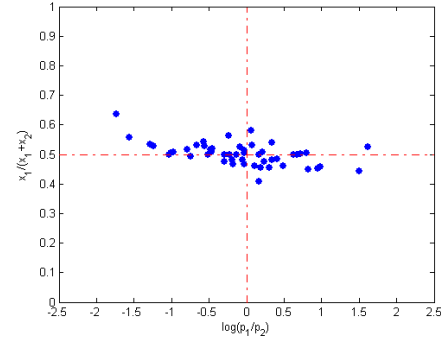
The Token Shares for Subject ID 17



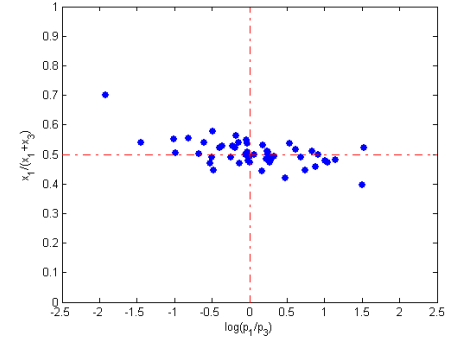
The Expenditure Shares for Subject ID 17



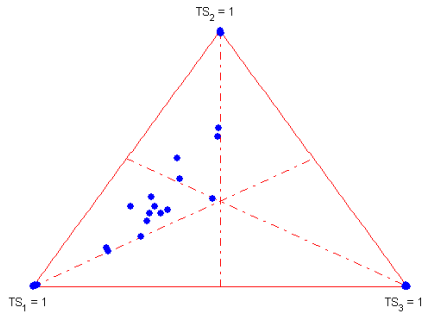
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 17



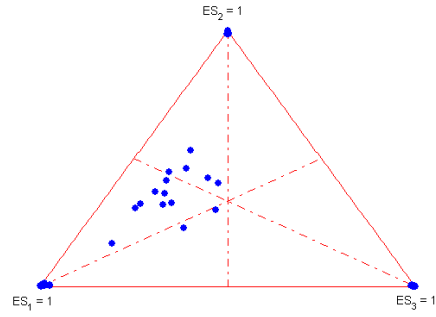
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 17



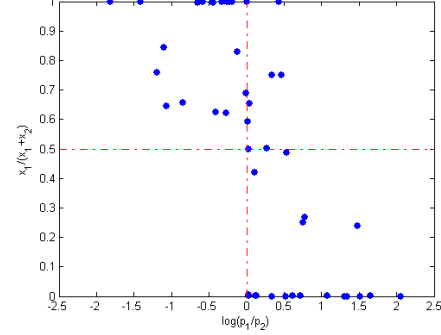
The Token Shares for Subject ID 18



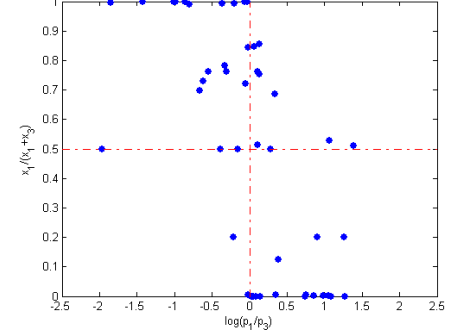
The Expenditure Shares for Subject ID 18



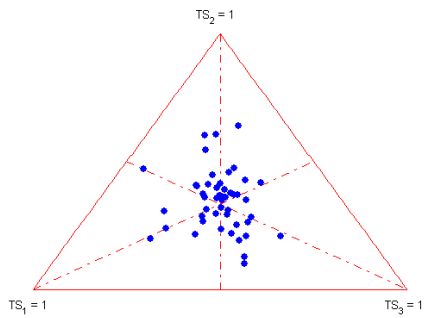
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 18



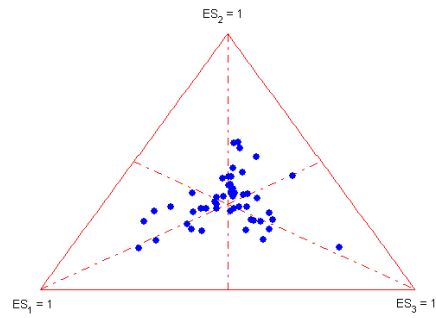
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 18



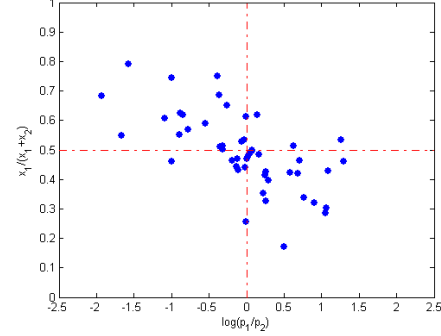
The Token Shares for Subject ID 19



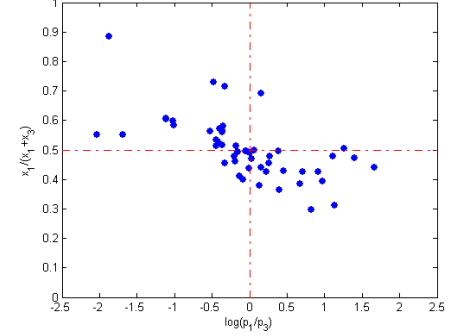
The Expenditure Shares for Subject ID 19



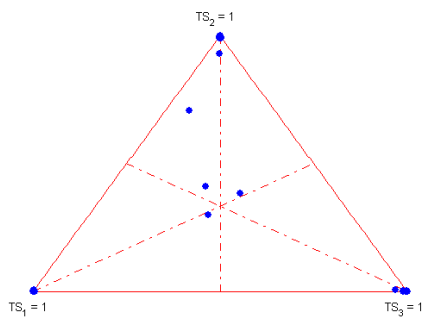
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 19



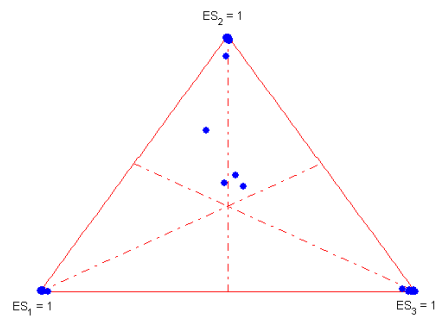
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 19



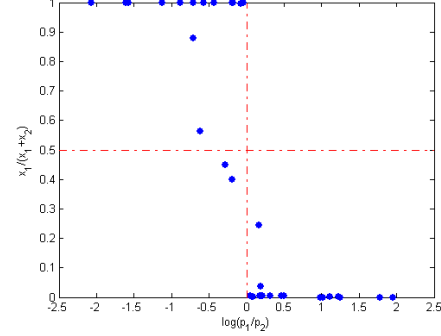
The Token Shares for Subject ID 20



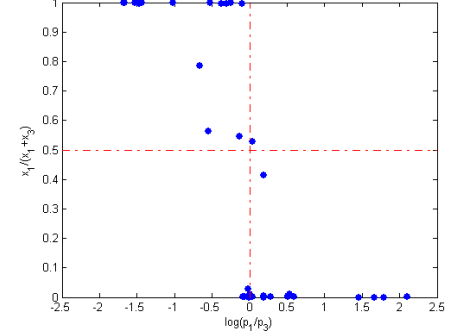
The Expenditure Shares for Subject ID 20



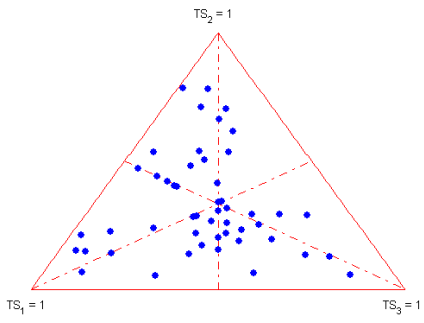
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 20



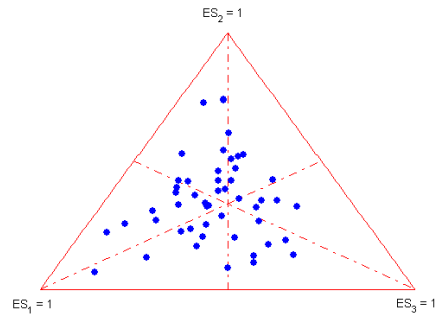
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 20



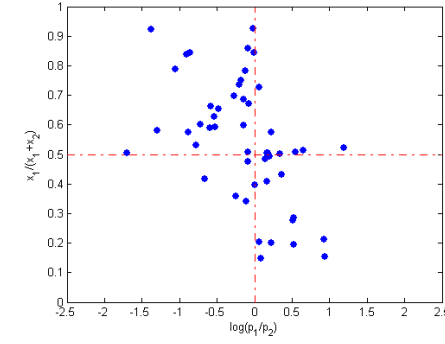
The Token Shares for Subject ID 21



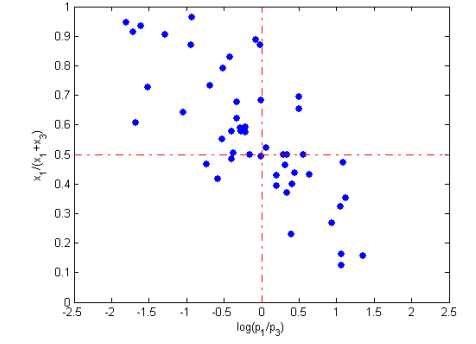
The Expenditure Shares for Subject ID 21



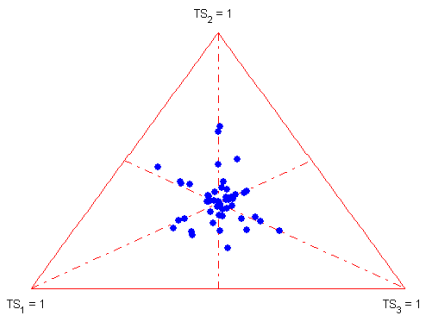
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 21



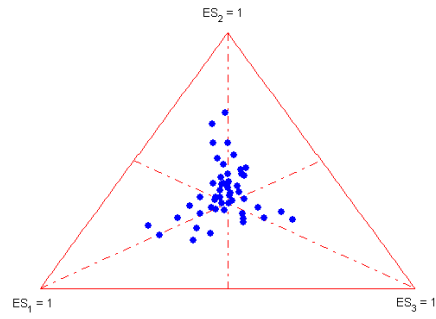
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 21



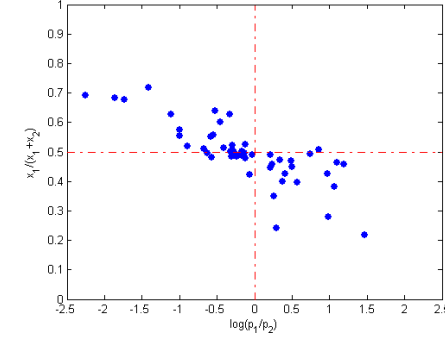
The Token Shares for Subject ID 22



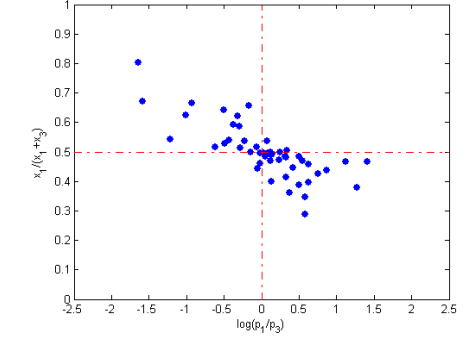
The Expenditure Shares for Subject ID 22



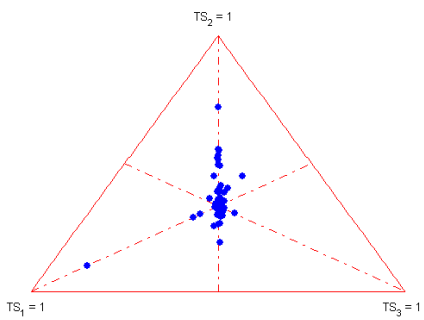
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 22



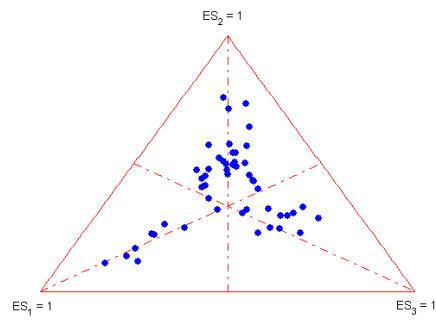
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 22



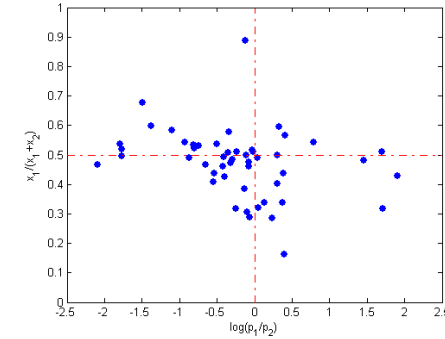
The Token Shares for Subject ID 23



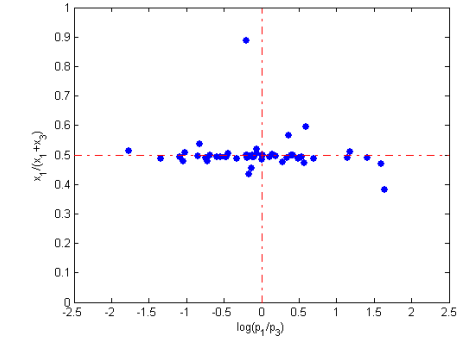
The Expenditure Shares for Subject ID 23



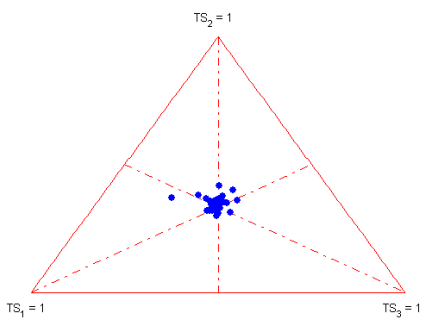
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 23



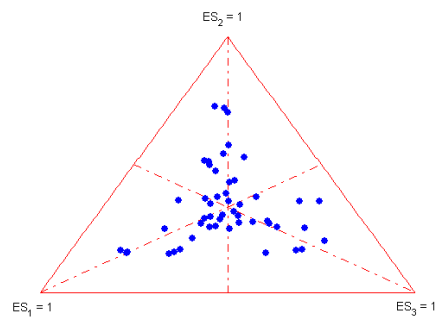
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 23



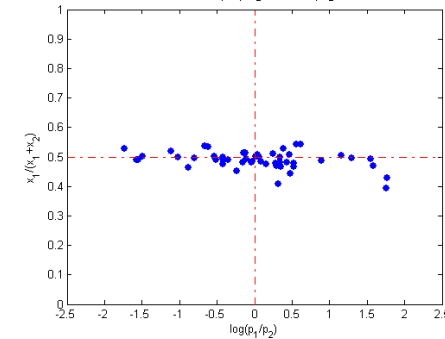
The Token Shares for Subject ID 24



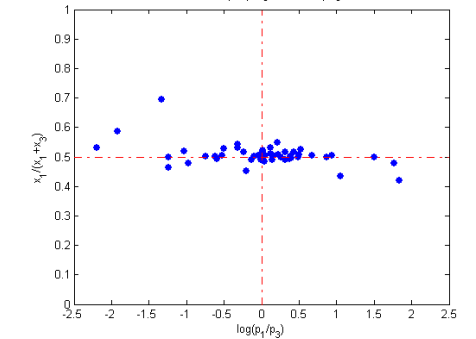
The Expenditure Shares for Subject ID 24



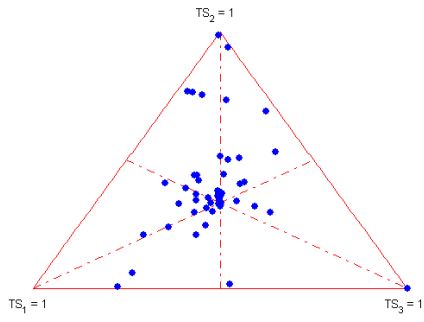
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 24



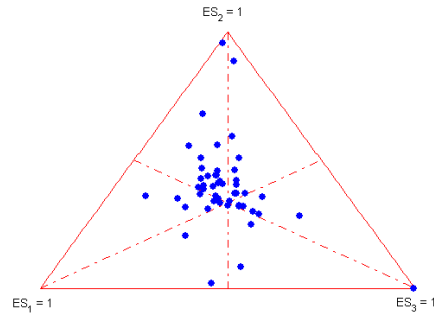
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 24



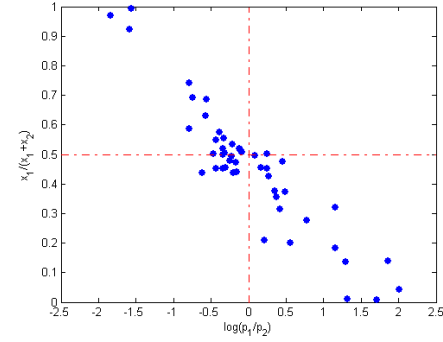
The Token Shares for Subject ID 25



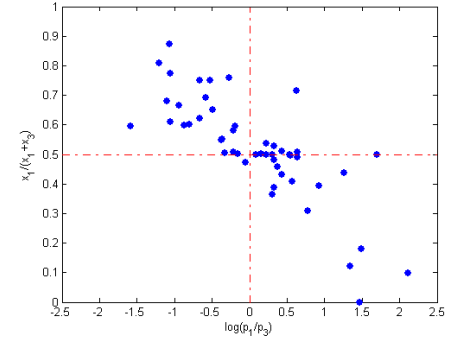
The Expenditure Shares for Subject ID 25



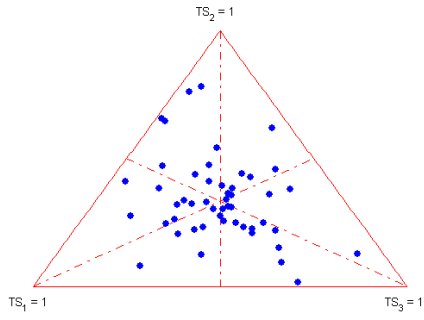
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 25



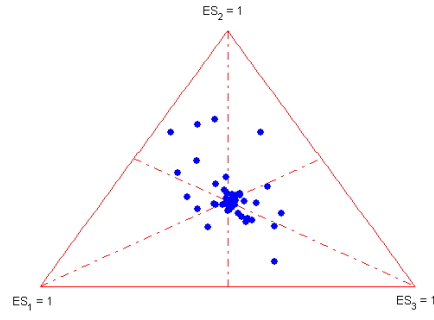
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 25



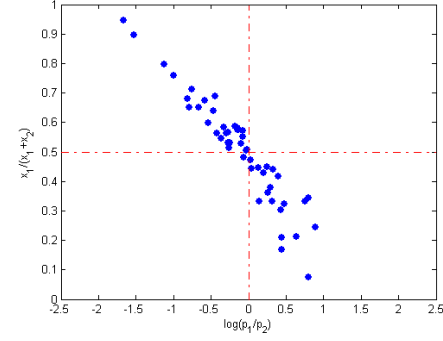
The Token Shares for Subject ID 26



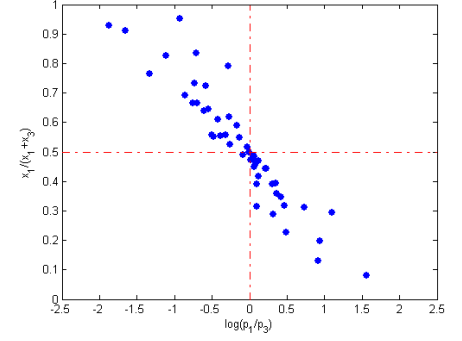
The Expenditure Shares for Subject ID 26



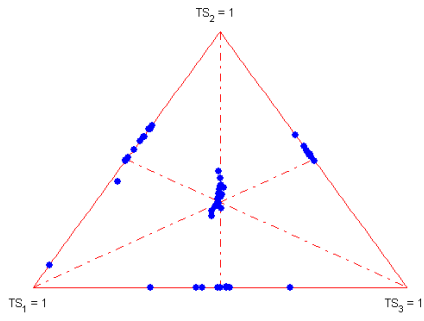
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 26



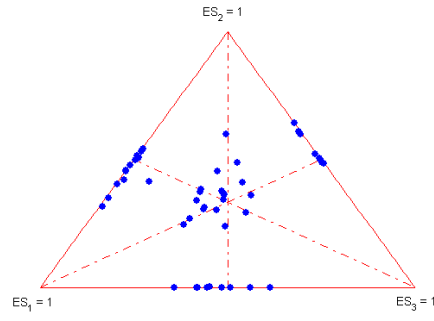
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 26



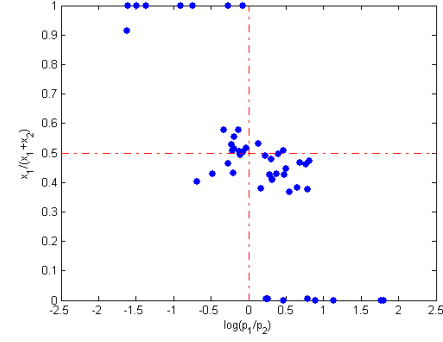
The Token Shares for Subject ID 27



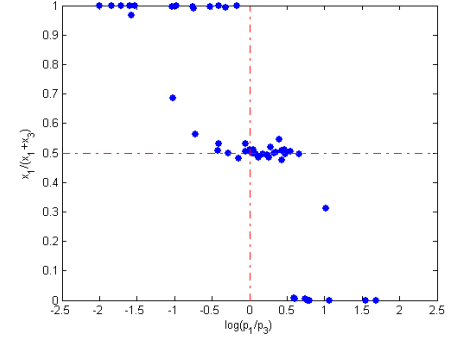
The Expenditure Shares for Subject ID 27



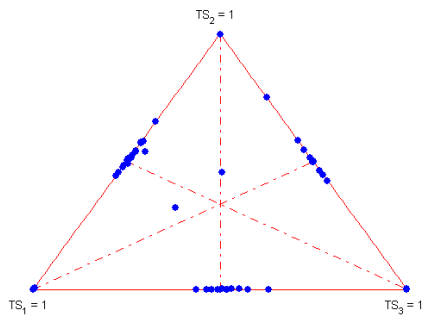
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 27



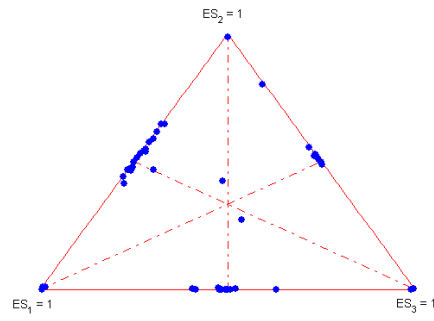
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 27



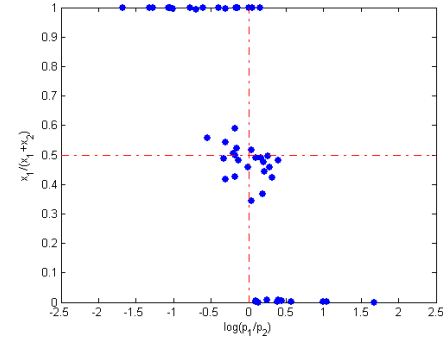
The Token Shares for Subject ID 28



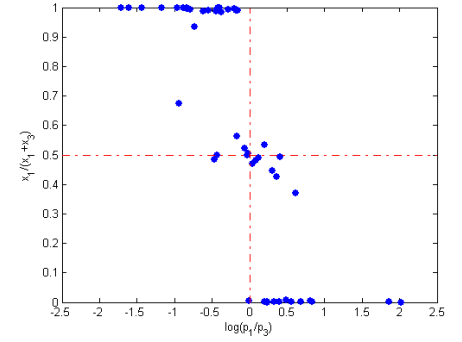
The Expenditure Shares for Subject ID 28



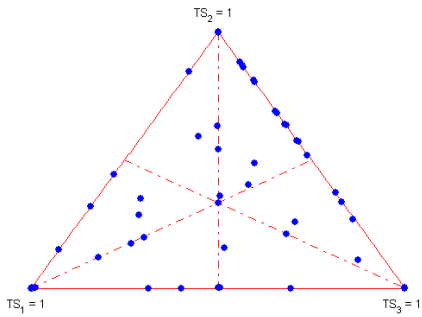
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 28



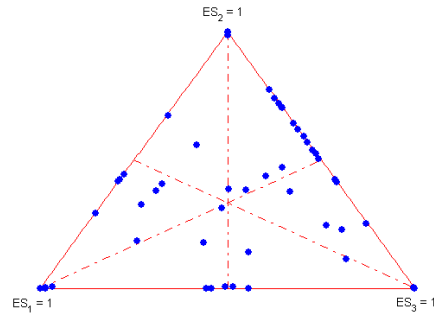
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 28



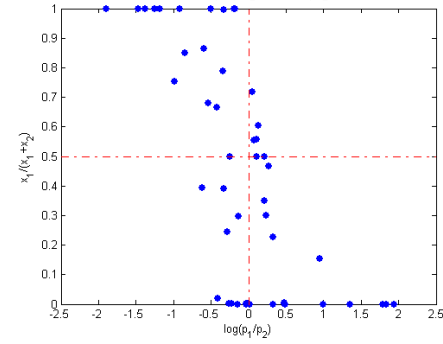
The Token Shares for Subject ID 29



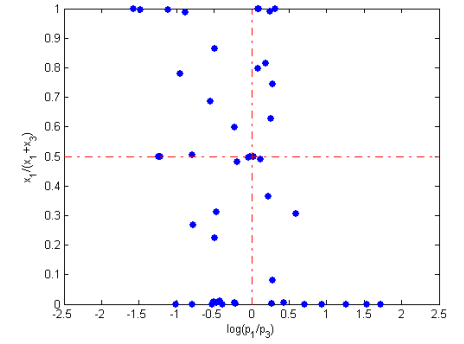
The Expenditure Shares for Subject ID 29



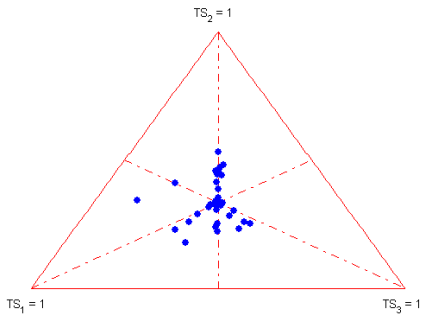
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 29



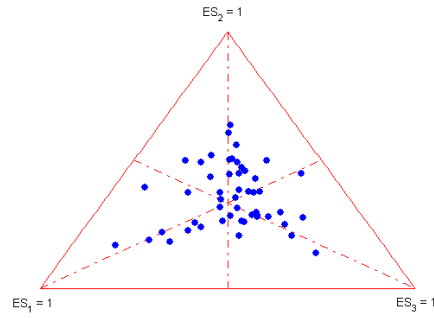
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 29



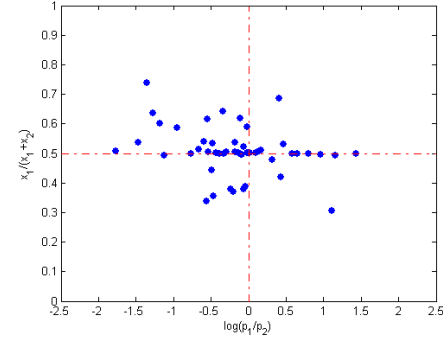
The Token Shares for Subject ID 30



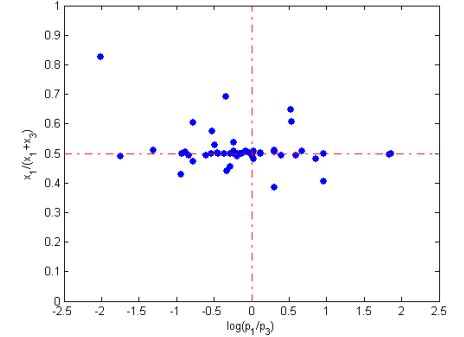
The Expenditure Shares for Subject ID 30



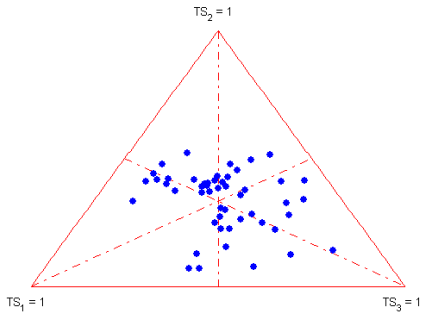
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 30



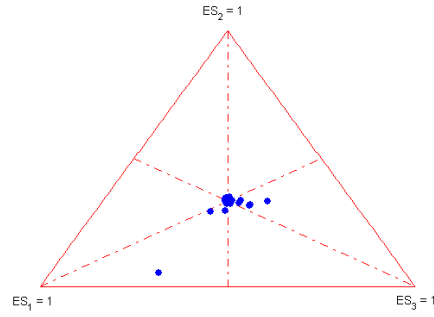
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 30



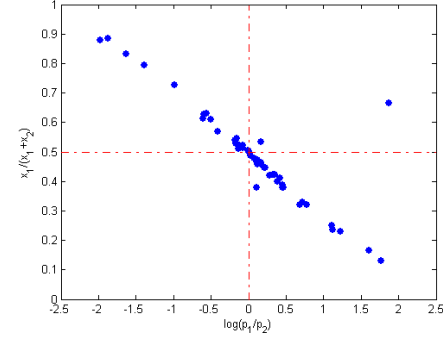
The Token Shares for Subject ID 31



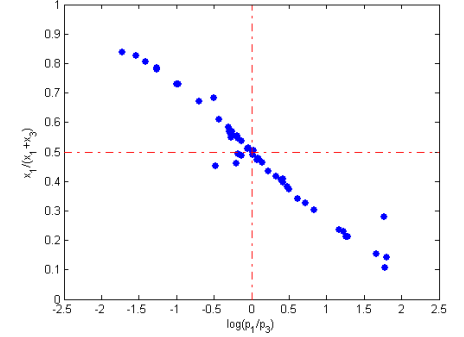
The Expenditure Shares for Subject ID 31



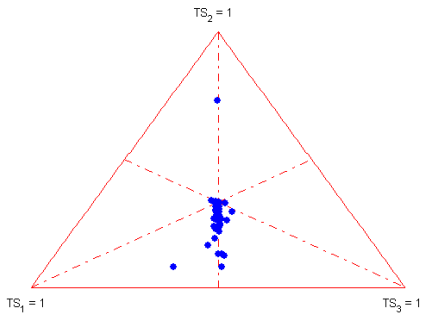
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 31



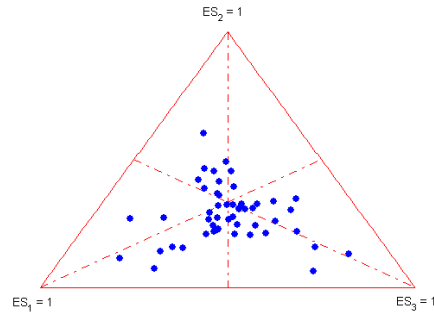
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 31



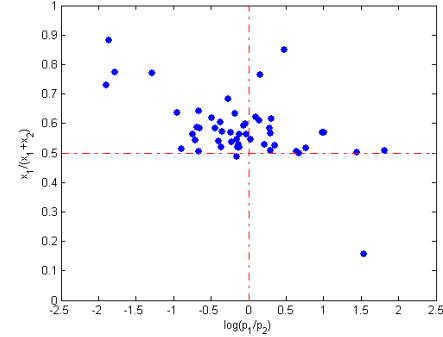
The Token Shares for Subject ID 32



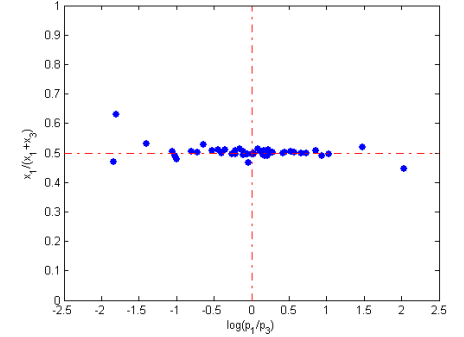
The Expenditure Shares for Subject ID 32



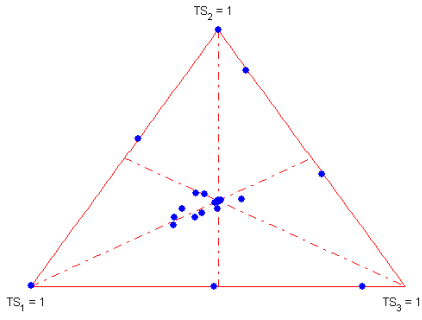
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 32



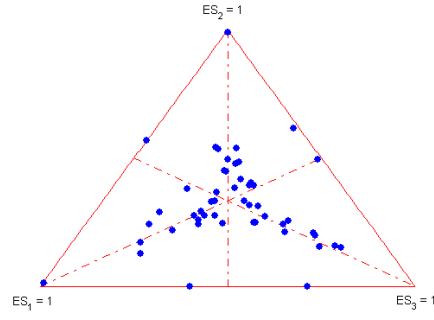
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 32



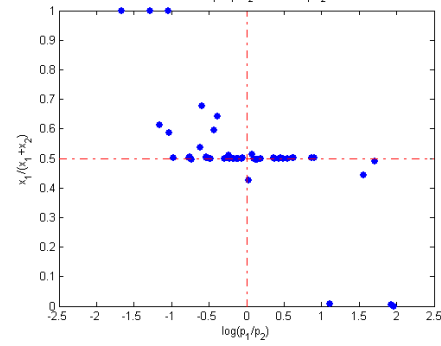
The Token Shares for Subject ID 33



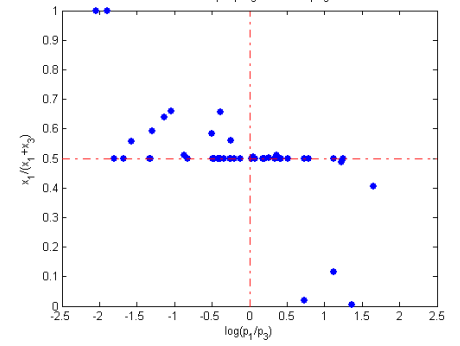
The Expenditure Shares for Subject ID 33



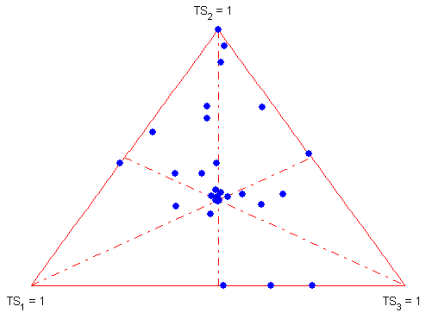
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 33



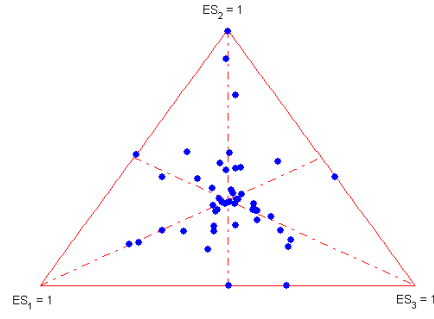
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 33



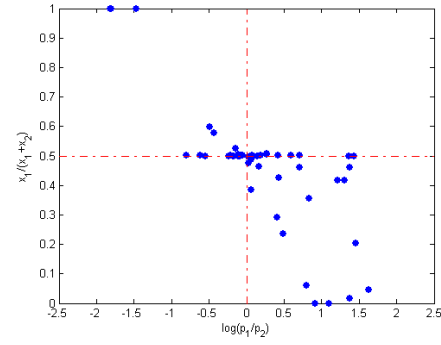
The Token Shares for Subject ID 34



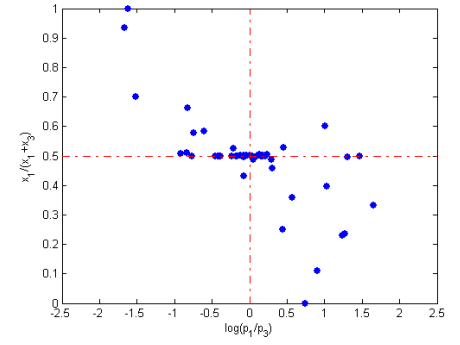
The Expenditure Shares for Subject ID 34



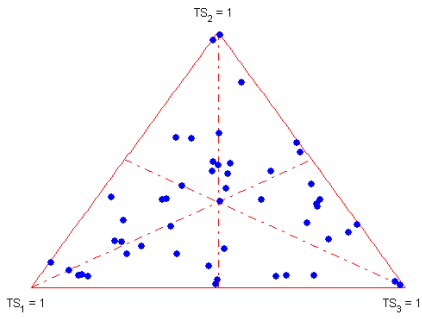
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 34



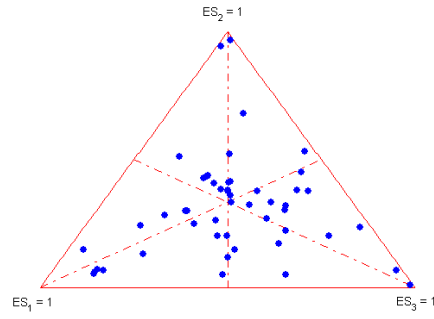
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 34



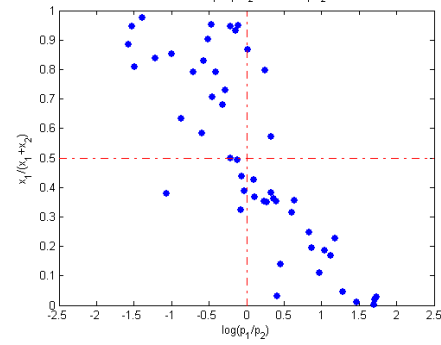
The Token Shares for Subject ID 35



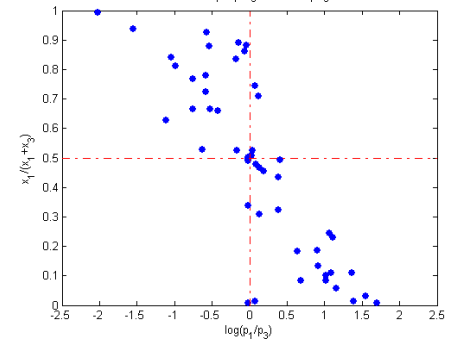
The Expenditure Shares for Subject ID 35



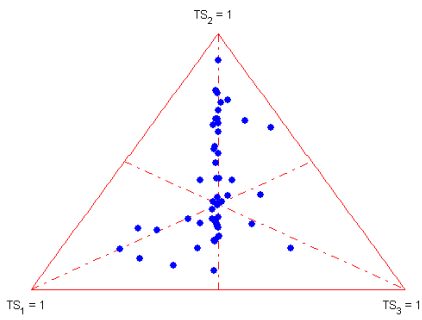
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 35



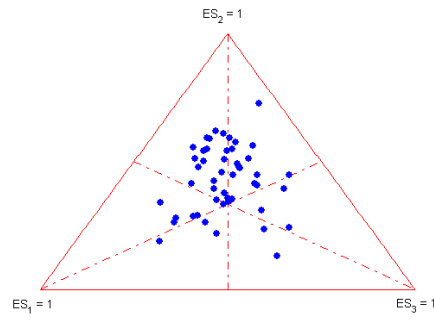
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 35



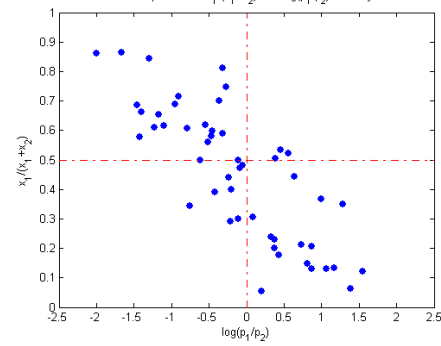
The Token Shares for Subject ID 36



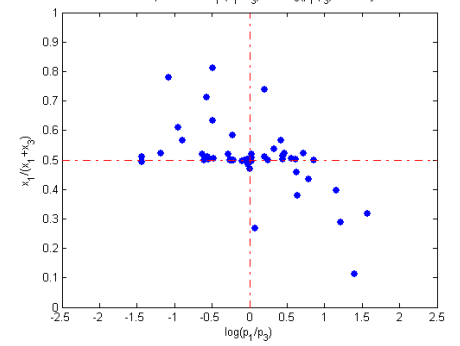
The Expenditure Shares for Subject ID 36



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 36



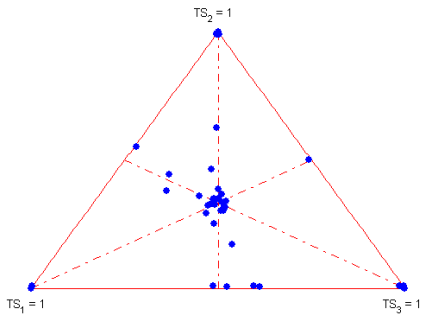
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 36



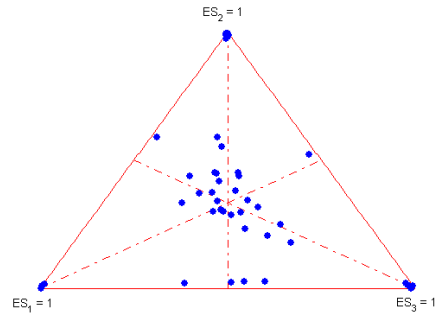




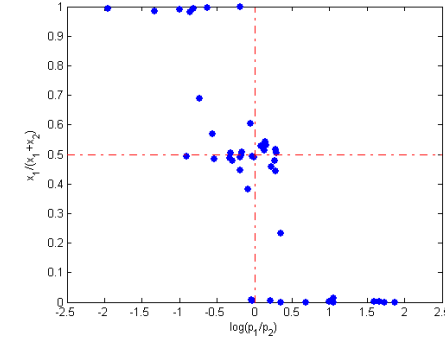
The Token Shares for Subject ID 41



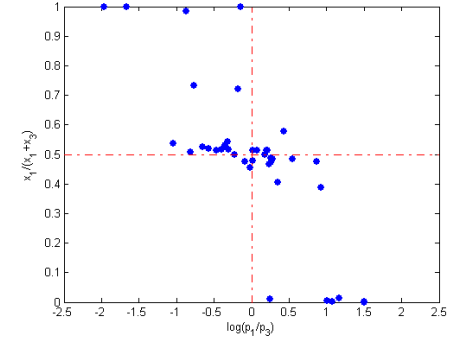
The Expenditure Shares for Subject ID 41



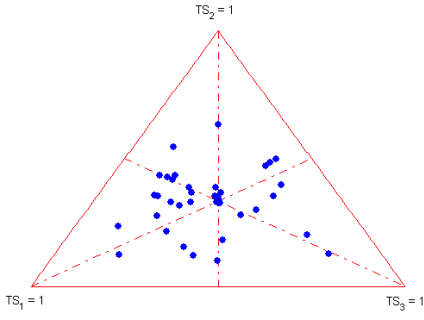
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 41



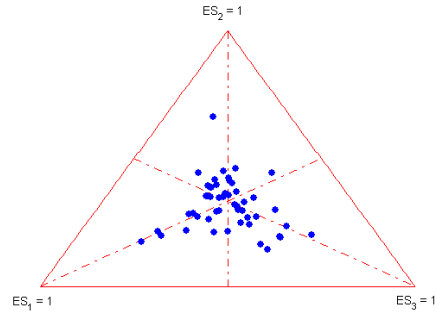
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 41



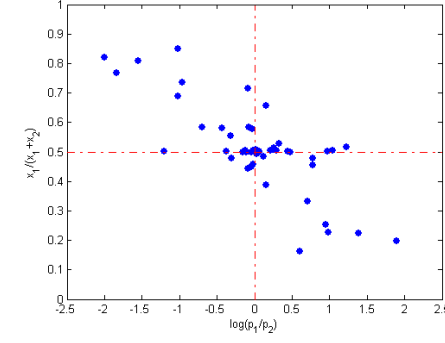
The Token Shares for Subject ID 42



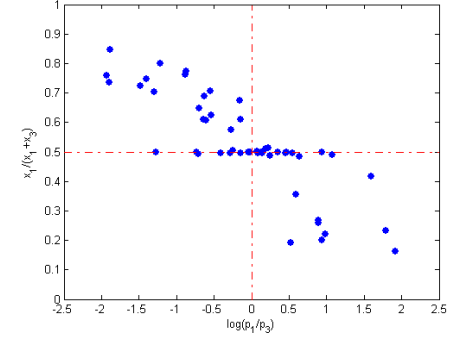
The Expenditure Shares for Subject ID 42



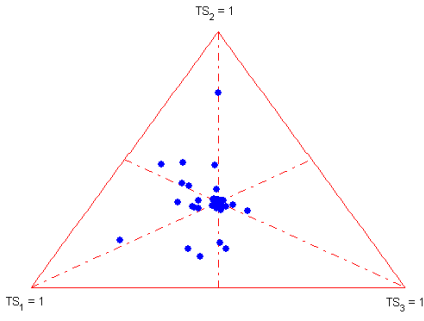
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 42



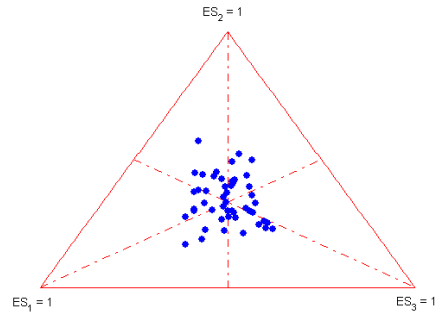
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 42



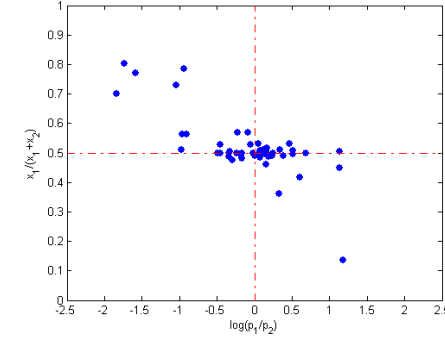
The Token Shares for Subject ID 43



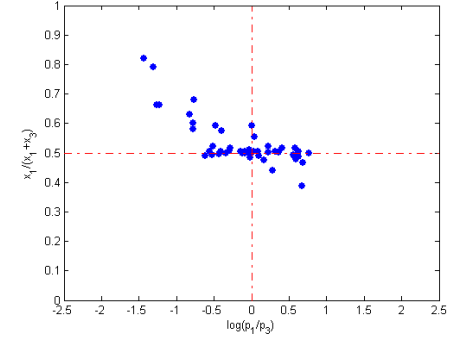
The Expenditure Shares for Subject ID 43



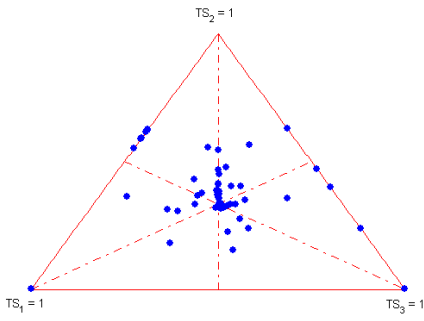
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 43



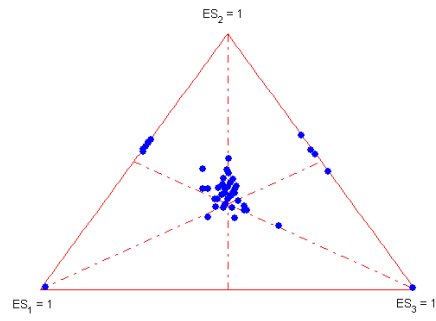
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 43



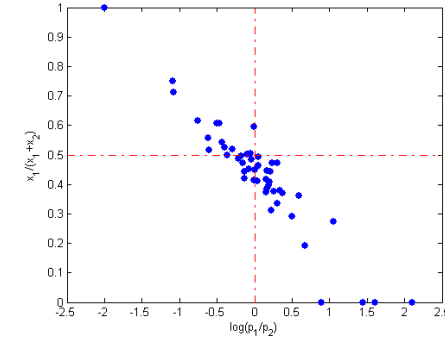
The Token Shares for Subject ID 44



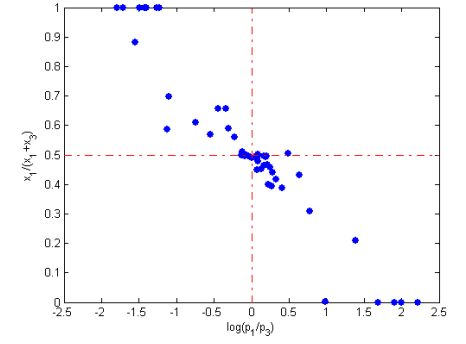
The Expenditure Shares for Subject ID 44



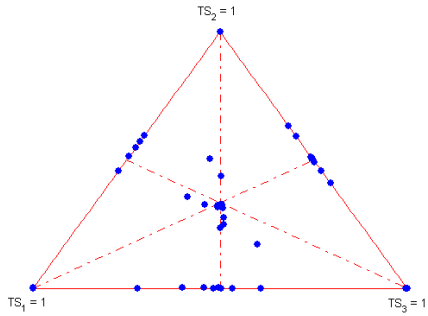
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 44



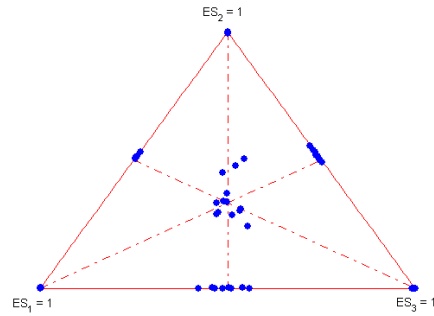
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 44



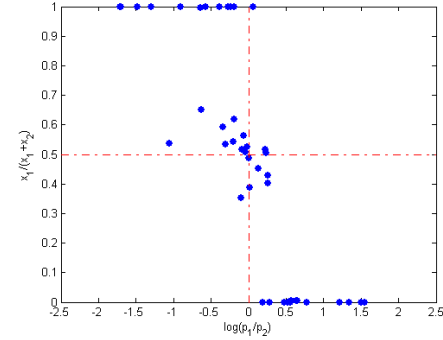
The Token Shares for Subject ID 45



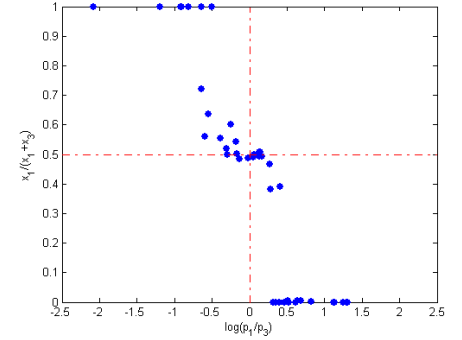
The Expenditure Shares for Subject ID 45



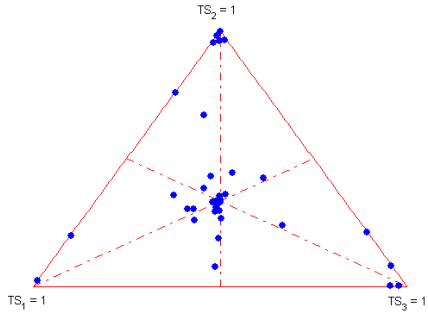
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 45



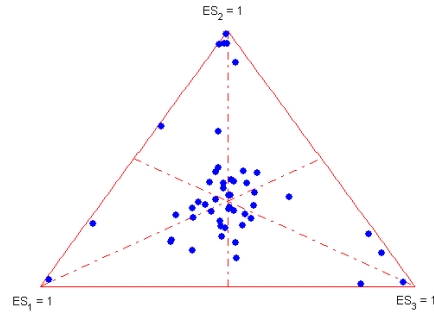
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 45



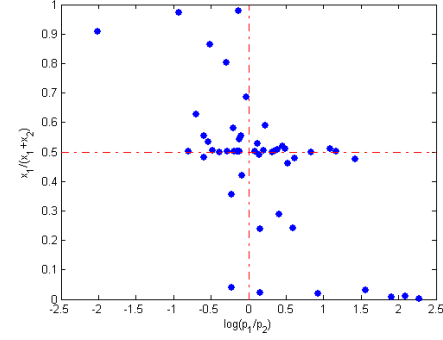
The Token Shares for Subject ID 46



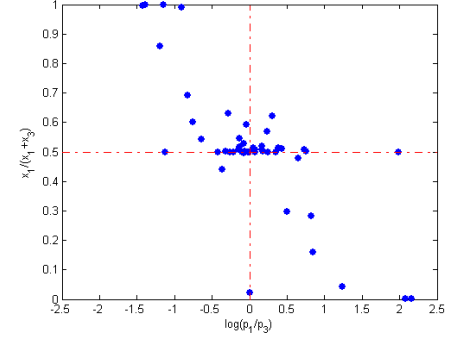
The Expenditure Shares for Subject ID 46



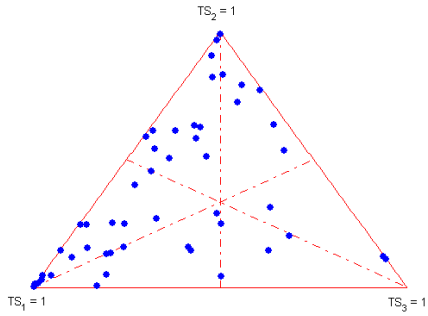
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 46



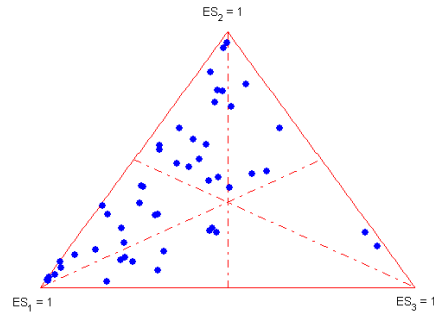
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 46



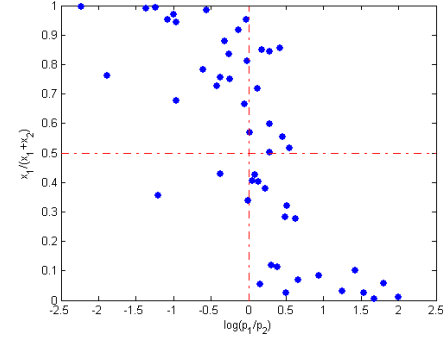
The Token Shares for Subject ID 47



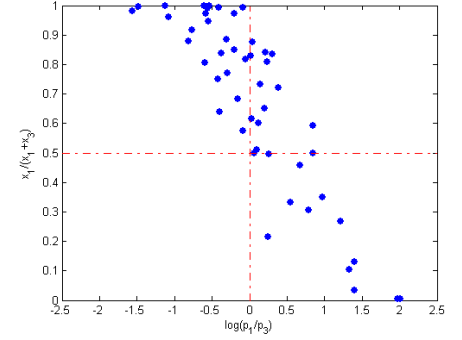
The Expenditure Shares for Subject ID 47



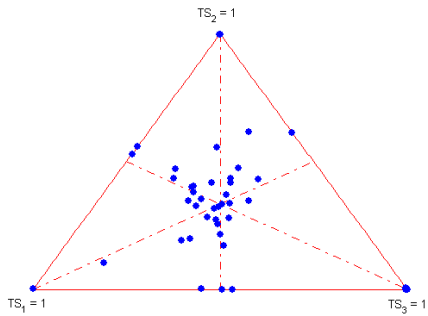
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 47



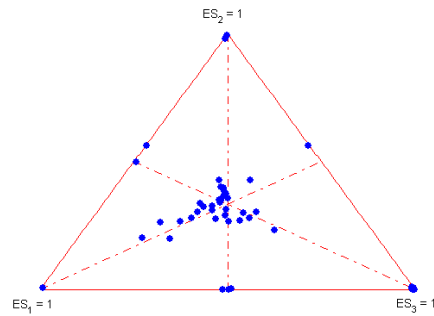
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 47



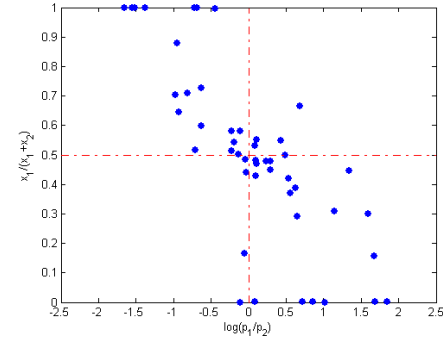
The Token Shares for Subject ID 48



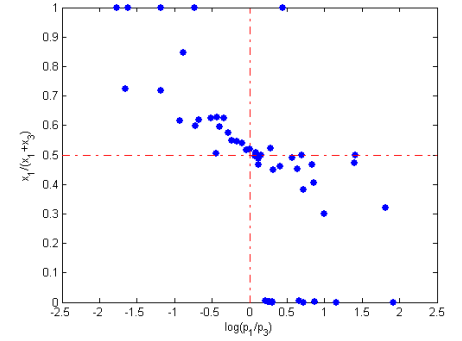
The Expenditure Shares for Subject ID 48



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 48

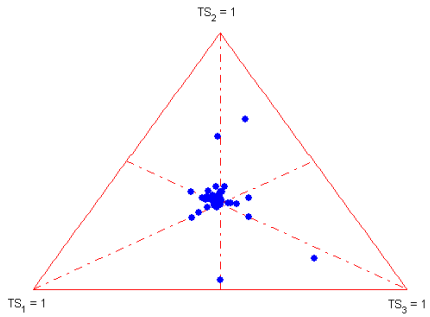


The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 48

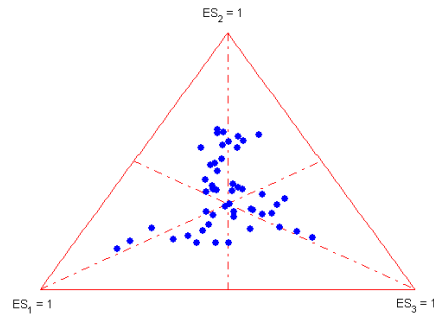




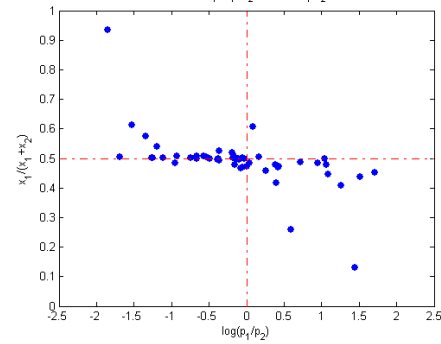
The Token Shares for Subject ID 53



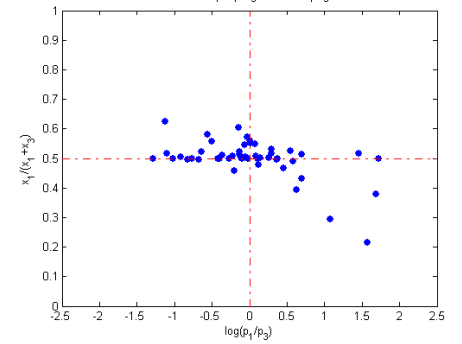
The Expenditure Shares for Subject ID 53



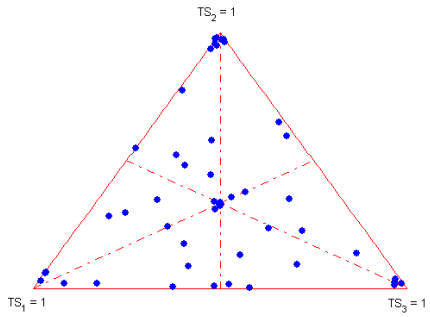
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 53



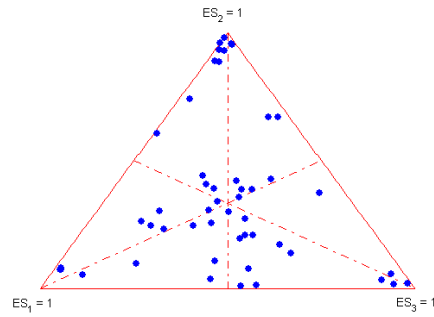
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 53



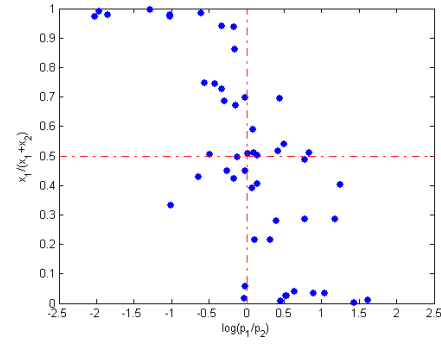
The Token Shares for Subject ID 54



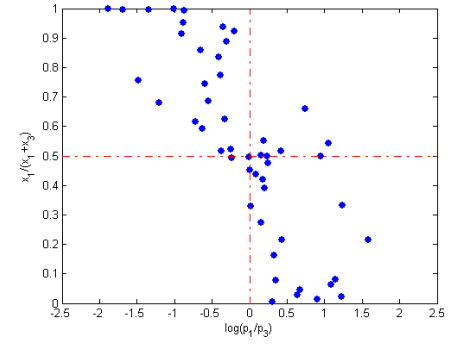
The Expenditure Shares for Subject ID 54



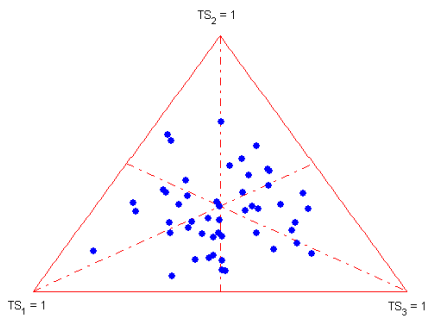
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 54



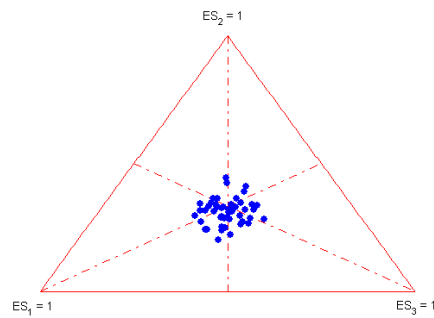
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 54



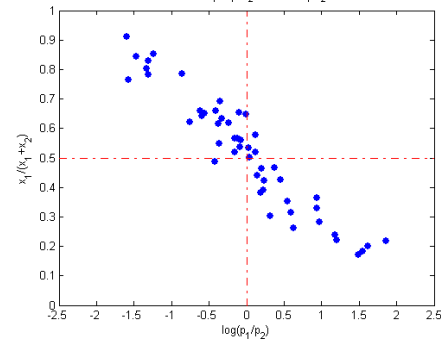
The Token Shares for Subject ID 55



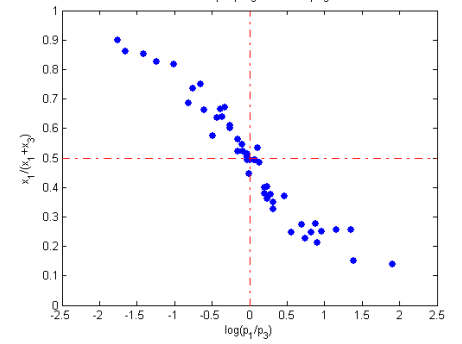
The Expenditure Shares for Subject ID 55



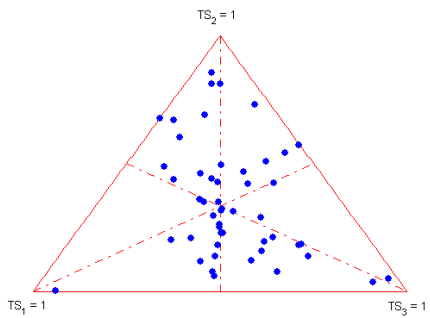
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 55



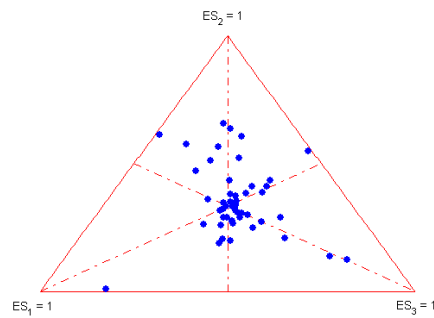
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 55



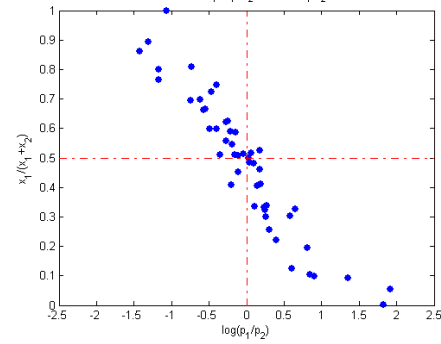
The Token Shares for Subject ID 56



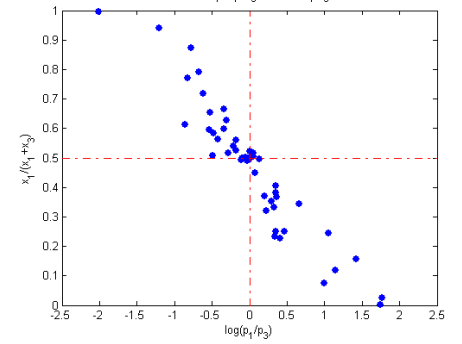
The Expenditure Shares for Subject ID 56



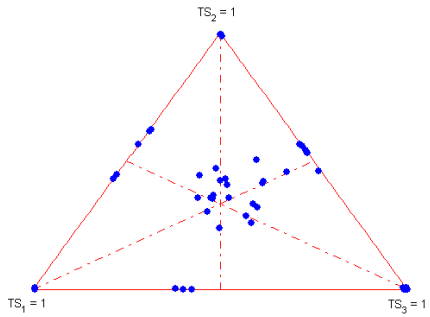
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 56



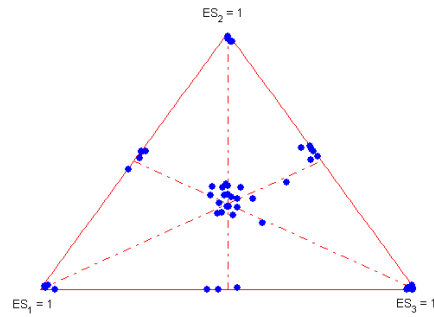
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 56



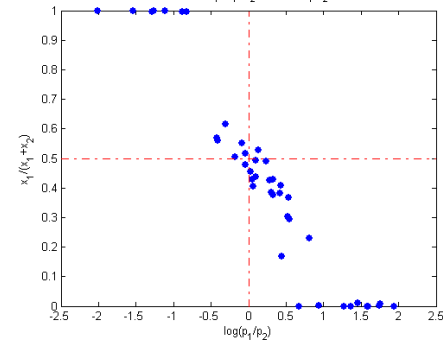
The Token Shares for Subject ID 57



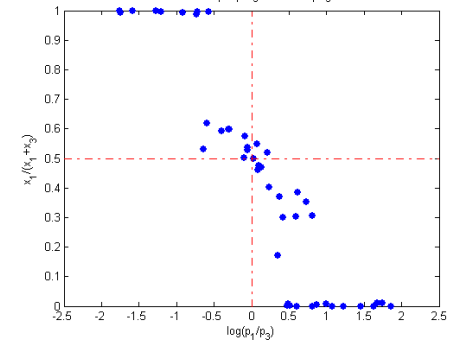
The Expenditure Shares for Subject ID 57



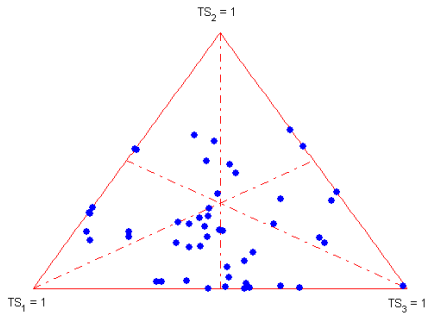
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 57



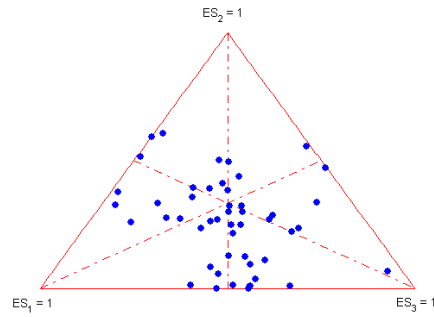
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 57



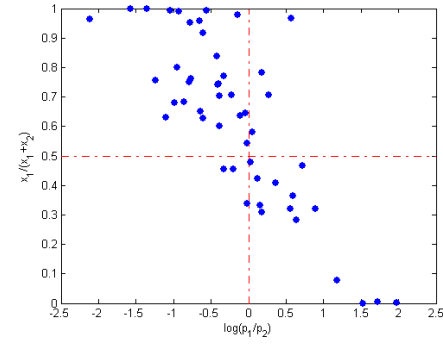
The Token Shares for Subject ID 58



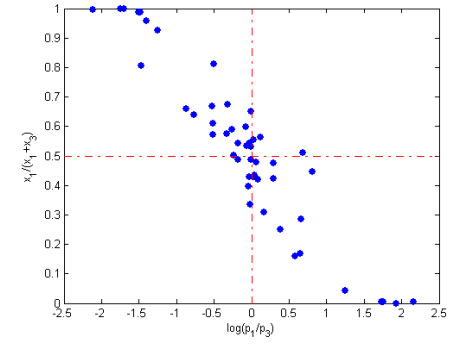
The Expenditure Shares for Subject ID 58



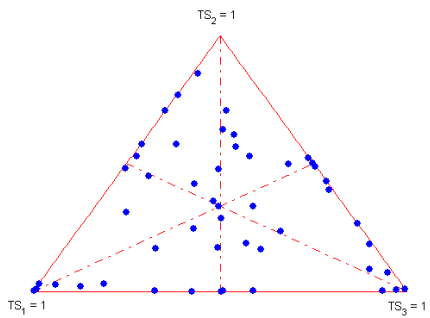
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 58



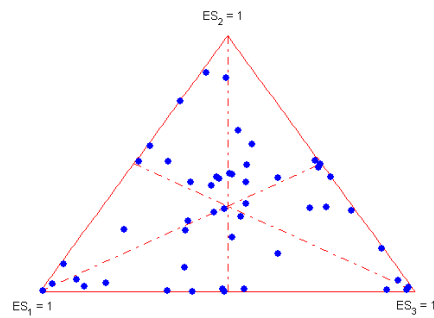
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 58



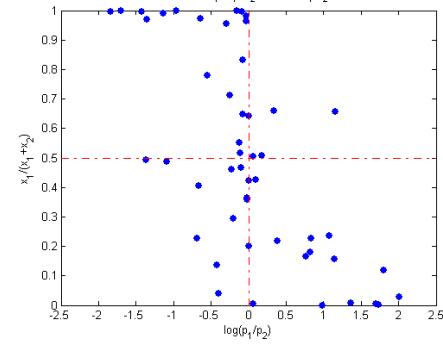
The Token Shares for Subject ID 59



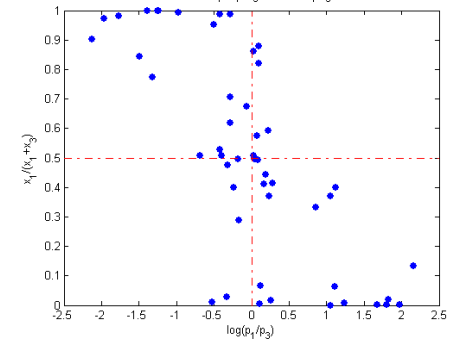
The Expenditure Shares for Subject ID 59



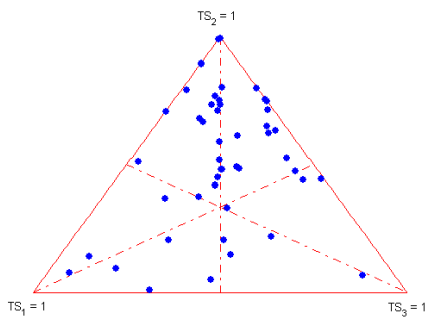
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 59



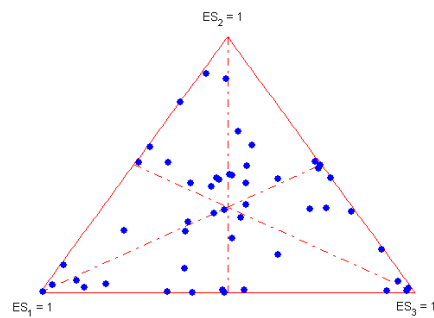
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 59



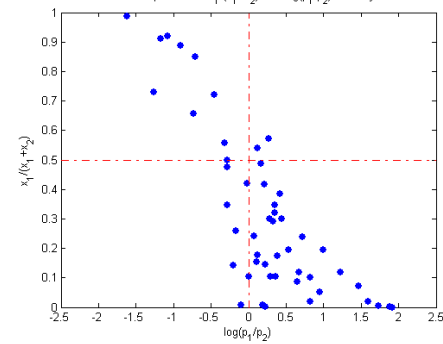
The Token Shares for Subject ID 60



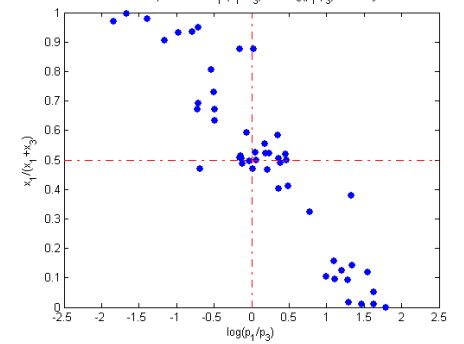
The Expenditure Shares for Subject ID 60



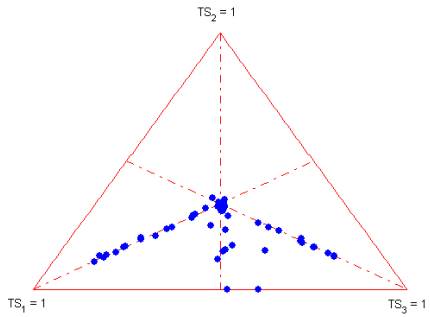
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 60



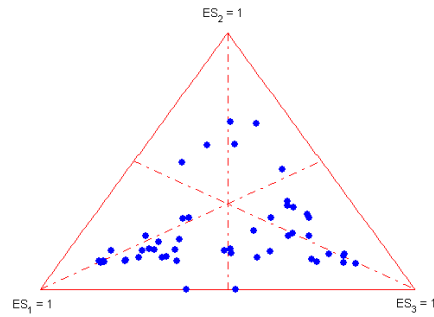
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 60



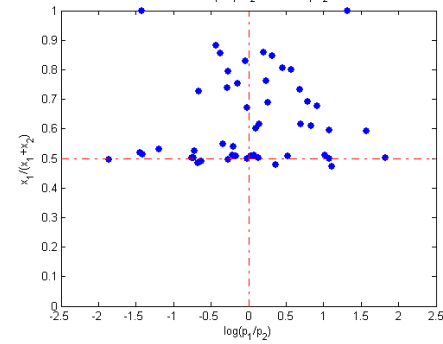
The Token Shares for Subject ID 61



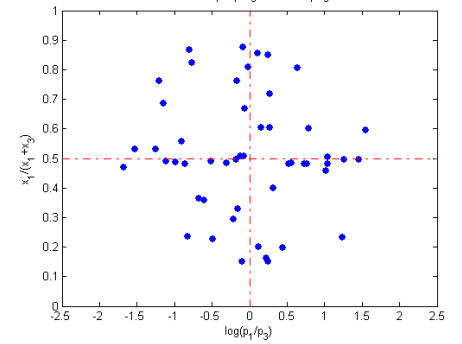
The Expenditure Shares for Subject ID 61



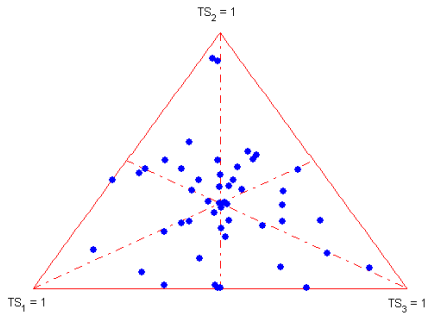
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 61



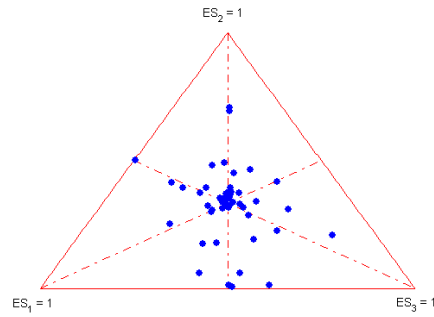
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 61



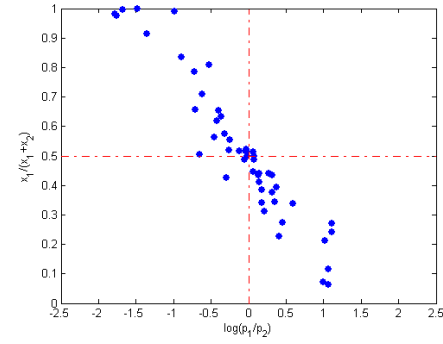
The Token Shares for Subject ID 62



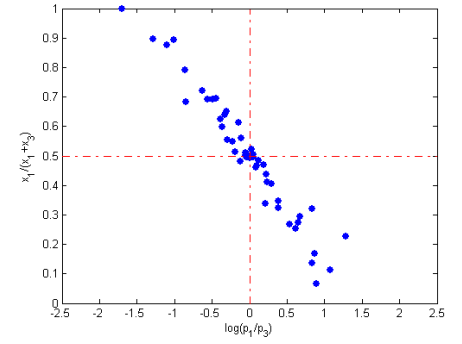
The Expenditure Shares for Subject ID 62



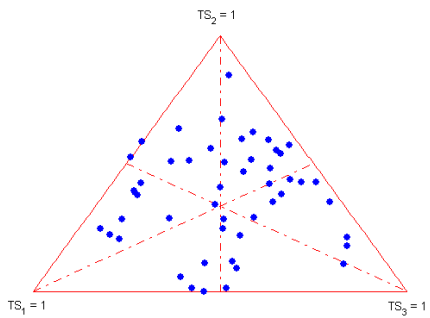
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 62



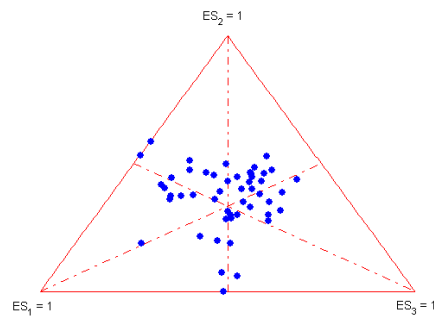
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 62



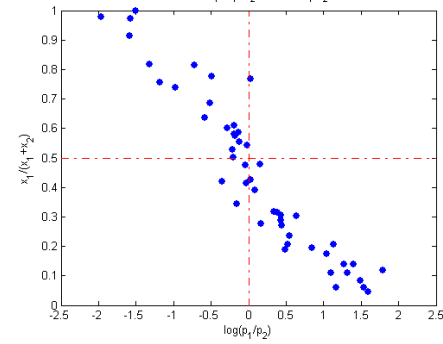
The Token Shares for Subject ID 63



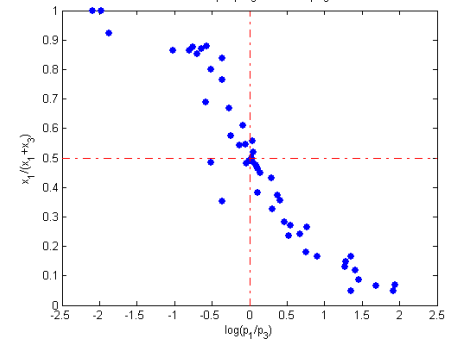
The Expenditure Shares for Subject ID 63



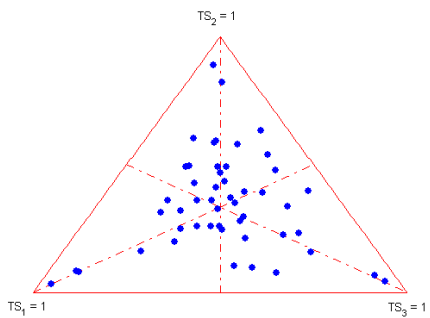
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 63



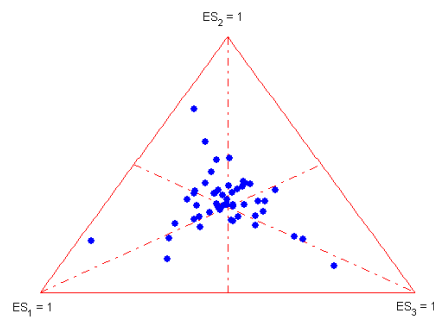
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 63



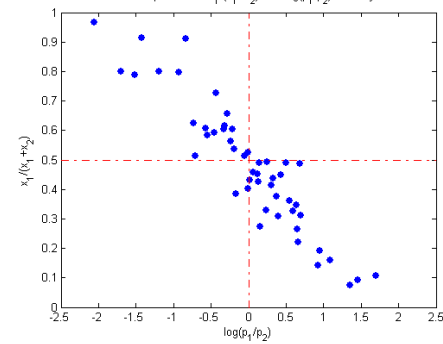
The Token Shares for Subject ID 64



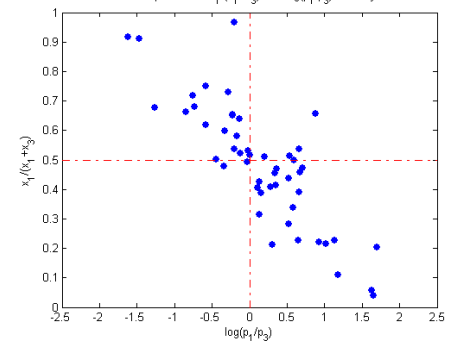
The Expenditure Shares for Subject ID 64



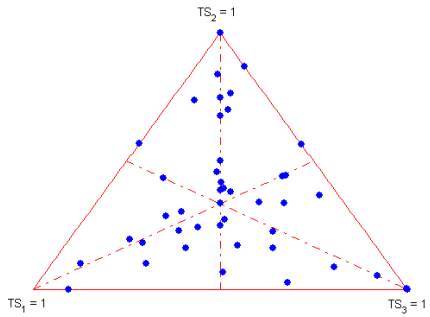
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 64



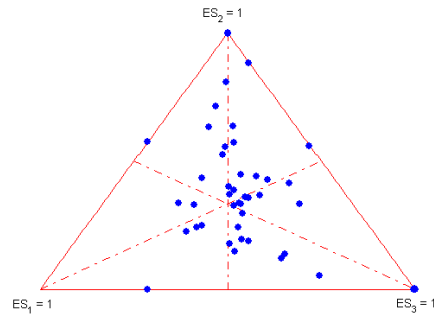
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 64



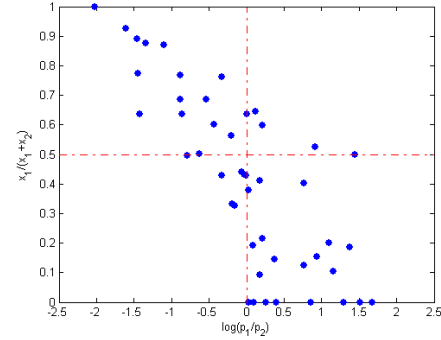
The Token Shares for Subject ID 65



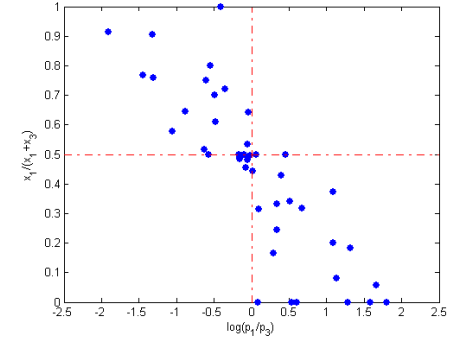
The Expenditure Shares for Subject ID 65



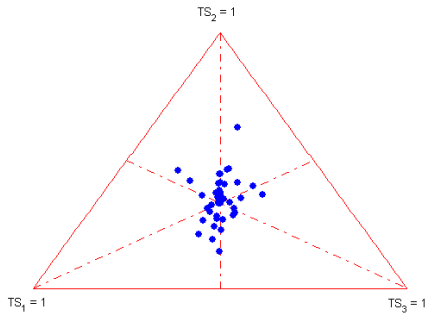
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 65



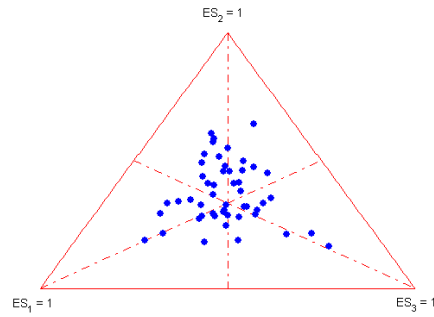
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 65



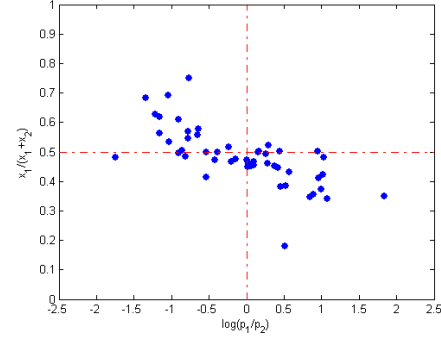
The Token Shares for Subject ID 66



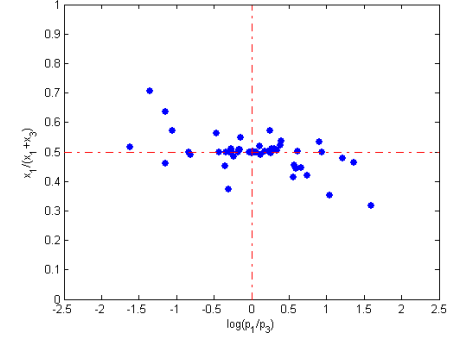
The Expenditure Shares for Subject ID 66



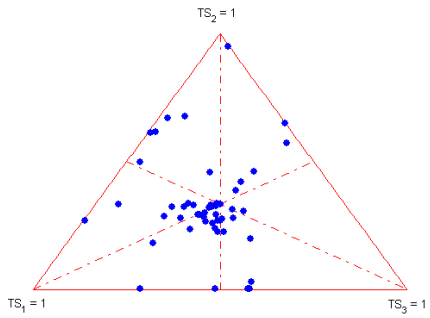
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 66



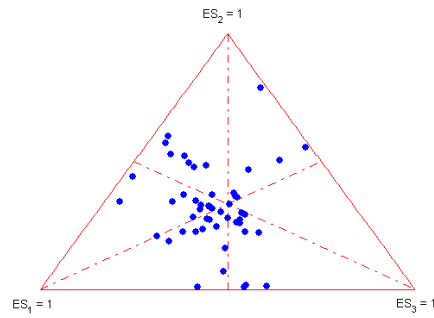
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 66



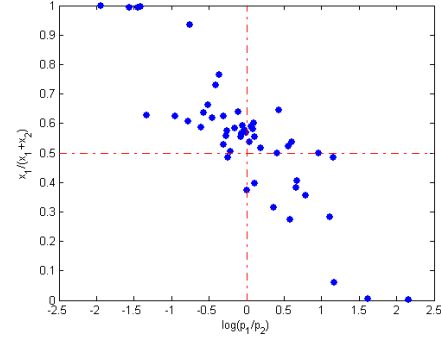
The Token Shares for Subject ID 67



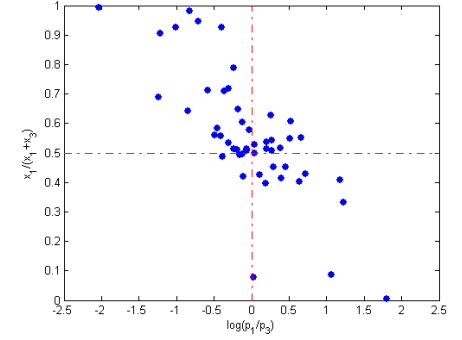
The Expenditure Shares for Subject ID 67



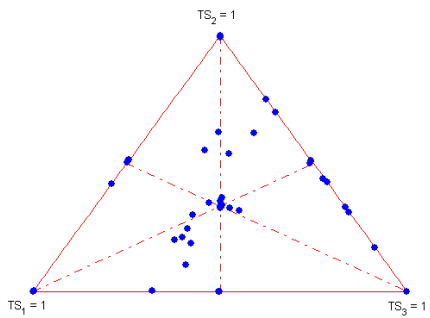
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 67



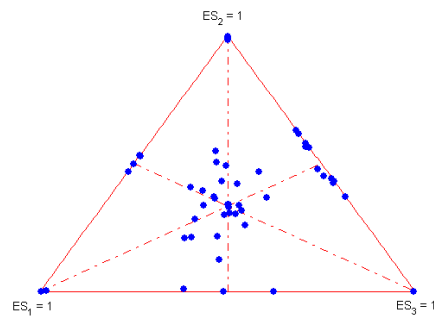
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 67



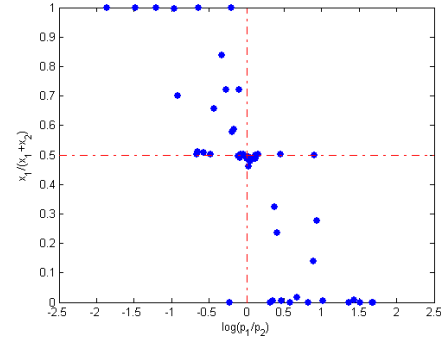
The Token Shares for Subject ID 68



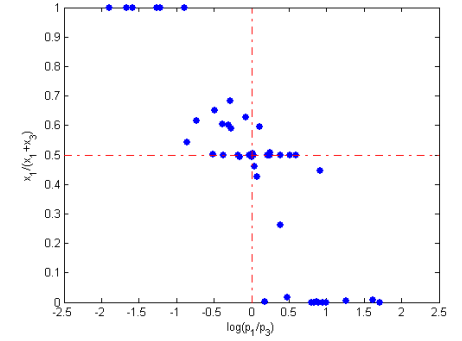
The Expenditure Shares for Subject ID 68



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 68



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_3)$  for Subject ID 68



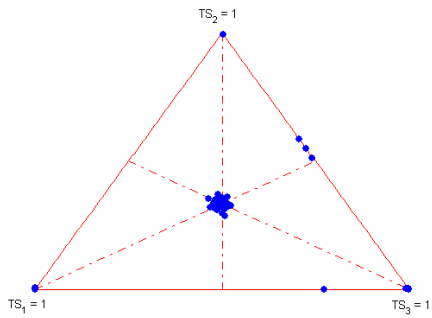




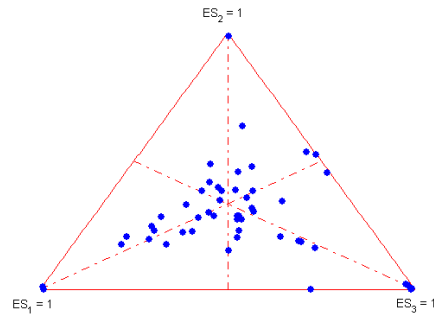




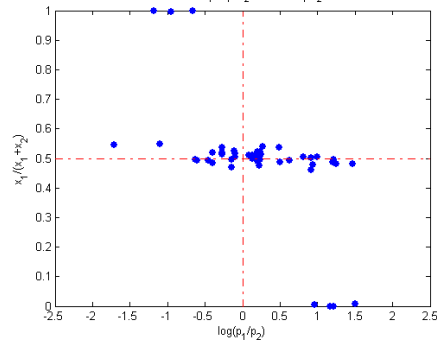
The Token Shares for Subject ID 81



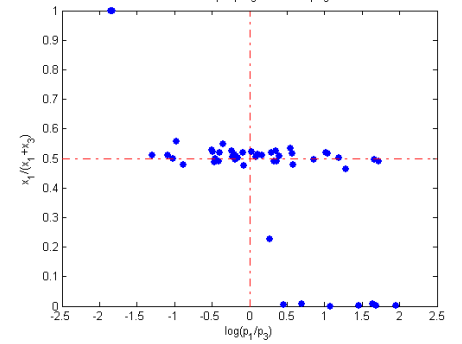
The Expenditure Shares for Subject ID 81



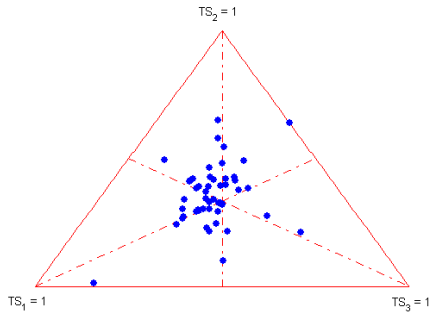
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 81



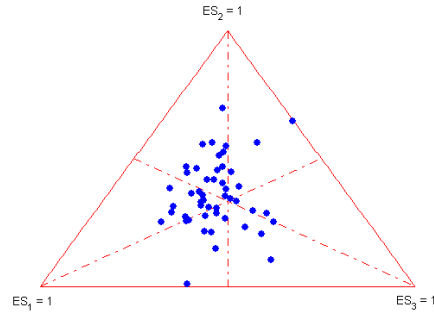
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 81



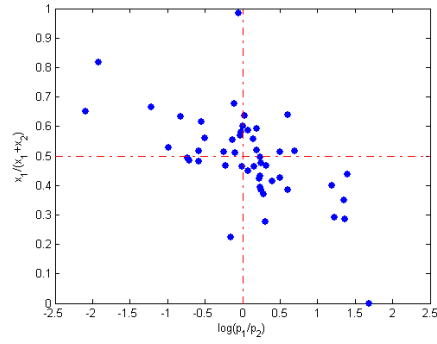
The Token Shares for Subject ID 82



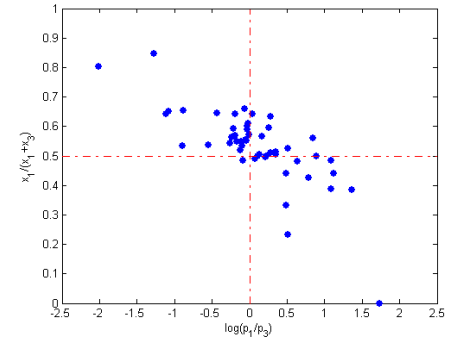
The Expenditure Shares for Subject ID 82



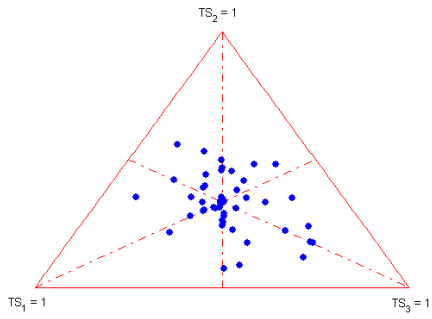
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 82



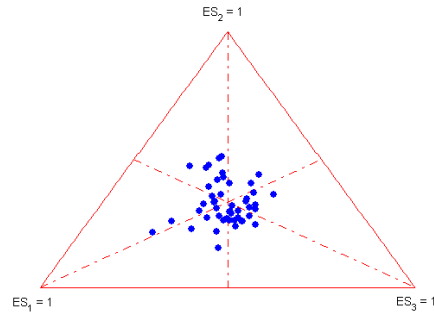
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 82



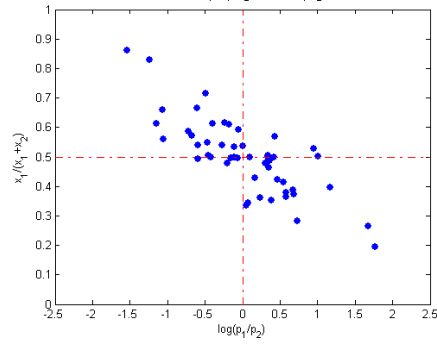
The Token Shares for Subject ID 83



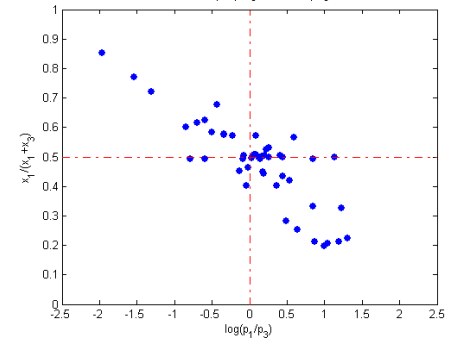
The Expenditure Shares for Subject ID 83



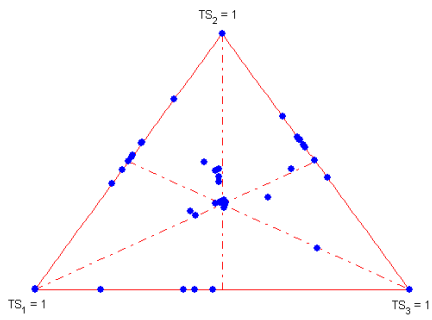
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 83



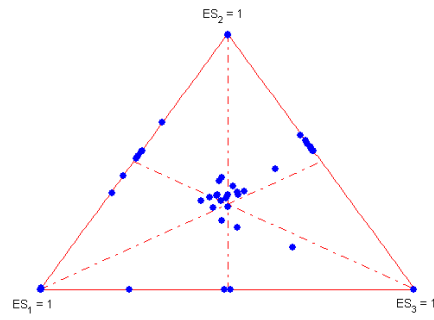
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 83



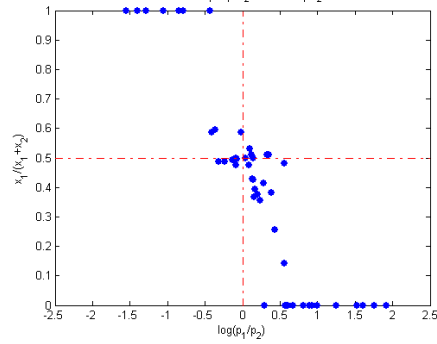
The Token Shares for Subject ID 84



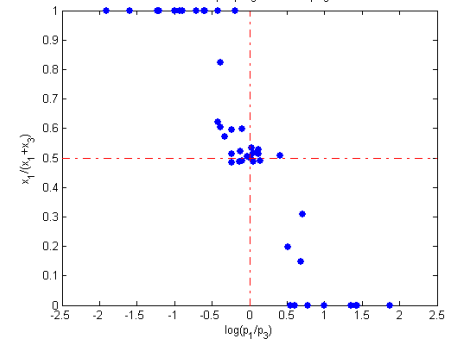
The Expenditure Shares for Subject ID 84



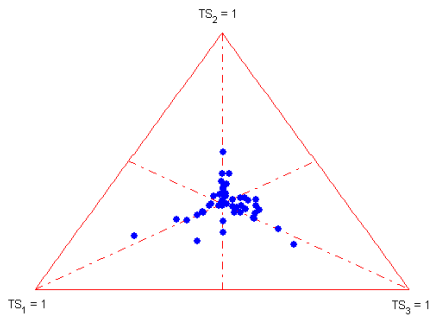
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 84



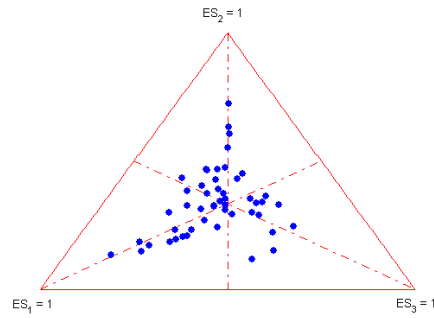
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 84



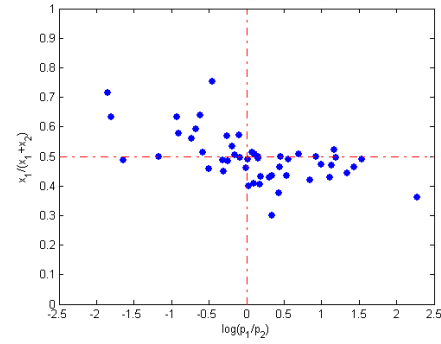
The Token Shares for Subject ID 85



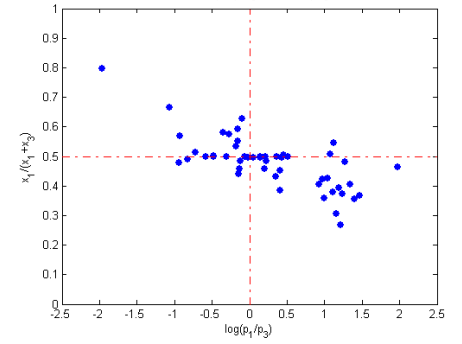
The Expenditure Shares for Subject ID 85



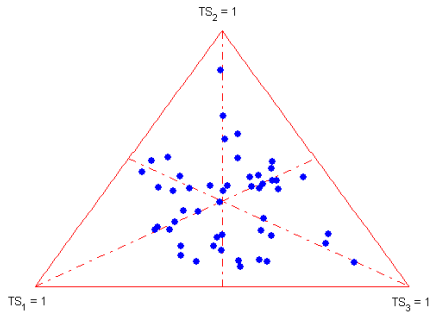
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 85



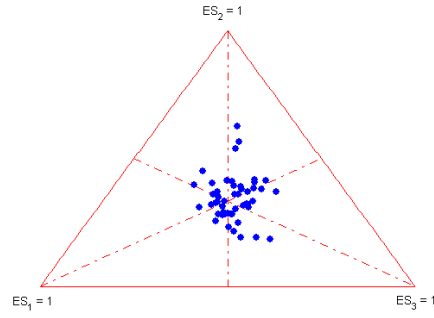
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 85



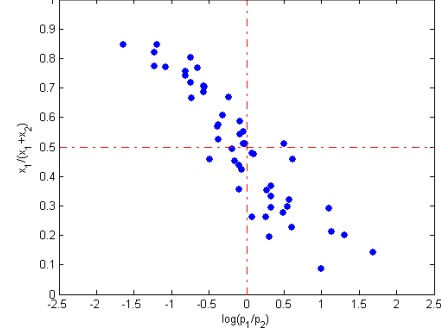
The Token Shares for Subject ID 86



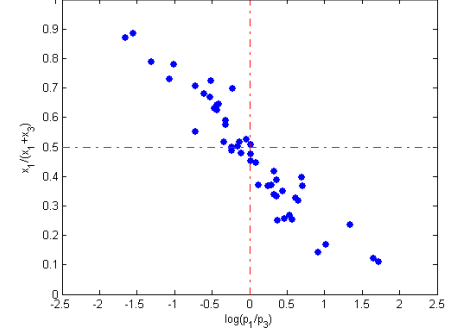
The Expenditure Shares for Subject ID 86



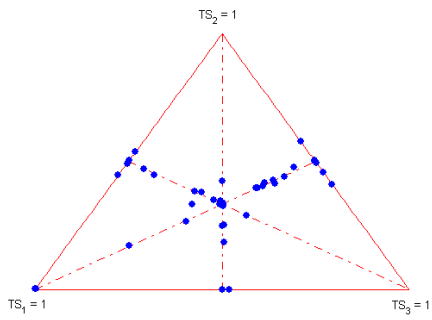
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 86



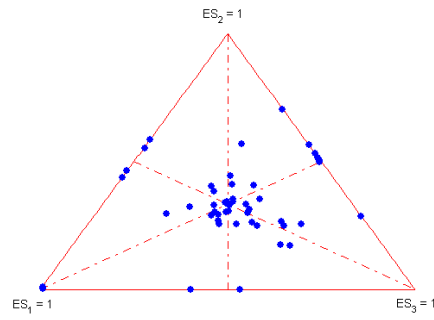
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 86



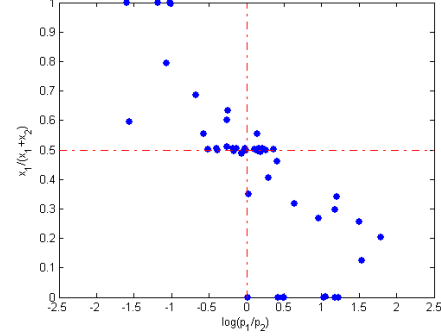
The Token Shares for Subject ID 87



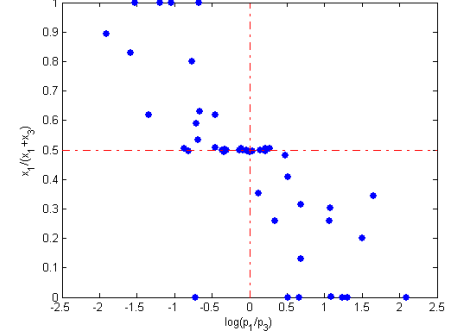
The Expenditure Shares for Subject ID 87



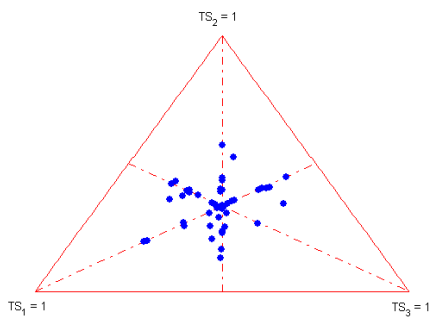
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 87



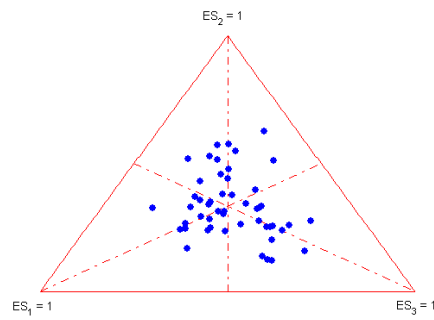
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 87



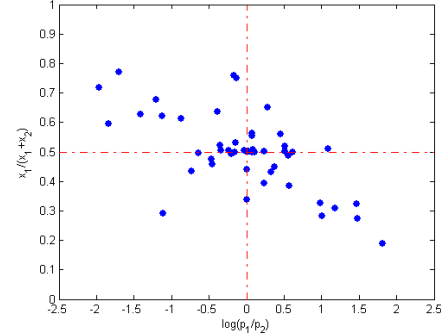
The Token Shares for Subject ID 88



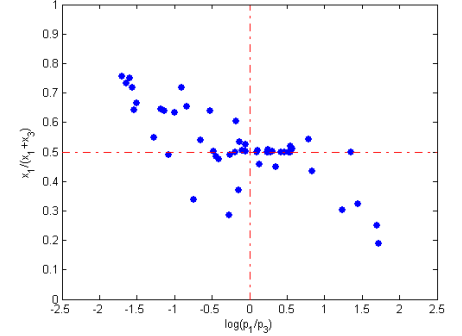
The Expenditure Shares for Subject ID 88



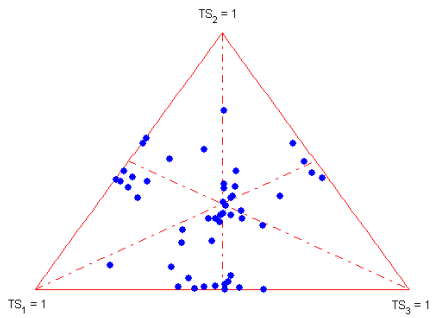
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 88



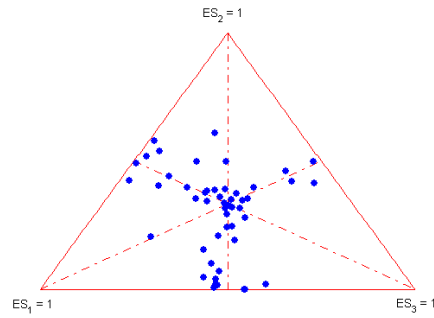
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 88



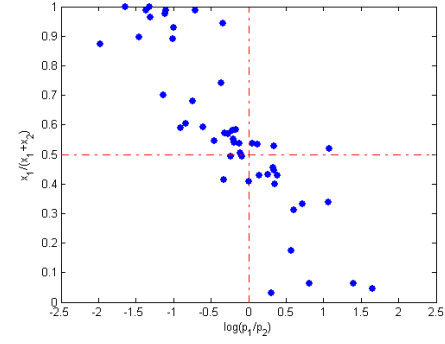
The Token Shares for Subject ID 89



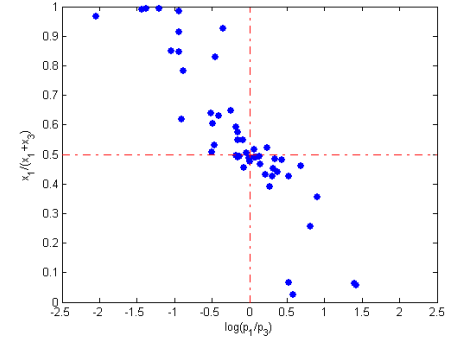
The Expenditure Shares for Subject ID 89



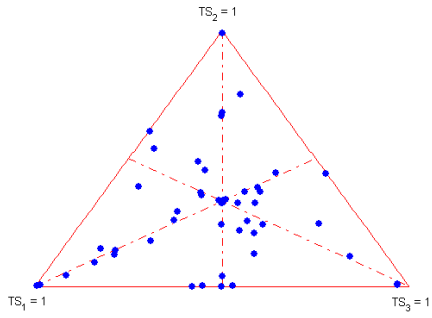
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 89



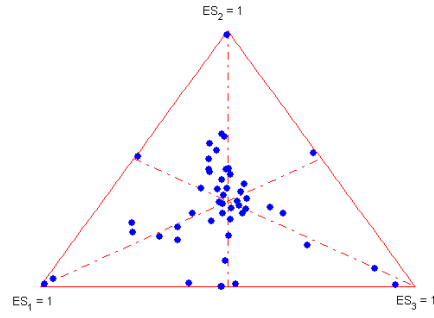
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 89



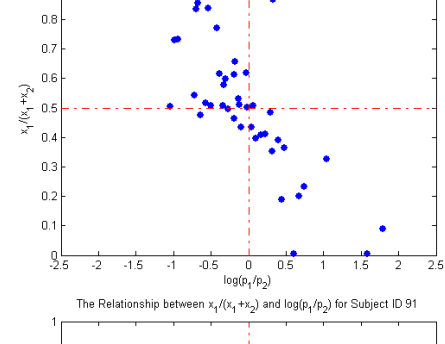
The Token Shares for Subject ID 90



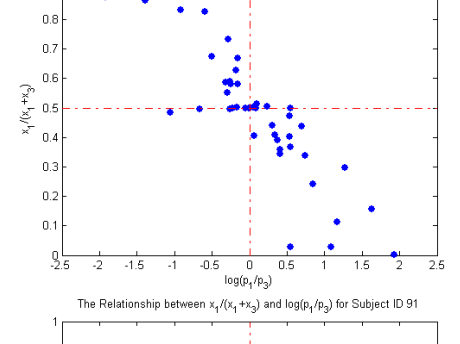
The Expenditure Shares for Subject ID 90



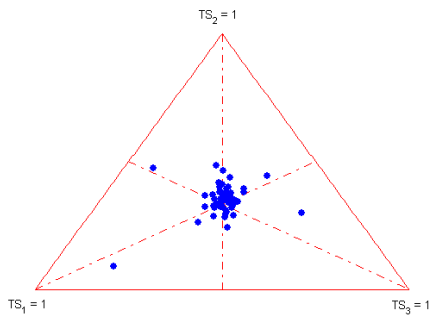
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 90



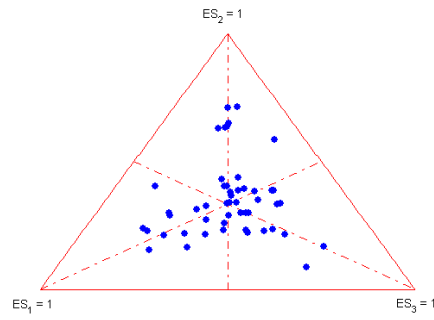
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 90



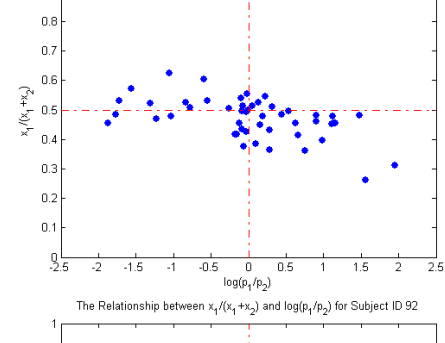
The Token Shares for Subject ID 91



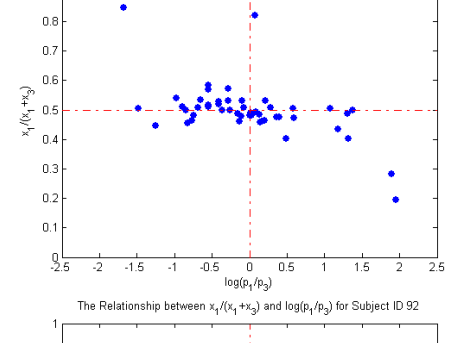
The Expenditure Shares for Subject ID 91



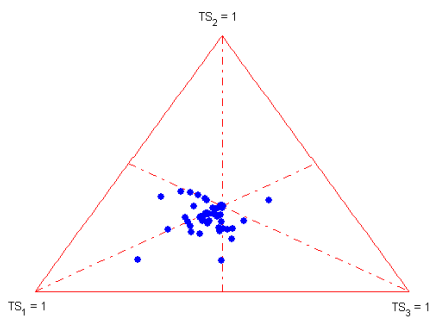
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 91



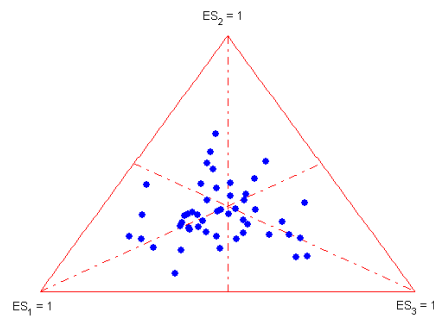
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 91



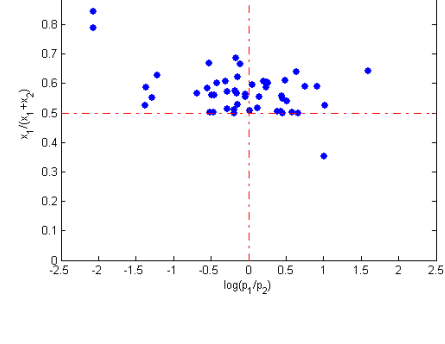
The Token Shares for Subject ID 92



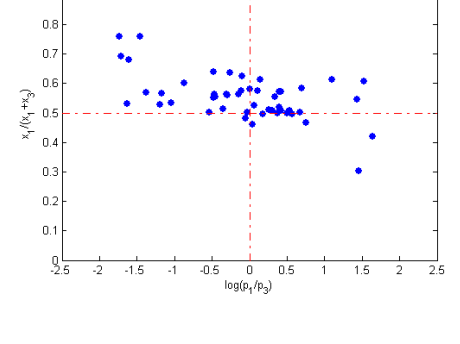
The Expenditure Shares for Subject ID 92



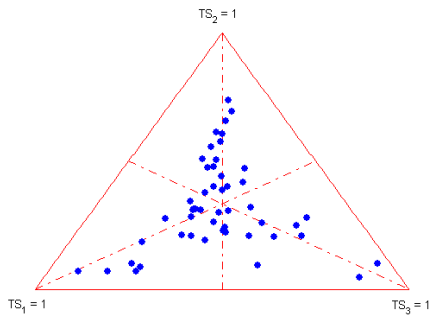
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 92



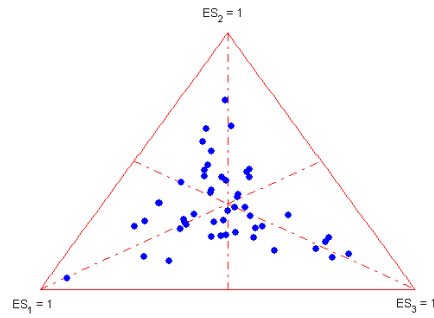
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 92



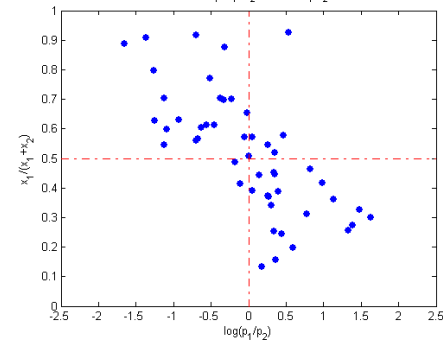
The Token Shares for Subject ID 93



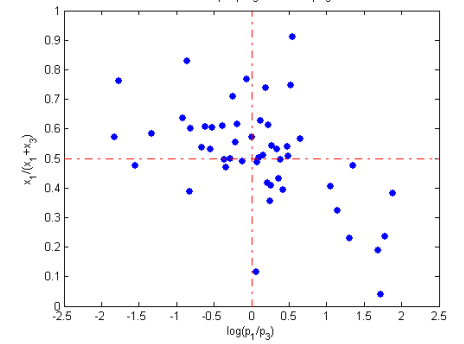
The Expenditure Shares for Subject ID 93



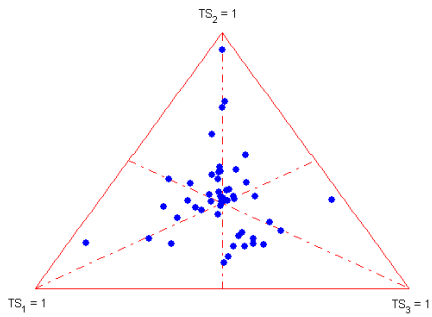
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 93



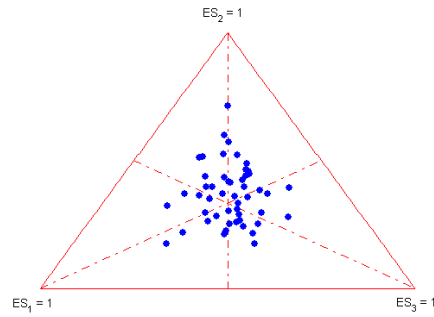
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 93



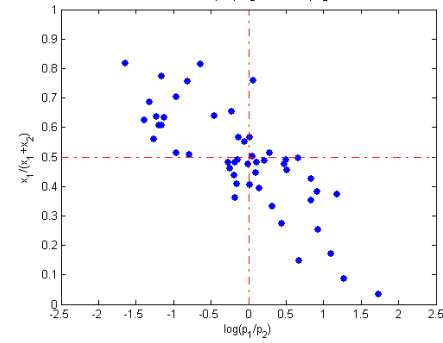
The Token Shares for Subject ID 94



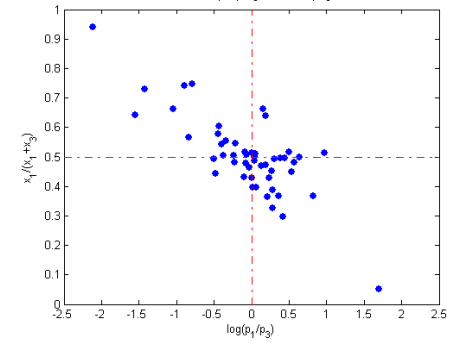
The Expenditure Shares for Subject ID 94



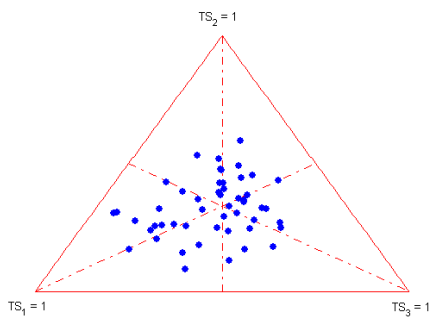
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 94



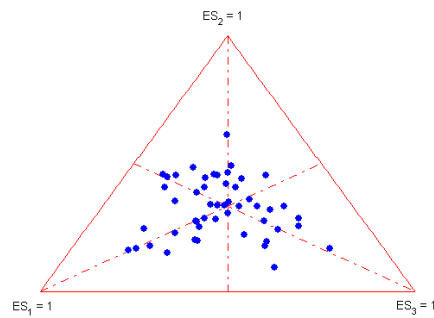
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 94



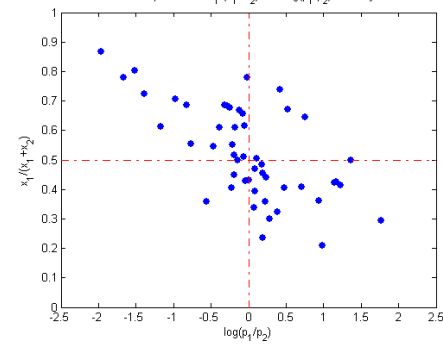
The Token Shares for Subject ID 95



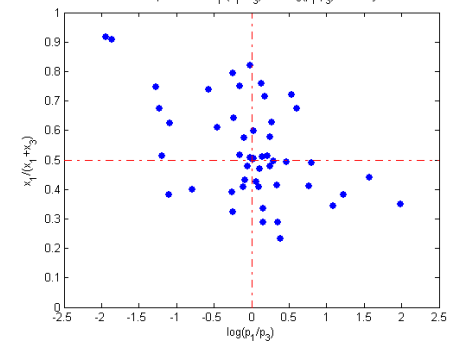
The Expenditure Shares for Subject ID 95



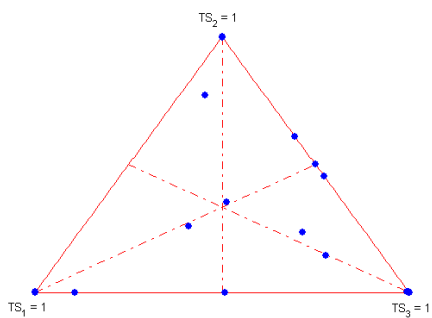
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 95



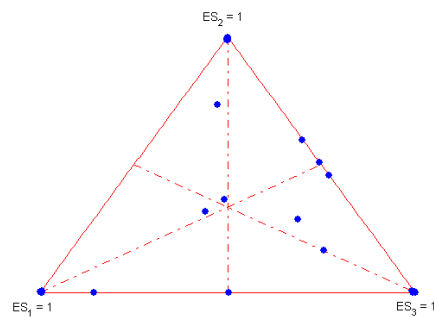
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 95



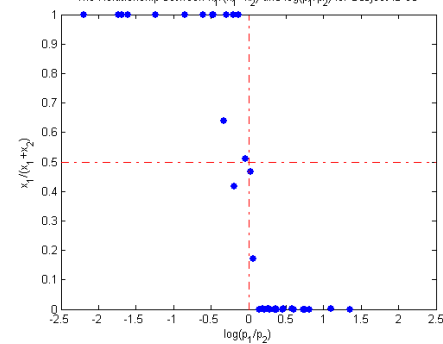
The Token Shares for Subject ID 96



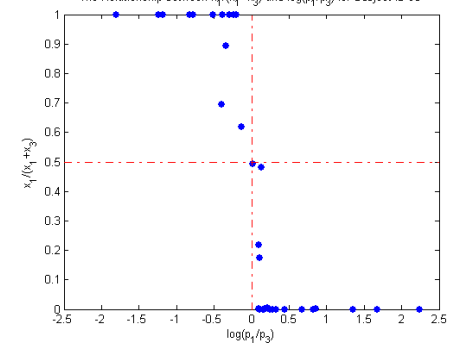
The Expenditure Shares for Subject ID 96



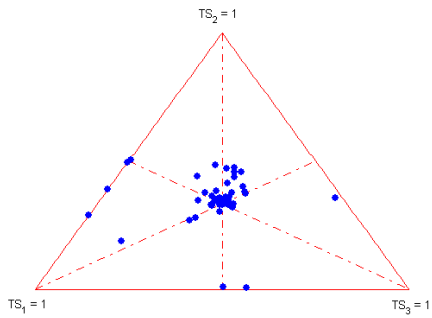
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 96



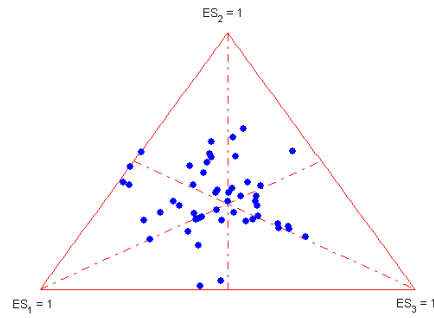
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 96



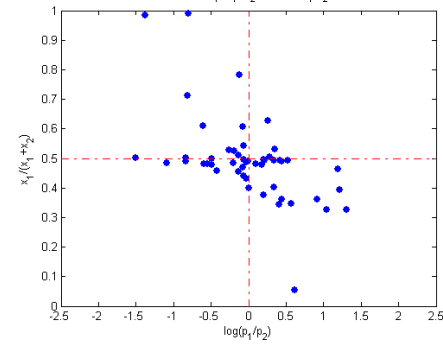
The Token Shares for Subject ID 97



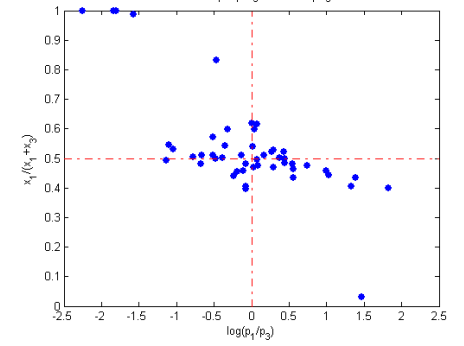
The Expenditure Shares for Subject ID 97



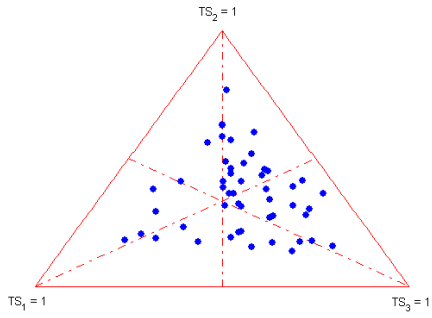
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 97



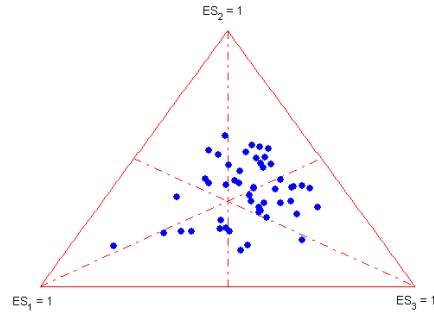
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 97



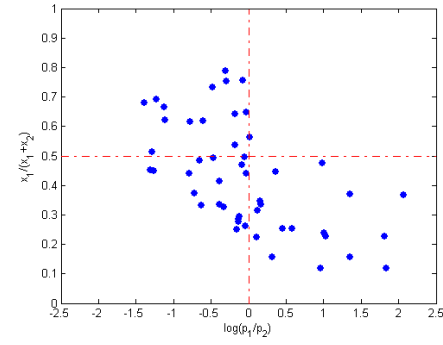
The Token Shares for Subject ID 98



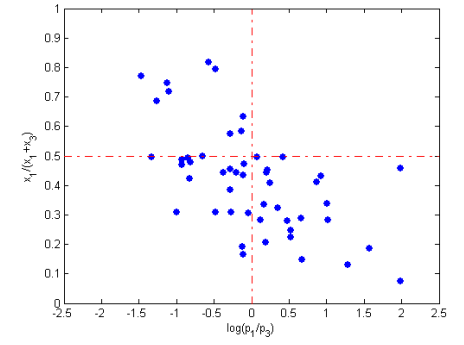
The Expenditure Shares for Subject ID 98



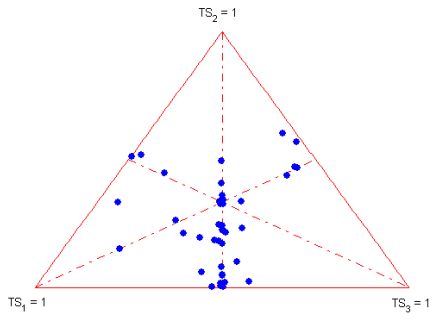
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 98



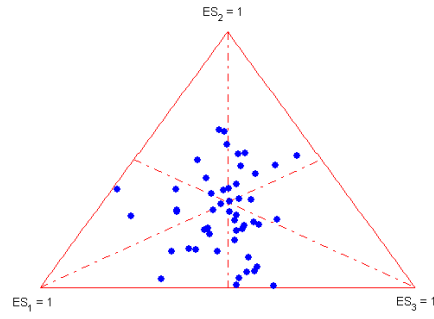
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 98



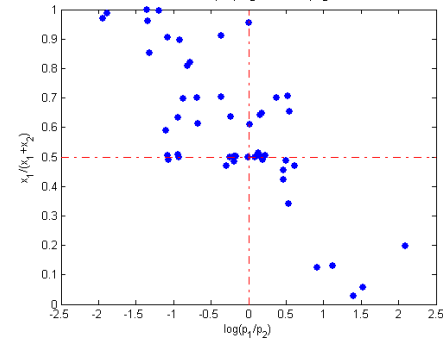
The Token Shares for Subject ID 99



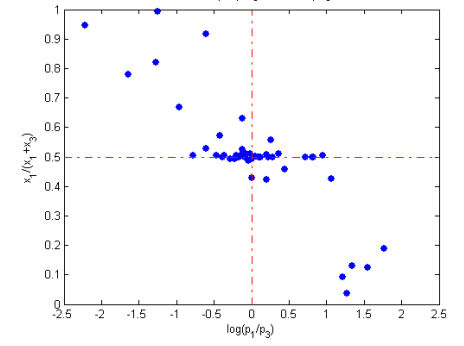
The Expenditure Shares for Subject ID 99



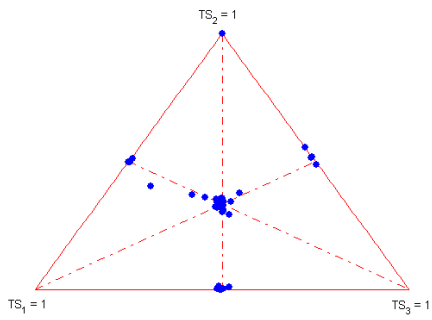
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 99



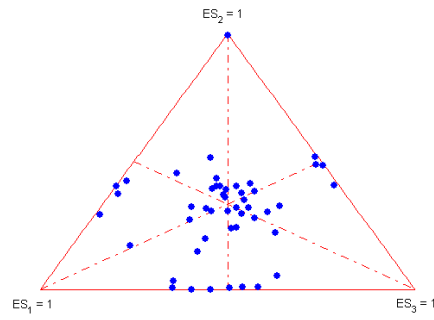
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 99



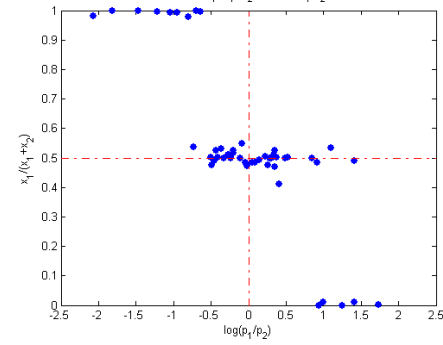
The Token Shares for Subject ID 100



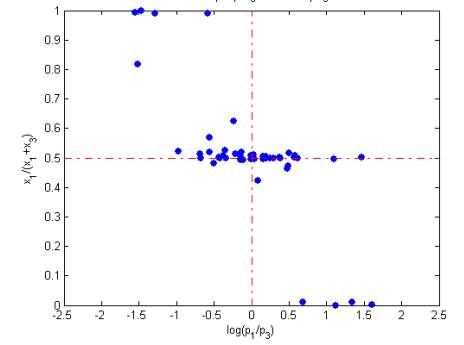
The Expenditure Shares for Subject ID 100



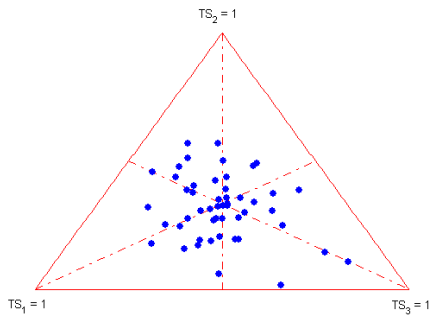
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 100



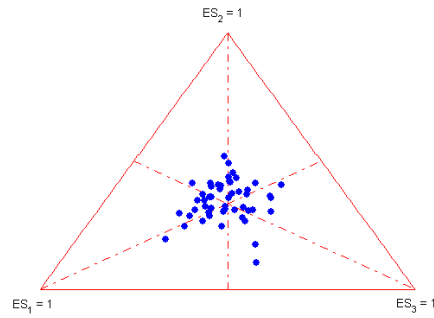
The Relationship between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  for Subject ID 100



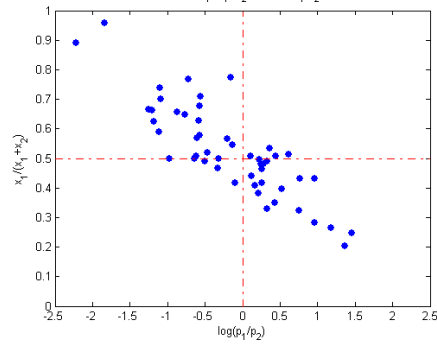
The Token Shares for Subject ID 101



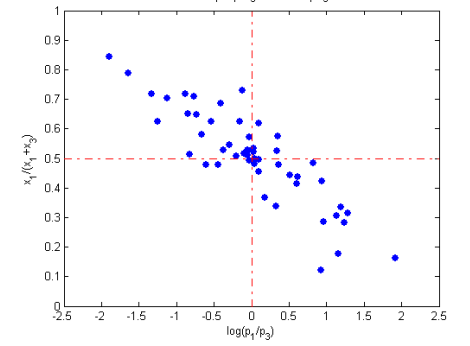
The Expenditure Shares for Subject ID 101



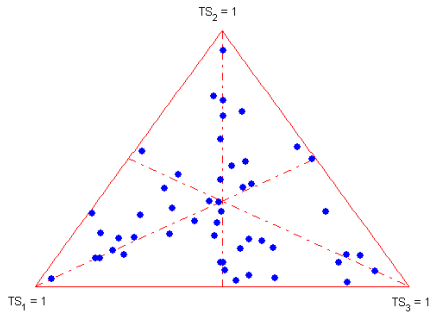
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 101



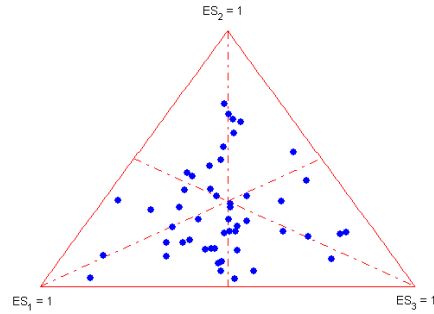
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 101



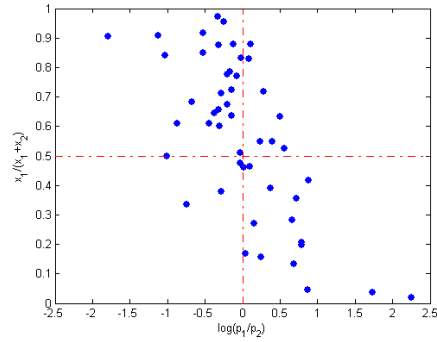
The Token Shares for Subject ID 102



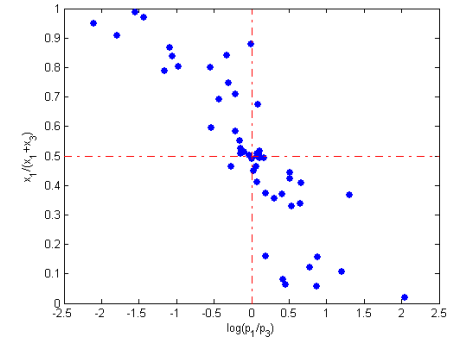
The Expenditure Shares for Subject ID 102



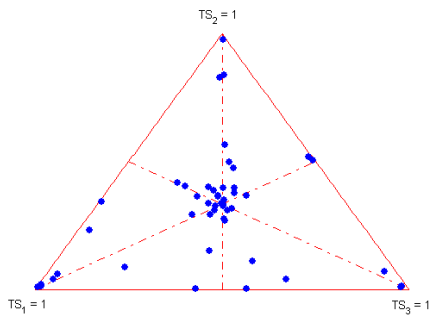
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 102



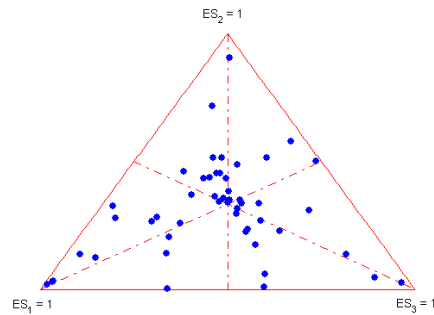
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 102



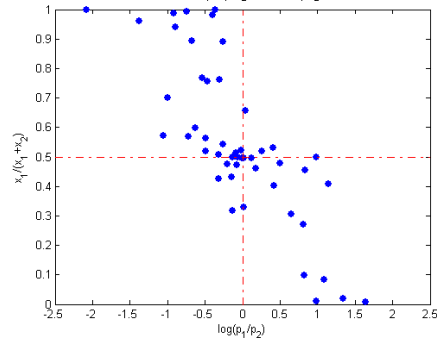
The Token Shares for Subject ID 103



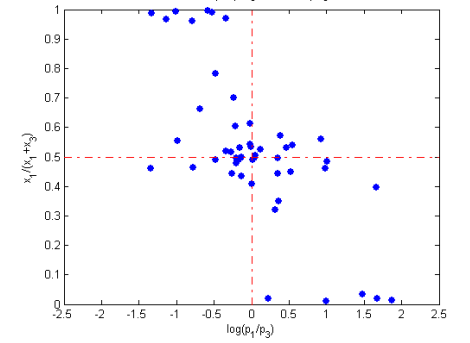
The Expenditure Shares for Subject ID 103



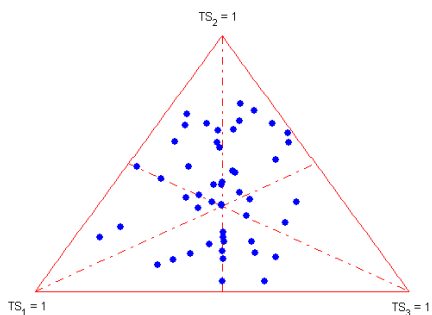
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 103



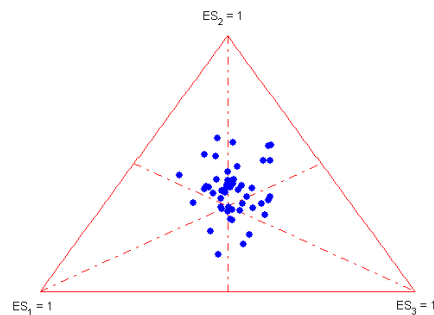
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 103



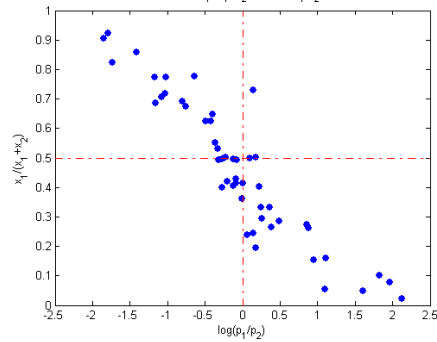
The Token Shares for Subject ID 104



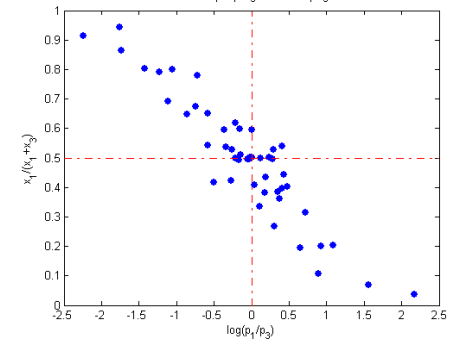
The Expenditure Shares for Subject ID 104



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 104

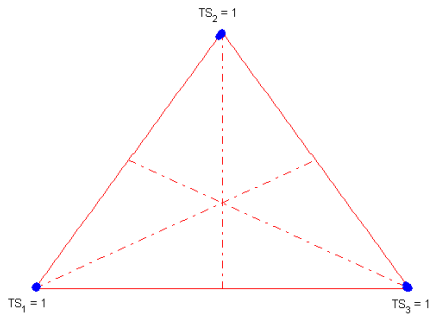


The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 104

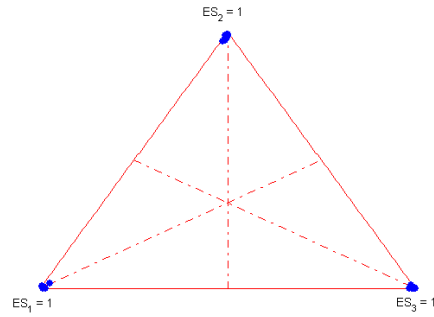




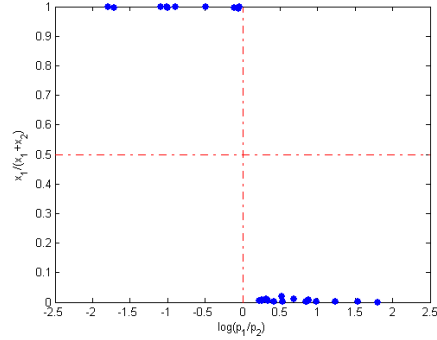
The Token Shares for Subject ID 105



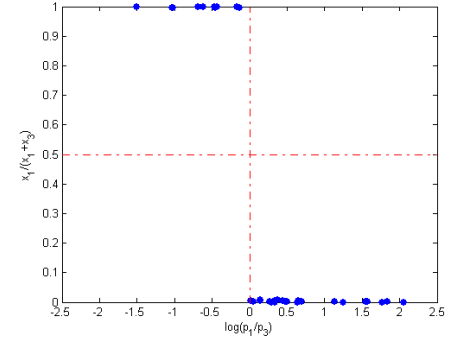
The Expenditure Shares for Subject ID 105



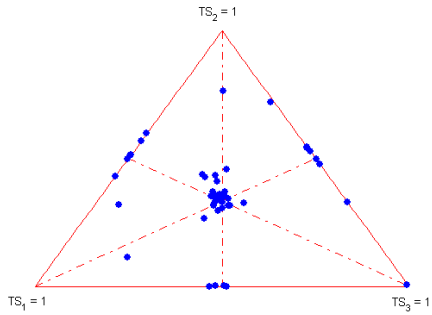
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 105



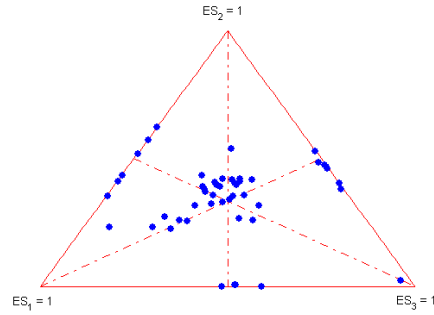
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 105



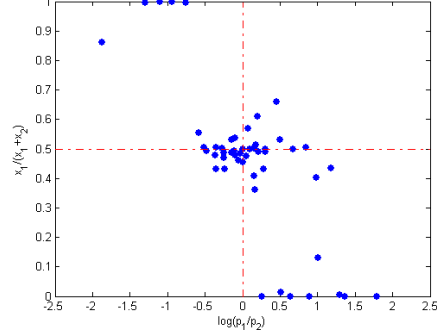
The Token Shares for Subject ID 106



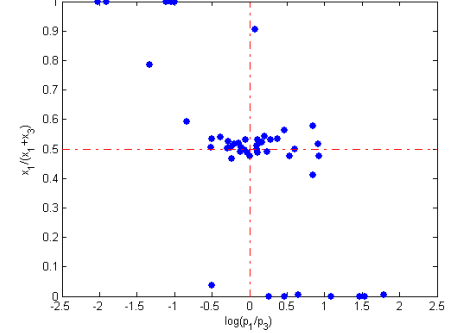
The Expenditure Shares for Subject ID 106



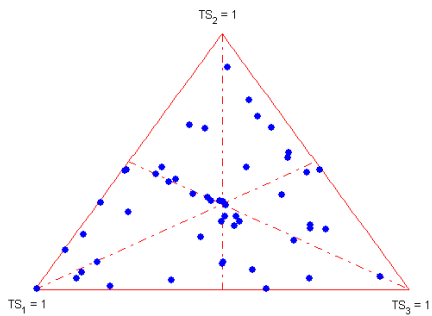
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 106



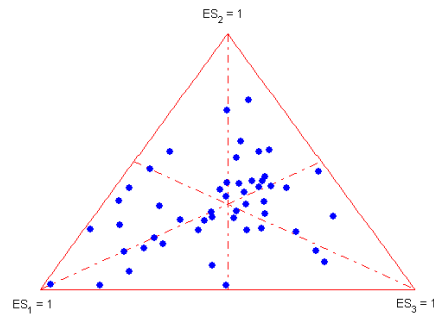
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 106



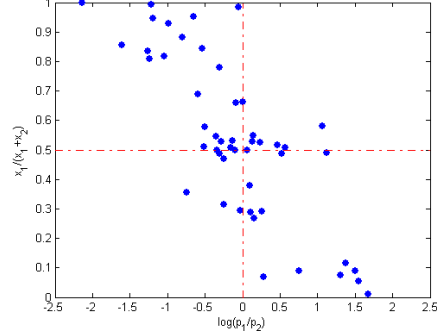
The Token Shares for Subject ID 107



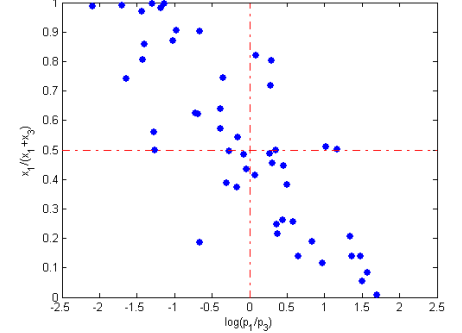
The Expenditure Shares for Subject ID 107



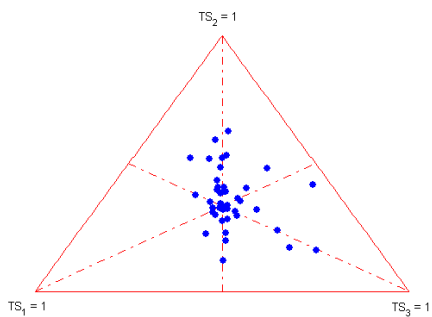
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 107



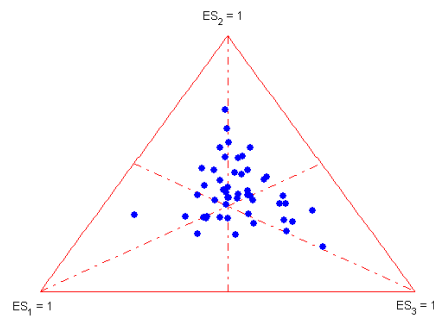
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 107



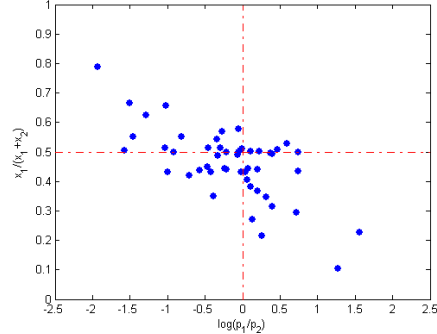
The Token Shares for Subject ID 108



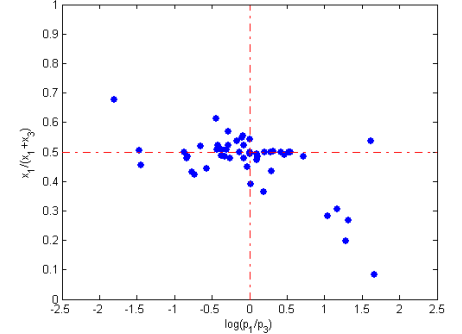
The Expenditure Shares for Subject ID 108



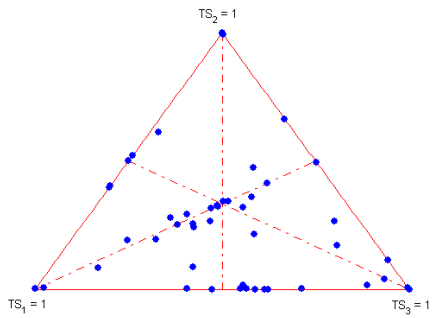
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 108



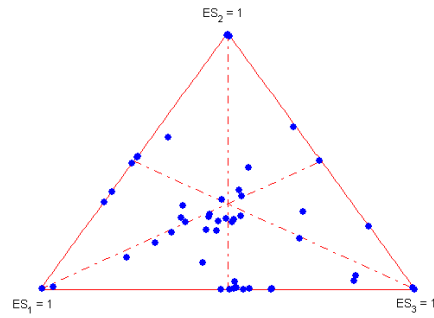
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 108



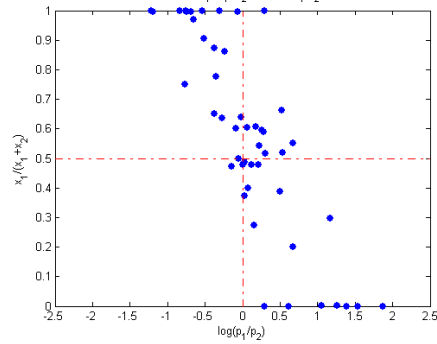
The Token Shares for Subject ID 109



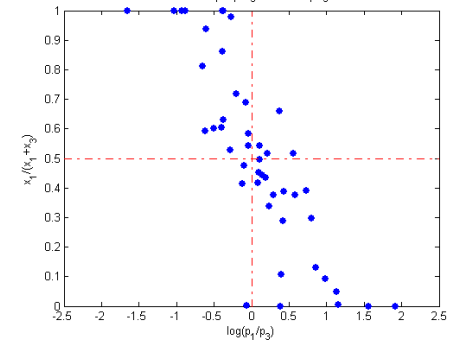
The Expenditure Shares for Subject ID 109



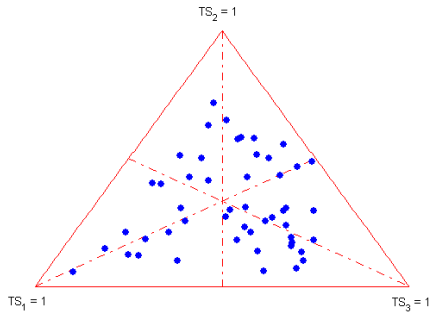
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 109



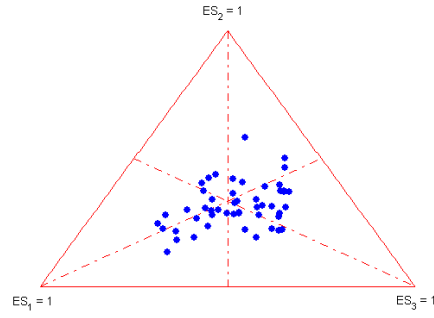
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 109



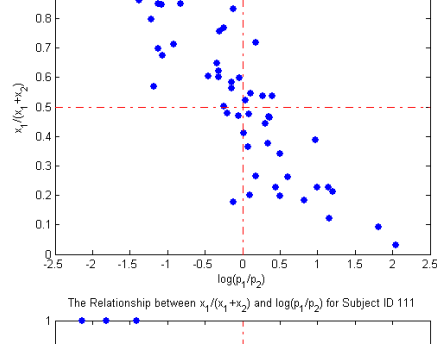
The Token Shares for Subject ID 110



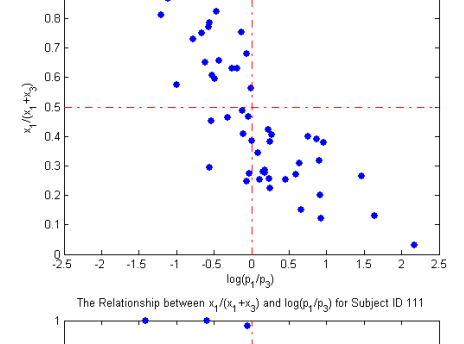
The Expenditure Shares for Subject ID 110



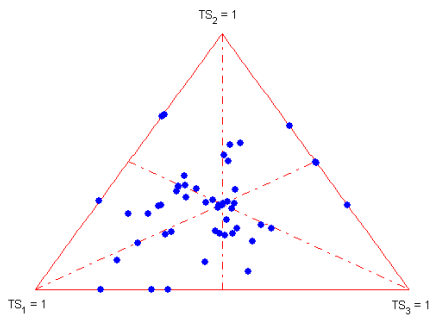
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 110



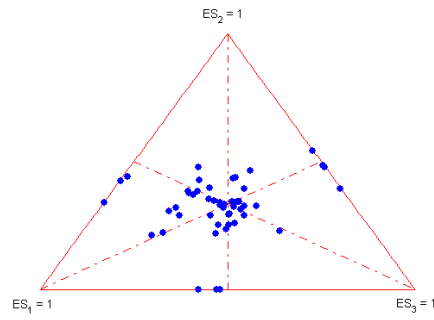
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 110



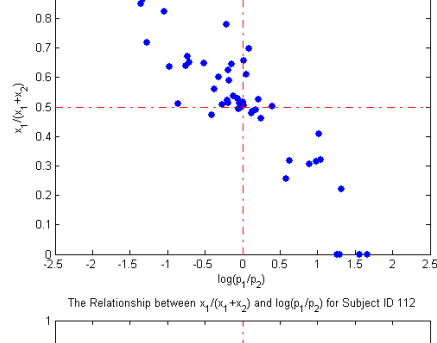
The Token Shares for Subject ID 111



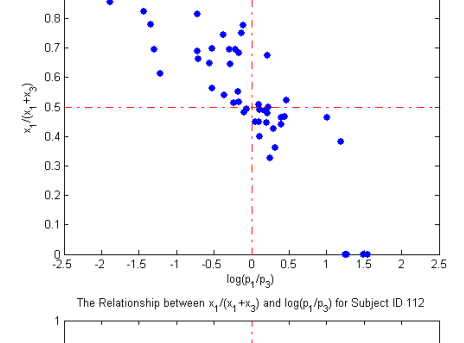
The Expenditure Shares for Subject ID 111



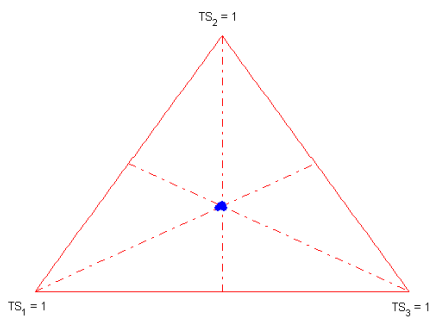
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 111



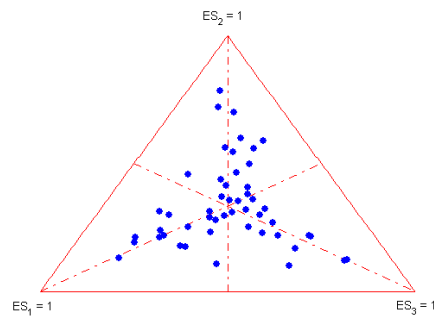
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 111



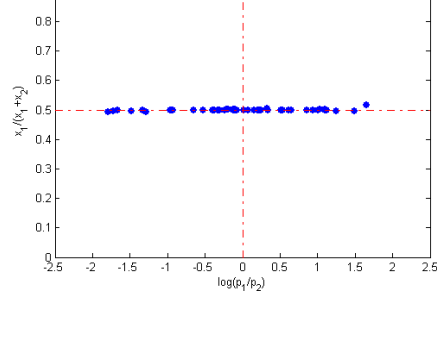
The Token Shares for Subject ID 112



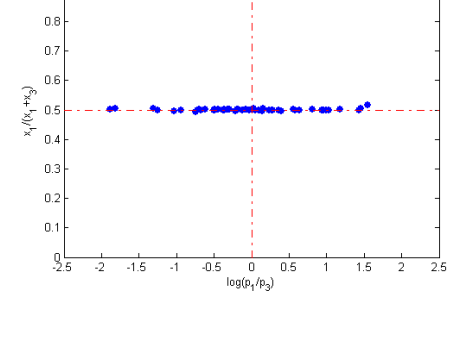
The Expenditure Shares for Subject ID 112



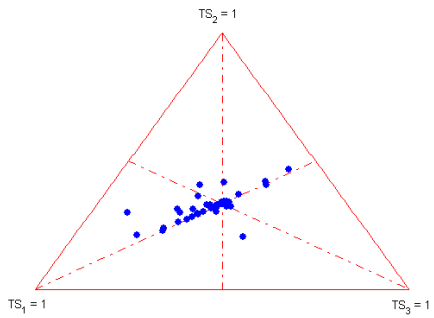
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 112



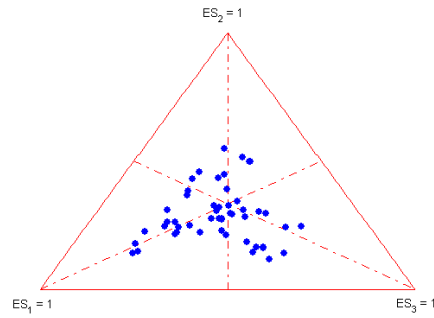
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 112



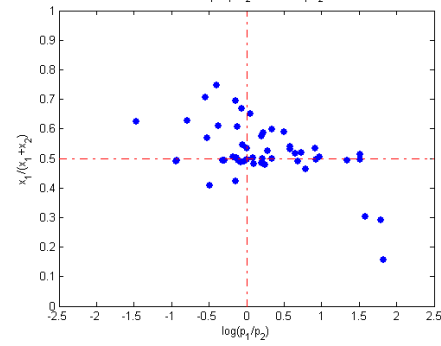
The Token Shares for Subject ID 113



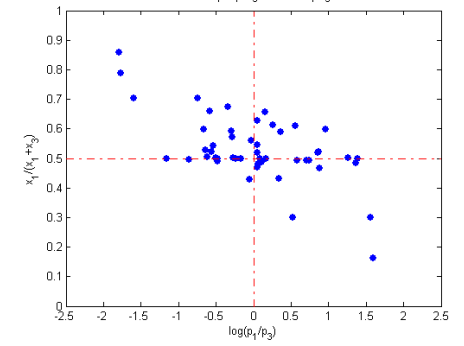
The Expenditure Shares for Subject ID 113



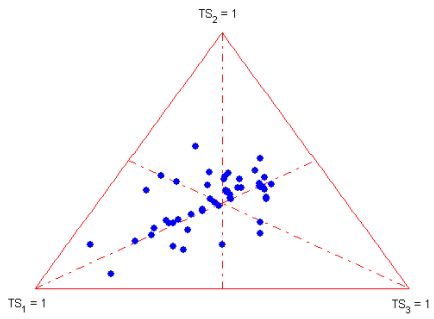
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 113



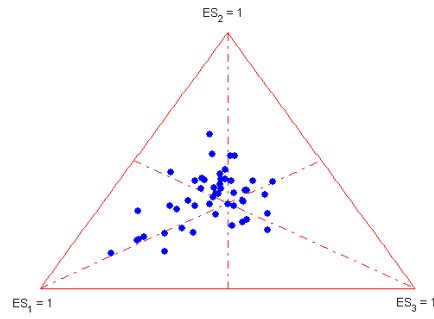
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 113



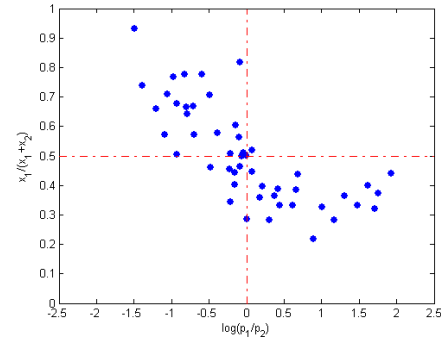
The Token Shares for Subject ID 114



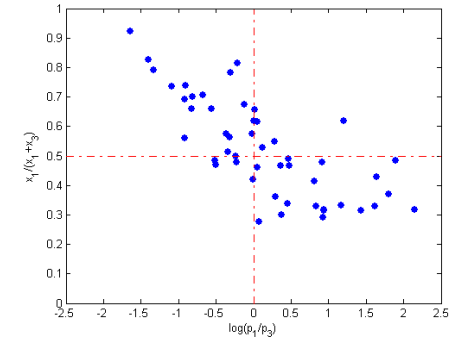
The Expenditure Shares for Subject ID 114



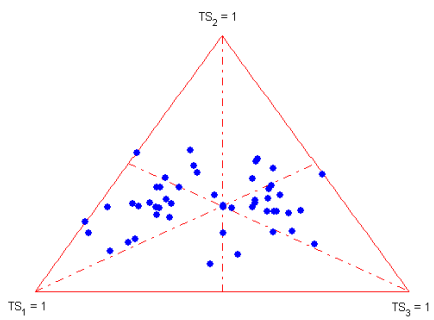
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 114



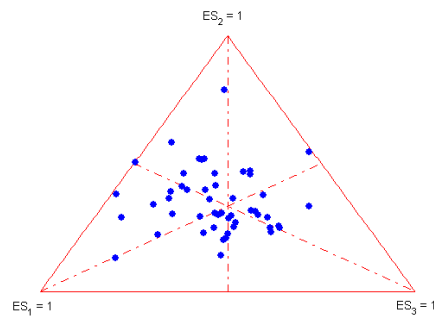
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 114



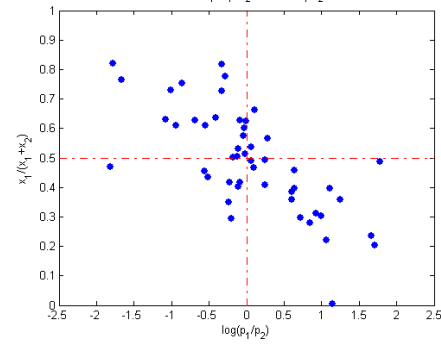
The Token Shares for Subject ID 115



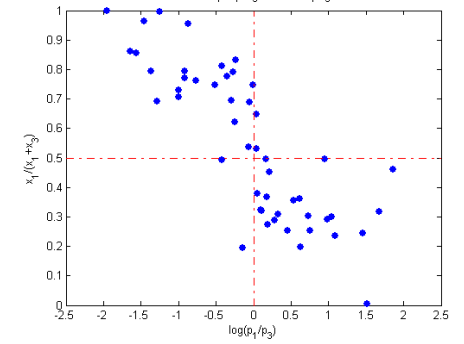
The Expenditure Shares for Subject ID 115



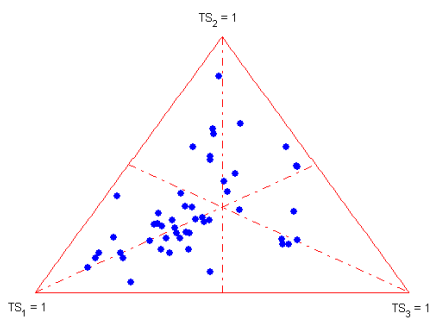
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 115



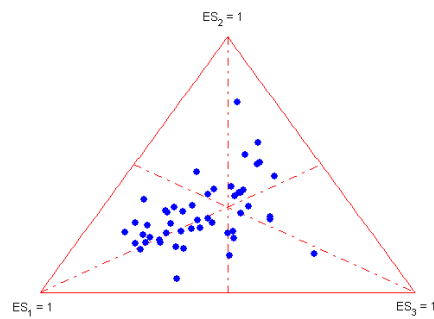
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 115



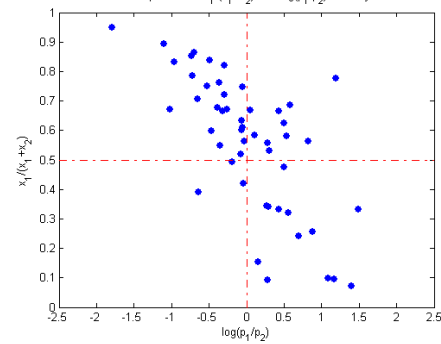
The Token Shares for Subject ID 116



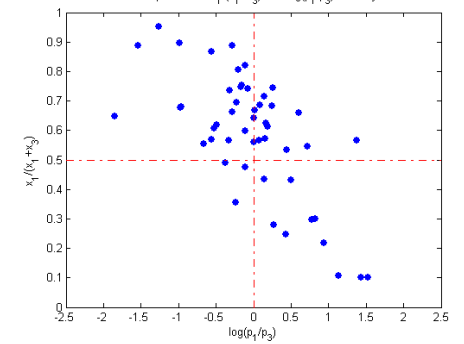
The Expenditure Shares for Subject ID 116



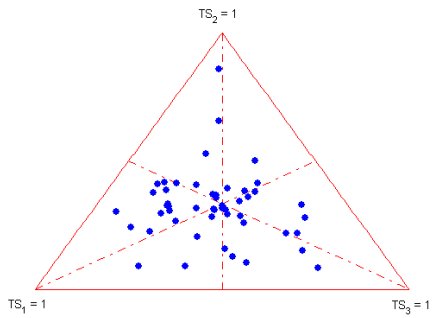
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 116



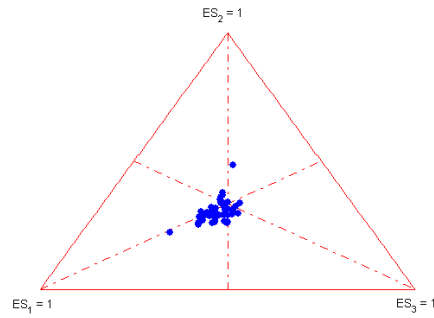
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 116



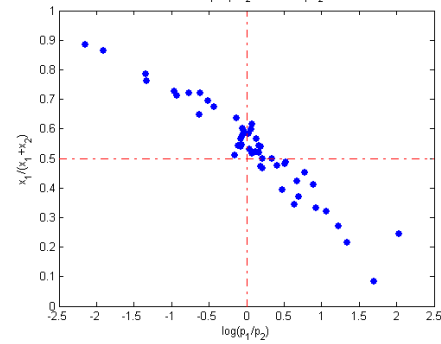
The Token Shares for Subject ID 117



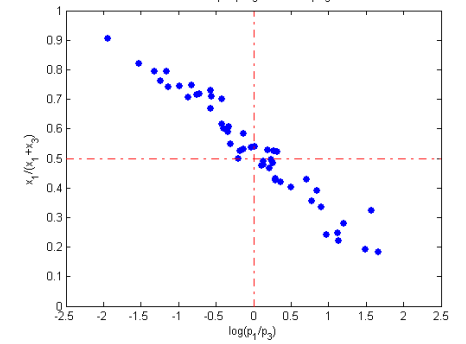
The Expenditure Shares for Subject ID 117



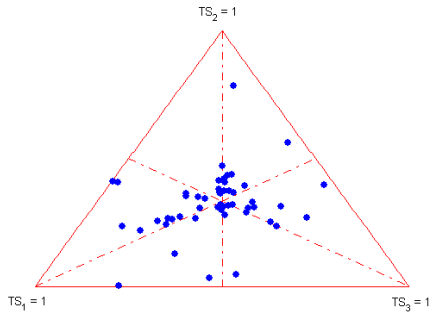
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 117



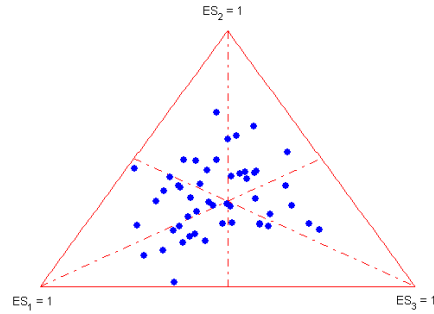
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 117



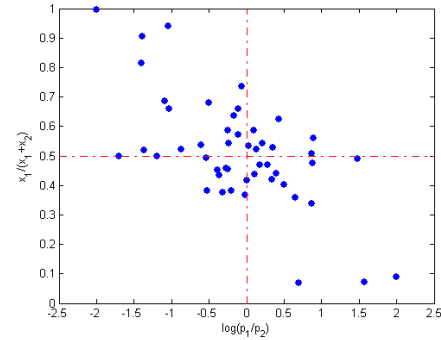
The Token Shares for Subject ID 118



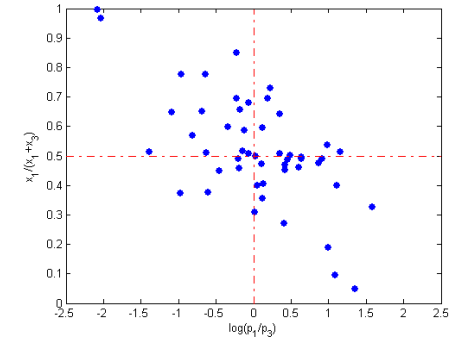
The Expenditure Shares for Subject ID 118



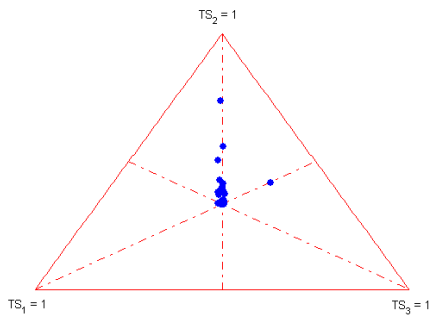
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 118



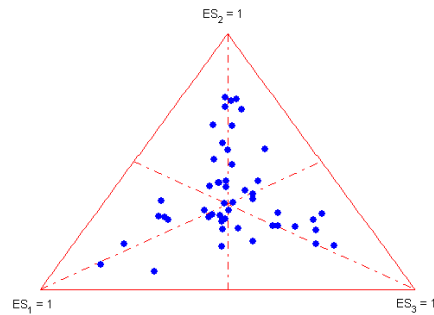
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 118



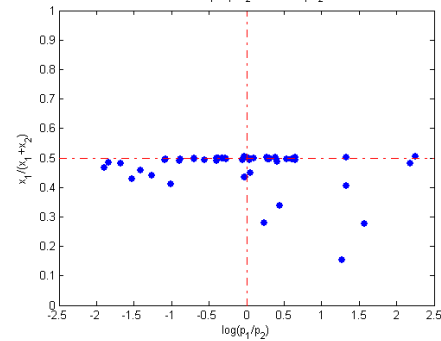
The Token Shares for Subject ID 119



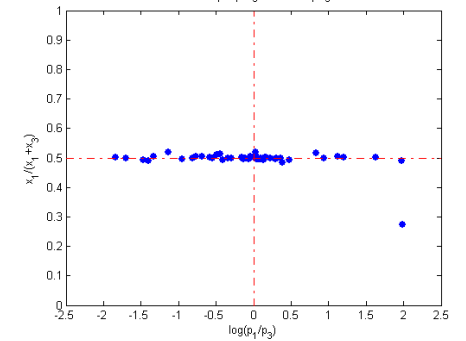
The Expenditure Shares for Subject ID 119



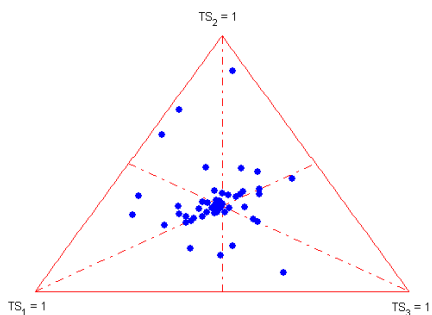
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 119



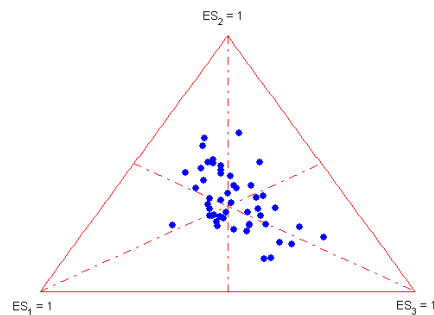
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 119



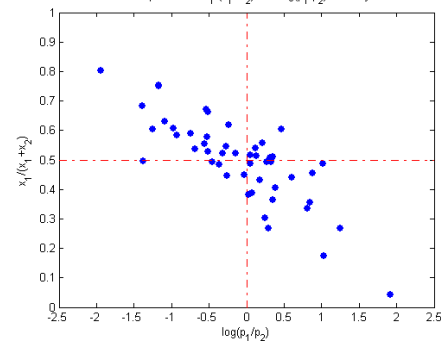
The Token Shares for Subject ID 120



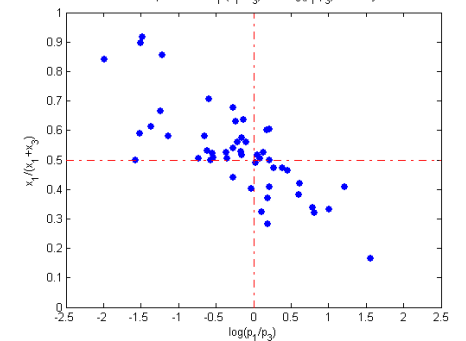
The Expenditure Shares for Subject ID 120



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 120

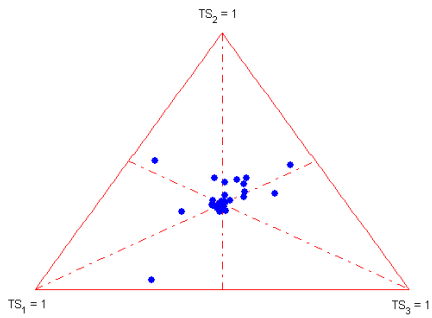


The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 120

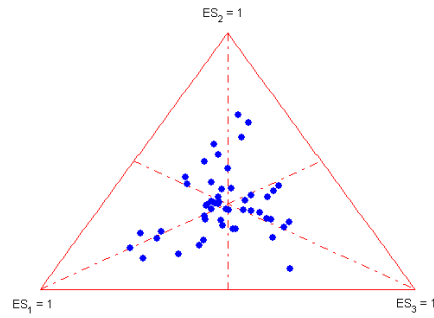




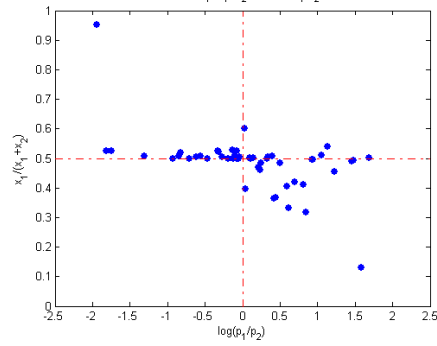
The Token Shares for Subject ID 125



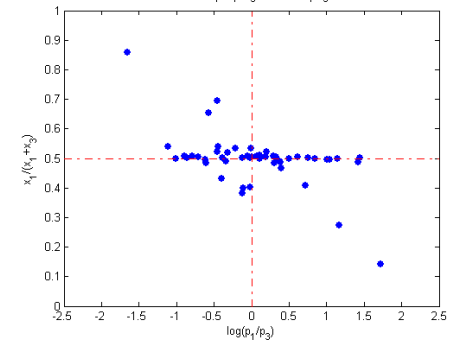
The Expenditure Shares for Subject ID 125



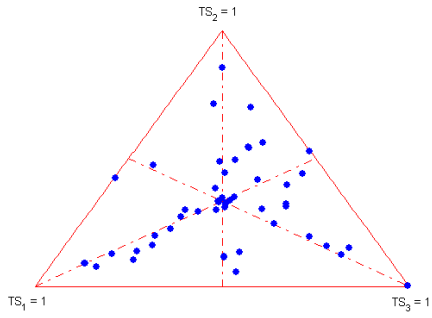
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 125



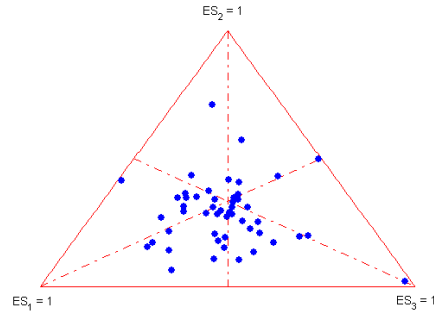
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 125



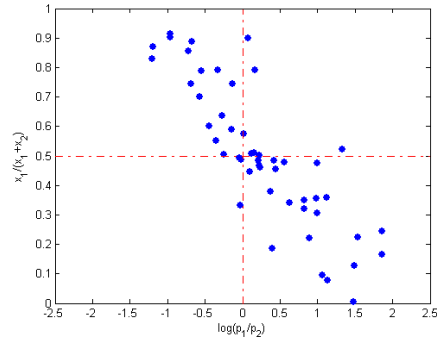
The Token Shares for Subject ID 126



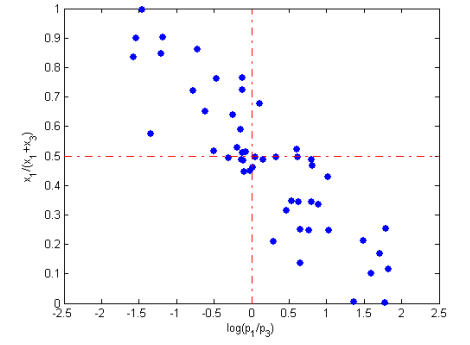
The Expenditure Shares for Subject ID 126



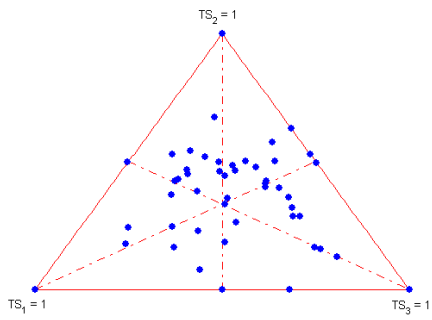
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 126



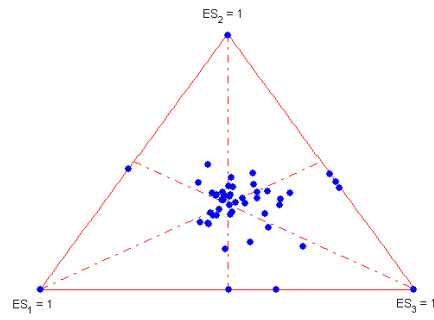
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 126



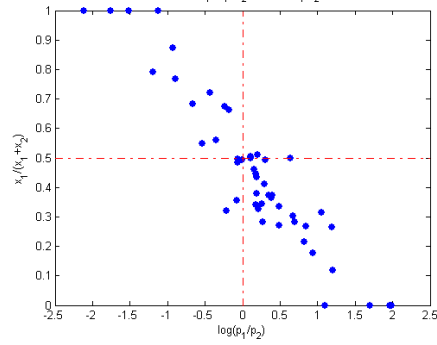
The Token Shares for Subject ID 127



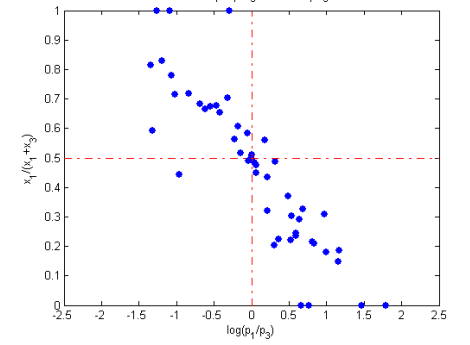
The Expenditure Shares for Subject ID 127



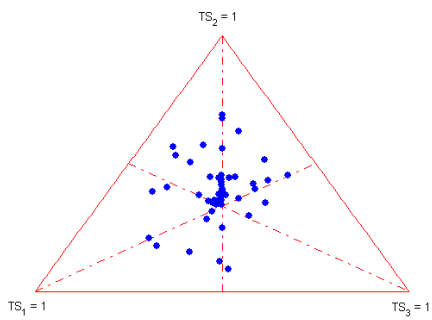
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 127



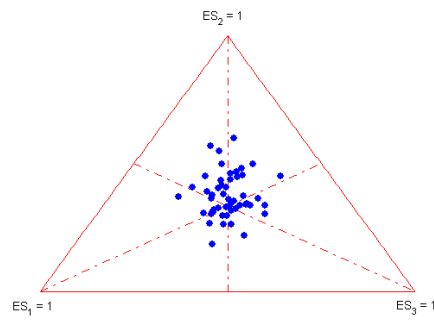
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 127



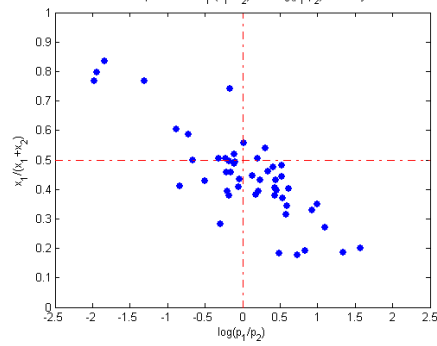
The Token Shares for Subject ID 128



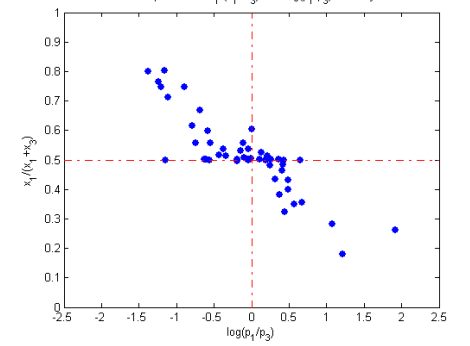
The Expenditure Shares for Subject ID 128



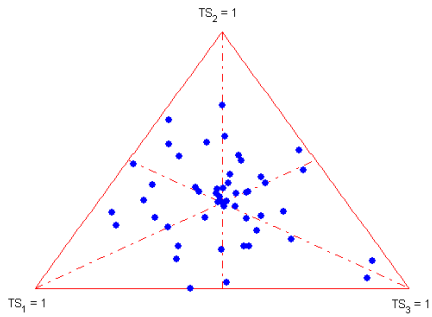
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 128



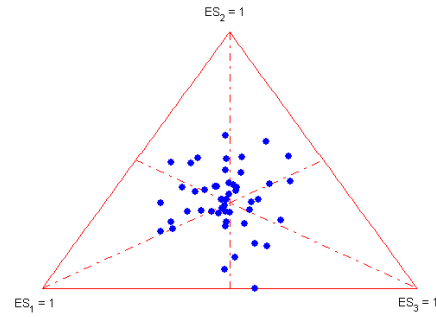
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 128



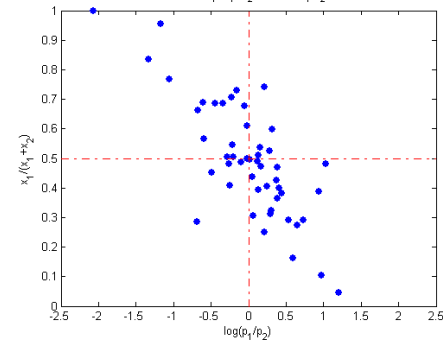
The Token Shares for Subject ID 129



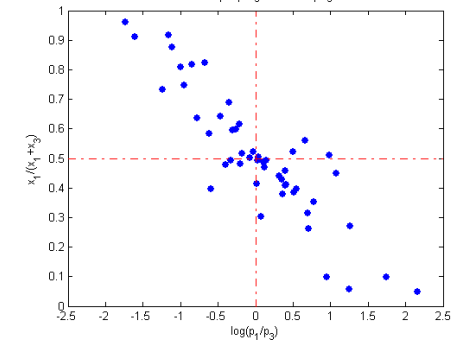
The Expenditure Shares for Subject ID 129



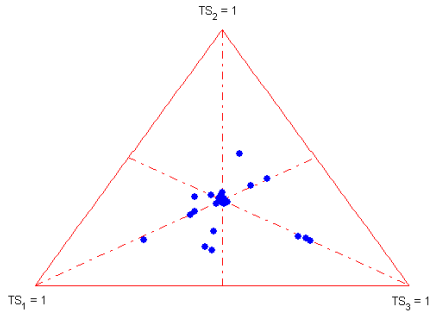
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 129



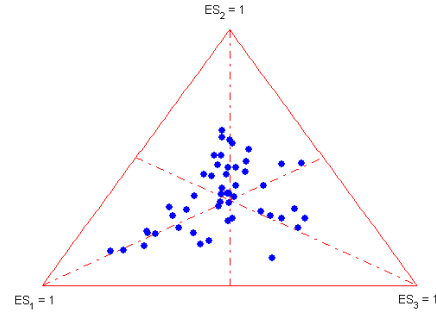
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 129



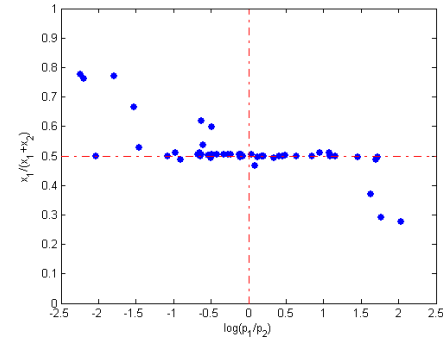
The Token Shares for Subject ID 130



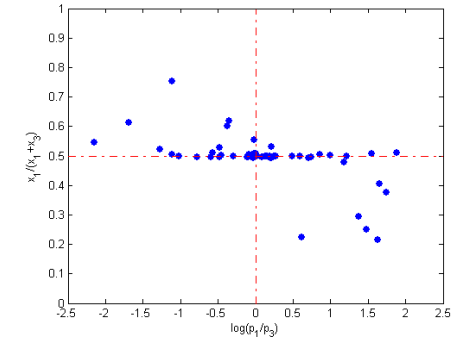
The Expenditure Shares for Subject ID 130



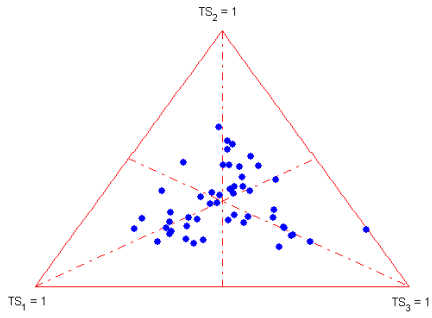
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 130



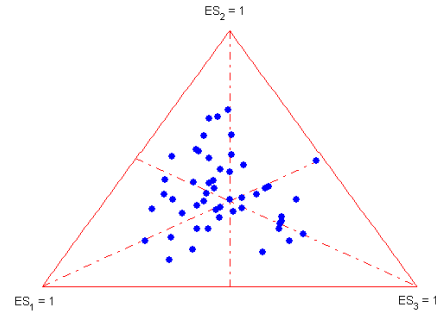
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 130



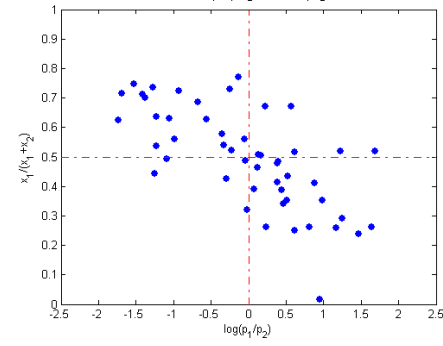
The Token Shares for Subject ID 131



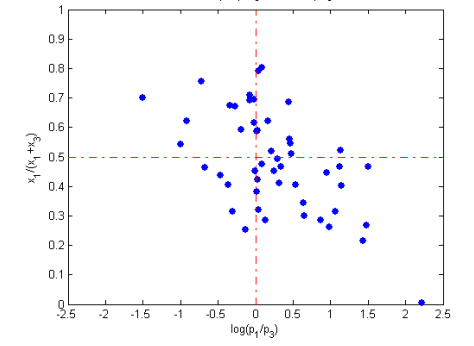
The Expenditure Shares for Subject ID 131



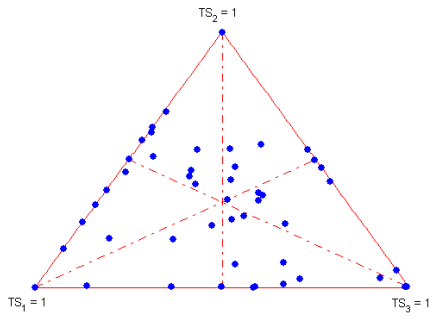
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 131



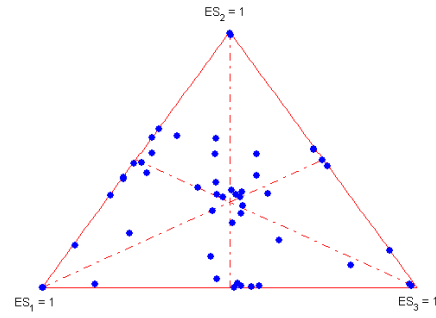
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 131



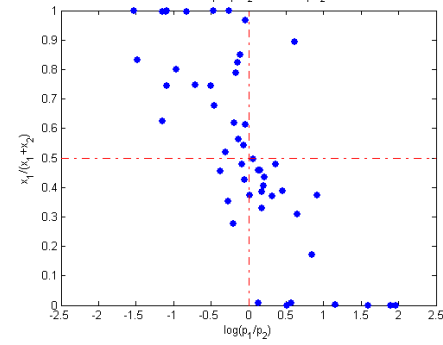
The Token Shares for Subject ID 132



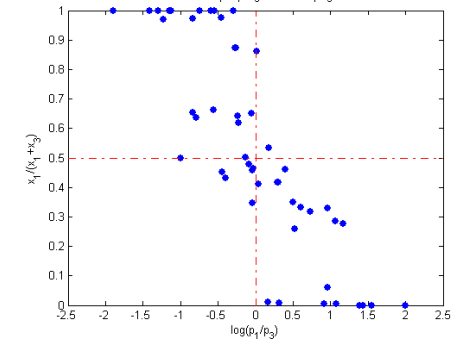
The Expenditure Shares for Subject ID 132



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 132



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 132



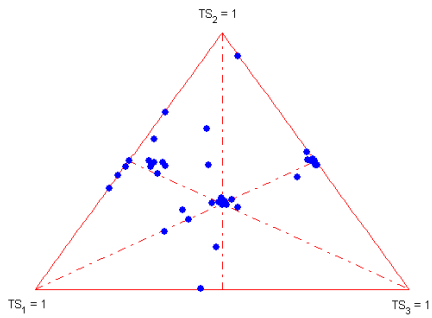




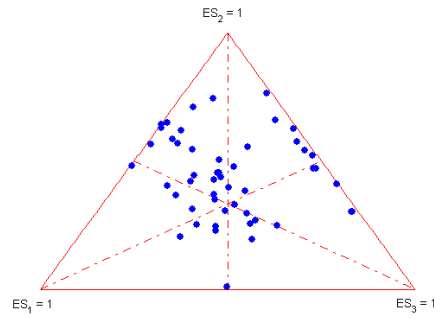




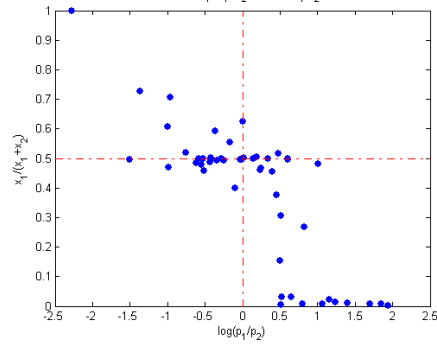
The Token Shares for Subject ID 145



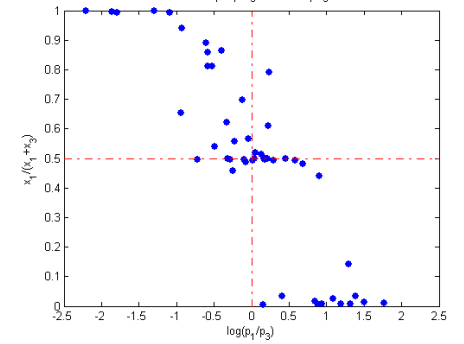
The Expenditure Shares for Subject ID 145



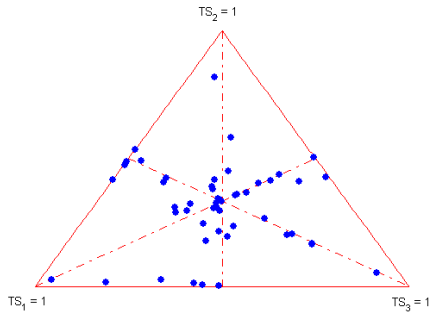
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 145



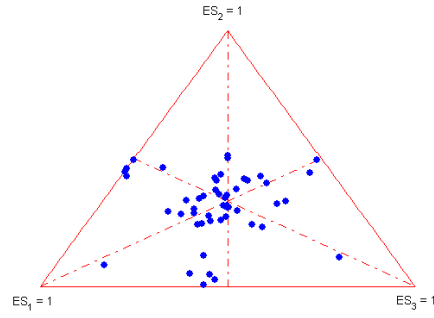
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 145



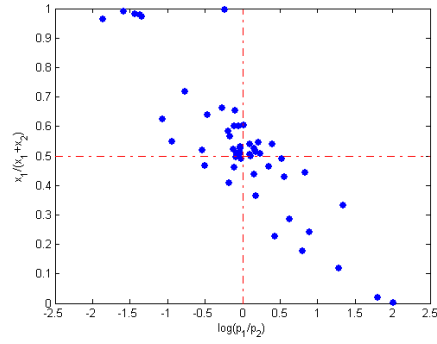
The Token Shares for Subject ID 146



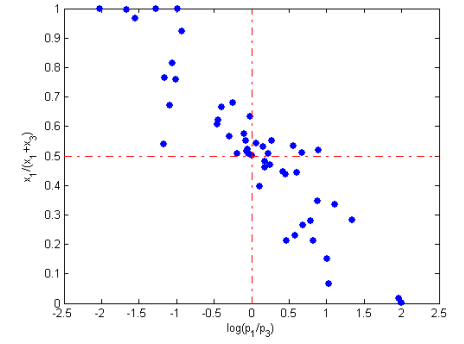
The Expenditure Shares for Subject ID 146



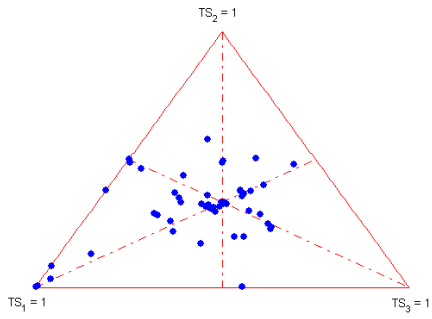
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 146



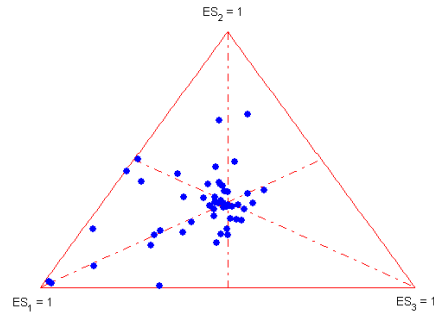
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 146



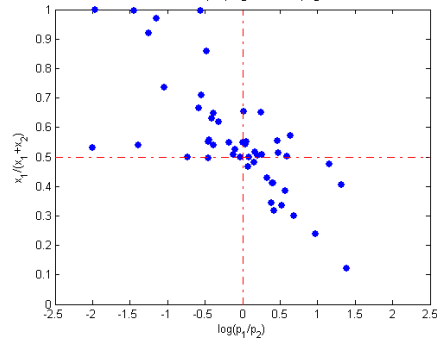
The Token Shares for Subject ID 147



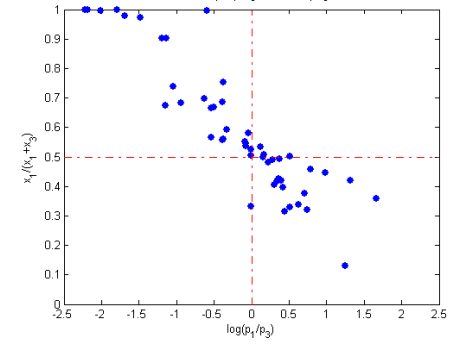
The Expenditure Shares for Subject ID 147



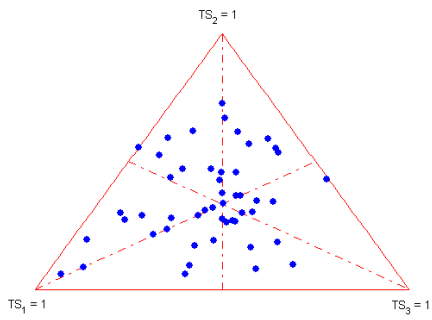
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 147



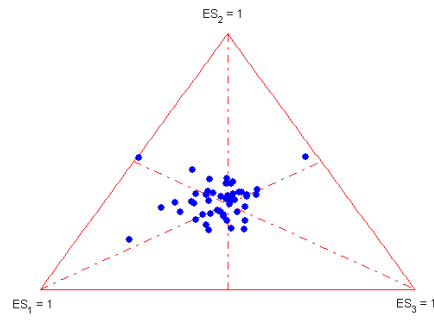
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 147



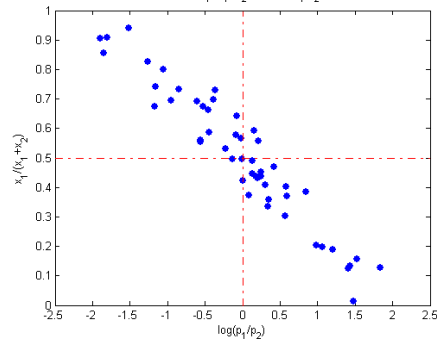
The Token Shares for Subject ID 148



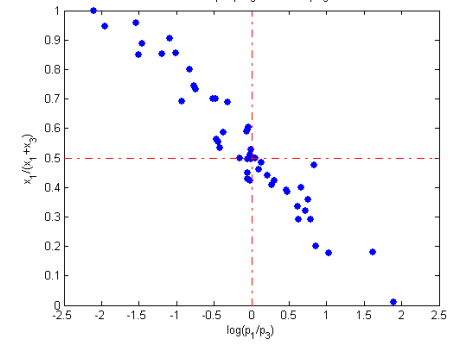
The Expenditure Shares for Subject ID 148



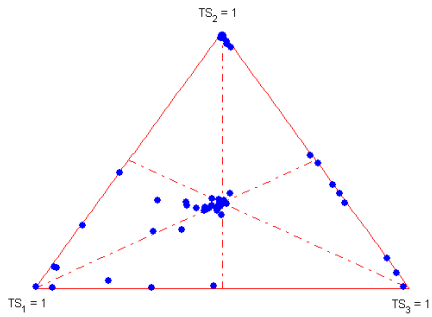
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 148



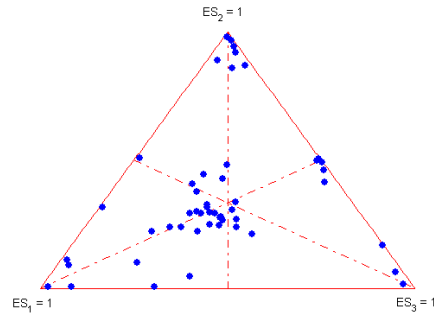
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 148



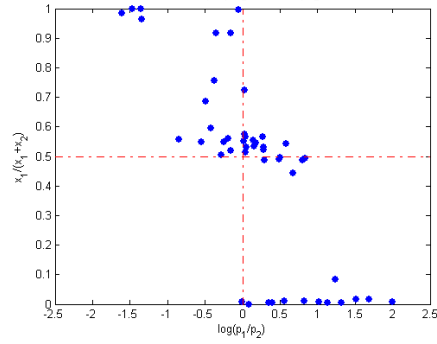
The Token Shares for Subject ID 149



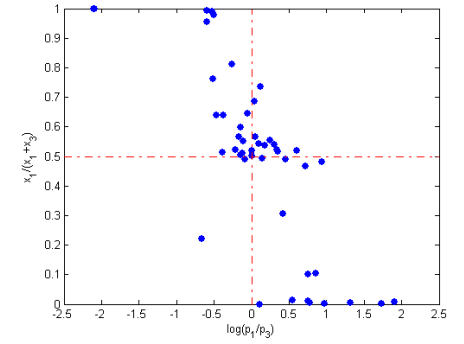
The Expenditure Shares for Subject ID 149



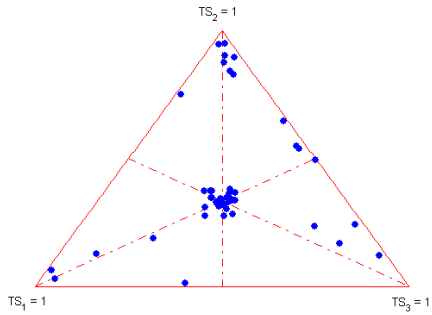
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 149



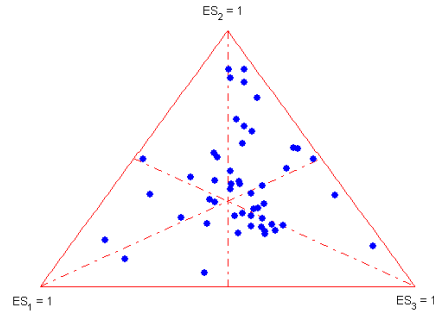
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 149



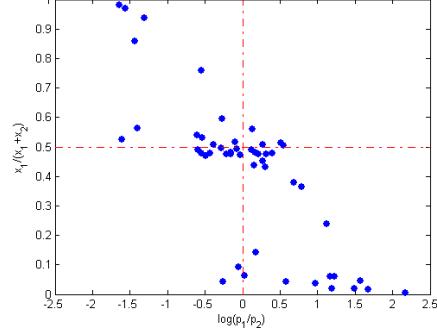
The Token Shares for Subject ID 150



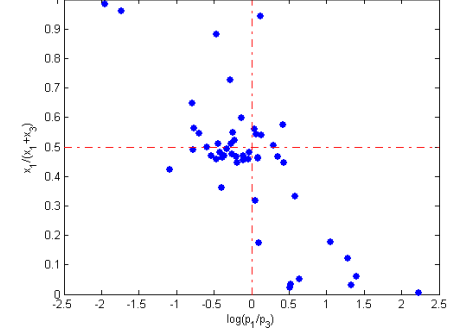
The Expenditure Shares for Subject ID 150



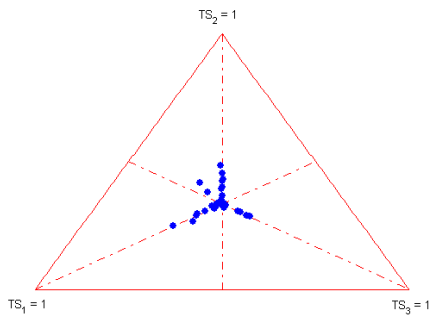
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 150



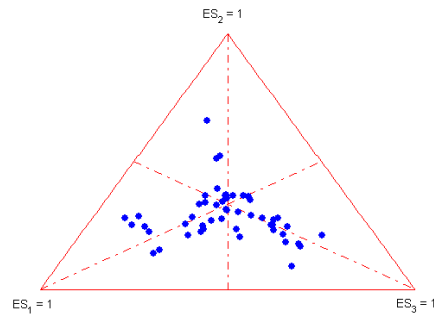
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 150



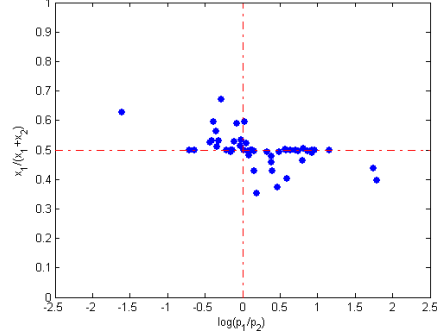
The Token Shares for Subject ID 151



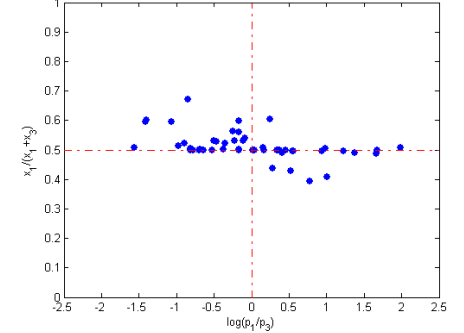
The Expenditure Shares for Subject ID 151



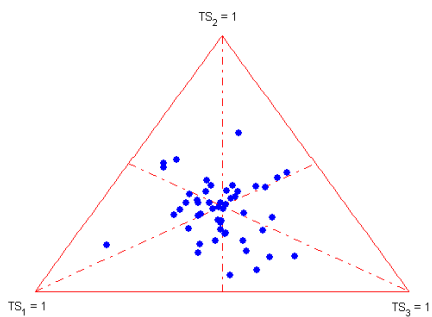
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 151



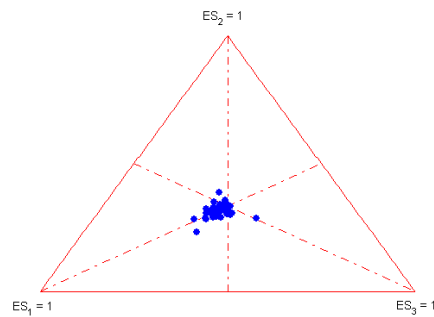
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 151



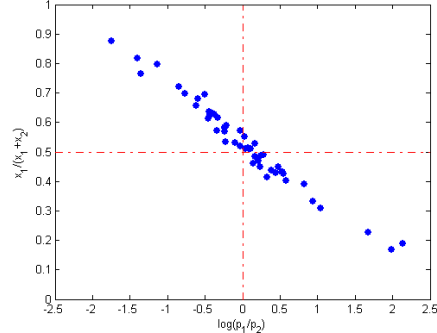
The Token Shares for Subject ID 152



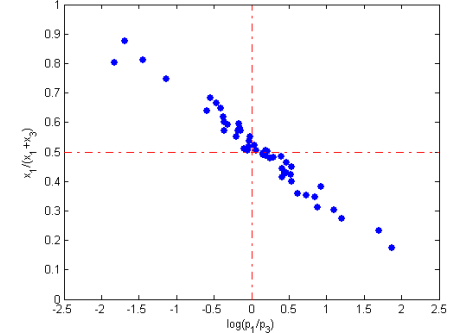
The Expenditure Shares for Subject ID 152



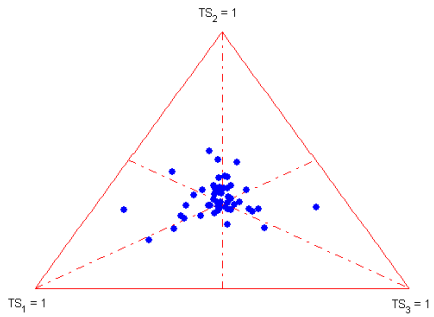
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 152



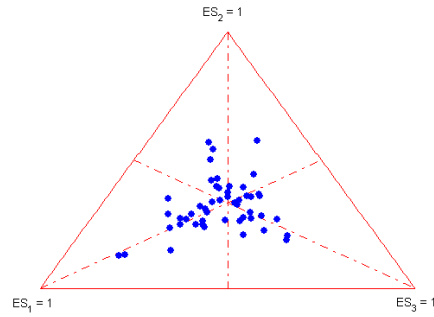
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(\rho_1/\rho_2)$  for Subject ID 152



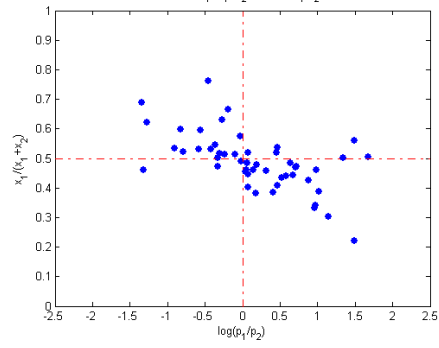
The Token Shares for Subject ID 153



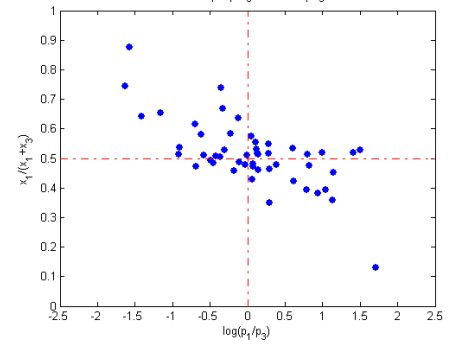
The Expenditure Shares for Subject ID 153



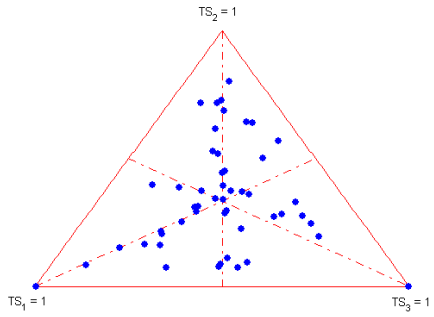
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 153



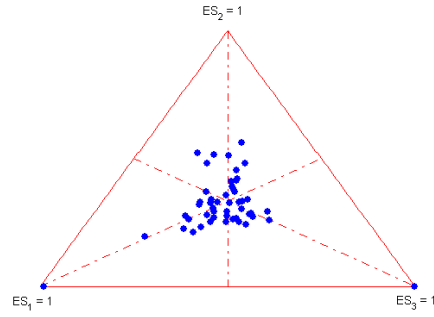
The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 153



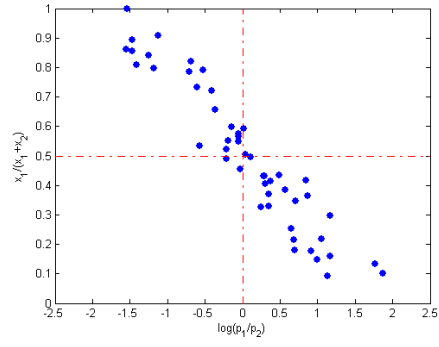
The Token Shares for Subject ID 154



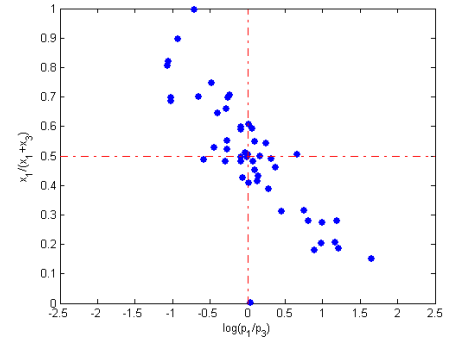
The Expenditure Shares for Subject ID 154



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 154



The Relationship between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  for Subject ID 154



**Appendix IV**  
**GARP violations and goodness-of-fit indices by subject**

ID	# of GARP violations	CCEI	Varian 1991	HM*	ID	# of GARP violations	CCEI	Varian 1991	HM*
1	754	0.761	0.408	15	27	130	0.930	0.740	43
2	2	1.000	0.998	49	28	34	0.905	0.832	42
3	167	0.898	0.635	39	29	717	0.822	0.393	21
4	8	0.984	0.966	46	30	95	0.913	0.770	42
5	80	0.939	0.799	43	31	10	0.951	0.848	48
6	6	0.973	0.911	48	32	68	0.941	0.628	43
7	261	0.834	0.524	29	33	0	1.000	1.000	50
8	2	0.994	0.974	49	34	14	0.935	0.779	45
9	10	0.974	0.960	46	35	54	0.958	0.865	39
10	5	0.979	0.810	49	36	16	0.963	0.925	44
11	0	1.000	1.000	50	37	12	0.980	0.935	46
12	15	0.976	0.920	45	38	124	0.917	0.518	32
13	0	1.000	1.000	50	39	12	0.975	0.934	47
14	0	1.000	1.000	50	40	8	0.980	0.925	46
15	254	0.904	0.485	26	41	64	0.884	0.650	42
16	178	0.852	0.690	36	42	4	0.985	0.952	49
17	7	0.988	0.984	47	43	4	0.992	0.983	49
18	350	0.857	0.605	27	44	2	0.998	0.995	49
19	15	0.986	0.890	45	45	4	0.986	0.924	48
20	58	0.946	0.885	41	46	45	0.854	0.632	45
21	92	0.946	0.797	36	47	30	0.893	0.683	42
22	2	0.997	0.990	49	48	282	0.882	0.614	29
23	43	0.958	0.882	42	49	18	0.959	0.873	44
24	17	0.960	0.942	45	50	14	0.958	0.955	46
25	4	0.995	0.984	49	51	129	0.838	0.616	44
26	2	0.996	0.995	49	52	0	1.000	1.000	50

ID	# of GARP violations	CCEI	Varian 1991	HM*
53	8	0.974	0.921	46
54	464	0.905	0.480	23
55	0	1.000	1.000	50
56	0	1.000	1.000	50
57	4	0.996	0.983	48
58	26	0.938	0.857	47
59	587	0.760	0.438	25
60	151	0.924	0.690	34
61	813	0.772	0.405	14
62	2	0.999	0.983	49
63	4	0.998	0.933	48
64	4	0.985	0.953	49
65	127	0.931	0.705	38
66	11	0.980	0.961	47
67	11	0.969	0.877	46
68	37	0.947	0.876	46
69	81	0.931	0.813	35
70	84	0.943	0.728	36
71	44	0.958	0.751	43
72	59	0.902	0.810	43
73	12	0.933	0.827	48
74	24	0.968	0.802	47
75	17	0.915	0.876	44
76	42	0.860	0.808	44
77	6	0.979	0.960	47
78	4	0.990	0.969	48

ID	# of GARP violations	CCEI	Varian 1991	HM*
79	40	0.955	0.891	46
80	0	1.000	1.000	50
81	19	0.794	0.783	47
82	196	0.790	0.572	34
83	8	0.985	0.981	46
84	6	0.998	0.969	47
85	17	0.957	0.930	46
86	0	1.000	1.000	50
87	205	0.854	0.525	32
88	90	0.932	0.727	38
89	16	0.882	0.867	48
90	104	0.879	0.620	35
91	5	0.998	0.956	49
92	18	0.926	0.892	45
93	564	0.745	0.446	21
94	20	0.978	0.936	44
95	131	0.927	0.659	37
96	2	0.994	0.988	49
97	37	0.976	0.927	39
98	21	0.977	0.921	42
99	80	0.866	0.789	41
100	29	0.884	0.778	45
101	4	0.996	0.987	48
102	35	0.956	0.753	44
103	102	0.946	0.660	37
104	8	0.962	0.900	49

ID	# of GARP violations	CCEI	Varian 1991	HM*
105	2	0.999	0.994	49
106	175	0.902	0.677	37
107	387	0.851	0.496	20
108	46	0.958	0.915	43
109	77	0.933	0.597	45
110	6	0.996	0.859	48
111	2	0.969	0.951	49
112	0	1.000	1.000	50
113	75	0.959	0.788	41
114	4	0.991	0.959	48
115	6	0.971	0.880	47
116	138	0.877	0.668	32
117	0	1.000	1.000	50
118	66	0.941	0.780	37
119	18	0.961	0.897	47
120	9	0.951	0.870	47
121	20	0.936	0.892	43
122	164	0.921	0.798	32
123	73	0.831	0.825	39
124	13	0.972	0.928	46
125	2	0.990	0.942	49
126	16	0.975	0.901	46
127	14	0.983	0.930	48
128	4	0.986	0.979	48
129	112	0.952	0.685	39

ID	# of GARP violations	CCEI	Varian 1991	HM*
130	0	1.000	1.000	50
131	64	0.963	0.842	37
132	50	0.903	0.719	43
133	4	0.988	0.986	48
134	399	0.895	0.497	36
135	21	0.950	0.922	45
136	2	1.000	0.997	49
137	6	0.966	0.946	49
138	93	0.906	0.734	40
139	6	0.983	0.961	47
140	13	0.966	0.944	46
141	0	1.000	1.000	50
142	18	0.990	0.980	46
143	34	0.923	0.860	44
144	88	0.909	0.690	37
145	32	0.987	0.745	45
146	4	0.999	0.987	48
147	4	0.980	0.958	48
148	0	1.000	1.000	50
149	82	0.861	0.818	37
150	63	0.931	0.673	44
151	7	0.990	0.961	49
152	2	0.999	0.999	49
153	23	0.979	0.956	43
154	0	1.000	1.000	50



\* The test proposed by Houtman and Maks (1985) (HM), finds the largest subset of choices that is consistent with GARP. This method is computationally very intensive. As a result, we were unable to calculate the HM indices for a small number of subjects who often violated GARP, and we therefore report only lower bounds. The Varian (1991) index is a lower bound on the CCEI. The reasons for this discrepancy are discussed in CFGKb.

## Appendix V

### The kinked and smooth specifications

#### [1] The kinked specification

The kinked utility function is so-called because the indifference curves have a “kink” at all portfolios where  $x_1 = x_3$ . The parametric specification we use has the form

$$U(\mathbf{x}; \alpha, \rho) = -\frac{2}{3}\alpha \exp\{-\rho x_{\min}\} - \frac{1}{3} \exp\{-\rho x_2\} - \frac{2}{3}(1 - \alpha) \exp\{-\rho x_{\max}\}, \quad (1)$$

where  $0 \leq \alpha \leq 1$  is the ambiguity parameter and  $\rho$  is the coefficient of risk aversion. The distinguishing feature of this specification is its dependence on the minimum and maximum payoffs,  $x_{\min} = \min\{x_1, x_3\}$  and  $x_{\max} = \max\{x_1, x_3\}$ , between the two ambiguous states, 1 and 3. The agent knows that the probabilities of states 1 and 3 lie between 0 and  $\frac{2}{3}$ . In the best case scenario, the probability of the state in which he receives  $x_{\max}$  is  $\frac{2}{3}$ ; in the worst case scenario, it is zero. What equation 1 says is that the agents’s utility is a weighted average, with weights  $\alpha$  and  $1 - \alpha$ , of the expected utility in the worst-case and best-case scenarios:  $\frac{1}{2} < \alpha \leq 1$  indicate preferences that are ambiguity averse,  $0 \leq \alpha < \frac{1}{2}$  indicate preferences that are ambiguity seeking, and if  $\alpha = \frac{1}{2}$  we have the standard SEU representation.

We next demonstrate how this kinked functional form can be generated by different classes of preferences.

#### [1.1] Maxmin Expected Utility with flexible priors

The Maxmin Expected Utility (MEU) model of Gilboa and Schmeidler (1989) evaluates a portfolio by its minimal expected utility over a set of subjective prior beliefs. This minimization over a non-singleton set can be interpreted as aversion to ambiguity. The general form of the MEU model is

$$U(\mathbf{x}) = \min_{\pi \in \Pi} \int_S u(x_s) d\pi(s),$$

where  $\Pi \subseteq \Delta S$  is a closed convex set of prior beliefs over states.

Connecting the general MEU model to our kinked specification assumes that the utility over tokens takes the CARA form and that the set of priors is symmetric about  $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ . In particular, the set of priors is

$$\Pi_\theta = \left\{ \pi : \pi_2 = \frac{1}{3}, \frac{1}{3} - \theta \leq \pi_1 \leq \frac{1}{3} + \theta, \pi_3 = \frac{2}{3} - \pi_1 \right\}$$

for some  $0 \leq \theta \leq \frac{1}{3}$ . Larger values of  $\theta$  indicate a larger set of priors, hence more ambiguity. This reduces the general MEU model to the following two-parameter formula:

$$U(\mathbf{x}; \delta, \rho) = -\left(\frac{1}{3} + \delta\right) \exp\{-\rho x_{\min}\} - \frac{1}{3} \exp\{-\rho x_2\} - \left(\frac{1}{3} - \delta\right) \exp\{-\rho x_{\max}\}.$$

This equation is exactly equation 1 with a change of variables, letting  $\alpha = \frac{1}{2} + \frac{3}{2}\theta$ .

### [1.2] Choquet Expected Utility with flexible capacity

The Choquet Expected Utility (CEU) model of Schmeidler (1989) is related to MEU and takes the following general form:

$$U(\mathbf{x}) = \int_S u(x_s) d\nu(s),$$

where  $\nu$  is a nonadditive capacity over the state space. Ambiguity in the CEU model is captured by the convexity of the capacity  $\nu$ .<sup>1,2</sup>

Any CEU representation with a convex capacity can be rewritten as an MEU representation where the set of priors is the core of the capacity. Correspondingly, if we assume CARA utility over tokens and that the capacity is symmetric over the two ambiguous states, then the CEU model reduces to the parameterized MEU model with symmetric priors presented in the previous section. In particular, if the capacity obeys:

$$\begin{aligned} \nu(\{1\}) = \nu(\{3\}) &= \frac{1}{3} - \theta, & \nu(\{2\}) &= \frac{1}{3}, \\ \nu(\{1, 2\}) = \nu(\{2, 3\}) &= \frac{2}{3} - \theta, & \nu(\{1, 3\}) &= \frac{2}{3}, \end{aligned}$$

for some  $0 \leq \theta \leq \frac{1}{3}$ , then the implied Choquet integral reduces to equation 1, via the same change of variables  $\alpha = \frac{1}{2} + \frac{3}{2}\theta$ .

### [1.3] Contraction Expected Utility with fixed information

The contraction model of Gajdos et al. (2008) incorporates objective information about the set of possible prior distributions over states. It enriches the standard subjective setup by considering acts or portfolios paired with some set of objectively known possible priors. The agent partially contracts this set towards its center and then applies the MEU criterion to this smaller set of priors. The general representation is

$$U(\mathbf{x}) = \min \left\{ \int_S u(x_s) d\pi(s) : \pi \in (1 - \epsilon)\{s(\Pi)\} + \epsilon\Pi \right\},$$

where  $s(\Pi) \in \Delta S$  is the Steiner point (a geometric notion of the center) of the set  $\Pi$  of objectively specified priors.<sup>3</sup> Larger values of  $\epsilon \in [0, 1]$  place more weight on the entire set of possible priors  $\Pi$  and, hence, suggest more ambiguity.

The experimental choice problem can be represented in this form, where every portfolio is paired with the same set of objective priors, namely  $\Pi = \{\pi : \pi_2 = \frac{1}{3} \text{ and } \pi_1 + \pi_3 = \frac{2}{3}\}$ . Its Steiner point is  $s(\Pi) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ . As Hayashi and Wada (2010) mention, the contraction model with a fixed set of possible priors is identical to a special form of the MEU model. To be specific, maintaining the CARA form for utility over tokens, the contraction model reduces to:

$$U(\mathbf{x}; \epsilon, \rho) = -\left(\frac{1+\epsilon}{3}\right) \exp\{-\rho x_{\min}\} - \frac{1}{3} \exp\{-\rho x_2\} - \left(\frac{1-\epsilon}{3}\right) \exp\{-\rho x_{\max}\}.$$

This is exactly the MEU model above with  $\theta = \frac{\epsilon}{3}$  and is the kinked specification in equation 1 with  $\alpha = \frac{1-\epsilon}{2}$ .

<sup>1</sup>The exact formula for integration with respect to a capacity can be found in Schmeidler (1989).

<sup>2</sup>A capacity is convex if  $\nu(A \cup B) + \nu(A \cap B) \geq \nu(A) + \nu(B)$  for any sets  $A$  and  $B$ .

<sup>3</sup>The convex combination of two sets  $A$  and  $B$  is defined as the union of their pointwise convex combinations:  $\lambda A + (1 - \lambda)B = \{\lambda a + (1 - \lambda)b : a \in A, b \in B\}$ .

**[1.4]  $\alpha$ -Maxmin Expected Utility with fixed priors**

A proposed generalization of MEU is  $\alpha$ -Maxmin Expected Utility ( $\alpha$ -MEU) characterized by Ghirardato et al. (2004) and Olszewski (2006), which evaluates each portfolio by a convex combination of its minimal and maximal expected utilities over some set of subjective prior beliefs over states.

The general form of the  $\alpha$ -MEU model is

$$U(\mathbf{x}) = \alpha \cdot \min_{\pi \in \Pi} \int u(x_s) d\pi(s) + (1 - \alpha) \cdot \max_{\pi \in \Pi} \int u(x_s) d\pi(s),$$

where  $\Pi \subseteq \Delta S$  is a closed convex set of distributions over states and  $\alpha \in [0, 1]$  reflects the relative weight of the worst versus the best possible expected utility of  $\mathbf{x}$  given  $\Pi$ . Hence,  $\alpha$  serves as a parameter reflecting ambiguity aversion. (In the most general case, the  $\alpha$ -MEU parameter could depend on the portfolio under consideration  $\alpha(\mathbf{x})$ .)

If we assume that  $u$  has the CARA form and that the set of priors  $\Pi$  is the entire set of distributions consistent with the objective information in the experiment,  $\Pi = \{\pi : \pi_2 = \frac{1}{3}\}$ , this reduces to the two-parameter formula in equation 1. The weight  $\alpha$  and the set of priors  $\Pi$  in the  $\alpha$ -MEU model cannot be separately identified. In fact, Siniscalchi (2006) proves that the  $\alpha$ -MEU and MEU models are generally confounded in the symmetric case: any MEU representation with some fixed symmetric set of priors can be rewritten as one of a continuum of  $\alpha$ -MEU representations with arbitrarily small alternative sets of priors.

In all of the models described above, the parameter  $\alpha$  that appears in equation 1 depends on the set  $\Pi$  (or the capacity  $\nu$  in the case of CEU). Unless the set  $\Pi$  is objectively known, knowledge of the estimated parameter  $\alpha$  does not allow us to characterize the degree of ambiguity aversion independently of the degree of ambiguity in the decision problem. In any case, the lack of identification is inherent in these theoretical models, rather than a feature of our data. When we adopt the MEU interpretation, we are fixing  $\alpha = 1$  and allowing the set of priors to vary; when we adopt the  $\alpha$ -MEU interpretation, we are fixing the set of priors and allowing  $\alpha$  to vary. To simplify the exposition and facilitate comparisons, we adopted the second convention as our main interpretation.

**[2] The smooth specification**

Our second utility specification is differentiable everywhere. The utility of a portfolio  $\mathbf{x} = (x_1, x_2, x_3)$  takes the form

$$U(\mathbf{x}; \alpha, \rho) = \frac{1}{\alpha} \int_0^{\frac{2}{3}} -\exp \left\{ -\alpha \left( \begin{array}{l} -\pi_1 \exp\{-\rho x_1\} - \frac{1}{3} \exp\{-\rho x_2\} \\ -(\frac{2}{3} - \pi_1) \exp\{-\rho x_3\} \end{array} \right) \right\} d\pi_1, \tag{2}$$

This specification involves two iterated integrals. First, the formula inside the parentheses is the expected value of the CARA utility of the portfolio  $\mathbf{x}$  when the probability of the first state is known to be  $\pi_1$ . Next, the integral ranging from 0 to  $\frac{2}{3}$  takes the expectation of these expected utilities with respect to the uniform distribution for  $\pi_1$ , with each expected utility transformed using a CARA aggregator. The utility function is normalized by  $\frac{1}{\alpha}$  so that utility does not go to zero as  $\alpha$  approaches zero.

While the kinked specification can be interpreted using a variety of different models, the smooth specification is really motivated by a single model. A recent view of ambiguity aversion (Ergin and

Gul, 2004; Klibanoff et al., 2005; Nau, 2005; and Seo, 2007; as well as related work by Halevy and Feltkamp, 2005; Giraud, 2006; and Ahn, 2008) assumes the agent has a subjective (second-order) distribution  $\mu$  over the possible (first-order) prior beliefs  $\pi$  over states. Unsure which of the possible first-order prior beliefs actually governs the states, the agent transforms the expected utilities for all prior beliefs  $\pi$  by a concave function  $\varphi$  before integrating these utilities with respect to his second-order distribution  $\mu$ . This procedure is entirely analogous to the transformation of wealth into cardinal utility before computing expected utility under risk. The concavity of this transformation captures ambiguity aversion. We follow Halevy (2007) in referring to this model as Recursive Expected Utility (REU), owing to its recursive double expectation.

The general form of the REU model is

$$U(\mathbf{x}) = \int_{\Delta S} \varphi \left( \int_S u(x_s) d\pi(s) \right) d\mu(\pi),$$

where  $\mu \in \Delta(\Delta(S))$  is a (second-order) distribution over possible priors  $\pi$  on  $S$  and  $\varphi : u(\mathbf{R}_+) \rightarrow \mathbf{R}$  is a possibly nonlinear transformation over expected utility levels.<sup>4</sup>

To facilitate comparison with the kinked specification, we reduce the REU model to two parameters. Assuming that

$$\varphi(z) = -e^{-\alpha z},$$

which replicates the constant curvature of  $u$ , and that  $\mu$  is uniformly distributed over the set of priors consistent with the objective information  $\Pi = \{\pi : \pi_2 = \frac{1}{3}\}$ , this specializes to the two-parameter formula in equation 2. Here,  $\alpha$  reflects the curvature of the aggregator  $\varphi$  and hence measures the degree of ambiguity aversion/seeking: any  $\alpha > 0$  indicate preferences that are ambiguity averse,  $\alpha < 0$  indicate preferences that are ambiguity seeking, and as  $\alpha \rightarrow 0$  we approach the standard SEU representation.

One of the crucial features of the REU specification is its reliance on a cardinal utility indicator. Unlike the preferences generated by SEU, MEU and  $\alpha$ -MEU, which are invariant to affine transformations of the utility function  $u(\cdot)$ , the preferences generated by REU are not independent of a change in the scale of utility. For example, if we introduce a scale parameter and set  $u(x) = -Ae^{-\rho x}$ , the concavity of the transformation  $\varphi$  implies that the ranking of uncertain prospects will not be invariant to changes in  $A$ . Since the parameters  $\alpha$  and  $A$  enter equation 2 only in the form of the product  $\alpha A$ , we can estimate  $\alpha A$  but cannot identify the values of  $\alpha$  and  $A$  separately. If we assume a common scale factor for all subjects, say  $A = 1$ , interpersonal comparisons of ambiguity aversion will still be affected by risk aversion. A higher coefficient of absolute risk aversion,  $\rho$ , will reduce the range of the function  $u(x) = -e^{-\rho x}$  and, hence, will reduce the ambiguity to which the agent is exposed. We can normalize the ambiguity parameters to take into account the different ranges of expected utility for different subjects, but the meaning of such comparisons is not clear.

### [3] Restricted specifications

In addition to the kinked and smooth specifications, we consider two important special cases. The first corresponds to SEU in the sense of Savage, while the second corresponds to an extreme form of MEU. Each is derived by setting the ambiguity parameter equal to some extreme value.

---

<sup>4</sup>Here,  $\Delta(\Delta(S))$  denotes the space of all probability measures over  $\Delta(S)$ , the set of all probability distributions on  $S$ .

### [3.1] Ambiguity neutrality: Subjective Expected Utility with a fixed prior

Subjective expected utility (SEU) is a special case of both the kinked and smooth formulations:

$$U(\mathbf{x}; \rho) = -\frac{1}{3} \exp\{-\rho x_1\} - \frac{1}{3} \exp\{-\rho x_2\} - \frac{1}{3} \exp\{-\rho x_3\}.$$

This corresponds to the kinked specification in equation 1 with  $\alpha = \frac{1}{2}$  and to the smooth specification in equation 2 with  $\alpha = 0$  and provides a benchmark for probabilistic sophistication within these specifications.

To derive this formula directly, recall that the general SEU model of Savage (1954) consists of a utility function  $u$  which is integrated with respect to a single subjective probability distribution  $\pi$ . The general form for the utility of a portfolio  $\mathbf{x} = (x_1, x_2, x_3)$  is:

$$U(\mathbf{x}) = \int_S u(x_s) d\pi(s)$$

where  $\pi$  is a subjective probability over states of the world and  $u$  is a cardinal utility index over tokens. If we assume that the agent believes the ambiguous states in our experimental choice problem are equally probable, that is, her prior belief over states is  $\pi = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$  and has CARA utility over tokens, this specializes to the above formula.

### [3.2] Extreme ambiguity aversion: Maxmin Expected Utility with maximal priors

The opposite special case for both the kinked and smooth specifications is the following restricted formulation:

$$U(\mathbf{x}; \rho) = -\frac{2}{3} \exp\{-\rho x_{\min}\} - \frac{1}{3} \exp\{-\rho x_2\}.$$

This corresponds to the kinked specification in equation 1 with  $\alpha = 1$  and to the smooth specification in equation 2 as  $\alpha \rightarrow \infty$  and provides the opposite benchmark of the most ambiguity averse subspecification within these models.

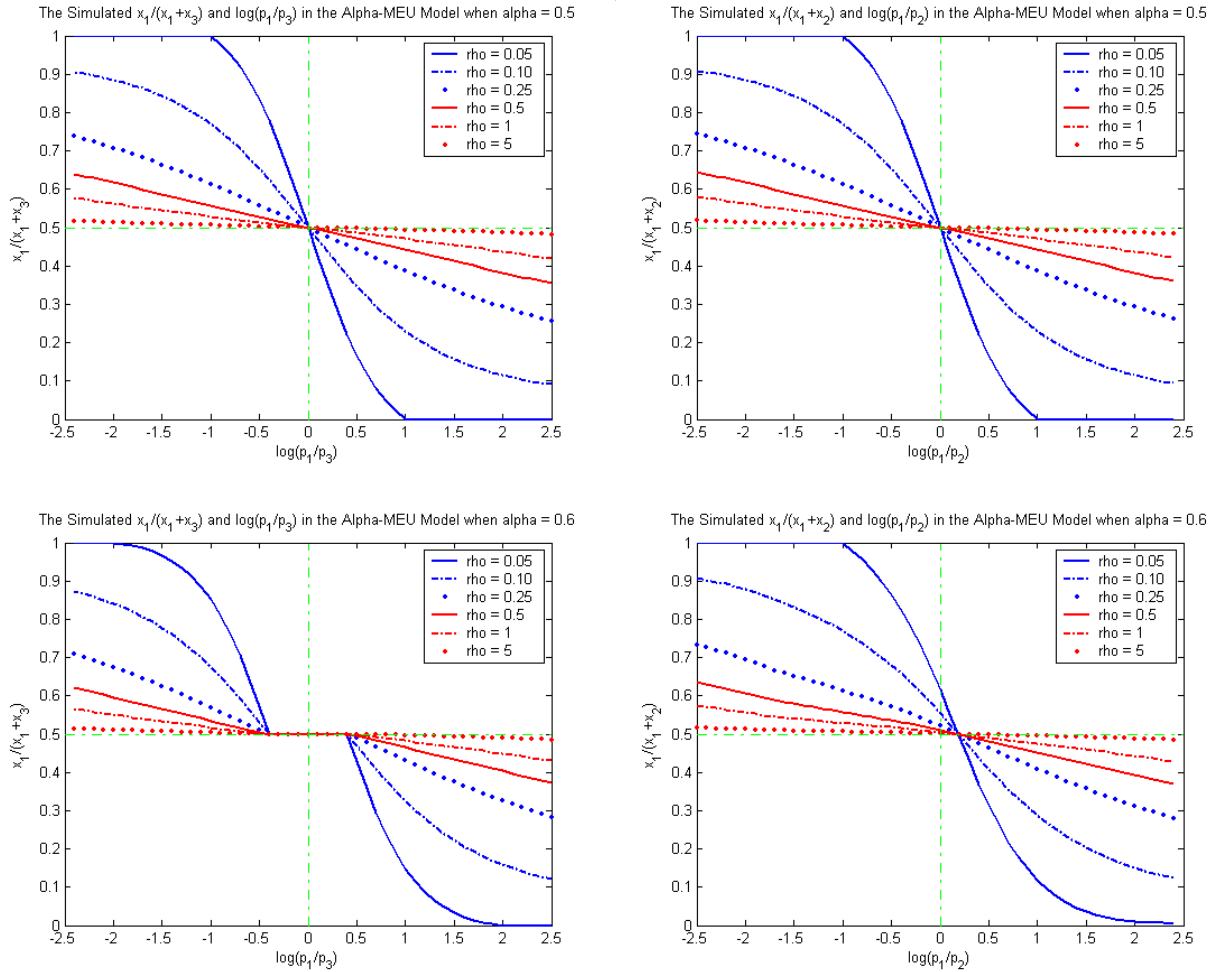
### [4] Additional references

1. Hayashi T. and R. Wada (2010) "Choice with Imprecise Information: An Experimental Approach," *Theory and Decision*, 69, pp. 355-373.
2. Siniscalchi, M. (2006) "A behavioral characterization of plausible priors," *Journal of Economic Theory*, 128, pp. 91-135.

## Appendix VI

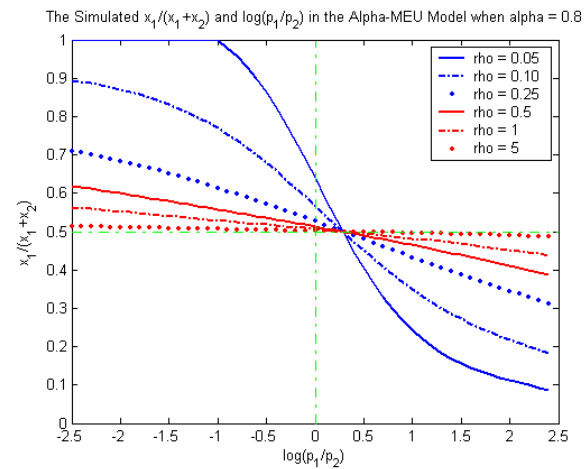
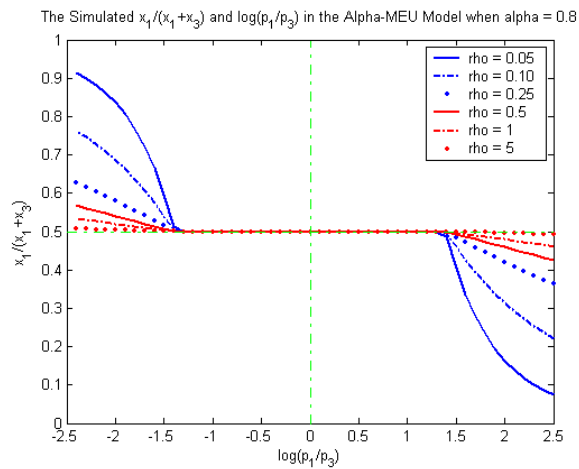
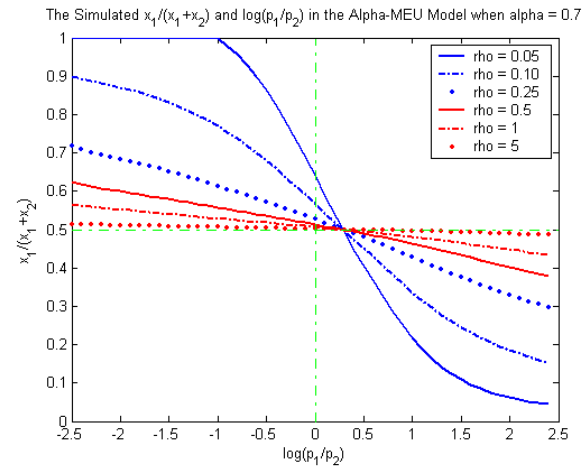
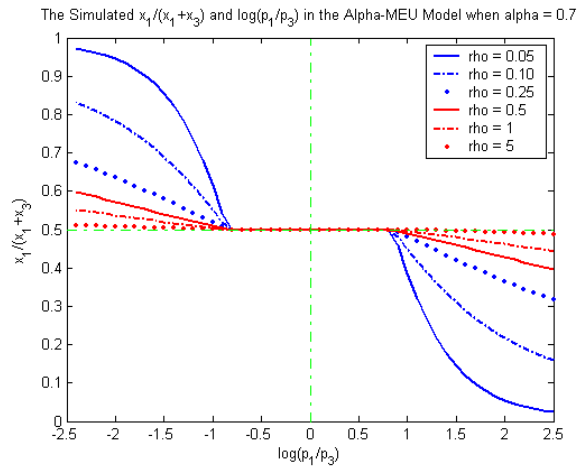
### An illustration of the relationships between log-price ratio and optimal token share

#### A: Kinked specification (equation 1)



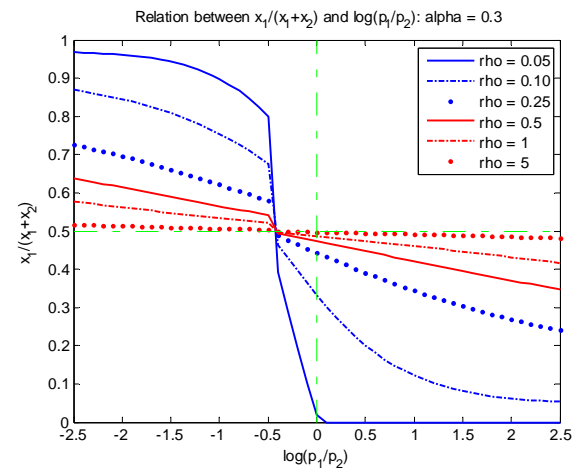
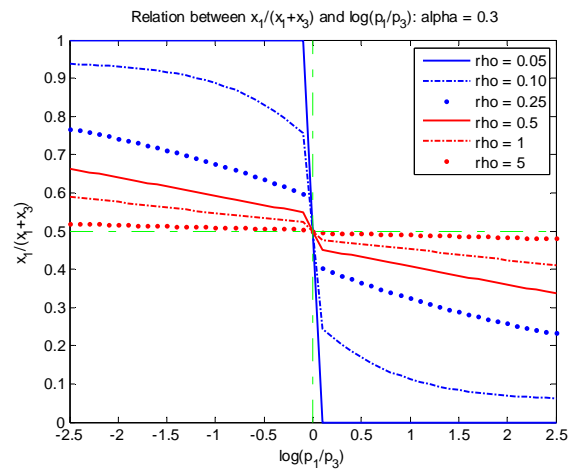
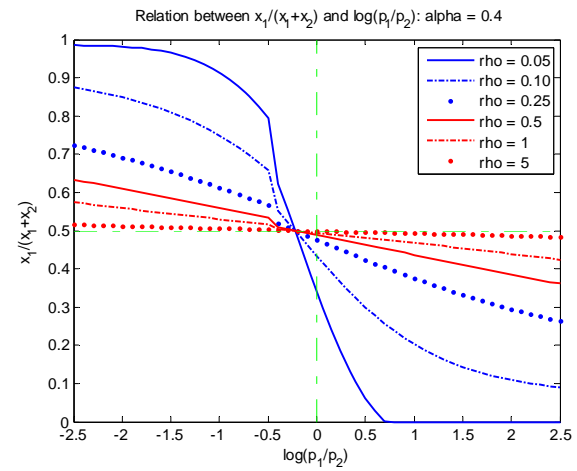
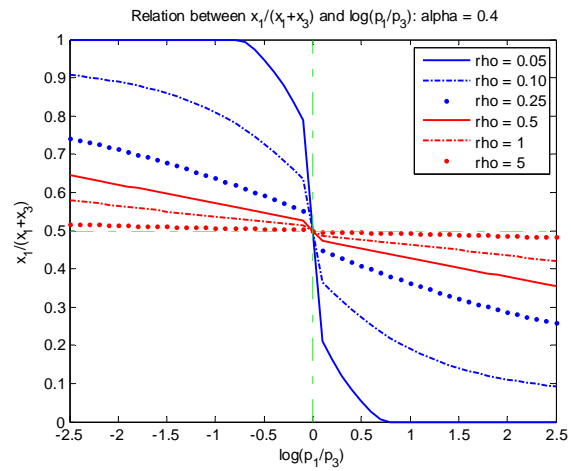
Note: we restrict the parameters so that preferences are risk averse ( $\rho \geq 0$ ) in both specifications and ambiguity averse ( $\alpha \geq 0$ ) in the smooth specification.

## Kinked specification (cont.)

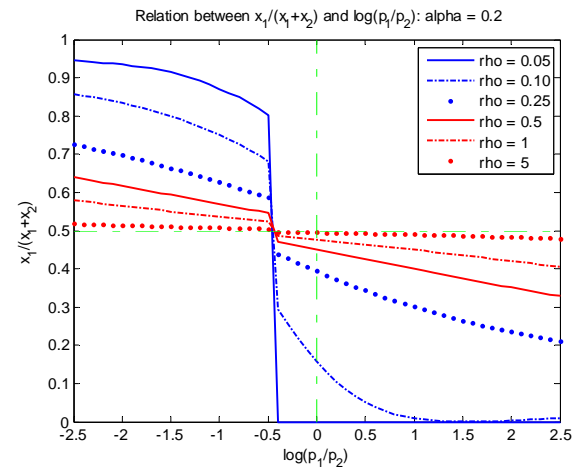
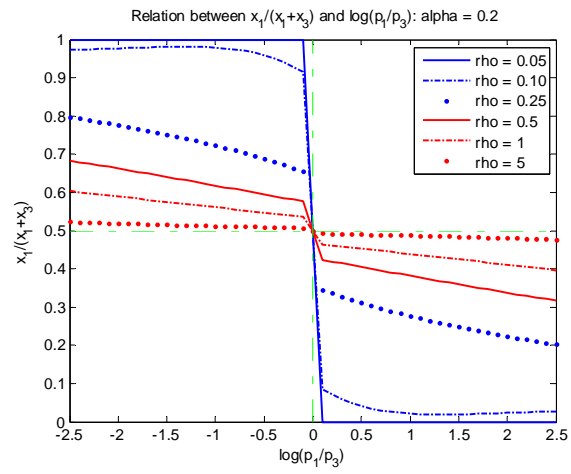




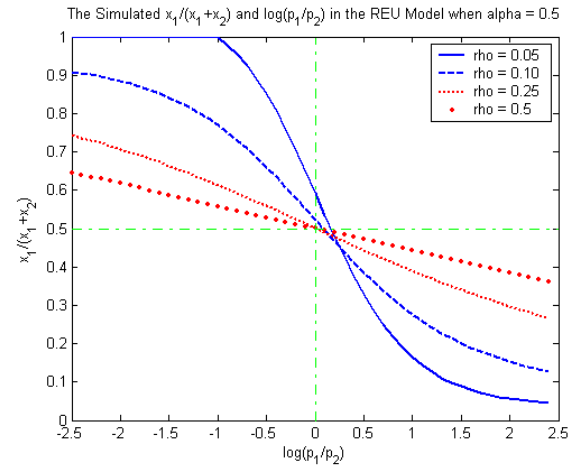
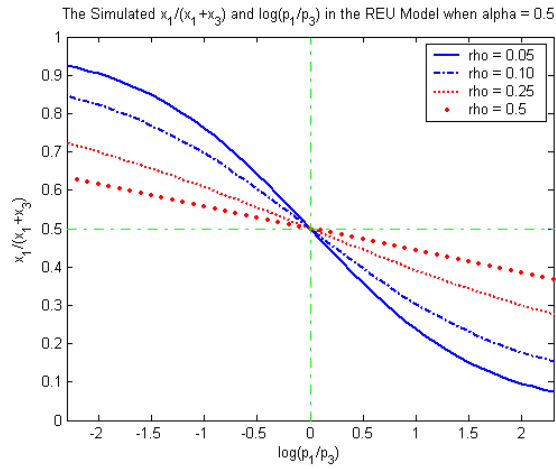
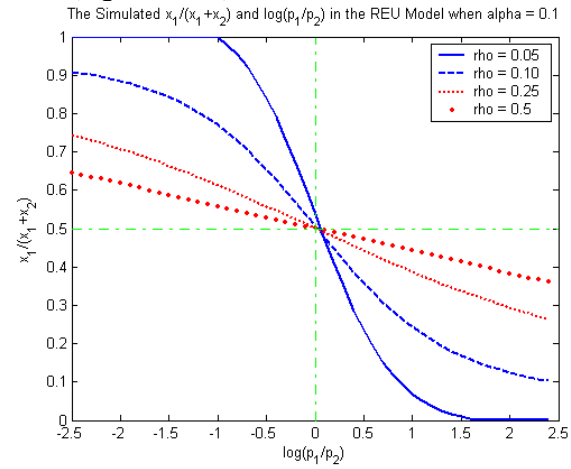
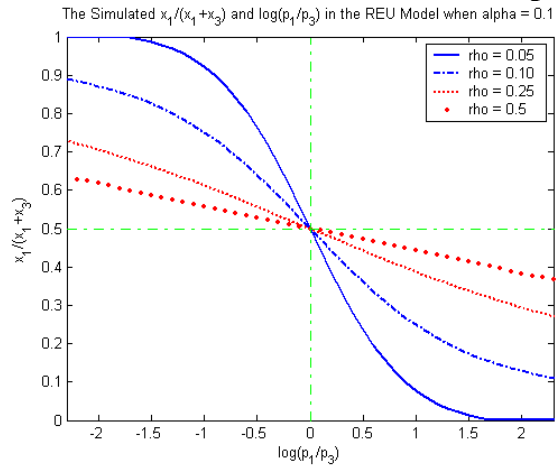
## Kinked specification (cont.)



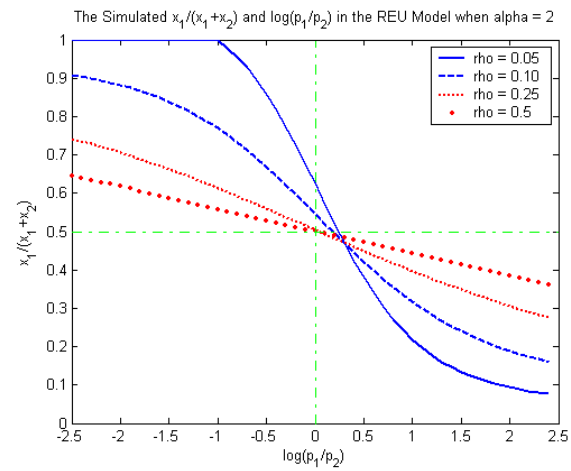
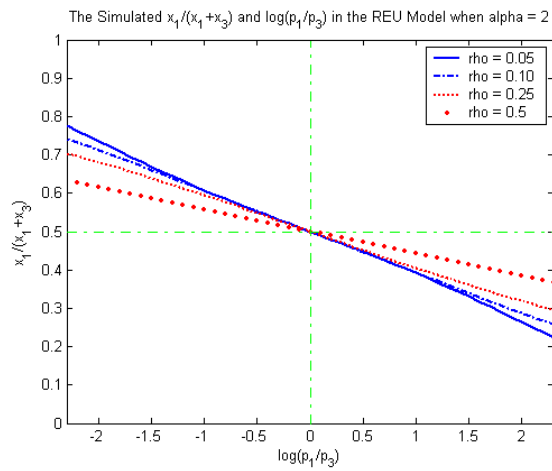
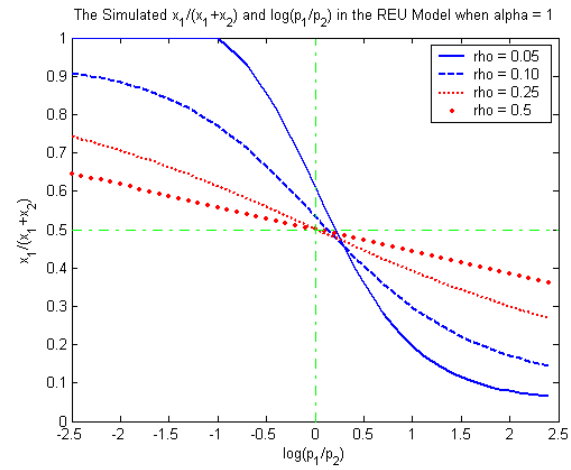
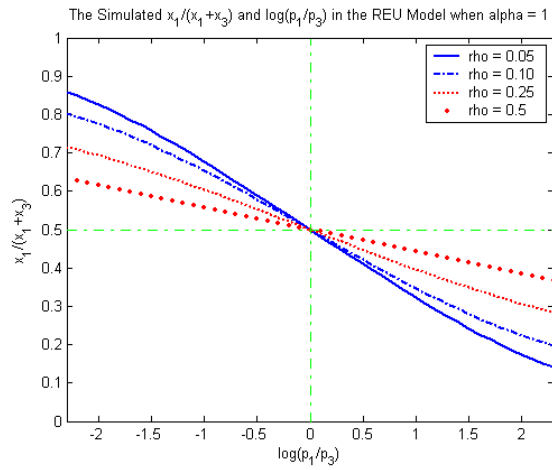
## Kinked specification (cont.)



## B: Smooth specification (equation 2)



## Smooth specification (cont.)



## Appendix VII

### Individual-level estimation results -- kinked and smooth specifications

ID	Kinked specification (equation 1)					Smooth specification (equation 2)				
	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR
1	0.501	0.023	0.162	-0.025	1.9E+04	0.000	0.804	0.236	0.075	19301.8
2	0.696	0.009	0.457	-0.056	673.2	0.010	0.539	0.490	0.059	701.7
3	0.569	0.007	0.121	-0.002	12111.9	0.688	0.824	0.122	0.048	12188.4
4	0.566	0.003	0.036	-0.001	11727.7	0.148	0.095	0.037	0.006	11571.6
5	0.424	0.004	0.072	0.000	14809.1	0.000	0.098	0.071	0.007	15064.2
6	0.562	0.001	0.037	0.000	9479.9	0.140	0.043	0.034	0.006	9096.8
7	0.448	0.007	0.025	0.000	63893.9	0.005	0.036	0.022	0.005	64862.3
8	0.420	0.001	0.104	0.000	4243.7	0.000	0.099	0.090	0.011	4374.4
9	0.543	0.001	0.024	0.000	15274.2	0.081	0.029	0.021	0.005	14875.9
10	0.564	0.001	0.022	0.000	19311.3	0.070	0.025	0.023	0.005	19161.0
11	0.765	0.001	5.000	0.000	6.9	0.003	0.780	0.499	0.001	273.9
12	0.569	0.002	0.054	0.000	6048.8	0.000	0.182	0.059	0.005	6336.1
13	0.599	0.001	0.058	0.000	9864.2	0.558	0.573	0.055	0.016	9901.6
14	0.513	0.000	0.003	0.000	3419.0	0.009	0.004	0.003	0.001	4241.2
15	0.850	0.004	0.114	-0.003	8574.1	2.000	0.777	0.110	0.144	8487.8
16	0.582	0.001	0.021	0.000	25713.4	0.086	0.055	0.021	0.004	26367.2
17	0.477	0.005	0.427	-0.010	476.1	0.010	0.469	0.467	0.073	452.5
18	0.503	0.000	0.003	0.000	58422.0	0.007	0.024	0.002	0.002	56416.2
19	0.533	0.004	0.129	-0.002	5031.5	1.749	0.518	0.106	0.015	4843.0
20	0.521	0.000	0.000	0.000	41268.7	0.009	0.003	0.001	0.000	44166.9
21	0.365	0.010	0.080	-0.003	17855.4	0.000	0.047	0.062	0.011	18689.3
22	0.439	0.001	0.166	-0.001	1887.3	0.000	0.631	0.153	0.024	1918.5
23	0.010	0.034	1.971	-0.038	7684.0	0.004	0.199	0.487	0.045	7270.8
24	0.549	0.012	1.235	-0.017	199.5	0.008	0.119	0.496	0.012	246.2
25	0.671	0.001	0.039	0.000	10841.8	0.435	0.259	0.036	0.007	9861.3
26	0.507	0.000	0.036	0.000	4592.0	0.112	0.040	0.031	0.007	4170.7

ID	Kinked specification (equation 1)					Smooth specification (equation 2)				
	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR
27	0.517	0.000	0.033	0.000	7067.6	0.000	0.009	0.034	0.003	7107.0
28	0.484	0.000	0.014	0.000	20676.9	0.000	0.009	0.013	0.002	20815.3
29	0.622	0.001	0.013	0.000	42644.8	0.119	0.316	0.013	0.003	46711.0
30	0.653	0.035	0.357	-0.084	1612.2	0.010	0.006	0.474	0.051	1479.7
31	0.463	0.000	0.081	0.000	1306.4	0.000	0.029	0.076	0.006	1348.6
32	0.779	0.055	0.179	-0.150	2466.8	0.012	1.000	0.346	0.166	2608.1
33	0.619	0.003	0.060	0.000	13379.7	0.323	0.495	0.062	0.019	13796.5
34	0.665	0.002	0.049	0.000	11705.1	0.535	0.262	0.047	0.011	11418.3
35	0.500	0.002	0.026	0.000	32352.7	0.041	0.034	0.023	0.007	31991.0
36	0.717	0.003	0.064	0.000	11851.2	1.899	0.477	0.064	0.015	11689.5
37	0.880	0.001	0.075	0.000	4451.0	2.000	0.000	0.074	0.033	4529.8
38	0.872	0.002	0.027	-0.001	26978.6	0.974	0.408	0.027	0.005	26477.1
39	0.510	0.002	0.073	0.000	6791.2	0.251	0.200	0.066	0.010	6628.5
40	0.544	0.001	0.033	0.000	6263.1	0.015	0.023	0.035	0.004	6611.9
41	0.632	0.007	0.011	0.000	42246.1	0.078	0.193	0.012	0.003	39806.7
42	0.404	0.003	0.137	0.000	3253.1	0.189	0.202	0.113	0.014	3388.8
43	0.556	0.001	0.115	0.000	2059.9	0.000	0.000	0.128	0.004	1952.3
44	0.521	0.001	0.038	0.000	7366.4	0.042	0.037	0.037	0.006	7351.9
45	0.525	0.000	0.007	0.000	18931.5	0.021	0.008	0.007	0.001	19775.8
46	0.580	0.002	0.034	0.000	27171.6	0.073	0.173	0.040	0.008	27963.4
47	0.492	0.003	0.021	0.000	47053.5	0.000	0.000	0.021	0.001	46965.0
48	0.476	0.003	0.018	-0.001	45195.6	0.031	0.022	0.020	0.005	45292.3
49	0.646	0.001	0.052	0.000	5710.6	0.359	0.151	0.052	0.008	5676.9
50	0.608	0.004	0.059	0.000	6562.8	0.288	0.350	0.059	0.014	6738.1
51	0.476	0.001	0.056	0.000	17428.4	0.086	0.121	0.051	0.004	17404.4
52	0.527	0.000	0.021	0.000	8670.0	0.030	0.020	0.020	0.002	8723.2

ID	Kinked specification (equation 1)					Smooth specification (equation 2)				
	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR
53	0.619	0.008	0.213	-0.004	1743.4	2.000	0.819	0.239	0.109	1779.9
54	0.535	0.002	0.013	0.000	41072.5	0.029	0.019	0.014	0.004	40387.5
55	0.434	0.000	0.072	0.000	1988.5	0.000	0.000	0.065	0.003	2270.2
56	0.527	0.000	0.023	0.000	5274.8	0.055	0.028	0.023	0.003	5096.0
57	0.512	0.000	0.012	0.000	14786.4	0.000	0.000	0.013	0.002	14963.4
58	0.498	0.001	0.036	0.000	11056.5	0.000	0.000	0.036	0.004	10779.2
59	0.507	0.002	0.031	0.000	38749.0	0.013	0.048	0.031	0.010	38757.2
60	0.649	0.002	0.016	0.000	30672.5	0.201	0.050	0.017	0.003	29021.8
61	0.469	0.006	0.426	0.004	21083.8	0.008	0.000	0.497	0.000	20343.1
62	0.507	0.000	0.029	0.000	3662.4	0.032	0.025	0.028	0.003	3594.5
63	0.471	0.001	0.037	0.000	4925.5	0.000	0.011	0.034	0.004	5070.4
64	0.447	0.001	0.060	0.000	7680.9	0.167	0.079	0.035	0.007	7153.4
65	0.501	0.002	0.021	0.000	57470.0	0.072	0.037	0.014	0.008	53661.8
66	0.598	0.002	0.178	-0.001	1632.1	2.000	0.100	0.188	0.052	1679.0
67	0.541	0.001	0.047	0.000	7416.4	0.123	0.094	0.046	0.009	7404.5
68	0.529	0.001	0.022	0.000	15444.3	0.019	0.023	0.023	0.005	15597.7
69	0.664	0.013	0.181	-0.018	1696.9	2.000	0.901	0.215	0.067	1796.3
70	0.443	0.002	0.047	0.000	17018.4	0.037	0.041	0.039	0.007	17362.9
71	0.492	0.000	0.016	0.000	20208.6	0.022	0.012	0.014	0.002	19926.7
72	0.621	0.010	0.085	-0.002	5632.8	0.401	0.580	0.095	0.016	5884.8
73	0.503	0.005	0.056	0.000	11999.9	0.000	0.008	0.056	0.006	11912.1
74	0.505	0.001	0.007	0.000	61017.7	0.018	0.009	0.006	0.001	55991.3
75	0.533	0.000	0.024	0.000	10395.5	0.000	0.029	0.026	0.005	10487.3
76	0.479	0.001	0.034	0.000	9681.1	0.032	0.028	0.032	0.008	9742.9
77	0.503	0.003	0.097	-0.001	3446.6	0.000	0.111	0.094	0.017	3434.7
78	0.541	0.001	0.025	0.000	16917.9	0.021	0.032	0.026	0.005	17207.3

ID	Kinked specification (equation 1)					Smooth specification (equation 2)				
	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR
79	0.545	0.003	0.051	0.000	10694.4	0.260	0.148	0.043	0.009	10180.1
80	0.456	0.000	0.047	0.000	3361.5	0.044	0.035	0.037	0.004	3514.2
81	0.531	0.004	0.048	0.000	30058.7	0.000	0.000	0.052	0.010	29691.7
82	0.398	0.018	0.151	-0.005	5866.9	0.000	0.000	0.126	0.029	6001.5
83	0.503	0.001	0.088	0.000	3072.5	0.110	0.146	0.086	0.008	3063.2
84	0.531	0.000	0.012	0.000	13711.4	0.016	0.008	0.014	0.001	14506.3
85	0.412	0.004	0.311	-0.002	1100.6	0.003	0.516	0.268	0.036	1124.8
86	0.454	0.000	0.057	0.000	4028.5	0.000	0.000	0.053	0.006	4193.8
87	0.427	0.003	0.064	0.000	13038.2	0.000	0.000	0.056	0.010	13362.8
88	0.523	0.004	0.176	-0.001	1939.1	0.000	0.200	0.181	0.014	1839.8
89	0.472	0.000	0.045	0.000	5900.6	0.000	0.000	0.039	0.005	5954.6
90	0.415	0.010	0.052	-0.001	20910.8	0.048	0.036	0.040	0.011	21745.1
91	0.517	0.023	0.254	-0.011	2017.1	0.017	0.693	0.264	0.056	2021.7
92	0.528	0.106	0.271	-0.187	1864.3	0.008	0.732	0.448	0.069	1766.7
93	0.618	0.011	0.085	-0.005	14910.9	0.394	0.650	0.093	0.015	14989.4
94	0.531	0.002	0.075	0.000	6347.5	0.000	0.000	0.079	0.011	6367.8
95	0.487	0.004	0.121	-0.001	8051.7	0.001	0.085	0.118	0.024	8055.2
96	0.506	0.000	0.001	0.000	10837.6	0.005	0.001	0.001	0.000	11615.6
97	0.548	0.001	0.074	0.000	5904.8	0.000	0.000	0.077	0.003	5692.6
98	0.478	0.002	0.096	-0.001	12117.8	0.288	0.278	0.084	0.011	12052.5
99	0.564	0.001	0.067	0.000	5041.3	0.197	0.145	0.067	0.009	5119.7
100	0.572	0.001	0.047	0.000	6350.9	0.054	0.198	0.052	0.007	6747.0
101	0.496	0.000	0.081	0.000	3839.5	0.221	0.100	0.074	0.008	3771.9
102	0.420	0.006	0.041	-0.001	21916.4	0.000	0.004	0.035	0.005	23102.1
103	0.482	0.001	0.029	0.000	28582.4	0.000	0.051	0.028	0.009	28682.8
104	0.567	0.001	0.049	0.000	5552.9	0.270	0.041	0.043	0.005	5108.7



ID	Kinked specification (equation 1)					Smooth specification (equation 2)				
	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR
105	0.502	0.000	0.000	0.000	94.2	0.001	0.001	0.000	0.000	2974.5
106	0.549	0.000	0.036	0.000	7888.4	0.021	0.041	0.040	0.006	8327.6
107	0.506	0.002	0.042	0.000	18701.2	0.000	0.000	0.043	0.005	18390.4
108	0.585	0.003	0.151	-0.002	2955.2	0.000	0.002	0.170	0.024	2966.8
109	0.514	0.001	0.015	0.000	28576.3	0.003	0.013	0.017	0.003	28479.6
110	0.411	0.001	0.060	0.000	8031.6	0.031	0.042	0.049	0.003	8643.4
111	0.451	0.002	0.057	0.000	5646.3	0.000	0.000	0.053	0.005	5860.3
112	0.824	0.001	4.987	0.000	4.1	0.006	0.000	0.500	0.000	206.8
113	0.512	0.013	0.212	-0.038	2395.5	0.000	0.000	0.232	0.000	2336.1
114	0.397	0.013	0.141	-0.002	6109.0	0.502	0.499	0.105	0.016	6181.7
115	0.196	0.005	0.141	-0.001	6572.8	0.000	0.000	0.084	0.011	8502.3
116	0.484	0.007	0.063	-0.001	16169.0	0.155	0.164	0.054	0.011	16062.9
117	0.453	0.001	0.081	0.000	1832.0	0.000	0.000	0.075	0.005	1895.3
118	0.456	0.002	0.098	0.000	7506.9	0.000	0.000	0.092	0.018	7572.9
119	0.010	0.009	4.058	0.026	644.4	0.008	0.000	0.498	0.000	709.4
120	0.492	0.002	0.113	0.000	3265.6	0.317	0.370	0.104	0.020	3237.8
121	0.314	0.010	0.212	-0.002	3545.8	0.002	0.019	0.165	0.019	3864.2
122	0.238	0.054	0.458	-0.107	9399.4	0.010	0.896	0.314	0.110	9536.5
123	0.467	0.003	0.035	-0.001	19281.0	0.000	0.000	0.030	0.009	19529.1
124	0.444	0.006	0.062	-0.001	9099.8	0.041	0.050	0.054	0.005	9355.8
125	0.529	0.007	0.223	-0.003	1789.8	0.001	0.000	0.251	0.000	1790.5
126	0.469	0.003	0.059	0.000	12892.0	0.047	0.056	0.052	0.008	12942.8
127	0.487	0.001	0.030	0.000	12421.3	0.027	0.054	0.028	0.006	12399.9
128	0.519	0.001	0.090	0.000	2659.7	0.093	0.273	0.091	0.012	2666.8
129	0.531	0.000	0.044	0.000	5791.7	0.068	0.052	0.043	0.005	5780.3
130	0.495	0.003	0.282	-0.001	1369.9	2.000	0.278	0.258	0.063	1352.3

ID	Kinked specification (equation 1)					Smooth specification (equation 2)				
	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR	$\alpha$	sd( $\alpha$ )	$\rho$	sd( $\rho$ )	SSR
131	0.432	0.013	0.149	-0.003	6621.5	0.000	0.090	0.137	0.024	6505.4
132	0.495	0.001	0.020	0.000	30343.0	0.000	0.005	0.017	0.004	28020.0
133	0.536	0.001	0.099	0.000	2016.8	0.446	0.214	0.091	0.014	1982.0
134	0.487	0.003	0.028	0.000	28170.8	0.000	0.019	0.027	0.005	28227.3
135	0.588	0.002	0.023	0.000	17132.2	0.047	0.059	0.034	0.008	17985.5
136	0.503	0.000	0.003	0.000	6137.8	0.004	0.000	0.002	0.000	5938.9
137	0.462	0.001	0.064	0.000	7406.5	0.120	0.081	0.056	0.005	7396.1
138	0.344	0.003	0.136	-0.001	7809.7	0.454	0.424	0.103	0.025	8273.1
139	0.440	0.003	0.066	0.000	6619.3	0.000	0.000	0.060	0.006	6874.3
140	0.562	0.001	0.062	0.000	5290.9	0.000	0.000	0.069	0.009	5492.0
141	0.472	0.000	0.071	0.000	1263.4	0.000	0.022	0.068	0.003	1310.5
142	0.617	0.001	0.130	0.000	992.7	1.681	0.817	0.134	0.019	1021.1
143	0.477	0.001	0.032	0.000	9859.4	0.059	0.046	0.029	0.004	9852.8
144	0.706	0.002	0.045	-0.001	12871.0	0.671	0.526	0.045	0.010	13159.2
145	0.460	0.002	0.062	0.000	8268.3	0.000	0.013	0.057	0.007	8372.4
146	0.501	0.000	0.047	0.000	4834.5	0.023	0.033	0.046	0.004	4826.0
147	0.444	0.003	0.053	0.000	14004.9	0.000	0.006	0.041	0.012	14300.0
148	0.458	0.001	0.057	0.000	4039.0	0.000	0.000	0.054	0.004	4193.1
149	0.510	0.004	0.023	0.000	33400.0	0.000	0.000	0.023	0.004	32773.9
150	0.575	0.001	0.031	0.000	14230.1	0.106	0.073	0.033	0.006	14706.8
151	0.400	0.011	0.558	-0.017	686.9	2.000	0.003	0.445	0.053	693.5
152	0.465	0.000	0.089	0.000	911.5	0.129	0.083	0.080	0.005	944.3
153	0.464	0.002	0.168	-0.002	2550.3	0.000	0.200	0.153	0.028	2557.4
154	0.473	0.001	0.045	0.000	11459.0	0.069	0.046	0.042	0.006	11458.1

## Appendix VIII

### The generalized kinked specification

We continue to assume that state 2 has an objectively known probability  $\pi_2 = \frac{1}{3}$ , whereas states 1 and 3 occur with unknown probabilities  $\pi_1$  and  $\pi_3$ . The utility of a portfolio  $\mathbf{x} = (x_1, x_2, x_3)$  takes the the following form:

$$\begin{aligned} \text{I. } x_2 \leq x_{\min} & \qquad \alpha_1^1 u(x_2) + \alpha_2^1 u(x_{\min}) + \alpha_3^1 u(x_{\max}) \\ \text{II. } x_{\min} \leq x_2 \leq x_{\max} & \qquad \alpha_1^2 u(x_{\min}) + \alpha_2^2 u(x_2) + \alpha_3^2 u(x_{\max}) \\ \text{III. } x_{\max} \leq x_2 & \qquad \alpha_1^3 u(x_{\min}) + \alpha_2^3 u(x_{\max}) + \alpha_3^3 u(x_2) \end{aligned}$$

where  $x_{\min} = \min\{x_1, x_3\}$  and  $x_{\max} = \max\{x_1, x_3\}$ . This formulation (equation 3) embeds the kinked specification (equation 1) as a special case. At the end of this note, we show that, through a suitable change of variables, the generalized kinked specification can also be interpreted as reflecting Recursive Nonexpected Utility (RNEU) where the ambiguity is modeled as an *equal* probability that  $\pi_1 = \frac{2}{3}$  or  $\pi_3 = \frac{2}{3}$ . We begin by deriving the optimality conditions.

#### [1] Parameter restrictions

##### [1.1] Consistency

When  $x_2 = x_{\min}$ , consistency requires that

$$(\alpha_1^1 + \alpha_2^1) u(x_{\min}) + \alpha_3^1 u(x_{\max}) = (\alpha_1^2 + \alpha_2^2) u(x_{\min}) + \alpha_3^2 u(x_{\max}).$$

Without loss of generality we can assume that

$$\alpha_1^1 + \alpha_2^1 + \alpha_3^1 = \alpha_1^2 + \alpha_2^2 + \alpha_3^2,$$

in which case the equation preceding the last implies that

$$(\alpha_1^1 + \alpha_2^1) [u(x_{\min}) - u(x_{\max})] = (\alpha_1^2 + \alpha_2^2) [u(x_{\min}) - u(x_{\max})]$$

or

$$\alpha_1^1 + \alpha_2^1 = \alpha_1^2 + \alpha_2^2.$$

Similarly, when  $x_2 = x_{\max}$  consistency requires that

$$\alpha_2^2 + \alpha_3^2 = \alpha_2^3 + \alpha_3^3.$$

We further normalize the coefficients so that

$$\alpha_1^j + \alpha_2^j + \alpha_3^j = 1 \text{ for all } j.$$

This leads to the following:

$$\alpha_3^1 = \alpha_3^2, \alpha_1^2 = \alpha_1^3.$$

## [1.2] Reparametrization

Let

$$\begin{aligned} \alpha_1^1 &= \beta_1, \quad \alpha_1^1 + \alpha_2^1 = \beta_2, \\ \alpha_1^2 &= \beta_3, \quad \alpha_1^3 + \alpha_2^3 = \beta_4. \end{aligned}$$

Using the consistency conditions, the original coefficients are reparametrized as follows:

$$\begin{aligned} \alpha_1^1 &= \beta_1, \quad \alpha_2^1 = \beta_2 - \beta_1, \quad \alpha_3^1 = 1 - \beta_2, \\ \alpha_1^2 &= \beta_3, \quad \alpha_2^2 = \beta_2 - \beta_3, \quad \alpha_3^2 = 1 - \beta_2, \\ \alpha_1^3 &= \beta_3, \quad \alpha_2^3 = \beta_4 - \beta_3, \quad \alpha_3^3 = 1 - \beta_4. \end{aligned}$$

Note that  $\beta_1 \leq \beta_2 \leq 1$ ,  $\beta_3 \leq \beta_2$  and  $\beta_3 \leq \beta_4$ . The utility of a portfolio  $\mathbf{x} = (x_1, x_2, x_3)$  can be written with parameters  $\beta_1, \dots, \beta_4$ :

I.  $x_2 \leq x_{\min}$

$$\beta_1 u(x_2) + (\beta_2 - \beta_1) u(x_{\min}) + (1 - \beta_2) u(x_{\max})$$

II.  $x_{\min} \leq x_2 \leq x_{\max}$

$$\beta_3 u(x_{\min}) + (\beta_2 - \beta_3) u(x_2) + (1 - \beta_2) u(x_{\max})$$

III.  $x_{\max} \leq x_2$

$$\beta_3 u(x_{\min}) + (\beta_4 - \beta_3) u(x_{\max}) + (1 - \beta_4) u(x_2)$$

We adopt a simpler three-parameter model, in which the parameter  $\delta$  measures the ambiguity attitudes, the parameter  $\gamma$  measures pessimism/optimism, and  $\rho$  is the coefficient of absolute risk aversion. The mapping from the two parameters  $\delta$  and  $\gamma$  to the four parameters  $\beta_1, \dots, \beta_4$  is given by the equations

$$\begin{aligned} \beta_1 &= \frac{1}{3} + \gamma \\ \beta_2 &= \frac{2}{3} + \gamma + \delta \\ \beta_3 &= \frac{1}{3} + \gamma + \delta \\ \beta_4 &= \frac{2}{3} + \gamma, \end{aligned}$$

with  $-\frac{1}{3} < \delta, \gamma < \frac{1}{3}$  and  $-\frac{1}{3} < \delta + \gamma < \frac{1}{3}$  so that the decision weight attached to each payoff in equation 3 is nonnegative.

## [2] Optimal solutions

By the symmetry property between  $x_1$  and  $x_3$ , we know that  $x_1 \leq x_3$  if and only if  $p_1 \geq p_3$ . We can use this fact to identify the price of  $x_{\min}$  as  $p_{\max} = \max\{p_1, p_3\}$ . Similarly, we can identify the price of  $x_{\max}$  as  $p_{\min} = \min\{p_1, p_3\}$ . For the rest of the note, we denote

$$\begin{aligned} x_i &= x_{\min} \text{ and } x_j = x_{\max}, \\ p_i &= p_{\max} \text{ and } p_j = p_{\min}. \end{aligned}$$

The maximization of the generalized kinked utility function can be broken down into three sub-problems:

- **SP1:**  $x_2 \leq x_i$

$$\begin{aligned} \max_{\mathbf{x}} & \left(\frac{1}{3} + \gamma\right) u(x_2) + \left(\frac{1}{3} + \delta\right) u(x_i) + \left(\frac{1}{3} - \gamma - \delta\right) u(x_j) \\ \text{s.t. } & \mathbf{p} \cdot \mathbf{x} = 1, x_j - x_i \geq 0 \text{ and } x_i - x_2 \geq 0. \end{aligned}$$

- **SP2:**  $x_i \leq x_2 \leq x_j$

$$\begin{aligned} \max_{\mathbf{x}} & \left(\frac{1}{3} + \gamma + \delta\right) u(x_i) + \left(\frac{1}{3}\right) u(x_2) + \left(\frac{1}{3} - \gamma - \delta\right) u(x_j) \\ \text{s.t. } & \mathbf{p} \cdot \mathbf{x} = 1, x_j - x_2 \geq 0 \text{ and } x_2 - x_i \geq 0. \end{aligned}$$

- **SP3:**  $x_j \leq x_2$

$$\begin{aligned} \max_{\mathbf{x}} & \left(\frac{1}{3} + \gamma + \delta\right) u(x_i) + \left(\frac{1}{3} - \delta\right) u(x_j) + \left(\frac{1}{3} - \gamma\right) u(x_2) \\ \text{s.t. } & \mathbf{p} \cdot \mathbf{x} = 1, x_j - x_i \geq 0, \text{ and } x_2 - x_j \geq 0. \end{aligned}$$

We adopt the CARA utility function  $u(x) = -\frac{1}{\rho}e^{-\rho x}$ . Instead of characterizing the exact conditions of prices and model parameters that tell which sub-problem the optimal solution of demands belongs to, we can adopt the following two-step algorithm computing a (globally) optimal demand:

**Step 1** Given a price vector  $\mathbf{p}$  and parameter values  $(\delta, \gamma, \rho)$ , compute a (locally) optimal solution in each of the three sub-problems.

**Step 2** Compare the utilities of locally optimal solutions of three sub-problems and choose one yielding the highest utility as a (globally) optimal solution of demand.

In what follows, we characterize optimal demand with conditions on parameters in each sub-problem. Due to the fact that the CARA utility function generates a boundary solution for certain price vectors, we first set up the Lagrangian function for optimal solutions without the non-negativity condition of demand and impose that condition later, for computational ease.

[2.1] **SP1:**  $x_2 \leq x_i$

The Lagrangian function without the non-negativity condition of demand is given by

$$\begin{aligned}\mathcal{L}(\mathbf{x}) &= \left(\frac{1}{3} + \gamma\right) u(x_2) + \left(\frac{1}{3} + \delta\right) u(x_i) + \left(\frac{1}{3} - \gamma - \delta\right) u(x_j) \\ &\quad + \lambda_1(x_i - x_2) + \lambda_2(x_j - x_i) + \mu(1 - p_1x_1 - p_2x_2 - p_3x_3).\end{aligned}$$

The necessary conditions for the maximization problem are given by

$$\begin{aligned}\mathcal{L}_2(\mathbf{x}) &= \left(\frac{1}{3} + \gamma\right) \exp\{-\rho x_2\} - \lambda_1 - \mu p_2 = 0, \\ \mathcal{L}_i(\mathbf{x}) &= \left(\frac{1}{3} + \delta\right) \exp\{-\rho x_i\} + \lambda_1 - \lambda_2 - \mu p_i = 0, \\ \mathcal{L}_j(\mathbf{x}) &= \left(\frac{1}{3} - \gamma - \delta\right) \exp\{-\rho x_j\} + \lambda_2 - \mu p_j = 0, \\ \lambda_1(x_i - x_2) &= 0 = \lambda_2(x_j - x_i), \lambda_1 \geq 0, \lambda_2 \geq 0, \\ x_i - x_2 &\geq 0, x_j - x_i \geq 0, \\ 1 &= p_1x_1 + p_2x_2 + p_3x_3, \mu > 0.\end{aligned}$$

[2.1.1]  $\lambda_1 > 0$  and  $\lambda_2 > 0$

This implies that  $x_i^* = x_2^* = x_j^*$ . Then the optimal demand is given by

$$x_1^* = x_2^* = x_3^* = \frac{1}{p_1 + p_2 + p_3}.$$

For the parameter conditions leading to this solution, we need to check the following:

$$\begin{aligned}\left(\frac{1}{3} + \gamma\right) \exp(-\rho x_2) &> \mu p_2, \\ \left(\frac{1}{3} - \gamma - \delta\right) \exp(-\rho x_j) &< \mu p_j, \\ \left(\frac{2}{3} + \gamma + \delta\right) \exp(-\rho x_i) &> \mu(p_2 + p_i), \\ \left(\frac{2}{3} - \gamma\right) \exp(-\rho x_j) &< \mu(p_1 + p_3),\end{aligned}$$

which yields the following inequality conditions under the optimal solution:

$$\begin{aligned}\ln\left(\frac{p_2}{p_j}\right) &< \ln\left(\frac{\frac{1}{3} + \gamma}{\frac{1}{3} - \gamma - \delta}\right), \\ \ln\left(\frac{p_2}{p_1 + p_3}\right) &< \ln\left(\frac{\frac{1}{3} + \gamma}{\frac{2}{3} - \gamma}\right), \\ \ln\left(\frac{p_2 + p_i}{p_j}\right) &< \ln\left(\frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta}\right).\end{aligned}$$

**[2.1.2]**  $\lambda_1 = 0$  and  $\lambda_2 > 0$

This implies that  $x_1^* = x_3^* > x_2^*$ . The solution without non-negativity condition is given by

$$x_2^* = \frac{1}{p_1 + p_2 + p_3} - \frac{(p_1 + p_3)}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_1 + p_3} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{2}{3} - \gamma} \right) \right],$$

$$x_1^* = x_3^* = \frac{1}{p_1 + p_2 + p_3} + \frac{p_2}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_1 + p_3} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{2}{3} - \gamma} \right) \right].$$

The inequality conditions for this solution are given by

$$\ln \left( \frac{p_2}{p_1 + p_3} \right) > \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{2}{3} - \gamma} \right),$$

$$\ln \left( \frac{p_i}{p_j} \right) < \ln \left( \frac{\frac{1}{3} + \delta}{\frac{1}{3} - \gamma - \delta} \right).$$

If  $x_2^* \geq 0$ , then the optimal demand is

$$x_2^* = \frac{1}{p_1 + p_2 + p_3} - \frac{(p_1 + p_3)}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_1 + p_3} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{2}{3} - \gamma} \right) \right],$$

$$x_1^* = x_3^* = \frac{1}{p_1 + p_2 + p_3} + \frac{p_2}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_1 + p_3} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{2}{3} - \gamma} \right) \right].$$

If  $x_2^* < 0$ , then the optimal demand is given by

$$x_2^* = 0 \text{ and } x_1^* = x_3^* = \frac{1}{p_2 + p_3}.$$

**[2.1.3]**  $\lambda_1 > 0$  and  $\lambda_2 = 0$

This implies that  $x_2^* = x_i^* < x_j^*$ . The solution without non-negativity condition is given by

$$x_2^* = x_i^* = \frac{1}{p_1 + p_2 + p_3} - \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2 + p_i}{p_j} \right) - \ln \left( \frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta} \right) \right],$$

$$x_j^* = \frac{1}{p_1 + p_2 + p_3} + \frac{p_2 + p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2 + p_i}{p_j} \right) - \ln \left( \frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta} \right) \right].$$

The inequality condition for this solution is given by

$$\ln \left( \frac{p_2 + p_i}{p_j} \right) > \ln \left( \frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta} \right),$$

$$\ln \left( \frac{p_2}{p_i} \right) < \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{1}{3} + \delta} \right).$$

If  $x_2^* = x_i^* \geq 0$ , the optimal demand will be the same as above:

$$x_2^* = x_i^* = \frac{1}{p_1 + p_2 + p_3} - \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2 + p_i}{p_j} \right) - \ln \left( \frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta} \right) \right],$$

$$x_j^* = \frac{1}{p_1 + p_2 + p_3} + \frac{p_2 + p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2 + p_i}{p_j} \right) - \ln \left( \frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta} \right) \right].$$

If  $x_2^* = x_i^* < 0$ , the optimal demand will be

$$x_2^* = x_i^* = 0 \text{ and } x_j^* = \frac{1}{p_j}.$$

[2.1.4]  $\lambda_1 = 0$  and  $\lambda_2 = 0$

This implies that  $x_j^* > x_i^* > x_2^*$ . The solution without non-negativity condition is given by

$$x_2^* = \frac{1}{p_1 + p_2 + p_3} - \frac{p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_i} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{1}{3} + \delta} \right) \right]$$

$$- \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_j} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{1}{3} - \gamma - \delta} \right) \right],$$

$$x_i^* = \frac{1}{p_1 + p_2 + p_3} + \frac{p_2 + p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_i} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{1}{3} + \delta} \right) \right]$$

$$- \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_j} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{1}{3} - \gamma - \delta} \right) \right],$$

$$x_j^* = \frac{1}{p_1 + p_2 + p_3} - \frac{p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_i} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{1}{3} + \delta} \right) \right]$$

$$+ \frac{p_2 + p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_j} \right) - \ln \left( \frac{\frac{1}{3} + \gamma}{\frac{1}{3} - \gamma - \delta} \right) \right].$$

If the non-negativity condition for each asset is satisfied, then the above solution is the optimal demand from the problem with the non-negativity condition of demands. Otherwise, we need to further refine the problem by setting an asset violating the non-negativity condition to be zero. There are two cases to consider: (i)  $x_2^* < x_i^* < 0$ , (ii)  $x_2^* < 0$  and  $x_i^* > 0$ .

(i)  $x_2^* < x_i^* < 0$

The optimal solution is then given by

$$x_j^* = \frac{1}{p_j} \text{ and } x_2^* = x_i^* = 0.$$

(ii)  $x_2^* < 0$  and  $x_i^* > 0$



The solution to the problem by imposing that  $x_2^* = 0$  is given by

$$\begin{aligned} x_i' &= \frac{1}{p_1 + p_3} - \frac{p_j}{\rho(p_1 + p_3)} \left[ \ln\left(\frac{p_i}{p_j}\right) - \ln\left(\frac{\frac{1}{3} + \delta}{\frac{1}{3} - \gamma - \delta}\right) \right], \\ x_j' &= \frac{1}{p_1 + p_3} + \frac{p_i}{\rho(p_1 + p_3)} \left[ \ln\left(\frac{p_i}{p_j}\right) - \ln\left(\frac{\frac{1}{3} + \delta}{\frac{1}{3} - \gamma - \delta}\right) \right]. \end{aligned}$$

If  $x_i' \geq 0$ , then the solution with  $x_2^* = 0$  is the optimal one in the original problem with the non-negativity condition of demands:

$$x_2^* = 0, x_i^* = x_i' \text{ and } x_j^* = x_j'.$$

If  $x_i' < 0$ , then the optimal solution is given by

$$x_2^* = x_i^* = 0 \text{ and } x_j^* = \frac{1}{p_j}.$$

**[2.2] SP2:**  $x_i \leq x_2 \leq x_j$

The Lagrangian function without the non-negativity condition of demand is given by

$$\begin{aligned} \mathcal{L}(\mathbf{x}) &= \left(\frac{1}{3} + \gamma + \delta\right) u(x_i) + \left(\frac{1}{3}\right) u(x_2) + \left(\frac{1}{3} - \gamma - \delta\right) u(x_j) \\ &\quad + \lambda_1(x_j - x_2) + \lambda_2(x_2 - x_i) + \mu(1 - p_1x_1 - p_2x_2 - p_3x_3). \end{aligned}$$

The necessary conditions for the maximization problem are given by

$$\begin{aligned} \mathcal{L}_i(\mathbf{x}) &= \left(\frac{1}{3} + \gamma + \delta\right) \exp(-\rho x_i) - \lambda_2 - \mu p_i = 0, \\ \mathcal{L}_2(\mathbf{x}) &= \left(\frac{1}{3}\right) \exp(-\rho x_2) - \lambda_1 + \lambda_2 - \mu p_2 = 0, \\ \mathcal{L}_j(\mathbf{x}) &= \left(\frac{1}{3} - \gamma - \delta\right) \exp(-\rho x_j) + \lambda_1 - \mu p_j = 0, \\ 0 &= \lambda_2(x_2 - x_i) = \lambda_1(x_j - x_2), \lambda_1 \geq 0, \lambda_2 \geq 0, \\ x_j - x_2 &\geq 0, x_2 - x_i \geq 0, \\ \mu &> 0, 1 - p_1x_1 - p_2x_2 - p_3x_3 = 0. \end{aligned}$$

**[2.2.1]**  $\lambda_1 > 0$  and  $\lambda_2 > 0$

This implies that  $x_i^* = x_2^* = x_j^*$ . Thus, the optimal demand is given by

$$x_1^* = x_2^* = x_3^* = \frac{1}{p_1 + p_2 + p_3}.$$

We need to check the following parameter conditions for the optimal demand:

$$\begin{aligned} \left(\frac{1}{3} + \gamma + \delta\right) \exp\{-\rho x_i\} &> \mu p_i, \\ \left(\frac{1}{3} - \gamma - \delta\right) \exp\{-\rho x_j\} &< \mu p_j, \\ \left(\frac{2}{3} + \gamma + \delta\right) \exp\{-\rho x_2\} &> \mu (p_i + p_2), \\ \left(\frac{2}{3} - \gamma - \delta\right) \exp\{-\rho x_2\} &< \mu (p_2 + p_j). \end{aligned}$$

Then we have the following inequality conditions for model parameters:

$$\begin{aligned} \ln\left(\frac{p_i}{p_j}\right) &< \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta}\right), \\ \ln\left(\frac{p_i}{p_2 + p_j}\right) &< \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{2}{3} - \gamma - \delta}\right), \\ \ln\left(\frac{p_i + p_2}{p_j}\right) &< \ln\left(\frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta}\right). \end{aligned}$$

[2.2.2]  $\lambda_1 = 0$  and  $\lambda_2 > 0$

This implies that  $x_2^* = x_i^* < x_j^*$ . The optimal demand without the non-negativity condition is given by

$$\begin{aligned} x_2^* = x_i^* &= \frac{1}{p_1 + p_2 + p_3} - \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_i + p_2}{p_j}\right) - \ln\left(\frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta}\right) \right], \\ x_j^* &= \frac{1}{p_1 + p_2 + p_3} + \frac{p_2 + p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_i + p_2}{p_j}\right) - \ln\left(\frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta}\right) \right]. \end{aligned}$$

The parameter condition for this solution is given by

$$\begin{aligned} \ln\left(\frac{p_i + p_2}{p_j}\right) &> \ln\left(\frac{\frac{2}{3} + \gamma + \delta}{\frac{1}{3} - \gamma - \delta}\right), \\ \ln\left(\frac{p_i}{p_2}\right) &< \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3}}\right). \end{aligned}$$

If  $x_2^* = x_i^* \geq 0$ , then the above solution is the optimal one from the original maximization problem. Otherwise, the optimal solution with the non-negativity condition is given by

$$x_2^* = x_i^* = 0 \text{ and } x_j^* = \frac{1}{p_j}.$$

**[2.2.3]**  $\lambda_1 > 0$  and  $\lambda_2 = 0$

This implies that  $x_j^* = x_2^* > x_i^*$ . The optimal demand without the non-negativity condition is given by

$$x_j^* = x_2^* = \frac{1}{p_1 + p_2 + p_3} + \frac{p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_i}{p_2 + p_j} \right) - \ln \left( \frac{\frac{1}{3} + \gamma + \delta}{\frac{2}{3} - \gamma - \delta} \right) \right],$$

$$x_i^* = \frac{1}{p_1 + p_2 + p_3} - \frac{p_2 + p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_i}{p_2 + p_j} \right) - \ln \left( \frac{\frac{1}{3} + \gamma + \delta}{\frac{2}{3} - \gamma - \delta} \right) \right].$$

The parameter condition for this solution is given by

$$\ln \left( \frac{p_i}{p_2 + p_j} \right) > \ln \left( \frac{\frac{1}{3} + \gamma + \delta}{\frac{2}{3} - \gamma - \delta} \right),$$

$$\ln \left( \frac{p_2}{p_j} \right) < \ln \left( \frac{\frac{1}{3}}{\frac{1}{3} - \gamma - \delta} \right).$$

If  $x_i^* \geq 0$ , the optimal demand from the original problem will be the same as above. Otherwise, the optimal demand with the non-negativity condition is

$$x_i^* = 0 \text{ and } x_2^* = x_j^* = \frac{1}{p_2 + p_j}.$$

**[2.2.4]**  $\lambda_1 = 0$  and  $\lambda_2 = 0$

This implies that  $x_j^* > x_2^* > x_i^*$ . The optimal solution without the non-negativity condition is given by

$$x_i^* = \frac{1}{p_1 + p_2 + p_3} - \frac{(p_2 + p_j)}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_i}{p_2} \right) - \ln \left( \frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3}} \right) \right]$$

$$- \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_j} \right) - \ln \left( \frac{\frac{1}{3}}{\frac{2}{3} - \gamma - \delta} \right) \right],$$

$$x_2^* = \frac{1}{p_1 + p_2 + p_3} + \frac{p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_i}{p_2} \right) - \ln \left( \frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3}} \right) \right]$$

$$- \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_j} \right) - \ln \left( \frac{\frac{1}{3}}{\frac{2}{3} - \gamma - \delta} \right) \right],$$

$$x_j^* = \frac{1}{p_1 + p_2 + p_3} + \frac{p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_i}{p_2} \right) - \ln \left( \frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3}} \right) \right]$$

$$+ \frac{p_i + p_2}{\rho(p_1 + p_2 + p_3)} \left[ \ln \left( \frac{p_2}{p_j} \right) - \ln \left( \frac{\frac{1}{3}}{\frac{2}{3} - \gamma - \delta} \right) \right].$$

If the non-negativity condition for each asset is satisfied, then the above solution is the optimal demand from the problem with the non-negativity condition of demands. Otherwise, we need to further refine the problem by setting an asset violating the non-negativity condition to be zero. There are two cases to consider: (i)  $x_i^* < x_2^* < 0$ , (ii)  $x_i^* < 0$  and  $x_2^* > 0$ .

(i)  $x_i^* < x_2^* < 0$

The optimal solution is then given by

$$x_i^* = x_2^* = 0 \text{ and } x_j^* = \frac{1}{p_j}.$$

(ii)  $x_i^* < 0$  and  $x_2^* > 0$

By imposing that  $x_i^* = 0$ , we have the new solution as

$$x_2' = \frac{1}{p_2 + p_j} - \frac{p_j}{\rho(p_2 + p_j)} \left[ \ln \left( \frac{p_2}{p_j} \right) - \ln \left( \frac{\frac{1}{3}}{\frac{2}{3} - \gamma - \delta} \right) \right],$$

$$x_j' = \frac{1}{p_2 + p_j} + \frac{p_2}{\rho(p_2 + p_j)} \left[ \ln \left( \frac{p_2}{p_j} \right) - \ln \left( \frac{\frac{1}{3}}{\frac{2}{3} - \gamma - \delta} \right) \right].$$

If  $x_2' \geq 0$ , then the optimal demand from the original problem will be

$$x_i^* = 0, x_2^* = x_2' \text{ and } x_j^* = x_j'.$$

If  $x_2' < 0$ , then the optimal demand will be

$$x_i^* = x_2^* = 0 \text{ and } x_j^* = \frac{1}{p_j}.$$

**[2.3] SP3:**  $x_j \leq x_2$

The Lagrangian function without the non-negativity condition is given by

$$\mathcal{L}(\mathbf{x}) = \left( \frac{1}{3} + \gamma + \delta \right) u(x_i) + \left( \frac{1}{3} - \delta \right) u(x_j) + \left( \frac{1}{3} - \gamma \right) u(x_2) \\ + \lambda_1 (x_2 - x_j) + \lambda_2 (x_j - x_i) + \mu (1 - p_1 x_1 - p_2 x_2 - p_3 x_3).$$

The necessary conditions for the maximization problem are given by

$$\mathcal{L}_i(\mathbf{x}) = \left( \frac{1}{3} + \gamma + \delta \right) \exp(-\rho x_i) - \lambda_2 - \mu p_i = 0,$$

$$\mathcal{L}_j(\mathbf{x}) = \left( \frac{1}{3} - \delta \right) \exp(-\rho x_j) - \lambda_1 + \lambda_2 - \mu p_j = 0,$$

$$\mathcal{L}_2(\mathbf{x}) = \left( \frac{1}{3} - \gamma \right) \exp(-\rho x_2) + \lambda_1 - \mu p_2 = 0,$$

$$0 = \lambda_1 (x_2 - x_j) = \lambda_2 (x_j - x_i), \lambda_1, \lambda_2 \geq 0,$$

$$\mu > 0 \text{ and } 1 - p_1 x_1 - p_2 x_2 - p_3 x_3 = 0.$$

**[2.3.1]**  $\lambda_1 > 0$  and  $\lambda_2 > 0$

This implies that  $x_2^* = x_j^* = x_i^*$ . The optimal solution from the original problem is then given by

$$x_1^* = x_2^* = x_3^* = \frac{1}{p_1 + p_2 + p_3}.$$

The parameter conditions for this solution are given by

$$\begin{aligned} \ln\left(\frac{p_i}{p_2}\right) &< \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3} - \gamma}\right), \\ \ln\left(\frac{p_i}{p_2 + p_j}\right) &< \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{2}{3} - \gamma - \delta}\right), \\ \ln\left(\frac{p_1 + p_3}{p_2}\right) &< \ln\left(\frac{\frac{2}{3} + \gamma}{\frac{1}{3} - \gamma}\right). \end{aligned}$$

**[2.3.2]**  $\lambda_1 = 0$  and  $\lambda_2 > 0$

This implies that  $x_j^* = x_i^* < x_2^*$ . The optimal solution without the non-negativity condition is given by

$$\begin{aligned} x_1^* = x_3^* &= \frac{1}{p_1 + p_2 + p_3} - \frac{p_2}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_1 + p_3}{p_2}\right) - \ln\left(\frac{\frac{2}{3} + \gamma}{\frac{1}{3} - \gamma}\right) \right], \\ x_2^* &= \frac{1}{p_1 + p_2 + p_3} + \frac{(p_1 + p_3)}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_1 + p_3}{p_2}\right) - \ln\left(\frac{\frac{2}{3} + \gamma}{\frac{1}{3} - \gamma}\right) \right]. \end{aligned}$$

The parameter conditions for this solution are given by

$$\begin{aligned} \ln\left(\frac{p_1 + p_3}{p_2}\right) &> \ln\left(\frac{\frac{2}{3} + \gamma}{\frac{1}{3} - \gamma}\right), \\ \ln\left(\frac{p_i}{p_j}\right) &< \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3} - \delta}\right). \end{aligned}$$

If  $x_1^* = x_3^* \geq 0$ , then the optimal solution from the original problem is the same as above. Otherwise, the optimal demand with the non-negativity condition is given by

$$x_1^* = x_3^* = 0 \text{ and } x_2^* = \frac{1}{p_2}.$$

**[2.3.3]**  $\lambda_1 > 0$  and  $\lambda_2 = 0$

This implies that  $x_2^* = x_j^* > x_i^*$ . The optimal demand without the non-negativity condition is given by

$$\begin{aligned} x_i^* &= \frac{1}{p_1 + p_2 + p_3} - \frac{(p_2 + p_j)}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_i}{p_2 + p_j}\right) - \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{2}{3} - \gamma - \delta}\right) \right], \\ x_2^* = x_j^* &= \frac{1}{p_1 + p_2 + p_3} + \frac{p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_i}{p_2 + p_j}\right) - \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{2}{3} - \gamma - \delta}\right) \right]. \end{aligned}$$

The parameter condition for this solution is given by

$$\begin{aligned}\ln\left(\frac{p_i}{p_2 + p_j}\right) &> \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{2}{3} - \gamma - \delta}\right), \\ \ln\left(\frac{p_j}{p_2}\right) &< \ln\left(\frac{\frac{1}{3} - \delta}{\frac{1}{3} - \gamma}\right).\end{aligned}$$

If  $x_i^* \geq 0$ , then the optimal demand from the original problem is the same as above. Otherwise, the optimal demand with the non-negativity condition is given by

$$x_i^* = 0 \text{ and } x_2^* = x_j^* = \frac{1}{p_2 + p_j}.$$

**[2.3.4]**  $\lambda_1 = 0$  and  $\lambda_2 = 0$

The conditions imply that  $x_2^* > x_j^* > x_i^*$ . The optimal demand without the non-negativity condition is given by

$$\begin{aligned}x_2 &= \frac{1}{p_1 + p_2 + p_3} + \frac{p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_i}{p_2}\right) - \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3} - \gamma}\right) \right] \\ &\quad + \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_j}{p_2}\right) - \ln\left(\frac{\frac{1}{3} - \delta}{\frac{1}{3} - \gamma}\right) \right], \\ x_j &= \frac{1}{p_1 + p_2 + p_3} + \frac{p_i}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_i}{p_2}\right) - \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3} - \gamma}\right) \right] \\ &\quad - \frac{(p_2 + p_i)}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_j}{p_2}\right) - \ln\left(\frac{\frac{1}{3} - \delta}{\frac{1}{3} - \gamma}\right) \right], \\ x_i &= \frac{1}{p_1 + p_2 + p_3} - \frac{(p_2 + p_j)}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_i}{p_2}\right) - \ln\left(\frac{\frac{1}{3} + \gamma + \delta}{\frac{1}{3} - \gamma}\right) \right] \\ &\quad + \frac{p_j}{\rho(p_1 + p_2 + p_3)} \left[ \ln\left(\frac{p_j}{p_2}\right) - \ln\left(\frac{\frac{1}{3} - \delta}{\frac{1}{3} - \gamma}\right) \right].\end{aligned}$$

If the non-negativity condition for each asset is satisfied, then the above solution is the optimal demand from the problem with the non-negativity condition of demands. Otherwise, we need to further refine the problem by setting an asset violating the non-negativity condition to be zero. There are two cases to consider: (i)  $x_i^* < x_j^* < 0$ , (ii)  $x_i^* < 0$  and  $x_j^* > 0$ .

(i)  $x_i^* < x_j^* < 0$

Then the optimal solution from the original problem is given by

$$x_1^* = x_3^* = 0 \text{ and } x_2^* = \frac{1}{p_2}.$$

(ii)  $x_i^* < 0$  and  $x_j^* > 0$

By imposing that  $x_i^* = 0$ , we have the following new solution as

$$\begin{aligned} x'_2 &= \frac{1}{p_2 + p_j} + \frac{p_j}{\rho(p_2 + p_j)} \left[ \ln \left( \frac{p_j}{p_2} \right) - \ln \left( \frac{\frac{1}{3} - \delta}{\frac{1}{3} - \gamma} \right) \right], \\ x'_j &= \frac{1}{p_2 + p_j} - \frac{p_2}{\rho(p_2 + p_j)} \left[ \ln \left( \frac{p_j}{p_2} \right) - \ln \left( \frac{\frac{1}{3} - \delta}{\frac{1}{3} - \gamma} \right) \right]. \end{aligned}$$

If  $x'_j \geq 0$ , then the optimal demand from the original problem is given by

$$x_i^* = 0, x_j^* = x'_j \text{ and } x_2^* = x'_2.$$

If  $x'_j < 0$ , then the optimal demand from the original problem is given by

$$x_1^* = x_3^* = 0 \text{ and } x_2^* = \frac{1}{p_2}.$$

#### [2.4] Non-uniqueness of the optimal demand

Finally we note that when  $\delta < 0$  and/or  $\gamma < 0$ , the optimal demand is not unique when  $p_k = p_{k'}$  for some  $k \neq k' = 1, 2, 3$  because the generalized kinked utility function is not quasi-convex everywhere. Nevertheless, the utility function is not quasi-convex in each sub-problem. The above characterization of the optimal demands incorporates the cases of non-uniqueness.

### [3] Recursive Nonexpected Utility (RNEU)

Finally, we show that the generalized kinked specification can also be interpreted as reflecting a special case of RNEU where there is an equal probability that  $\pi_1 = \frac{2}{3}$  or  $\pi_3 = \frac{2}{3}$ . Consider the following two-stage recursive Rank-Dependent Utility (RDU) model. Given a fixed underlying distribution  $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3)$ , the first-stage rank-dependent expected utility  $V_{\boldsymbol{\pi}}$  is given by

$$\begin{aligned} V_{(\frac{2}{3}, \frac{1}{3}, 0)}(\mathbf{x}) &= [1 - w(\frac{1}{3})] \max\{u(x_1), u(x_2)\} + w(\frac{1}{3}) \min\{u(x_1), u(x_2)\}, \\ V_{(0, \frac{1}{3}, \frac{2}{3})}(\mathbf{x}) &= [1 - w(\frac{1}{3})] \max\{u(x_2), u(x_3)\} + w(\frac{1}{3}) \min\{u(x_2), u(x_3)\}. \end{aligned}$$

The second stage takes the rank-dependent expectation of the first-stage rank-dependent expected utilities:

$$\begin{aligned} U(\mathbf{x}) &= [1 - w(\frac{1}{2})] \max \left\{ V_{(\frac{2}{3}, \frac{1}{3}, 0)}(\mathbf{x}), V_{(0, \frac{1}{3}, \frac{2}{3})}(\mathbf{x}) \right\} \\ &\quad + w(\frac{1}{2}) \min \left\{ V_{(\frac{2}{3}, \frac{1}{3}, 0)}(\mathbf{x}), V_{(0, \frac{1}{3}, \frac{2}{3})}(\mathbf{x}) \right\}, \end{aligned}$$

and the decision weights can be expressed as follows:

$$\begin{aligned} \beta_1 &= w(\frac{1}{3}), \\ \beta_2 - \beta_1 &= w(\frac{1}{2})[1 - w(\frac{1}{3})], \\ \beta_3 &= w(\frac{1}{2})w(\frac{1}{3}), \\ \beta_4 - \beta_3 &= [1 - w(\frac{1}{2})]w(\frac{1}{3}). \end{aligned}$$

Now consider the three relevant cases:

I.  $x_2 \leq x_{\min}$

$$\begin{aligned} U(\mathbf{x}) &= [1 - w(\frac{1}{2})] \left\{ [1 - w(\frac{1}{3})]u(x_{\max}) + w(\frac{1}{3})u(x_2) \right\} \\ &\quad + w(\frac{1}{2}) \left\{ [1 - w(\frac{1}{3})]u(x_{\min}) + w(\frac{1}{3})u(x_2) \right\}. \end{aligned}$$

Rearranging,

$$U(\mathbf{x}) = \beta_1 u(x_2) + (\beta_2 - \beta_3) u(x_{\min}) + (1 - \beta_2) u(x_{\max}).$$

II.  $x_{\min} \leq x_2 \leq x_{\max}$

$$\begin{aligned} U(\mathbf{x}) &= [1 - w(\frac{1}{2})] \left\{ [1 - w(\frac{1}{3})]u(x_{\max}) + w(\frac{1}{3})u(x_2) \right\} \\ &\quad + w(\frac{1}{2}) \left\{ [1 - w(\frac{1}{3})]u(x_2) + w(\frac{1}{3})u(x_{\min}) \right\}. \end{aligned}$$

Rearranging,

$$U(\mathbf{x}) = \beta_3 u(x_{\min}) + (\beta_2 - \beta_3) u(x_2) + (1 - \beta_2) u(x_{\max}).$$

III.  $x_{\max} \leq x_2$

$$\begin{aligned} U(\mathbf{x}) &= [1 - w(\frac{1}{2})] \left\{ [1 - w(\frac{1}{3})]u(x_2) + w(\frac{1}{3})u(x_{\max}) \right\} \\ &\quad + w(\frac{1}{2}) \left\{ [1 - w(\frac{1}{3})]u(x_2) + w(\frac{1}{3})u(x_{\min}) \right\}. \end{aligned}$$

Rearranging,

$$U(\mathbf{x}) = \beta_3 u(x_{\min}) + (\beta_4 - \beta_3) u(x_{\max}) + (1 - \beta_4) u(x_2).$$

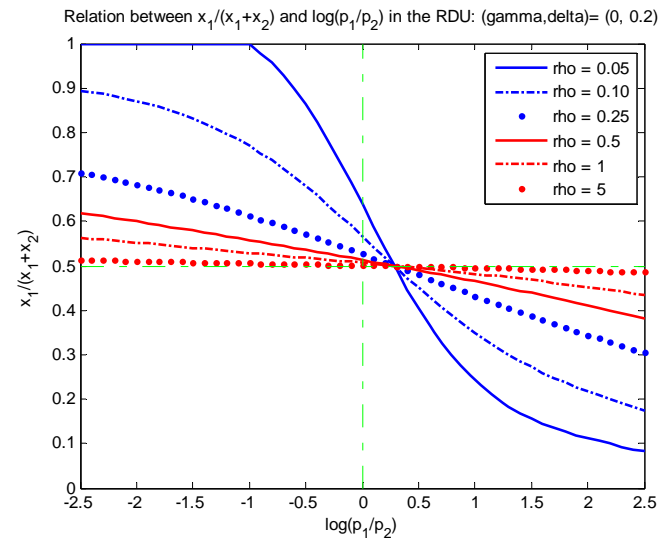
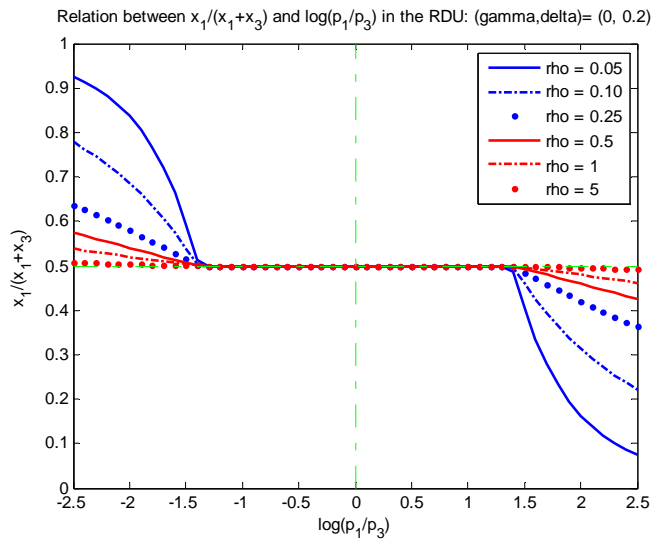
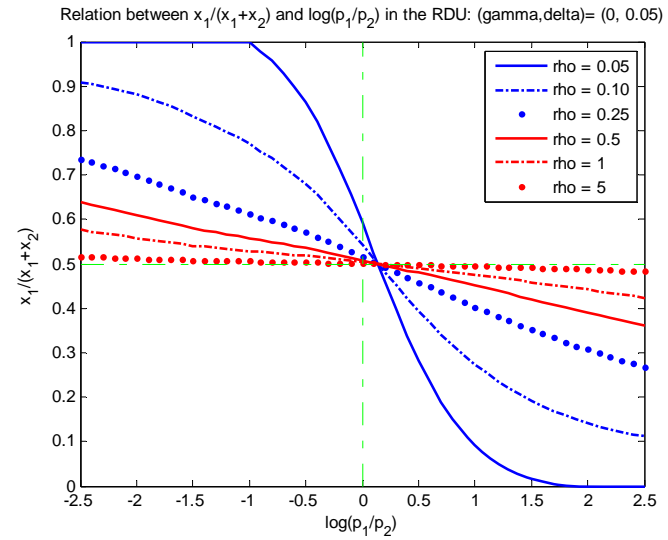
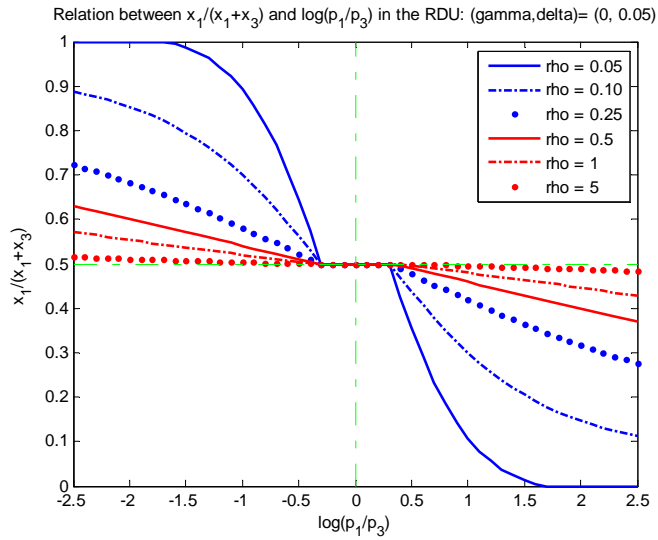


## Appendix IX

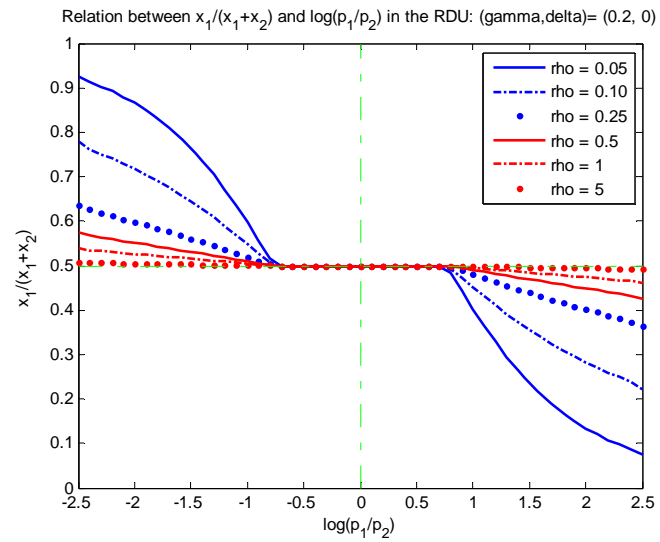
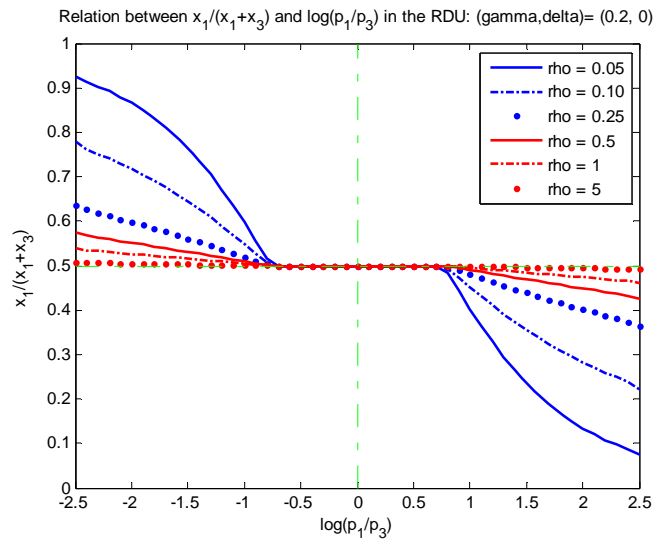
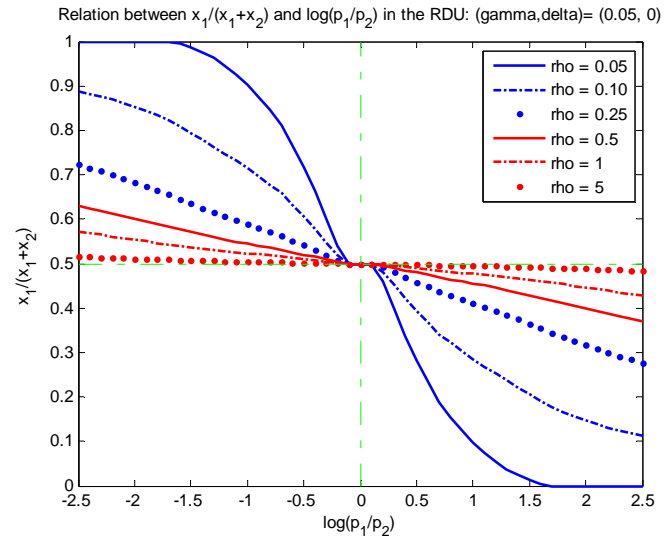
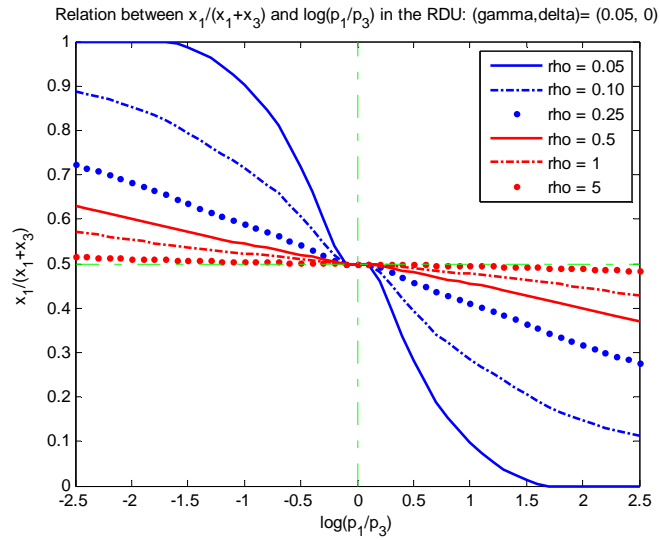
### An illustration of the relationships between log-price ratio and optimal token share

#### The generalized kinked specification (equation 3)

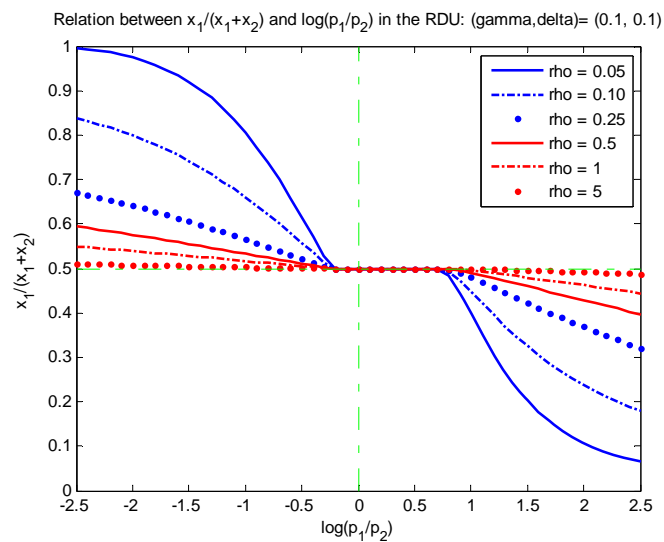
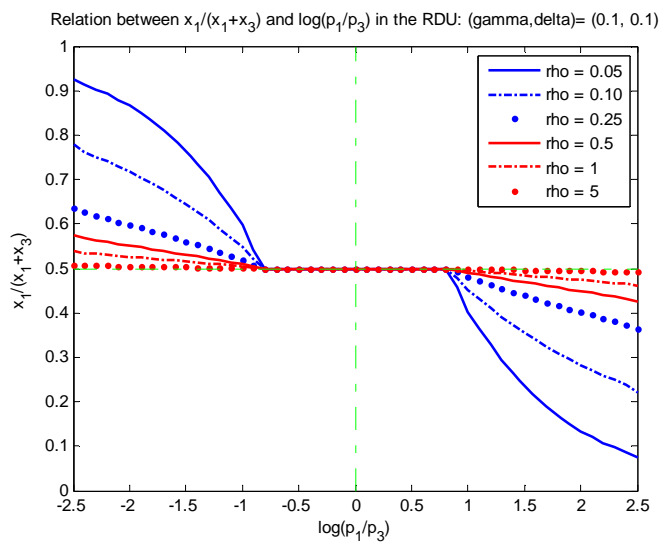
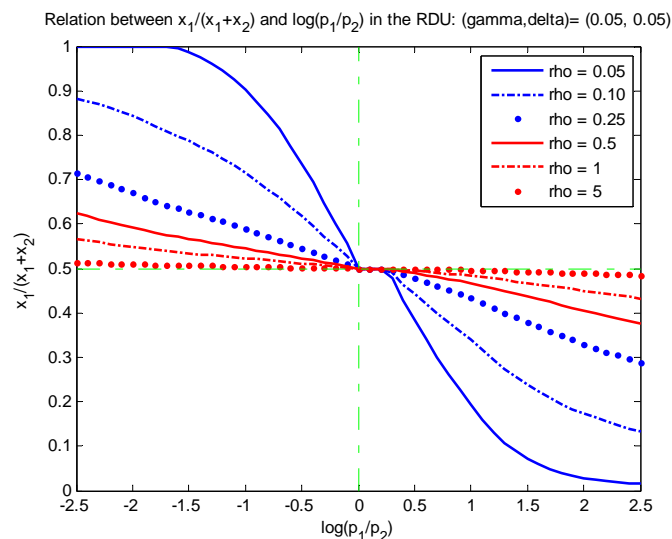
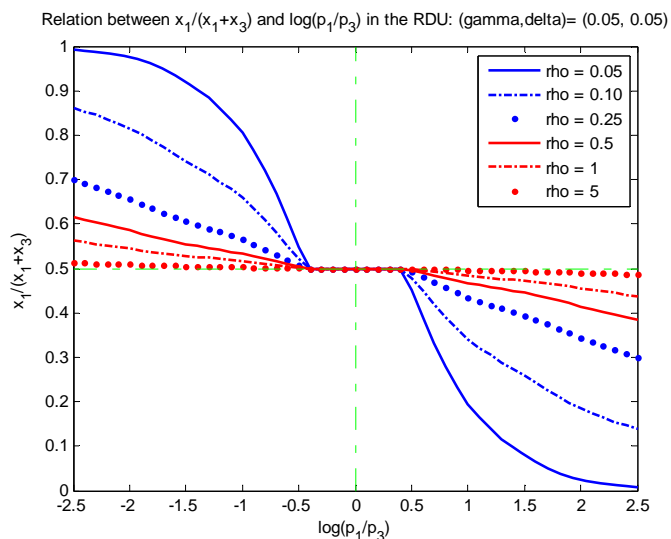
$$\gamma = 0 \text{ and } \delta > 0$$



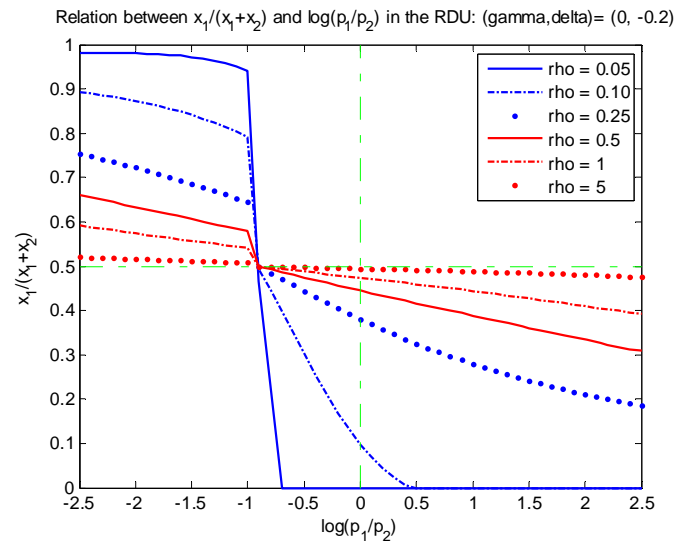
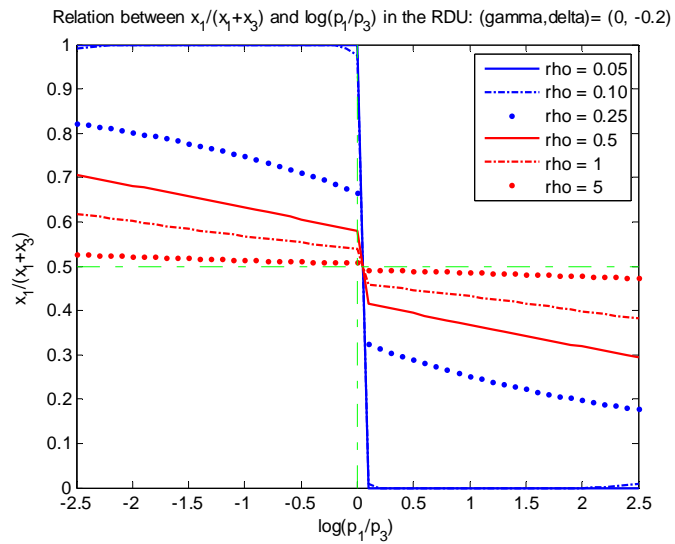
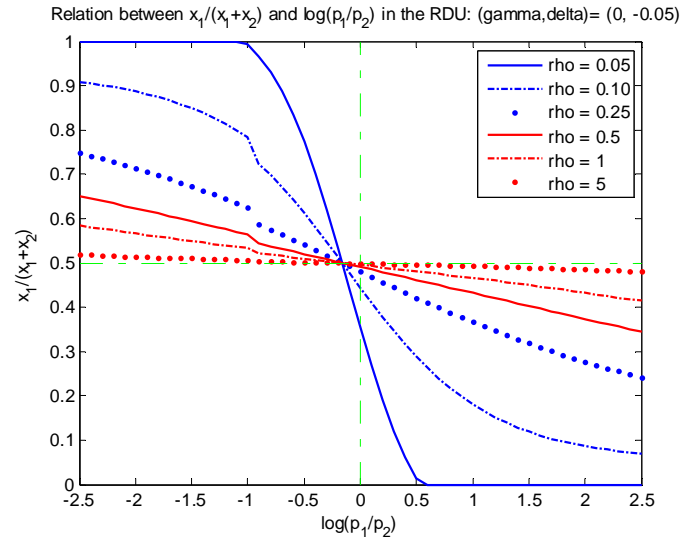
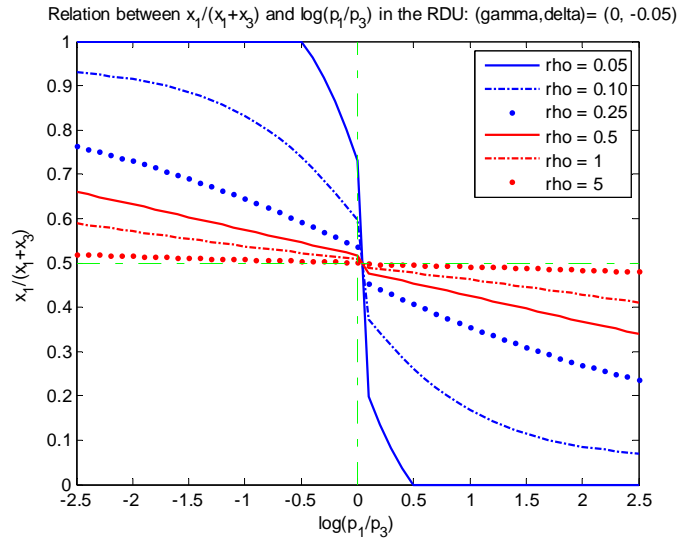
$\gamma > 0$  and  $\delta = 0$



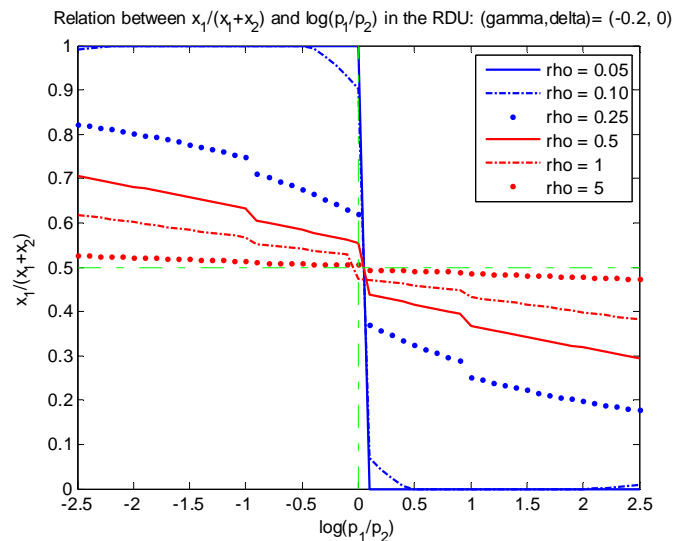
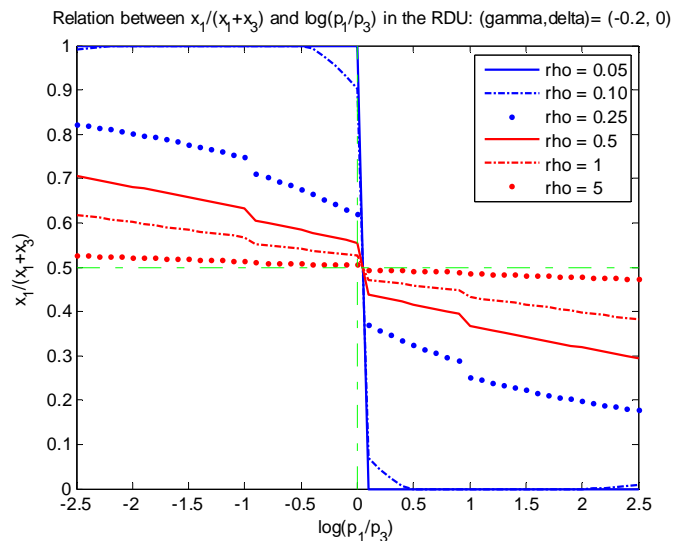
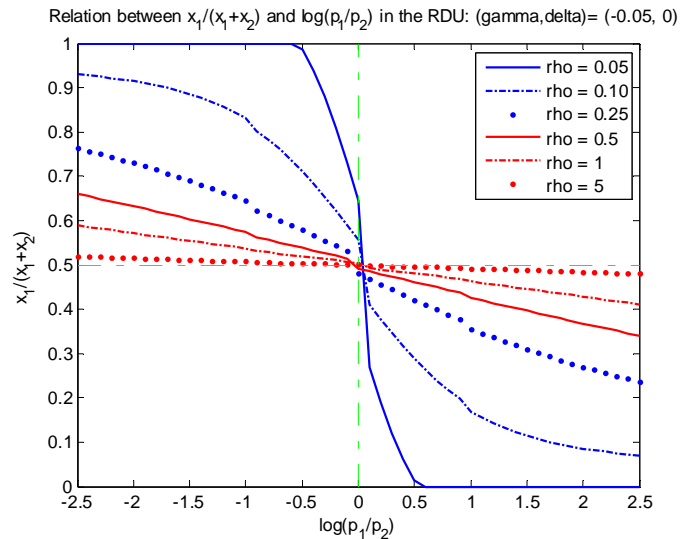
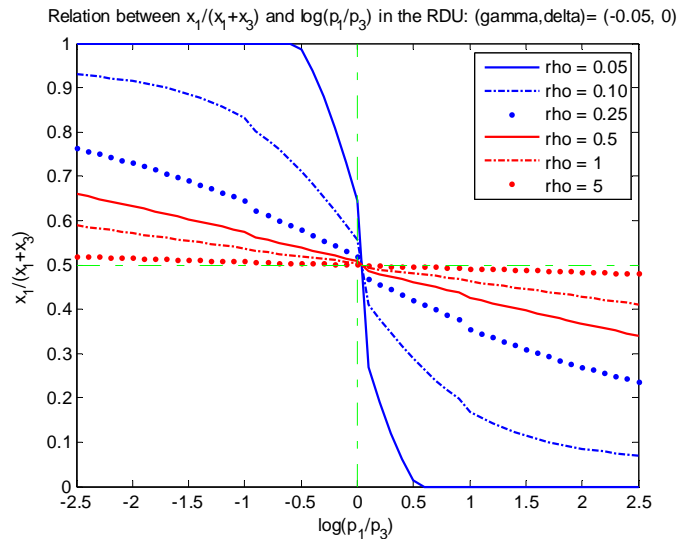
$\gamma > 0$  and  $\delta > 0$



$\gamma = 0$  and  $\delta < 0$

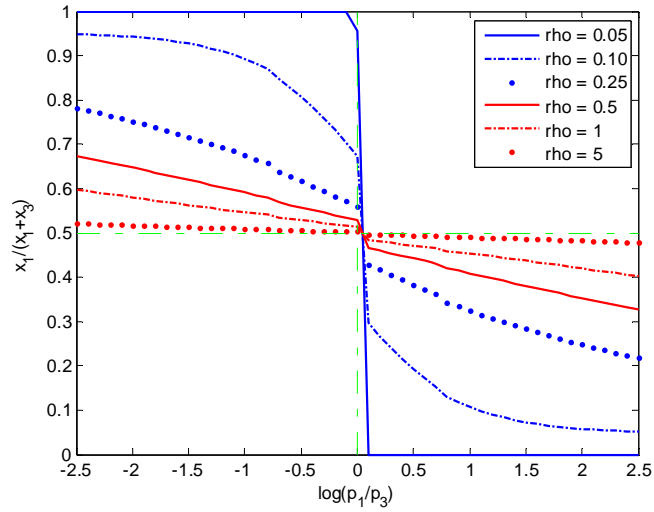


$\gamma < 0$  and  $\delta = 0$

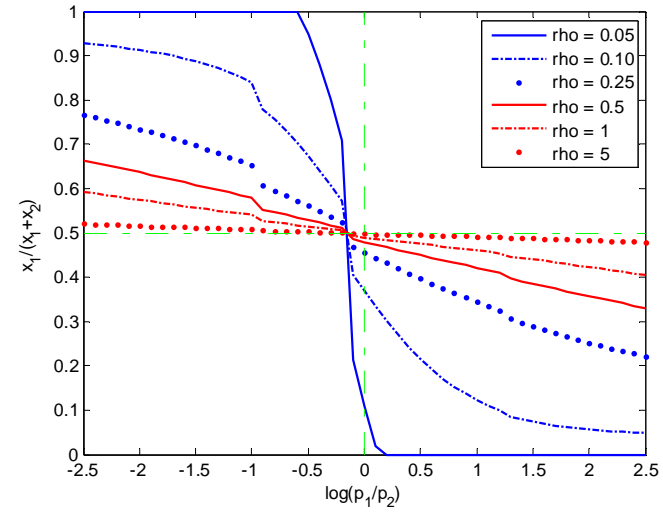


$\gamma < 0$  and  $\delta < 0$

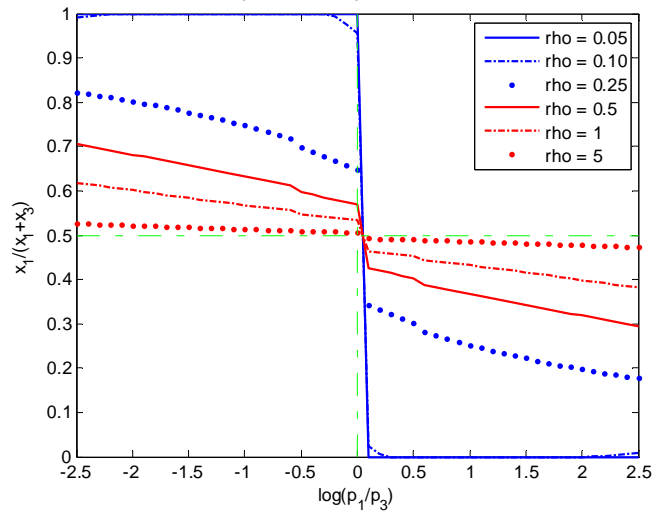
Relation between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  in the RDU:  $(\gamma, \delta) = (-0.05, -0.05)$



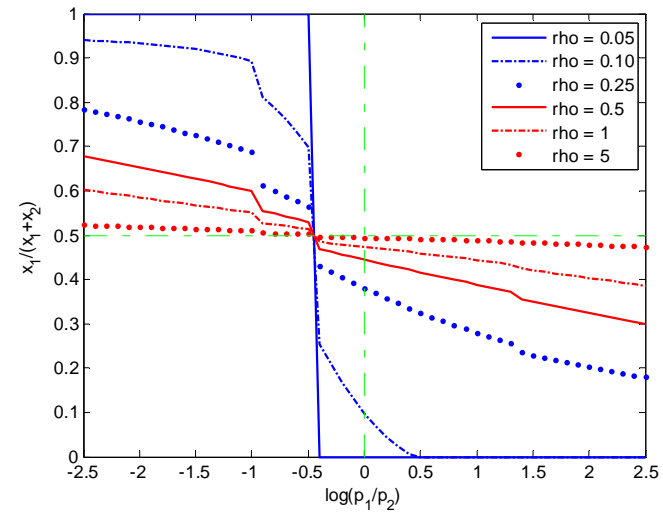
Relation between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  in the RDU:  $(\gamma, \delta) = (-0.05, -0.05)$



Relation between  $x_1/(x_1+x_3)$  and  $\log(p_1/p_3)$  in the RDU:  $(\gamma, \delta) = (-0.1, -0.1)$



Relation between  $x_1/(x_1+x_2)$  and  $\log(p_1/p_2)$  in the RDU:  $(\gamma, \delta) = (-0.1, -0.1)$



**Appendix X**  
**Individual-level estimation results -- generalized kinked specification (equation 3)**

ID	$\rho$	sd( $\rho$ )	$\gamma$	sd( $\gamma$ )	$\delta$	sd( $\delta$ )	SSR
1	0.293	0.029	-0.252	0.003	0.081	0.004	18442.9
2	0.109	0.692	0.286	0.008	-0.025	0.007	596.3
3	0.097	0.002	0.052	0.005	0.057	0.005	12042.9
4	0.032	0.001	0.021	0.010	0.049	0.001	11618.2
5	0.102	0.000	-0.122	0.002	-0.029	0.001	13805.5
6	0.039	0.000	-0.012	0.001	0.044	0.001	9453.9
7	0.031	0.000	-0.111	0.014	0.006	0.002	59411.6
8	0.139	0.001	-0.138	0.002	0.004	0.001	3885.3
9	0.027	0.000	-0.036	0.001	0.046	0.000	14628.9
10	0.021	0.000	0.012	0.004	0.040	0.002	19214.2
11	5.000	0.006	0.323	0.003	0.001	0.003	4.8
12	0.044	0.002	0.043	0.024	0.048	0.004	5776.1
13	0.037	0.000	0.118	0.003	-0.001	0.008	9296.8
14	0.002	0.000	0.015	0.000	0.005	0.000	2081.5
15	0.050	0.085	0.131	0.023	0.193	0.010	8377.8
16	0.023	0.000	-0.009	0.001	0.052	0.001	25665.0
17	0.057	0.703	0.323	0.056	0.000	0.004	457.6
18	0.010	0.000	-0.152	0.016	-0.006	0.000	57096.1
19	0.213	0.012	-0.188	0.003	0.062	0.002	4472.5
20	0.001	0.000	-0.003	0.003	0.013	0.000	41006.7
21	0.106	0.001	-0.141	0.002	-0.020	0.001	16494.4
22	0.220	0.003	-0.090	0.005	-0.023	0.001	1825.8
23	1.448	0.389	-0.299	0.018	-0.025	0.007	7520.6
24	0.276	1.204	0.323	0.023	-0.088	0.004	179.8
25	0.039	0.000	0.001	0.004	0.114	0.006	10841.7
26	0.047	0.000	-0.040	0.001	0.010	0.000	4406.2

ID	$\rho$	sd( $\rho$ )	$\gamma$	sd( $\gamma$ )	$\delta$	sd( $\delta$ )	SSR
27	0.030	0.000	0.017	0.002	0.009	0.000	7012.4
28	0.032	0.001	-0.166	0.010	0.013	0.000	17680.9
29	0.016	0.000	-0.017	0.001	0.092	0.001	42355.4
30	0.079	0.347	0.225	0.020	0.026	0.009	1480.4
31	0.100	0.000	-0.077	0.000	-0.003	0.000	1127.7
32	0.568	0.030	-0.316	0.006	0.305	0.002	1633.6
33	0.041	0.000	0.085	0.003	0.056	0.005	12664.6
34	0.015	0.000	0.130	0.003	0.159	0.003	9716.9
35	0.052	0.000	-0.129	0.004	0.015	0.001	30261.5
36	0.086	0.002	-0.070	0.003	0.107	0.004	11599.0
37	0.087	0.011	-0.032	0.003	0.255	0.002	4389.6
38	0.032	0.002	-0.053	0.003	0.253	0.006	24510.5
39	0.095	0.000	-0.096	0.002	0.036	0.001	6341.2
40	0.011	0.000	0.148	0.001	-0.022	0.000	4204.6
41	0.009	0.000	0.019	0.003	0.096	0.001	41385.3
42	0.137	0.001	0.002	0.003	-0.065	0.001	3252.9
43	0.086	0.000	0.063	0.005	0.012	0.003	1983.7
44	0.010	0.000	0.225	0.008	-0.111	0.003	6580.7
45	0.004	0.000	0.039	0.000	0.007	0.000	16132.3
46	0.015	0.000	0.101	0.006	0.063	0.013	26264.2
47	0.020	0.000	0.016	0.004	-0.010	0.002	46993.8
48	0.013	0.000	0.060	0.015	-0.056	0.004	44514.5
49	0.051	0.000	0.004	0.002	0.098	0.003	5707.7
50	0.022	0.000	0.130	0.001	0.139	0.001	5384.2
51	0.064	0.000	-0.047	0.012	0.002	0.004	17283.8
52	0.021	0.000	-0.002	0.000	0.018	0.000	8668.7



ID	$\rho$	sd( $\rho$ )	$\gamma$	sd( $\gamma$ )	$\delta$	sd( $\delta$ )	SSR
53	0.036	0.212	0.271	0.030	0.052	0.009	1651.0
54	0.016	0.000	-0.019	0.004	0.027	0.001	40802.5
55	0.091	0.000	-0.084	0.001	-0.021	0.000	1760.3
56	0.022	0.000	0.007	0.000	0.017	0.000	5259.3
57	0.005	0.000	0.080	0.001	0.006	0.000	10423.0
58	0.037	0.000	-0.003	0.004	0.000	0.001	11055.3
59	0.036	0.001	-0.042	0.022	0.016	0.003	38103.3
60	0.017	0.000	-0.009	0.001	0.102	0.001	30583.0
61	0.385	0.007	-0.323	0.001	0.200	0.005	17601.3
62	0.028	0.000	0.004	0.000	0.002	0.000	3654.5
63	0.051	0.000	-0.067	0.001	-0.009	0.000	4618.2
64	0.074	0.000	-0.088	0.010	0.003	0.008	7208.8
65	0.031	0.000	-0.144	0.007	0.048	0.001	50360.0
66	0.204	0.011	-0.035	0.003	0.078	0.002	1621.1
67	0.043	0.000	0.017	0.002	0.028	0.001	7362.4
68	0.022	0.000	-0.002	0.002	0.020	0.001	15442.5
69	0.140	0.040	0.062	0.017	0.096	0.008	1630.0
70	0.049	0.000	-0.016	0.005	-0.033	0.002	16992.4
71	0.021	0.000	-0.032	0.003	-0.005	0.001	19541.0
72	0.088	0.001	-0.007	0.014	0.081	0.005	5630.0
73	0.068	0.000	-0.073	0.002	0.025	0.001	11404.2
74	0.014	0.000	-0.180	0.011	0.013	0.001	54938.5
75	0.015	0.000	0.057	0.002	0.014	0.000	9666.1
76	0.027	0.000	0.033	0.002	-0.026	0.001	9555.1
77	0.105	0.001	-0.032	0.012	0.018	0.003	3425.0
78	0.014	0.000	0.089	0.002	-0.010	0.000	14459.0

ID	$\rho$	sd( $\rho$ )	$\gamma$	sd( $\gamma$ )	$\delta$	sd( $\delta$ )	SSR
79	0.059	0.000	-0.048	0.002	0.041	0.002	10206.0
80	0.071	0.000	-0.116	0.000	-0.009	0.000	2273.5
81	0.002	0.001	0.235	0.005	-0.059	0.005	26524.4
82	0.122	0.018	0.089	0.021	-0.112	0.004	5840.3
83	0.095	0.000	-0.022	0.001	0.011	0.000	3058.2
84	0.006	0.000	0.043	0.001	0.023	0.000	11495.4
85	0.385	0.012	-0.106	0.009	-0.009	0.004	1063.8
86	0.076	0.000	-0.085	0.001	-0.012	0.000	3630.4
87	0.061	0.001	0.018	0.008	-0.056	0.002	13027.5
88	0.077	0.003	0.217	0.008	-0.041	0.001	1803.0
89	0.037	0.000	0.020	0.003	-0.014	0.000	5846.2
90	0.062	0.000	-0.076	0.010	-0.023	0.004	20462.5
91	0.045	0.173	0.323	0.027	-0.026	0.002	1658.3
92	0.070	0.236	0.208	0.076	0.017	0.004	1643.6
93	0.103	0.002	-0.082	0.004	0.112	0.002	14092.0
94	0.080	0.001	-0.018	0.003	0.024	0.001	6326.2
95	0.145	0.003	-0.071	0.012	0.022	0.006	7955.6
96	0.001	0.000	0.008	0.003	0.007	0.000	10258.0
97	0.040	0.001	0.161	0.005	-0.024	0.004	4971.3
98	0.164	0.004	-0.185	0.004	0.021	0.001	11501.7
99	0.051	0.000	0.075	0.004	0.025	0.001	4938.7
100	0.028	0.000	0.094	0.002	0.033	0.001	5439.1
101	0.090	0.000	-0.036	0.002	0.011	0.001	3797.2
102	0.071	0.000	-0.168	0.021	0.058	0.008	20953.1
103	0.011	0.000	0.136	0.004	-0.077	0.002	26150.5
104	0.068	0.000	-0.068	0.001	0.041	0.001	5190.9

ID	$\rho$	sd( $\rho$ )	$\gamma$	sd( $\gamma$ )	$\delta$	sd( $\delta$ )	SSR
105	0.006	0.000	-0.225	0.002	0.003	0.000	94.8
106	0.020	0.000	0.081	0.002	0.037	0.001	6963.9
107	0.035	0.000	0.109	0.007	-0.074	0.003	18338.9
108	0.169	0.040	-0.027	0.028	0.056	0.010	2943.9
109	0.012	0.000	0.027	0.002	-0.001	0.000	28149.9
110	0.076	0.000	-0.108	0.002	-0.005	0.001	7592.4
111	0.060	0.000	-0.015	0.003	-0.027	0.001	5631.9
112	5.000	0.021	0.323	0.000	0.000	0.000	2.3
113	0.018	0.015	0.323	0.005	0.001	0.002	1675.7
114	0.270	0.018	-0.239	0.007	-0.013	0.001	5777.3
115	0.135	0.001	0.070	0.008	-0.263	0.006	6560.9
116	0.093	0.001	-0.126	0.004	0.019	0.001	15254.8
117	0.085	0.000	-0.014	0.001	-0.028	0.000	1821.5
118	0.045	0.001	0.192	0.008	-0.061	0.002	6828.8
119	0.388	2.875	0.323	0.033	-0.155	0.025	643.3
120	0.107	0.001	0.018	0.004	-0.014	0.004	3259.6
121	0.220	0.006	-0.024	0.012	-0.108	0.005	3544.2
122	0.496	0.709	-0.116	0.044	-0.074	0.011	9370.7
123	0.036	0.001	-0.012	0.005	-0.017	0.001	19257.3
124	0.085	0.000	-0.120	0.002	-0.003	0.001	8376.6
125	0.020	0.118	0.323	0.011	0.000	0.001	1192.5
126	0.068	0.000	-0.065	0.003	0.009	0.001	12654.0
127	0.030	0.000	-0.002	0.004	-0.008	0.001	12420.7
128	0.087	0.000	0.009	0.001	0.009	0.001	2655.7
129	0.046	0.000	-0.012	0.001	0.024	0.000	5771.6
130	0.262	0.013	0.029	0.008	-0.017	0.004	1366.6

ID	$\rho$	sd( $\rho$ )	$\gamma$	sd( $\gamma$ )	$\delta$	sd( $\delta$ )	SSR
131	0.207	0.008	-0.120	0.022	-0.010	0.006	6500.3
132	0.020	0.000	-0.002	0.005	-0.003	0.003	30341.6
133	0.105	0.001	-0.017	0.001	0.029	0.001	2007.0
134	0.033	0.000	-0.039	0.004	0.005	0.001	27904.7
135	0.015	0.000	0.027	0.003	0.061	0.002	16596.8
136	0.004	0.000	-0.013	0.000	0.003	0.000	5437.4
137	0.078	0.000	-0.076	0.001	-0.009	0.001	6980.8
138	0.172	0.004	-0.112	0.005	-0.054	0.002	7653.9
139	0.139	0.001	-0.255	0.005	0.013	0.000	5213.6
140	0.009	0.000	0.323	0.003	-0.053	0.006	2709.0
141	0.080	0.000	-0.043	0.000	-0.006	0.000	1186.6
142	0.135	0.001	-0.009	0.002	0.082	0.002	991.1
143	0.040	0.000	-0.053	0.002	-0.002	0.000	9482.9
144	0.038	0.000	0.033	0.003	0.133	0.002	12781.3
145	0.009	0.002	0.184	0.010	0.003	0.004	7311.1
146	0.046	0.000	0.005	0.001	-0.002	0.001	4831.9
147	0.031	0.000	0.260	0.006	-0.214	0.005	12179.9
148	0.061	0.000	-0.021	0.003	-0.020	0.001	4003.1
149	0.005	0.000	0.078	0.003	-0.008	0.002	32985.8
150	0.011	0.000	0.124	0.002	0.059	0.001	12584.9
151	0.842	0.043	-0.158	0.003	-0.010	0.002	651.0
152	0.110	0.000	-0.069	0.000	-0.009	0.000	771.2
153	0.215	0.007	-0.103	0.004	0.022	0.001	2441.1
154	0.054	0.000	-0.045	0.010	-0.008	0.005	11184.0