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Is idiosyncratic risk conditionally priced?

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In Merton (1987), idiosyncratic risk is priced in equilibrium as a consequence of incomplete diversification. We modify his model to allow the degree of diversification to vary with average idiosyncratic volatility. This simple recognition results in a *state-dependent* idiosyncratic risk premium that is higher when average idiosyncratic volatility is low, and vice versa. The data appear to be consistent a positive state-dependent premium for idiosyncratic risk both in the US and other developed markets.

Keywords. Idiosyncratic risk, factor models, risk premium asset pricing.

JEL classification. G11, G12.

1. Introduction

A major research initiative in finance focuses on the determinants of the cross-sectional and time series properties of asset returns. There are two prominent classes of asset pricing models with microeconomic foundations that address this issue, the CAPM and the consumption CAPM, (along with their numerous extensions). These models, while theoretically elegant, prove inadequate when confronted with data.[1] This has led to a third class of largely ad hoc empirical factor models that attempt to connect the expected re-

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[1] See, for example, Sharpe (1964), Lintner (1965), Mossin (1966), Black (1972), Rubinstein (1976), Lucas (1978), Breeden (1979), and Ross (1976). The list of empirical studies that reject these models is long and catalogued in numerous review papers.

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turns on the assets with their "betas".[2] While these models invoke some version of the arbitrage pricing theory (APT) and the ICAPM (Merton (1973)) for theoretical justification, it is not clear that they succeed as asset pricing models, in the sense of connecting returns to "risk premia".[3] For example, in constructing their empirical three- and five-factor models, Fama and French (1993, 2015) propose their factors as combinations of securities that provide exposure to unspecified state variables in the ICAPM but without any explicit linkage. A key abstraction common to the aforementioned models is that information is costless and investors hold fully diversified portfolios. A fourth model class, where asset prices are determined by individual preferences and beliefs but where investors have incomplete information (and hence hold underdiversified portfolios), has received less attention. This class has its genesis in the early work of Levy (1978) and Mayshar (1979, 1981). It forms the basis of the asset pricing model proposed by Robert Merton in his 1986 presidential address to the American Finance Association. The central insight in Merton (1987) is best clarified by contrasting two states of the world, where investors are either fully diversified or underdiversified. Investors are risk averse and prefer portfolios with lower variance. In a world where investors are fully diversified, adding a security to their portfolio only affects the variance through the covariance. In contrast, in a world where investors are underdiversified, adding a security to their portfolio affects the variance through both covariance and idiosyncratic volatility. Thus when investors hold under-diversified portfolios, idiosyncratic risk should be priced, leading to a positive premium for bearing idiosyncratic risk. This is in sharp contrast to the implications of CAPM and CCAPM, which are predicated on frictionless markets with no role for idiosyncratic risk.

In this paper, we modify Merton's model. Our point of departure is the empirical observation that average idiosyncratic volatility varies considerably over time. This is illustrated in Figure 1, which shows average idiosyncratic volatility from July 1931 to December 2014. The figure shows that the variation in average idiosyncratic volatility over the entire time series is large. Even within decades, average idiosyncratic volatility can vary significantly.[4] Our economic intuition is that since the marginal benefit from diversification is likely to be higher in states of the world characterized by high average idiosyncratic volatility, the idiosyncratic risk premium should be lower in such periods (and vice versa). This is perhaps most obvious in periods like the financial turmoil in late 2008 and early 2009 when diversification was especially valuable. The implication is that time series variation in average idiosyncratic volatility should lead to a state-dependent risk premium. In contrast to Merton's original model where the risk premium is positive and constant, in our modification, the risk premium varies inversely with the degree of

[2]Harvey, Liu, and Zhu (2016) catalogue 316 anomalies proposed as potential factors in asset-pricing models and they note that there are others that do not make their list.

[3]The APT was introduced by Ross (1976) and extended by Huberman (1982), Chamberlain (1983), Chamberlain and Rothschild (1983), Connor (1984), Reisman (1988), and Gilles and LeRoy (1991), among others.

[4]Campbell, Lettau, Malkiel, and Xu (2001) argued that average idiosyncratic volatility has increased over time, although that conclusion is controversial since much of the attributed increase occurred in the 1990s. For our purpose, what matters is not an increase in average idiosyncratic volatility but economically meaningful time series variation.

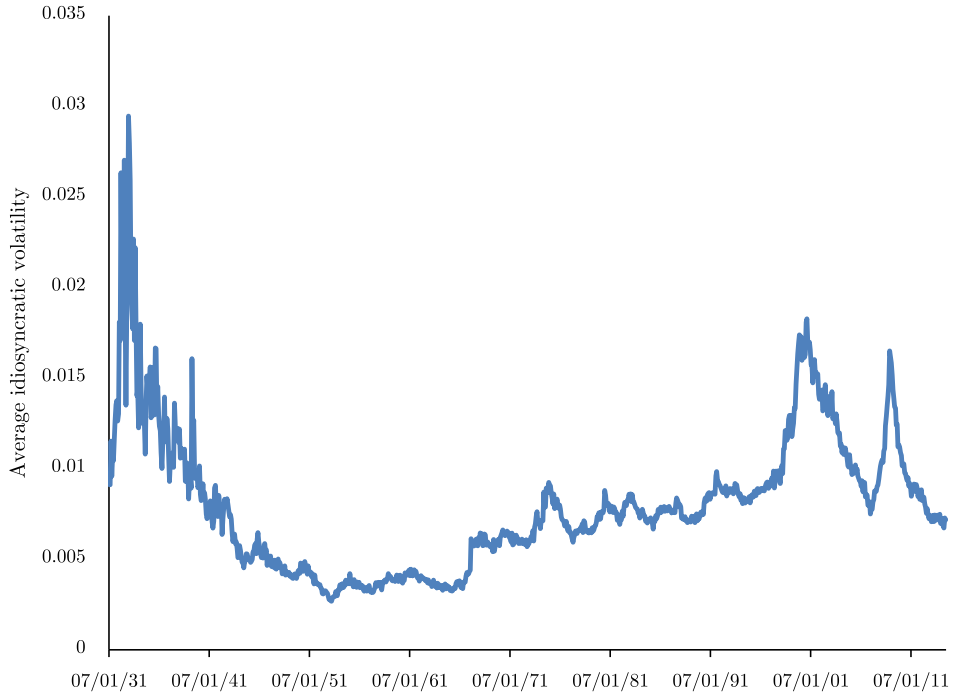

FIGURE 1. We compute the value-weighted average idiosyncratic volatility for small and large capitalization stocks, and then calculate a simple average of the two to obtain average idiosyncratic volatility for each month. We use NYSE median breaks to separate small and large cap stocks.

diversification, which in turn, varies with average idiosyncratic volatility. This leads to an asset pricing model where the time series variation in the idiosyncratic risk premium is linked to average idiosyncratic volatility. We illustrate this in Figure 2, highlighting the difference between Merton's (1987) formulation and our modification.

Our aim in the empirical section is to test the implications of the model developed in this paper; an additional outcome is that our results shed light on the volatility literature. All prior investigations of the Merton model, whether reexamining the theory, or its empirical content, ignore time series variation in average idiosyncratic volatility. For instance, Wu, Li, and Wei (1996) allow for heterogeneous expectations and short-sale restrictions, which generate offsetting effects, but remain unconditional. Empirical investigations are far more voluminous. Most prominent and puzzling is Ang, Hodrick, Xing, and Zhang (2006) who found that contrary to Merton's (1987) prediction, there is a negative relation between expected returns and lagged idiosyncratic volatility. Stambaugh, Yu, and Yuan (2015) suggested that this relation may be due to short sale constraints. But Fu (2009) questions the negative relation entirely, claiming that it is expected, rather than lagged, idiosyncratic volatility that should matter. He finds a positive relation between contemporaneous returns and expected idiosyncratic volatility. Guo, Kassa, and Ferguson (2014) pointed out that Fu's (2009) findings are driven by a look-ahead bias

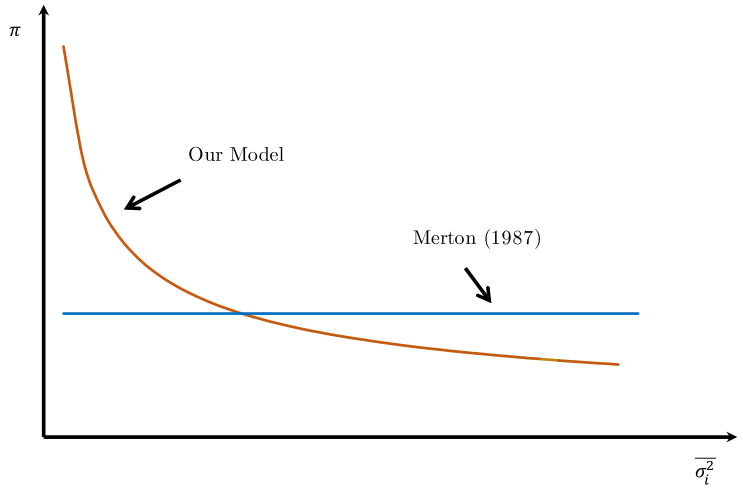

FIGURE 2. The x-axis shows average idiosyncratic volatility. The y-axis shows the premium associated with idiosyncratic volatility.

in his tests, and that there is in fact no statistically discernible relation between average returns and expected idiosyncratic volatility.

All of these tests, as well as numerous others that seek to understand this connection, are unconditional.[5] Our formal model says that the premium should be positive and, for the model to be meaningful, pricing should be conditional. If the relevant state variable in conditional pricing was persistent, or deviated by small amounts in the time series, the economic impact of conditional versus unconditional pricing would be empirically unimportant. Indeed, this is precisely the point that Lewellen and Nagel (2006) make with respect to tests of the conditional CAPM—that CAPM betas move so slowly that conditional tests are not very different from unconditional tests. That is clearly not the case for idiosyncratic volatility; Figure 1 shows that average idiosyncratic volatility varies substantially over time. Moreover, since an unconditional model does not necessarily imply a conditional model, the existing empirical evidence cannot be used to draw inferences about our conditional version of Merton's model.

The most direct way to determine if the model has any traction in the data is to ask whether in the cross-section, the risk premium on idiosyncratic risk is positive and depends on average idiosyncratic volatility. Before doing so, we first replicate the "standard" results that monthly returns are negatively related to lagged idiosyncratic volatility and unrelated to expected (unconditional) idiosyncratic volatility. With that as the baseline, we then estimate Fama–MacBeth regressions of monthly returns on contemporaneous expected idiosyncratic volatility, scaled by expected average idiosyncratic volatility. Scaling by expected average idiosyncratic volatility is important from both a

[5]The results in Ang et al. (2006) spawned a large literature attempting to explain this idiosyncratic risk "puzzle," mostly absent guiding theory. A partial list of papers includes Bali and Cakici (2008), Chen and Petkova (2012), Detzel, Duarte, Kamara, and Siegel (forthcoming), Han and Lesmond (2011), Herskovic, Kelly, Lustig, and Nieuwerburgh (2016), Hou and Loh (2016), and Spiegel and Wang (2005). Idiosyncratic risk is also often invoked as an impediment to arbitrage.

theoretical and empirical standpoint. Conceptually, the scaling variable is not ad hoc and follows directly from our theory—it lies at the very heart of the model which says that the marginal benefit of diversification is high when expected average idiosyncratic volatility is high. The fact that the model identifies the relevant state variable is a significant advantage, particularly in light of Cochrane's (2001) refrain that the conditional CAPM is technically not testable because the econometrician cannot know the "right" state variable. Empirically, the scaling allows us to test the conditional model in a unified framework without resorting to subsample tests with limited power.

In US data from 1931 to 2014, controlling for conditional market betas, the slope on expected idiosyncratic volatility scaled by expected average idiosyncratic volatility is positive. This positive slope is in stark contrast to the unconditional idiosyncratic volatility literature which finds a negative or no relation between idiosyncratic volatility and expected returns. We also estimate similar regressions in markets outside the US. Since the tests require an adequate cross-section and a time series of returns, we restrict our attention to Canada, France, Japan and the UK. In these markets, too, the slopes on scaled expected idiosyncratic volatility are positive and statistically significant. Overall, the data appear to be consistent with a conditional version of Merton (1987) in which the positive premium for idiosyncratic risk varies over time with average idiosyncratic risk.

A natural question that arises is whether the slopes on scaled expected idiosyncratic volatility are sensitive to the inclusion of size, book-to-market ratios and other such empirically motivated variables. Our purpose is to evaluate the theoretical model developed in this paper. The null hypothesis against which our (and Merton's) model should be judged is the CAPM, not an empirically motivated ad hoc factor model. This is because the CAPM becomes a special case of our model when information costs go to zero. One could potentially generate any number of variables from the so-called factor zoo that drive out a theoretically motivated construction. We take the position that there is something to be learned from the conditional model, even if one can find factors that dominate it empirically.

The paper is organized as follows: in Section 2, we present the model, both in summary and in detail. Section 3 describes our sample, measurement approach, and results. Section 4 concludes.

2. The model

We provide a summary of the model and its intuition below, followed by a formal derivation, where we describe both Merton's (1987) original formulation and our critical modifications.

2.1 *Model summary*

Merton (1987) presented a model where investors are under-diversified, the market portfolio is not mean—variance efficient, the CAPM does not hold and idiosyncratic risk is priced in equilibrium. In this paper, we extend the Merton model by making two modifications:

1. We assume that the fraction of all investors who "know" about a security is proportional to its market value relative to the value of the market portfolio. An intuitively appealing implication of this is that the idiosyncratic risk premium varies inversely with the average number of securities in an investor's portfolio.

2. In addition to the conditions in Merton (1987), we require that, in equilibrium, there is no incentive for investors to further diversify. We achieve this by imposing the condition that the marginal increase in utility due to increased diversification is offset by the marginal disutility due to the (implicit) costs, I, of information acquisition. As a result, the degree of diversification varies inversely with the costs of diversification and directly with average idiosyncratic volatility.

The rest of our model follows Merton (1987): investors are risk averse, have identical preferences, are price-takers, have the same initial wealth, and are mean variance optimizers. Investors are less than fully diversified as they only invest in a security if they "know" about that security in the sense that they know the mean and variance of its return distribution. Investors have conditional homogenous beliefs in the sense that all investors who "know" about a security have the same information about the security. This leads to an asset-pricing model with clear testable implications.

The equilibrium expected return on security i in this model is

$$\overline{R}_i = R_f + b_i \overline{b} \delta + \frac{\sigma_i^2 \delta}{\overline{Q}^*}, \tag{1}$$

where δ is the coefficient of risk aversion, R_f is the risk-free rate, σ_i^2 is the idiosyncratic volatility of security i, b_i is its beta, $\overline{Q}^*$ is the average number of stocks held by an investor in equilibrium, and $\overline{b}$ is the average beta of the investor's portfolio. As in Merton (1987), there is a positive premium for idiosyncratic volatility. The key deviation from his model is that the parameter $\overline{Q}^*$, representing portfolio diversification, is determined in equilibrium as follows:

$$\overline{Q}^* = \sqrt{\frac{\delta \overline{\sigma_i^2}}{2I}}. \tag{2}$$

$\overline{Q}^*$ is determined by risk aversion, average idiosyncratic volatility ($\overline{\sigma_i^2}$) and the cost of information acquisition (I), which accords with our intuition. Combining equations (1) and (2), we can express (1) as

$$\overline{R}_i = R_f + b_i \overline{b} \delta + \pi \sigma_i^2, \tag{3}$$

where

$$\pi = \sqrt{\frac{2I\delta}{\overline{\sigma_i^2}}}$$

is the state dependent idiosyncratic risk premium. Equation (3) highlights the role of both average idiosyncratic volatility ($\overline{\sigma_i^2}$) and the costs of information (I) in portfolio

diversification. In the limiting case, with perfect information ($I = 0$), investors are fully diversified and the idiosyncratic risk premium π disappears. When I is not zero, changes in average idiosyncratic volatility influence the disutility of under-diversification and, therefore, the idiosyncratic risk premium.[6]

It is reasonable to ask is why investors could not simply diversify by holding passive mutual funds or ETFs. We do not claim that information costs (I) are the sole reason why investors in real world are not fully diversified. A variety of other frictions or behavioral biases could also account for underdiversification (see, e.g., Goetzmann and Kumar (2008)). Referring to incomplete diversification, Merton (1987) notes "a numbers of other factors, for example, market segmentation and institutional restrictions including limitations on short sales, taxes, tractions costs, liquidity, imperfect divisibility of securities, in addition to incomplete information that in varying degrees, could contribute to this observed behavior." Thus, the parameter (I), while formally referred to as information costs, subsumes other restrictions that cause underdiversification.[7]

2.2 *The formal model*

The economy has N firms, $N \gg 1$. The return $\tilde{R}_i$ from investing in firm i has a factor structure:

$$\tilde{R}_i = \overline{R}_i + b_i \tilde{Y} + \sigma_i \tilde{\varepsilon}_i, \quad i = 1, \ldots, N, \tag{4}$$

where $\tilde{Y}$ is a common factor with $E(\tilde{Y}) = 0$, $E(\tilde{Y}^2) = 1$, b_i is the factor loading of security i, $\tilde{\varepsilon}_i$ is a firm-specific random variable with

$$E(\tilde{\varepsilon}_i) = E(\tilde{\varepsilon}_i \mid \tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_{i-1}, \tilde{\varepsilon}_{i+1}, \ldots, \tilde{\varepsilon}_N, Y) = 0, \quad i = 1, \ldots, N. \tag{5}$$

$E(\tilde{\varepsilon}_i^2) = 1$, σ_i^2 is the idiosyncratic volatility of security i, and $\overline{\sigma^2}$ is the value weighted average idiosyncratic volatility across the N securities. $\overline{R}_M$ denotes the value weighted expected return of the N securities.

In addition to the N securities issued by firms, the economy has two "inside" securities with zero net supply:

(a) a "*factor mimicking*" security with return, $\tilde{R}_{N+1} = \overline{R}_{N+1} + \tilde{Y}$,

(b) a riskless security with return R_f.

The economy has K investors, $K \gg N$. Investors are risk averse, with identical mean-variance preferences:

$$U_k = E\big(\tilde{R}^k\big) - \frac{\delta}{2} \operatorname{Var}\big(\tilde{R}^k\big), \quad k = 1, \ldots, K. \tag{6}$$

$\tilde{R}^k$ denotes the portfolio return, and δ is the coefficient of risk aversion. Investors are price takers and assumed to have identical initial wealth W_0, which we normalize to 1.

[6] Bekaert, Hodrick, and Zhang (2012), Brown and Kapadia (2007), and others offered explanations for the source of time series variation in average idiosyncratic volatility, but for our purpose, it is exogenous and outside the model.

[7] Merton (1987), pages 488–490.

An investor only includes security i in his portfolio if he is "informed" in the sense that he knows $(\overline{R}_i, b_i, \sigma_i^2)$. Information is costly and as a consequence investor k selects only a subset of the N available securities to include in his portfolio.[8] We assume that the securities he selects, Q_k are much smaller than N ($Q_k \ll N$) and that the probability of selecting a firm is proportional to its value relative to the market portfolio. Θ_k is the set of integers that index the Q_k firms selected by investor k.[9]

In addition to firm-specific knowledge, each investor's information set contains common knowledge: $(R_f, \overline{R}_{N+1}, \overline{R}_M, \overline{\sigma^2})$.

Equilibrium in capital markets is characterized as follows:

(a) Given the set of securities selected, each investor chooses an optimal portfolio.

(b) Markets clear.

The optimal portfolio holdings for any investor k is determined as follows:
From (4) and (6), an investor's portfolio return can be specified as

$$\tilde{R}^k = \overline{R}^k + b^k \tilde{Y} + \sigma^k \tilde{\varepsilon}^k, \tag{7}$$

where

$$b^k = \sum_{i \in \Theta_k} w_i^k b_i + w_{N+1}^k, \tag{8}$$

$$\left(\sigma^k\right)^2 = \sum_{i \in \Theta_k} \left(w_i^k\right)^2 \sigma_i^2. \tag{9}$$

w_i^k and w_{N+1}^k denote the fraction of investor k's wealth allocated to security i and $N+1$. The expected portfolio return and variance are:

$$E\left(\tilde{R}^k\right) = R_f + b^k(\overline{R}_{N+1} - R_f) + \sum_{i \in \Theta_k} w_i^k \Delta_i, \tag{10}$$

$$\operatorname{Var}\left(\tilde{R}^k\right) = \left(b^k\right)^2 + \sum_{i \in \Theta_k} \left(w_i^k\right)^2 \sigma_i^2, \tag{11}$$

where

$$\Delta_i = (\overline{R}_i - R_f) - b_i(\overline{R}_{N+1} - R_f), \quad i \in \Theta_k. \tag{12}$$

The investor's optimal portfolio choice is the solution to the following problem:

$$\max_{\{b^k, w_i^k\}} \left[E\left(\tilde{R}^k\right) - \frac{\delta}{2} \operatorname{Var}\left(\tilde{R}^k\right) \right], \quad i \in \Theta_k$$

$$\text{Subject to } \sum_{i \in \Theta_k} w_i^k + w_{N+1}^k + w_f^k = 1. \tag{13}$$

[8]These subsets will in general differ across the K investors.

[9]They are a subset of the first N natural numbers.

From (10) and (11), the first-order conditions for (13) are

$$\overline{R}_{N+1} - R_f - b^k \delta = 0, \tag{14}$$

$$\Delta_i - w_i^k \sigma_i^2 \delta = 0, \quad i \in \Theta_k. \tag{15}$$

From (8), (14), and (15), the investor's optimal portfolio solution is

$$b^k = \frac{\overline{R}_{N+1} - R_f}{\delta}, \tag{16}$$

$$w_i^k = \frac{\Delta_i}{\sigma_i^2 \delta}, \quad i \in \Theta_k, \tag{17}$$

$$w_{N+1}^k = b^k - \sum_{i \in \Theta_k} w_i^k b_i, \tag{18}$$

$$w_f^k = 1 - b^k + \sum_{i \in \Theta_k} w_i^k (b_i - 1). \tag{19}$$

We aggregate to determine equilibrium expected returns. From (16), all investors choose the same b^k. Let $b^k = B$, $k = 1, \ldots, K$. Thus, from (16), we have

$$\overline{R}_{N+1} = R_f + B\delta. \tag{20}$$

From (17), the aggregate demand for security i is:

$$D_i = \sum_{k=1}^{K_i} W_0 w_i^k = \sum_{k=1}^{K_i} W_0 \frac{\Delta_i}{\sigma_i^2 \delta}. \tag{21}$$

In the equation above, K_i is the number of investors who know about the firm i.

From (18) and (19), the aggregate demand for "inside" securities is

$$D_{N+1} = \sum_{k=1}^{K} W_0 w_{N+1}^k = \sum_{k=1}^{K} W_0 B - \sum_{i=1}^{N} b_i D_i, \tag{22}$$

$$D_f = \sum_{k=1}^{K} W_0 w_f^k = \sum_{k=1}^{K} W_0 - \sum_{i=1}^{N+1} D_i. \tag{23}$$

In equilibrium, the demand for these securities is zero: $D_{N+1} = D_f = 0$. As a consequence,

$$B = \frac{\sum_{i=1}^{N} b_i D_i}{\sum_{k=1}^{K} W_0} = \sum_{i=1}^{N} x_i b_i = \overline{b}, \tag{24}$$

where x_i is the fraction of investors' total wealth allocated to security i. Using (24), we can rewrite (20) as

$$\overline{R}_{N+1} = R_f + \overline{b}\delta. \tag{25}$$

If V_i denotes the equilibrium value of firm i, then

$$x_i = \frac{V_i}{\sum_{k=1}^{K} W_0} \tag{26}$$

is the fraction of investors' total wealth invested in firm i. Market clearing implies that $V_i = D_i$, and hence,

$$x_i = \frac{V_i}{\sum_{k=1}^{K} W_0} = \frac{D_i}{\sum_{k=1}^{K} W_0} = q_i \frac{\Delta_i}{\sigma_i^2 \delta} \tag{27}$$

and

$$q_i = \frac{\sum_{k=1}^{K_i} W_0}{\sum_{k=1}^{K} W_0} = \frac{K_i}{K}, \tag{28}$$

where q_i is the fraction of investors who invest in firm i. Equation (27) corresponds to equation (15) in Merton (1987).

Our first modification is that we assume that the fraction of all investors who know about a security is proportional to the weight of the security in the market portfolio. This implies that q_i is proportional to x_i

$$x_i = \phi q_i. \tag{29}$$

Using (17), (27), and (29),

$$w_i^k = \frac{x_i}{q_i} = \phi, \quad i \in \Theta_k. \tag{30}$$

Since

$$\sum_{k=1}^{K} \left[\sum_{i \in \Theta_k} w_i^k + w_{N+1}^k + w_f^k \right] = K \tag{31}$$

using (30) we get

$$K = \sum_{k=1}^{K} \sum_{i \in \Theta_k} \phi + \sum_{k=1}^{K} \left(w_{N+1}^k + w_f^k \right) = \sum_{k=1}^{K} \phi Q_k = \phi \sum_{k=1}^{K} Q_k. \tag{32}$$

Here, we have used the observation that the number of firms in Θ_k is Q_k and that the holdings of security $N+1$ and the risk-free asset sum to zero across all investors.

Hence

$$\phi = 1 \Big/ \left(\frac{\sum_{k=1}^{K} Q_k}{K} \right) = 1/\overline{Q}, \tag{33}$$

where $\overline{Q} = \frac{1}{K} \sum_{k=1}^{K} Q_k$ is the average number of securities in a portfolio.

From (18), (19), (24), (30), and (33), we have

$$w_i^k = \frac{1}{\overline{Q}}, \tag{34}$$

$$w_{N+1}^k = \overline{b} - \sum_{i \in \Theta_k} \frac{b_i}{\overline{Q}}, \tag{35}$$

$$w_f^k = 1 - \overline{b} + \sum_{i \in \Theta_k} \frac{b_i - 1}{\overline{Q}}. \tag{36}$$

As noted in (34), w_i^k is the same for each investor in firm i, while w_{N+1}^k, w_f^k can be different across investors.

From (10), (25)–(29) and (33), we observe that the expected security returns

$$\overline{R}_i = R_f + b_i \overline{b} \delta + \frac{\sigma_i^2 \delta}{\overline{Q}}, \quad i = 1, \ldots, N \tag{37}$$

are linear in idiosyncratic volatility and the idiosyncratic risk premium ($\frac{\delta}{\overline{Q}}$) varies inversely with the average number of securities.[10]

Our second modification is that we assume that in equilibrium investors have no incentive to increase their holdings Q_k. We achieve this by imposing the condition that the marginal increase in utility due to increased diversification is offset by the disutility due to the (implicit) costs of information acquisition, I.

[10]The "betas" in equation (37) are with respect to a common factor $\tilde{Y}$ and *not the market portfolio* (equation (4)). To transform the beta coefficients in equation (37) to their counterparts with respect to the market, we use the fact that the return on the market portfolio, $\tilde{R}_m = \sum_{i=1}^{N} x_i \tilde{R}_i$, $\overline{b} = \sum_{i=1}^{N} x_i b_i$, hence $\beta_i = \mathrm{Cov}(\tilde{R}_i, \tilde{R}_m)/\sigma_m^2 = (b_i \overline{b} + x_i \sigma_i^2)/\sigma_m^2$. Since $x_i = q_i/\overline{Q}$ (using (30) and (33)) we get:

$$\overline{R}_i = R_f + \beta_i \sigma_m^2 \delta + \frac{\sigma_i^2 \delta}{\overline{Q}} (1 - q_i).$$

Here, $1 - q_i$ is the fraction of investors *who do not know about security i*. When all investors know all securities and hold all securities $1 - q_i = 0$ and indeed the above equation reduces to the CAPM. (To see this, substitute $\overline{R}_m$ for $\overline{R}_i$ and use the fact that $\beta_m = 1$ to conclude that $\delta = (\overline{R}_m - R_f)/\sigma_m^2$.) Of course, as emphasized by Merton (p. 488) and by us, the *raison d'etre* of our papers to examine the case when investors only know a small fraction of all securities, that is, $N \gg Q_k$ or $1 - q_i \approx 1$.

From (10)–(12) and (34)–(37), the expected portfolio return and portfolio variance are

$$E\big(\tilde{R}^k\big) = R_f + \overline{b}^2\delta + \frac{\delta}{\overline{Q}^2}\sum_{i\in\Theta_k}\sigma_i^2, \tag{38}$$

$$\mathrm{Var}\big(\tilde{R}^k\big) = \overline{b}^2 + \frac{1}{\overline{Q}^2}\sum_{i\in\Theta_k}\sigma_i^2. \tag{39}$$

Thus, the utility of investor k is

$$U_k = E\big(\tilde{R}^k\big) - \frac{\delta}{2}\mathrm{Var}\big(\tilde{R}^k\big) = R_f + \frac{\overline{b}^2\delta}{2} + \frac{\delta}{2\overline{Q}^2}\sum_{i\in\Theta_k}\sigma_i^2. \tag{40}$$

With access to an additional security a, where a is an element of $\{N\}\backslash\Theta_k$, the investor's new optimal portfolio allocation is the solution to the maximization problem:

$$\max_{\{b^k, w_i^k\}}\left[E\big(\tilde{R}^k\big) - \frac{\delta}{2}\mathrm{Var}\big(\tilde{R}^k\big)\right], \quad i \in \Theta_k \cup \{a\}. \tag{41}$$

The resulting expected portfolio return and variance *conditional* on selecting security a are

$$E\big(\tilde{R}^k \mid a\big) = R_f + \overline{b}^2\delta + \frac{\delta}{\overline{Q}^2}\left(\sum_{i\in\Theta_k}\sigma_i^2 + \sigma_a^2\right), \tag{42}$$

$$\mathrm{Var}\big(\tilde{R}^k \mid a\big) = \overline{b}^2 + \frac{1}{\overline{Q}^2}\left(\sum_{i\in\Theta_k}\sigma_i^2 + \sigma_a^2\right). \tag{43}$$

Since the probability of selecting an additional security is proportional to its market capitalization, the expected idiosyncratic volatility of the additional security is the value weighted average idiosyncratic volatility across the securities he does not hold:

$$E\big[\sigma_a^2\big] = \sum_{i\in\{N\}\backslash\Theta_k} x_i\sigma_i^2, \tag{44}$$

where $\{N\}$ is the set of integers $1,\ldots,N$. Since $N \gg Q_k$ (the investor only knows a small fraction of all securities) $E[\sigma_a^2]$ can be approximated as

$$E\big[\sigma_a^2\big] \cong \sum_{i=1}^{N} x_i\sigma_i^2 = \overline{\sigma^2}. \tag{45}$$

Using (45), the *unconditional* expected portfolio return and variance can be written as

$$E\big(\tilde{R}^k\big) = R_f + \overline{b}^2\delta + \frac{\delta}{\overline{Q}^2}\left(\sum_{i\in\Theta_k}\sigma_i^2 + \overline{\sigma^2}\right), \tag{46}$$

$$\operatorname{Var}(\tilde{R}^k) = \overline{b}^2 + \frac{1}{\overline{Q}^2}\left(\sum_{i\in\Theta_k}\sigma_i^2 + \overline{\sigma^2}\right) \tag{47}$$

and the expected utility of investor k becomes

$$U_k' = E(\tilde{R}^k) - \frac{\delta}{2}\operatorname{Var}(\tilde{R}^k) = R_f + \frac{\overline{b}^2\delta}{2} + \frac{\delta}{2\overline{Q}^2}\left(\sum_{i\in\Theta_k}\sigma_i^2 + \overline{\sigma^2}\right) \tag{48}$$

Comparing (40) with (48), we see that the expected increase in utility ΔU_k is

$$\Delta U_k = U_k' - U_k = \frac{\delta}{2\overline{Q}^2}\overline{\sigma^2}. \tag{49}$$

Note that as a result of the approximation in (45) ΔU_k is same for all investors and we have

$$\Delta U = \Delta U_k = \frac{\delta}{2\overline{Q}^2}\overline{\sigma^2} \quad \forall k.$$

For investors to have no incentive to learn about an additional security, ΔU must be no greater than the disutility of the cost of information acquisition I:[11]

$$\Delta U - I \le 0. \tag{50}$$

From (49) and (50), we have

$$\frac{\delta}{2\overline{Q}^{*2}}\overline{\sigma^2} = I \tag{51}$$

where $\overline{Q}^*$ is the average number of stocks held by investor k in equilibrium.

Hence

$$\overline{Q}^* = \sqrt{\frac{\delta\overline{\sigma^2}}{2I}}. \tag{52}$$

Using (37) and (52), the expected return on asset i can be written as

$$\overline{R}_i = R_f + b_i\overline{b}\delta + \frac{\sigma_i^2\delta}{\overline{Q}^*}. \tag{53}$$

After substituting for $\overline{Q}^*$, we can rewrite equation (53) as

$$\overline{R}_i = R_f + b_i\overline{b}\delta + \pi\sigma_i^2, \tag{54}$$

where

$$\pi = \sqrt{\frac{2I\delta}{\overline{\sigma_i^2}}}$$

[11] In this framework, $U(I) =$ a constant $\times I$. We have normalized the constant to be 1 as it does not affect the subsequent analysis.

is the state dependent idiosyncratic risk premium. This corresponds to equation (3) in the model summary above.

3. Empirical tests

3.1 *Testing strategy*

To test the model, we transform the beta coefficients in equation (53) to their counterparts with respect to the market, resulting in the following equation:

$$\overline{R}_i = R_f + \beta_i \sigma_m^2 \delta + \frac{\sigma_i^2 \delta}{\overline{Q}^*}(1 - q_i), \tag{55}$$

where $\beta_i = \text{Cov}(\tilde{R}_i, \tilde{R}_m)/\sigma_m^2$ and $\sigma_m^2 = \text{Var}(\tilde{R}_m)$.

We make the approximation that $1 - q_i \approx 1$ where $1 - q_i$ is the fraction of investors who *do not* hold asset i.[12] This results in the following equation that we test:

$$\overline{R}_i = R_f + \beta_i \sigma_m^2 \delta + \frac{\sigma_i^2 \delta}{\overline{Q}^*}. \tag{56}$$

Our interest is in the risk premium in the last term of equation (55). The most straightforward way to test the model is to recognize that $\overline{Q}^* = \sqrt{\frac{\delta \overline{\sigma^2}}{2I}}$. Substituting this expression into the last term of equation (55) we get

$$\overline{R}_i = R_f + \beta_i \sigma_m^2 \delta + \gamma \frac{\sigma_i^2}{\sqrt{\overline{\sigma^2}}}, \tag{57}$$

where

$$\gamma = \sqrt{2I\delta}.$$

Equations (56) and (57) say that controlling for conditional betas, the average slope of regressions of stock returns on the ratio of expected idiosyncratic volatility to the (square root of) expected average idiosyncratic volatility should be positive. More precisely, the regression coefficient is identified as γ.

3.2 *Sample construction*

Our sample of US stocks is derived from the CRSP-Compustat universe with CRSP share codes 10 or 11 that restrict the universe to common stocks, and with exchange codes 1, 2, and 3 corresponding to NYSE, Amex. and Nasdaq listed securities. We eliminate

[12]Alternatively, we could assume that $\beta_i \sigma_m^2 - x_i \sigma_i^2 \approx \beta_i \sigma_m^2$. to get equation (56). This is similar to the approximation in equation (9) in Dybvig and Ross (1985). Quantitatively, the approximation is innocuous. x_i, the fraction of the market portfolio invested in asset i, is of the order of 10^{-2} or less. Using typical values for the other parameters, ($\beta_i \approx 1$, $\sigma_m^2 \approx 0.04$, $\sigma_i^2 \approx 0.16$), we see that while $\beta_i \sigma_m^2$ is of the order 10^{-2}, the neglected term $x_i \sigma_i^2$ is of order 10^{-3}. For the indicated parameters, the approximation involves using 0.04 instead of the exact value 0.0384.

stocks with a share price below $1 at the beginning of the month. The tests are based on a sample period from 1931 to 2014 because we need at least 5 years of data to calculate expected idiosyncratic volatility.

For the international sample, we obtain a time series of market information from Datastream. We start with an unconstrained universe of all firms in the following developed markets between 1990 and 2014: Canada, France, Japan, and the United Kingdom. We restrict our attention to these countries because the tests require an adequate cross-section of securities as well as a reasonable time series. The universe of stocks includes live as well as dead stocks. We apply the sequence of filters described in Goyal and Wahal (2015), retaining only equity issues from the primary exchange of the country, and ensuring that we only sample local (not cross-listed) stocks. US dollar returns are computed by converting local currency returns using the conversion function built into Datastream, which uses spot rates. Market values are similarly converted to US dollar equivalents.

3.3 *Measurement*

For each security-month, we estimate daily time series market model regressions of excess stock returns on the excess market return. We use the market to generate residuals because the CAPM serves as the natural theoretical counterpart to Merton's (1987) model and our modification. The idiosyncratic component ε_i from these regressions is assumed to be normally distributed. The model says that that conditional expected returns should be positively related to *expected* (not lagged) idiosyncratic volatility. We model expected idiosyncratic volatility for stock i in month t using the EGARCH process used by Guo, Kassa, and Ferguson (2014) as follows:

$$\ln \sigma_{i,t}^2 = a_{i,t-1} + \sum_{l=1}^{p} b_{i,l,t-1} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k,t-1} \left\{ \theta_{i,t-1} \left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right) + \gamma_{i,t-1} \left[\left| \frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}} \right| - \left(\frac{2}{\pi} \right)^{1/2} \right] \right\}. \tag{58}$$

In estimating the above, we ensure that the sample used stops in month $t-1$ so that there is no look-ahead bias in the estimates. As in Guo, Kassa, and Ferguson (2014), we require at least 60 monthly observations to estimate month t idiosyncratic volatility. We consider nine EGARCH specifications, corresponding to values of p and q from $\{1, 2, 3\}$ and choose the one that converges with the lowest Akaike Information Criterion (AIC). Estimates of expected idiosyncratic volatility are trimmed at the 95th percentile to prevent outliers from influencing the tests.

We calculate the empirical counterpart of the market-wide average expected idiosyncratic volatility ($\overline{\sigma^2}$) using a two-step process as follows:

$$\sqrt{\overline{\sigma_{\mathrm{SL}}^2}} = \sqrt{\frac{1}{2} \left(\sum_{s=1}^{S} w_s \sigma_s^2 + \sum_{l=1}^{L} w_l \sigma_l^2 \right)}, \tag{59}$$

where the subscripts s and l refer to small and large stocks, respectively, the weights w_s and w_l are market capitalization weights within small and large stocks, and the expected idiosyncratic volatility estimates (σ_i^2) are derived from equation (58) above. We use the NYSE median market capitalization in the prior month to designate each security into small and large stock groups. This process of value-weighting expected idiosyncratic volatility for small and large stocks separately, and then taking a simple average of the two, ensures a balance between small and large stocks. As a robustness check, we also compute average expected idiosyncratic volatility using market-wide equal- and value-weights as below:

$$\sqrt{\overline{\sigma_{\text{EW}}^2}} = \sqrt{\frac{1}{N}\left(\sum_{i=1}^{N} \sigma_i^2\right)}, \tag{60}$$

$$\sqrt{\overline{\sigma_{\text{VW}}^2}} = \sqrt{\sum_{i=1}^{N} w_i \sigma_i^2}. \tag{61}$$

We caution, however, that the former is heavily influenced by the large number of more volatile small stocks. In the latter, a small number of large less volatile stocks dominate the calculation.

3.4 *Results*

Before proceeding to tests of our model, we first replicate the existing results in the literature. Panel A of Table 1 reports average slopes and standard errors from monthly Fama–MacBeth regressions of returns on market betas and 1-month-lagged realized idiosyncratic volatility. As in Ang et al. (2006), the slopes on lagged idiosyncratic volatility are reliably negative with a t-statistic of −2.39. Panel B of Table 1 reports average slopes and standard errors from Fama–MacBeth regressions of returns on market betas and unscaled expected idiosyncratic volatility (Guo, Kassa, and Ferguson (2014)). Consistent with prior findings in Guo, Kassa, and Ferguson (2014), the slopes on unscaled expected idiosyncratic volatility is statistically indistinguishable from zero. The implication is that the unconditional Merton (1987) model is not consistent with the data.

Panel A of Table 2 contains average slopes and standard errors from monthly Fama–MacBeth regressions of returns on conditional market betas measured over the prior 3 months using daily returns (Lewellen and Nagel (2006)), and expected idiosyncratic volatility scaled by the square root of average expected idiosyncratic volatility ($\frac{\sigma_i^2}{\sqrt{\overline{\sigma_{\text{SL}}^2}}}$). The slopes are multiplied by 100 for expositional convenience. Conditional betas are statistically indistinguishable from zero. This is inconsistent with the model as specified in equation (55). It is, however, consistent with existing evidence that the conditional CAPM does not perform much better than the unconditional CAPM. More importantly from our perspective, the slopes on expected idiosyncratic volatility scaled by average expected idiosyncratic volatility are positive. In equal-weighted regressions, the slope is 0.91 with a t-statistic of 2.04. In value-weighted regressions, which are less subject to

TABLE 1. Fama–MacBeth regressions of returns on CAPM beta and realized idiosyncratic volatility, on CAPM beta and expected idiosyncratic volatility, for US markets, 1931–2014.

	Fama–MacBeth Regressions
Panel A	
β	0.03 (0.07)
$\sigma^2_{i,t-1}$	−0.98 (0.41)
Panel B	
β	0.02 (0.07)
σ^2_i	0.10 (0.06)

Note: The table reports average slope estimates and standard errors from monthly standard Fama and MacBeth (1973) regressions on stock returns on conditional market betas and prior-month realized idiosyncratic volatility. The sample is from July 1931 through 2014. Conditional market betas are measured over the prior 3 months using daily returns. Realized idiosyncratic volatility is measured over the prior month using daily returns. Expected idiosyncratic volatility is measured over the prior 60 months using an EGARCH (1,3) model but employing the lowest Akaike Information Criterion (AIC) to generate estimates. Coefficients on betas are multiplied by 100. Standard errors are based on Newey–West method with 4 lags.

TABLE 2. Fama–MacBeth regressions of returns on conditional CAPM beta and scaled expected idiosyncratic volatility for US markets, 1931–2014.

	Equal-weighted Regressions	Value-weighted Regressions
Panel A		
β	0.02	−0.10
	(0.07)	(0.09)
$\frac{\sigma^2_i}{\sqrt{\sigma^2_{\text{SL}}}}$	0.91	2.22
	(0.45)	(0.56)
Panel B		
β	0.02	−0.10
	(0.07)	(0.09)
$\frac{\sigma^2_i}{\sqrt{\sigma^2_{\text{EW}}}}$	1.00	2.67
	(0.51)	(0.64)
Panel C		
β	0.02	−0.10
	(0.07)	(0.09)
$\frac{\sigma^2_i}{\sqrt{\sigma^2_{\text{VW}}}}$	0.70	1.68
	(0.36)	(0.44)

Note: The table reports average slope estimates and standard errors from monthly Fama and MacBeth (1973) regressions on stock returns on conditional market betas and expected idiosyncratic volatility scaled by average expected idiosyncratic volatility. The sample is from July 1931 through 2014. Conditional market betas are measured over the prior 3 months using daily returns. Expected idiosyncratic volatility is measured over the prior 60 months using an EGARCH (1,3) model but employing the lowest Akaike Information Criterion (AIC) to generate estimates. All coefficients are multiplied by 100. Standard errors are based on th Newey–West method with 4 lags.

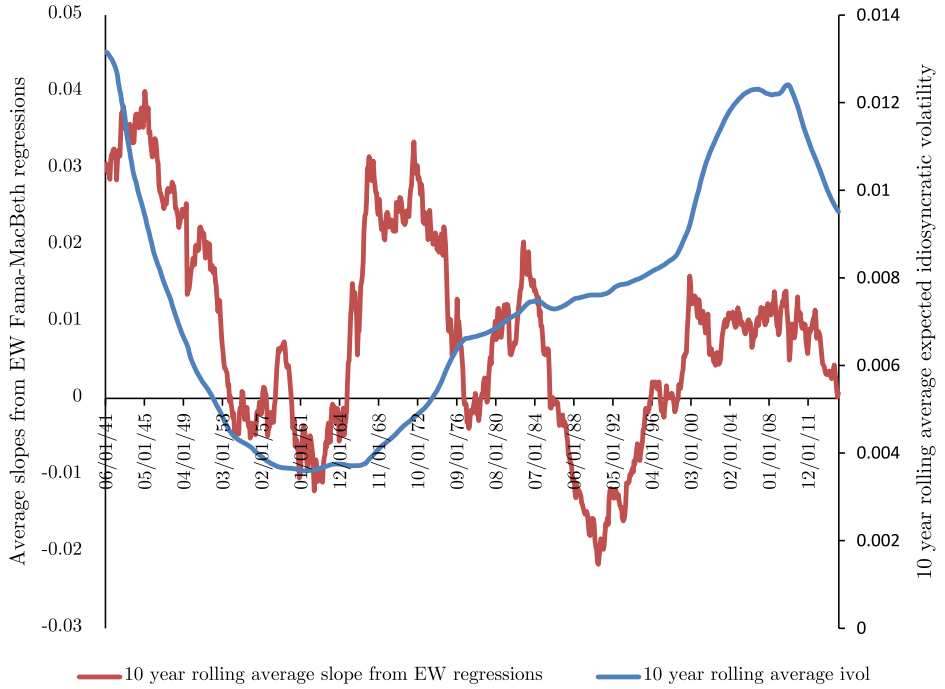

FIGURE 3. The figure reports 10-year rolling average slopes from the equal-weighted Fama–MacBeth regressions in Table 1, along with 10-year rolling average expected idiosyncratic volatility over the same period.

the presence of outliers and to the large number of stocks in the sample, the slope on expected volatility rises to 2.22 with a t-statistic of 4.00.[13]

Panels B and C show estimates when average expected idiosyncratic volatility is measured using equal- or value-weighted averages ($\sqrt{\overline{\sigma^2_{\text{EW}}}}$ and $\sqrt{\overline{\sigma^2_{\text{VW}}}}$ resp.) These approaches do not appear to make a difference to inferences. The coefficients on conditional betas do not move and the slopes on scaled expected idiosyncratic volatility are quite similar.

It is also interesting to examine the variation in the regression slopes over time. Individual coefficients from monthly Fama–MacBeth regressions are quite noisy so we compute 10-year rolling averages. These, along with a 10-year rolling average of average expected idiosyncratic volatility, are plotted in Figure 3. Simple visual inspection suggests an inverse relation between the risk premium and average idiosyncratic volatility.

Prima facie, these results suggest that the data are consistent with a conditional version of Merton's model. Models are parsimonious descriptions of the behavior of homo economicus and are agnostic to countries. It is therefore useful to test them in other countries as a crude out of sample test. Since power is an important consideration, we

[13]Since the slopes are equal to $\sqrt{2I\delta}$, it is tempting to make assumptions about either the cost of information (I) or risk reversion (δ), and infer the other. We resist this temptation because the cost of information and risk aversion *jointly* determine the slope.

TABLE 3. Fama–MacBeth regressions of returns on conditional CAPM beta and scaled expected idiosyncratic volatility for international markets, 1990–2014.

	Canada	France	Japan	UK
Value-Weighted Regressions				
β	0.20	−0.31	0.31	0.32
	(0.71)	(0.52)	(0.20)	(0.32)
$\frac{\sigma_i^2}{\sqrt{\sigma_{\text{SL}}^2}}$	4.28	3.77	2.61	5.33
	(1.96)	(2.26)	(1.22)	(1.82)
Equal-Weighted Regressions				
β	−0.18	−0.18	0.11	0.12
	(0.13)	(0.17)	(0.18)	(0.13)
$\frac{\sigma_i^2}{\sqrt{\sigma_{\text{SL}}^2}}$	1.33	1.10	0.85	−0.15
	(0.58)	(0.67)	(0.91)	(0.68)

Note: The table reports average slope estimates and standard errors from monthly value-weighted Fama and MacBeth (1973) regressions on stock returns on conditional market betas and expected idiosyncratic volatility scaled by average expected idiosyncratic volatility for four international markets. The sample period is from July 1990 through 2014. Conditional market betas are measured over the prior 3 months using daily returns. Expected idiosyncratic volatility is measured over the prior 60 months using an EGARCH $(1, 3)$ model but employing the lowest Akaike Information Criterion (AIC) to generate estimates. All coefficients are multiplied by 100. Standard errors are based on the Newey–West method with 4 lags.

can only do so in markets that have a sufficiently large cross-section of securities and a long enough time series. Four countries for which we have data meet that threshold: Canada, France, Japan, and the UK. Table 3 contains similar regressions for these countries. As in the US data, the slopes on conditional betas are statistically insignificant. In value-weighted regressions, the slopes on scaled expected idiosyncratic volatility are reliably positive in Canada, Japan, and the UK, with t-statistics above 2.00. In France, the slope is positive but with a t-statistic of only 1.67. In equal-weighted regressions, the slopes on scaled idiosyncratic volatility are positive for Canada and France with t-statistics of 2.29 and 1.65, respectively. In Japan and the UK, the slopes are insignificantly different from zero, suggesting that the relation is weaker in small stocks.

While unconditional tests of Merton (1987) provide little empirical support for his model, our regressions suggest that a conditional version of Merton's model has a footprint in the data. One could complain that these regressions ignore the existing evidence on size, value, profitability, investment, accruals, and other such variables that have explanatory power for returns. This omission is willful. Empirically motivated variables may have explanatory power but do not constitute tests of asset pricing models and are subject to the factor zoo problem. We avoid the inclusion of ad hoc variables to maintain the integrity of the test of the theory in Section 2. We include conditional betas because the CAPM generates that natural equilibrium counter to the underdiversification that is at the heart of both Merton's original model and our modification.

4. CONCLUSION

The key insight in Merton's (1987) model of asset pricing under incomplete diversification is that there should be a positive premium for bearing idiosyncratic risk. We propose a simple, yet important modification to his model—the premium for bearing idiosyn-

cratic risk should vary with average idiosyncratic risk. When average idiosyncratic risk is high, the marginal benefit from diversification is also high, implying a lower risk premium (and vice versa). This simple intuition delivers a conditional asset pricing model in the spirit of Merton (1987), where the relevant state variable, average idiosyncratic risk, is identified by the theory.

We test the model in the spirit of classical tests of the CAPM (Fama and MacBeth (1973)): regressions of returns on expected idiosyncratic volatility scaled by average expected idiosyncratic risk. The coefficient on this scaled variable is positive in the US between 1931 and 2014. This variable also has a positive slope Canada, France, Japan, and the UK between 1990 and 2014. These results are in stark contrast to the negative relation between lagged idiosyncratic volatility and expected returns documented by Ang et al. (2006) and explored by numerous others. The key difference is that the theory demands conditional tests because the state variable (average idiosyncratic risk) is economically meaningful and readily identifiable. Both the model and our results suggest that a rejection of the ideas in Merton (1987) may be premature.

References

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006), "The cross-section of volatility and expected returns." *Journal of Finance*, 51, 259–299. [627, 628, 640, 644]

Bali, T. and N. Cakici (2008), "Idiosyncratic volatility and the cross-section of expected returns." *Journal of Financial and Quantitative Analysis*, 43, 29–58. [628]

Bekaert, G., R. J. Hodrick, and X. Zhang (2012), "Aggregate idiosyncratic volatility." *Journal of Financial and Quantitative Analysis*, 47, 1155–1185. [631]

Black, F. (1972), "Capital market equilibrium with restricted borrowing." *Journal of Business*, 45, 444–455. [625]

Breeden, D. T. (1979), "An intertemporal asset pricing model with stochastic consumption and investment opportunities." *Journal of Financial Economics*, 7, 265–296. [625]

Brown, G. and N. Kapadia (2007), "Firm-specific risk and equity market development." *Journal of Financial Economics*, 84, 358–388. [631]

Campbell, J., M. Lettau, B. Malkiel, and Y. Xu (2001), "Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk." *Journal of Finance*, 56, 1–43. [626]

Chamberlain, G. (1983), "Funds, factors and diversification in arbitrage pricing models." *Econometrica*, 51, 1305–1323. [626]

Chamberlain, G. and M. Rothschild (1983), "Arbitrage, factor structure and mean variance analysis in large asset markets." *Econometrica*, 51, 1281–1304. [626]

Chen, Z. and R. Petkova (2012), "Does idiosyncratic volatility proxy for risk exposure?" *The Review of Financial Studies*, 25, 2745–2787. [628]

Cochrane, J. (2001), *Asset Pricing*. Princeton University Press, Princeton, NJ, p. 145. [629]

Connor, G. (1984), "A unified beta pricing theory." *Journal of Economic Theory*, 34, 13–31. [626]

Detzel, A., J. Duarte, A. Kamara, and S. Siegel (forthcoming), "The cross-section of volatility and expected returns: Then and now." *Critical Finance Review*. [628]

Dybvig, P. and S. Ross (1985), "Differential information and performance measurement using a security market line." *The Journal of Finance*, 40, 383–399. [638]

Fama, E. and K. French (1993), "Common risk factors in the returns on stocks and bonds." *Journal of Financial Economics*, 33, 3–56. [626]

Fama, E. and K. French (2015), "A five-factor asset pricing model." *Journal of Financial Economics*, 116, 1–22. [626]

Fama, E. and J. MacBeth (1973), "Risk, return, and equilibrium: Empirical tests." *Journal of Political Economy*, 81, 607–636. [641, 643, 644]

Fu, F. (2009), "Idiosyncratic risk and the cross-section of expected stock returns." *Journal of Financial Economics*, 91, 24–37. [627]

Gilles, C. and S. F. LeRoy (1991), "On the arbitrage pricing theory." *Economic Theory*, 1, 213–229. [626]

Goetzmann, W. and A. Kumar (2008), "Equity portfolio diversification." *Review of Finance*, 12, 433–463. [631]

Goyal, A. and S. Wahal (2015), "Is momentum an echo?" *Journal of Financial and Quantitative Analysis*, 50, 1237–1267. [639]

Guo, H., H. Kassa, and M. F. Ferguson (2014), "On the relation between EGARCH idiosyncratic volatility and expected stock returns." *Journal of Financial and Quantitative Analysis*, 49, 271–296. [627, 639, 640]

Han, Y. and D. Lesmond (2011), "Liquidity biases and the pricing of cross-sectional idiosyncratic volatility." *The Review of Financial Studies*, 24, 1590–1629. [628]

Harvey, C. R., Y. Liu, and H. Zhu (2016), "... and the cross-section of expected returns." *The Review of Financial Studies*, 29, 5–68. [626]

Herskovic, B., B. Kelly, H. Lustig, and S. V. Nieuwerburgh (2016), "The common factor in idiosyncratic volatility: Quantitative asset pricing implications." *Journal of Financial Economics*, 199, 249–283. [628]

Hou, K. and R. K. Loh (2016), "Have we solved the idiosyncratic volatility puzzle?" *Journal of Financial Economics*, 121, 167–194. [628]

Huberman, G. (1982), "A simple approach to arbitrage pricing theory." *Journal of Economic Theory*, 28, 183–192. [626]

Levy, H. (1978), "Equilibrium in an imperfect market: A constraint on the number of securities in a portfolio." *American Economic Review*, 68, 643–658. [626]

Lewellen, J. and S. Nagel (2006), "The conditional CAPM does not explain asset-pricing anomalies." *Journal of Financial Economics*, 82, 289–314. [628, 640]

Lintner, J. (1965), "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets." *Review of Economics and Statistics*, 47, 13–37. [625]

Lucas, R. (1978), "Asset prices in an exchange economy." *Econometrica*, 46, 1429–1445. [625]

Mayshar, J. (1979), "Transaction costs in a model of capital market equilibrium." *Journal of Political Economy*, 87, 673–700. [626]

Mayshar, J. (1981), "Transaction costs and the pricing of assets." *Journal of Finance*, 36, 583–597. [626]

Mehra, R., S. Wahal, and D. Xie (2021), "Supplement to 'Is idiosyncratic risk conditionally priced?"' *Quantitative Economics Supplemental Material*, 12, https://doi.org/10.3982/QE1528. [625]

Merton, R. C. (1973), "An intertemporal capital asset pricing model." *Econometrica*, 41, 867–887. [626]

Merton, R. C. (1987), "A simple model of capital market equilibrium with incomplete information." *Journal of Finance*, 42, 483–510. [625, 626, 627, 629, 630, 631, 634, 639, 640, 643, 644]

Mossin, J. (1966), "Equilibrium in a capital asset market." *Econometrica*, 35, 768–783. [625]

Reisman, H. (1988), "A general approach to the arbitrage pricing theory (APT)." *Econometrica*, 56, 473–476. [626]

Ross, S. (1976), "The arbitrage theory of capital asset pricing." *Journal of Economic Theory*, 13, 341–360. [625, 626]

Rubinstein, M. (1976), "The valuation of uncertain income streams and the pricing of options." *The Bell Journal of Economics*, 7, 407–425. [625]

Sharpe, W. (1964), "Capital asset prices: A theory of market equilibrium under conditions of risk." *Journal of Finance*, 19, 425–442. [625]

Spiegel, M. and X. Wang (2005), "Cross-sectional variation in stocks returns: Liquidity and idiosyncratic risk." Unpublished Manuscript, Yale School of Management. [628]

Stambaugh, R., J. Yu, and Y. Yuan (2015), "Arbitrage asymmetry and the idiosyncratic volatility puzzle." *Journal of Finance*, 60, 1903–1948. [627]

Wu, C., Q. Li, and K. C. J. Wei (1996), "Incomplete-information capital market equilibrium with heterogeneous expectations and short sale restrictions." *Review of Quantitative Finance and Accounting*, 7, 119–136. [627]

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