

# Supplement to “A job ladder model with stochastic employment opportunities”

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## APPENDIX A: ONLINE APPENDIX

### A.1 *Distribution of match quality*

**A.1.1 *Steady-state distribution of vacancy stock*** We denote the number of vacancies in the stock by  $j \in \mathbb{N}_+$ . The probability that a worker in employment state  $s$  has  $j$  vacancies in the stock is denoted by  $p_s(j)$ . The inflow of employed workers with  $j \geq 1$  offers comes from two sources: (i) those with  $j - 1$  offers who receive an additional offer and (ii) those with  $j + 1$  offers who lose an offer. The inflow for employed workers with no offers  $j = 0$  is from two sources: employed workers with one offer, which they lose and workers who just matched with the stock, independent of employment state

$$\begin{aligned} \text{inflow} &= \lambda_e p_e(j - 1) + v(j + 1) p_e(j + 1) \quad \forall j \geq 1, \\ \text{inflow} &= v p_e(1) + \gamma_e + \gamma_u(1 - p_u(0)) \frac{u}{1 - u}, \quad j = 0. \end{aligned}$$

Similarly, the outflow can be due to separation from a job, losing or losing an offer in hand or because the worker was matched with the stock. The outflow is then given below

$$\text{outflow} = (\lambda_e + \gamma_e + \mu + \delta + vj) p_e(j).$$

The steady-state distributions are given by equalizing the outflow and inflow of a given number of job offers  $j$ . The number of outstanding offers is then

$$(\lambda_e + \gamma_e + \mu + \delta + vj) p_e(j) = \lambda_e p_e(j - 1) + v(j + 1) p_e(j + 1) \quad \forall j \geq 1, \quad (13)$$

$$(\lambda_e + \gamma_e + \mu + \delta) p_e(0) = v p_e(1) + \gamma_e + \gamma_u(1 - p_u(0)) \frac{u}{1 - u}, \quad j = 0. \quad (14)$$

The inflow of unemployed workers with the stock of  $j \geq 1$  vacancies can either be because a worker had a stock of  $j - 1$  vacancies and accrues one more, or because a worker

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with a stock  $j + 1$  loses one or an employed worker with that number of opportunities is hit by a job destruction shock. The inflow for  $j = 0$  is from unemployed workers who lose an offer and employed workers with no offers that are hit by a destruction shock

$$\text{inflow} = \lambda_u p_u(j-1) + v(j+1)p_u(j+1) + (\delta + \mu) \frac{u}{1-u} p_e(j) \quad \forall j \geq 1,$$

$$\text{inflow} = v p_u(1) + (\delta + \mu) \frac{u}{1-u} p_e(0).$$

For the unemployed, the outflow can be due to workers taking job offers when they match with the stock at a rate  $\gamma_u$ . In addition, they also acquire new offers at a rate  $\lambda_u$  and lose offers at rate  $\delta$ ,

$$\text{outflow} = (\lambda_u + \gamma_u + vj) p_u(j) \quad \forall j \geq 1,$$

$$\text{outflow} = \lambda_u p_u(0), \quad j = 0.$$

The steady-state distribution solves the equations

$$\begin{aligned} (\lambda_u + \gamma_u + vj) p_u(j) &= \lambda_u p_e(j-1) + v(j+1) p_u(j+1) \\ &+ (\delta + \mu) \frac{u}{1-u} p_e(j) \quad \forall j \geq 1, \end{aligned} \quad (15)$$

$$\lambda_u p_u(0) = v p_u(1) + (\delta + \mu) \frac{u}{1-u} p_e(0), \quad j = 0. \quad (16)$$

A.1.2 *Derivations of  $\Sigma$  Employed  $\Sigma_e$ .* Define the probability generating function (pgf) of the stationary distribution as

$$\Sigma_e(F) = \sum_{j=0}^{\infty} F^j p_e(j). \quad (17)$$

Summing equations (13) and (14) over  $j$  and using the definition of  $\Sigma_e(F)$  give

$$0 = -(\lambda_e(1-F) + \gamma_e + \mu + \delta) \Sigma_e(F) + v(1-F) \Sigma_e'(F) + \gamma_e + \mu + \delta.$$

Solving the differential equation gives

$$\Sigma_e(F) = \frac{1}{1-F} \int_F^1 \exp[-\lambda_e/v(\tilde{F} - F)] \left( \frac{1-\tilde{F}}{1-F} \right)^{\frac{\gamma_e + \mu + \delta}{v} - 1} \frac{\gamma_e + \mu + \delta}{v} d\tilde{F}.$$

The limits are

$$\Sigma_e(1) = 1, \quad (18)$$

$$\left. \frac{\partial \Sigma_e(F)}{\partial F} \right|_{F=1} = \frac{\frac{\lambda_e}{v}}{1 + \frac{\gamma_e + \mu + \delta}{v}} = \frac{\lambda_e}{(\gamma_e + \mu + \delta + v)}, \quad (19)$$

$$\left. \frac{\partial^2 \Sigma_e(F)}{\partial F^2} \right|_{F=1} = \frac{2\lambda_e^2}{(\gamma_e + \mu + \delta + v)(\gamma_e + \mu + \delta + 2v)}. \quad (20)$$

*Unemployed  $\Sigma_u$ .* Define the pgf for the average unemployed as

$$\Sigma_u(F, t) = \sum_{j=0}^{\infty} F^j p_u(j, t). \quad (21)$$

Summing equations (15) and (16) over  $j$  and using the definition of  $\Sigma_u(F)$  give

$$0 = -(\lambda_u(1-F) + \gamma_u)\Sigma_u(F) + v(1-F)\Sigma'_u(F) + (\delta + \mu)(1-u)/u\Sigma_e(F) + \gamma_u\Sigma_u(0).$$

Solving the differential equation using  $\Sigma_u(1) = 1$  gives

$$\begin{aligned} \Sigma_u(0) &= \frac{\int_0^1 \exp[-\lambda_u/v\tilde{F}](1-\tilde{F})^{\frac{\gamma_u}{v}-1} \left[ \frac{\gamma_u\Sigma_e(\tilde{F})}{v} \right] d\tilde{F}}{1 - \int_0^1 \exp[-\lambda_u/v\tilde{F}](1-\tilde{F})^{\frac{\gamma_u}{v}-1} \left[ \frac{\gamma_u}{v}(1-\Sigma_e(\tilde{F})) \right] d\tilde{F}}, \\ \Sigma_u(F) &= \frac{1}{1-F} \int_F^1 \exp[-\lambda_u/v(\tilde{F}-F)] \left( \frac{1-\tilde{F}}{1-F} \right)^{\frac{\gamma_u}{v}-1} \\ &\quad \times \left[ \frac{\gamma_u\Sigma_u(0)}{v} + \frac{(\delta + \mu)(1-u)/u\Sigma_e(\tilde{F})}{v} \right]. \end{aligned}$$

*Unemployed  $\Sigma_{uu}$ .* Lastly, we derive the distribution of wages that a worker expects who starts in unemployment with no prospects. The flow equations are then given by

$$\begin{aligned} 0 &= -(\lambda_u + \gamma_u + vj)p_{uu}(j) + \lambda_u p_{uu}(j-1) + v(j+1)p_e(j+1) \quad \forall j \geq 1, \\ 0 &= -(\lambda_u + \gamma_u)p_{uu}(0) + vp_{uu}(1) + \gamma_u. \end{aligned}$$

Rewriting in terms of the probability generating function gives

$$0 = -(\lambda_u(1-F) + \gamma_u)\Sigma_{uu}(F) + v(1-F)\Sigma'_{uu}(F) + \gamma_u.$$

Again, solving the differential equation gives

$$\Sigma_{uu}(F) = \frac{1}{1-F} \int_F^1 \exp[-\lambda_u/v(\tilde{F}-F)] \left( \frac{1-\tilde{F}}{1-F} \right)^{\frac{\gamma_u}{v}-1} \frac{\gamma_u}{v} d\tilde{F}.$$

**A.1.3 Distribution of outstanding matches  $G$**  The first and second derivative of  $G(\cdot)$  are given by

$$u = \frac{(\delta + \mu)}{(\delta + \mu + \gamma_u(1 - \Sigma_u(0)))}, \quad (22)$$

$$G(F) = \frac{(\delta + \mu)(\Sigma_u(F) - \Sigma_u(0))}{(1 - \Sigma_u(0))(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))}, \quad (23)$$

$$G'(F) = \frac{(\delta + \mu)\Sigma'_u(F)}{(1 - \Sigma_u(0))(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))} + \frac{\gamma_e(\delta + \mu)(\Sigma_u(F) - \Sigma_u(0))\Sigma'_e(F)}{(1 - \Sigma_u(0))(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))^2}, \quad (24)$$

$$G''(F) = \frac{(\delta + \mu)\Sigma''_u(F)}{(1 - \Sigma_u(0))(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))} + \frac{\gamma_e(\delta + \mu)(\Sigma_u(F) - \Sigma_u(0))\Sigma''_e(F)}{(1 - \Sigma_u(0))(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))^2} + 2\gamma_e\Sigma'_e(F)\frac{(\delta + \mu)\Sigma'_u(F)}{(1 - \Sigma_u(0))(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))^2} + 2\frac{\gamma_e^2(\delta + \mu)(\Sigma_u(F) - \Sigma_u(0))\Sigma'_e(F)^2}{(1 - \Sigma_u(0))(\delta + \mu + \gamma_e(1 - \Sigma_e(F)))^3}. \quad (25)$$

### A.2 Value functions

The value function can be calculated using the (expected) average flow benefit and the (expected) average duration using the formula

$$(\text{Avg. Duration})W(w(F), 0) = \text{Avg. Flow benefit.}$$

The average duration in a job with quality  $F$  is  $\delta + \mu + \gamma_e(1 - \Sigma_e(F))$ . The average flow benefits consist of the following parts: the wage  $w(F)$ , the search option  $\gamma_e \int_F^1 W(w(\tilde{F}), 0) d\Sigma_e(\tilde{F})$ , and the separation value  $\delta U_{eu}$  where  $U_{eu}$  refers to the average value in unemployment for a worker when the separation shock  $\delta$  hits. (Note that this value is different from the average value among the employed due to the different distribution of  $j$ .) The value function at the time of hiring is then given by

$$W(w(F), 0) = \frac{w(F) + \gamma_e \int_F^1 W(w(\tilde{F}), 0) d\Sigma_e(\tilde{F}) + \delta U_{eu}}{\delta + \mu + \gamma_e(1 - \Sigma_e(F))}, \quad (26)$$

$$W(w(F), 0) - W(0, 0) = \int_0^F \frac{w'(\tilde{F})}{\delta + \mu + \gamma_e(1 - \Sigma_e(\tilde{F}))} d\tilde{F}. \quad (27)$$

Similarly, for the unemployed, we can calculate the value functions as follows:

$$U(0) = \frac{b + \gamma_u \int_0^1 W(w(\tilde{F}), 0) d\Sigma_{uu}(\tilde{F})}{\mu + \gamma_u(1 - \Sigma_{uu}(0))},$$

$$U_{ue} = \frac{b + \gamma_u \int_0^1 W(w(\tilde{F}), 0) d\Sigma_u(\tilde{F})}{\mu + \gamma_u(1 - \Sigma_u(0))}.$$

Evaluating the value function for an employed worker at the worst match ( $F = 0$ ) and using  $W(w(0), 0) = U(0)$  gives the expression for the reservation wage

$$b = w(0) + \gamma_e \int_0^1 \frac{w'(\tilde{F})(1 - \Sigma_e(\tilde{F}))}{\delta + \mu + \gamma_e(1 - \Sigma_e(\tilde{F}))} d\tilde{F} - \gamma_u \int_0^1 \frac{w'(\tilde{F})(1 - \Sigma_{uu}(\tilde{F}))}{\delta + \mu + \gamma_e(1 - \Sigma_e(\tilde{F}))} d\tilde{F} \\ + \delta \left( \frac{\gamma_u \int_0^1 \frac{w'(\tilde{F})}{\delta + \mu + \gamma_e(1 - \Sigma_e(\tilde{F}))} (\Sigma_{uu}(\tilde{F}) - \Sigma_u(\tilde{F})) d(\tilde{F})}{(\mu + \gamma_u(1 - \Sigma_u(0)))} \right).$$

### A.3 Proof of identification

The identification argument relies on two steps. First, a subset of the transition parameters can be estimated independently from any other parameters. Second, we show that, conditional on the parameters identified in the first step, the remaining transition parameters can be estimated on the basis of transition rates alone.

First, starting with the independent identification. The transition parameters  $\mu$  and  $\delta$  can be estimated using the rate at which the employed workers leave employment for unemployment and to be out of the labor force, respectively. Similarly,  $\nu$  is identified from the rate at which vacancies expire; see Appendix A.4.

Second, turning to the identification of the remaining transition parameters. Define the function  $\Sigma_{s0}(F, t)$  as the probability generating function (pgf) for a worker who has been in employment state  $s \in \{u, e\}$  for duration  $t$  and moved into employment in a firm of productivity rank  $F$  without any opportunities. The differential equation for the function  $\Sigma_{s0}(F, t)$  satisfies

$$\frac{\partial \Sigma_{s0}(F, t)}{\partial t} = \nu(1 - F) \frac{\partial \Sigma_{s0}(F, t)}{\partial F} - \lambda_s(1 - F) \Sigma_{s0}(F, t) + \gamma_s \Sigma_{s0}(0, t)(1 - \Sigma_{s0}(F, t)).$$

We will use the following derivatives:

$$\frac{\partial^2 \Sigma_{s0}(F, t)}{\partial t \partial F} = -\nu \frac{\partial \Sigma_{s0}(F, t)}{\partial F} + \nu(1 - F) \frac{\partial^2 \Sigma_{s0}(F, t)}{\partial F^2} - \lambda_s(1 - F) \frac{\partial \Sigma_{s0}(F, t)}{\partial F} \\ + \lambda_s \Sigma_{s0}(F, t) - \gamma_s \Sigma_{s0}(0, t) \frac{\partial \Sigma_{s0}(F, t)}{\partial F}, \\ \frac{\partial^2 \Sigma_{s0}(F, t)}{\partial t^2} = \nu(1 - F) \frac{\partial^2 \Sigma_{s0}(F, t)}{\partial F \partial t} - \lambda_s(1 - F) \frac{\partial \Sigma_{s0}(F, t)}{\partial t} \\ + \gamma_s \frac{\partial \Sigma_{s0}(0, t)}{\partial t} (1 - \Sigma_{s0}(F, t)) - \gamma_s \Sigma_{s0}(0, t) \frac{\partial \Sigma_{s0}(F, t)}{\partial t}, \\ \frac{\partial^3 \Sigma_{s0}(F, t)}{\partial t^2 \partial F} = \nu(1 - F) \frac{\partial^3 \Sigma_{s0}(F, t)}{\partial F^2 \partial t} - \nu \frac{\partial^2 \Sigma_{s0}(F, t)}{\partial F \partial t} - \lambda_s(1 - F) \frac{\partial^2 \Sigma_{s0}(F, t)}{\partial t \partial F} \\ + \lambda_s \frac{\partial \Sigma_{s0}(F, t)}{\partial t} - \gamma_s \frac{\partial \Sigma_{s0}(0, t)}{\partial t} \frac{\partial \Sigma_{s0}(F, t)}{\partial F} - \gamma_s \Sigma_{s0}(0, t) \frac{\partial^2 \Sigma_{s0}(F, t)}{\partial t \partial F},$$

$$\begin{aligned} \frac{\partial^3 \Sigma_{s0}(F, t)}{\partial t^3} &= v(1-F) \frac{\partial^3 \Sigma_{s0}(F, t)}{\partial F \partial t^2} - \lambda_s(1-F) \frac{\partial^2 \Sigma_{s0}(F, t)}{\partial t^2} + \gamma_s \frac{\partial^2 \Sigma_{s0}(0, t)}{\partial t^2} (1 - \Sigma_{s0}(F, t)) \\ &\quad - 2\gamma_s \frac{\partial \Sigma_{s0}(0, t)}{\partial t} \frac{\partial \Sigma_{s0}(F, t)}{\partial t} - \gamma_s \Sigma_{s0}(0, t) \frac{\partial^2 \Sigma_{s0}(F, t)}{\partial t^2}. \end{aligned}$$

For  $F = 0$  and  $t = 0$ , since the worker has no prospects,  $\Sigma_s(0, 0) = 1$  and  $\frac{\partial \Sigma_s(0, 0)}{\partial F} = \frac{\partial^2 \Sigma_s(0, 0)}{\partial F^2} = 0$ . The expressions therefore simplify to

$$\frac{\partial \Sigma_{s0}(0, 0)}{\partial t} = -\lambda_s,$$

$$\frac{\partial^2 \Sigma_{s0}(0, 0)}{\partial t \partial F} = \lambda_s,$$

$$\begin{aligned} \frac{\partial^2 \Sigma_{s0}(0, 0)}{\partial t^2} &= v \frac{\partial^2 \Sigma_{s0}(0, 0)}{\partial F \partial t} - \lambda_s \frac{\partial \Sigma_{s0}(0, 0)}{\partial t} - \gamma_s \frac{\partial \Sigma_{s0}(0, 0)}{\partial t} \\ &= \lambda_s(v + \lambda_s + \gamma_s), \end{aligned}$$

$$\frac{\partial^3 \Sigma_{s0}(0, 0)}{\partial F^2 \partial t} = 0,$$

$$\begin{aligned} \frac{\partial^3 \Sigma_{s0}(0, 0)}{\partial t^2 \partial F} &= -v \frac{\partial^2 \Sigma_{s0}(0, 0)}{\partial F \partial t} - \lambda_s \frac{\partial^2 \Sigma_{s0}(0, 0)}{\partial t \partial F} + \lambda_s \frac{\partial \Sigma_{s0}(0, 0)}{\partial t} - \gamma_s \Sigma_{s0}(0, 0) \frac{\partial^2 \Sigma_{s0}(0, 0)}{\partial t \partial F} \\ &= -\lambda_s(v + 2\lambda_s + \gamma_s) \end{aligned}$$

$$\begin{aligned} \frac{\partial^3 \Sigma_{s0}(0, 0)}{\partial t^3} &= v \frac{\partial^3 \Sigma_{s0}(0, 0)}{\partial F \partial t^2} - \lambda_s \frac{\partial^2 \Sigma_{s0}(0, 0)}{\partial t^2} \\ &\quad - 2\gamma_s \frac{\partial \Sigma_{s0}(0, 0)}{\partial t} \frac{\partial \Sigma_{s0}(0, 0)}{\partial t} - \gamma_s \frac{\partial^2 \Sigma_{s0}(0, 0)}{\partial t^2} \\ &= -\lambda_s(v + 2\lambda_s + \gamma_s)v - \lambda_s \lambda_s(v + \lambda_s + \gamma_s) - 2\gamma_s \lambda_s^2 - \gamma_s \lambda_s(v + \lambda_s + \gamma_s). \end{aligned}$$

From the data, we can estimate the job finding rate of someone employed in the lowest job  $F = 0$  as a function of tenure. We denote the quit rate in the lowest match quality by  $H_e(0, t) = \gamma_e(1 - \Sigma_{e0}(0, t))$ . Similarly, the job finding rate for someone with unemployment duration  $t$  (when they started unemployment without any prospects) is denoted by  $H_u(0, t) = \gamma_u(1 - \Sigma_{u0}(0, t))$ . The least productive firm  $F = 0$  can be identified by just finding the firm that pays the lowest wage. Similarly, we can identify workers how start unemployment without any prospects by those that laid off with from very short jobs. Thus, from the estimated function  $H_s(0, t) \forall s \in \{e, u\}$ , we get the equations

$$\frac{\partial H_s(0, 0)}{\partial t} = \lambda_s \gamma_s, \quad (28)$$

$$\frac{\partial^2 H_s(0, 0)}{\partial t^2} = -\gamma_s \lambda_s(v + \lambda_s + \gamma_s), \quad (29)$$

$$\begin{aligned} \frac{\partial^3 H_s(0, 0)}{\partial t^3} = & -\gamma(\lambda_s(v + 2\lambda_s + \gamma_s)v - \lambda_s\lambda_s(v + \lambda_s + \gamma_s) \\ & - 2\gamma_s\lambda_s^2 - \gamma_s\lambda_s(v + \lambda_s + \gamma_s)) \end{aligned} \quad (30)$$

substituting for  $\gamma_s$  in the second equation gives the quadratic equation for  $\lambda_s$

$$\left( \frac{\partial H_s(0, 0)}{\partial t} \right)^{-1} \frac{\partial^2 H_s(0, 0)}{\partial t^2} \lambda_s + v\lambda_s + \lambda_s^2 + \frac{\partial H_s(0, 0)}{\partial t} = 0. \quad (31)$$

This gives two potential solutions for  $\lambda_s$ . Since  $\gamma_s$  and  $\lambda_s$  enter the first and second derivative of  $H_s$  symmetrically, the solution for  $\gamma_s$  is the other root. However, since the third derivative is not symmetric with respect to  $\gamma_s$  and  $\lambda_s$ , there is a unique solution.

Notice, one could take higher order partial derivatives with respect to time of the function  $\Sigma_{s0}(F, t)$  to additionally identify  $v$ . Since the expressions are difficult to interpret, and in practice this would require placing a large burden on the data, we instead opt to calibrate it outside of estimation.

#### A.4 Vacancy duration

The data are taken from the “The Conference Board Help Wanted Online Data Series” (HWOL). The HWOL aims at an exhaustive coverage of all job vacancies advertised online. The data are collected from over 16,000 online job boards and contain two time series which start in May 2005. The first is “new ads,” that is, the number of unduplicated ads that did not appear in the previous reference period. An ad is only counted as “new” in the first reference point in which it appears. The second variable is “total ads.” This is the total number of unduplicated ads appearing in the reference period. This is the sum of “new ads” and reposted ads from previous periods. Finally, it is worth noting that a reference period is centered on the first of the months. For example, “total ads” for October is the sum of all posted ads from September 14th until October 13th.

*Expiry rate of a vacancy.* We use these data to infer the rate at which vacancies expire. A steady-state approximation implies that the inflow of new vacancies in month  $t$  ( $n_t$ ) is equal to the total amount of vacancies expiring, the product of the stock ( $v_t$ ) and the expiry rate ( $\sigma_t$ ).

$$n_t \approx \sigma_t v_t.$$

Unfortunately, we do not observe a snapshot of the stock of vacancies. Instead, we observe the total vacancies that have accumulated over that reference period, which we call  $V_t$ . Since the stock of vacancies is constant over a reference period, given our steady-state assumption, we can approximate  $v_t$  as

$$v_t \approx V_t - n_t.$$

Combining the above gives a straightforward approximation of the monthly rate at which vacancies expire for a reference period  $t$ ,

$$\sigma_t \approx \frac{n_t}{V_t - n_t}.$$

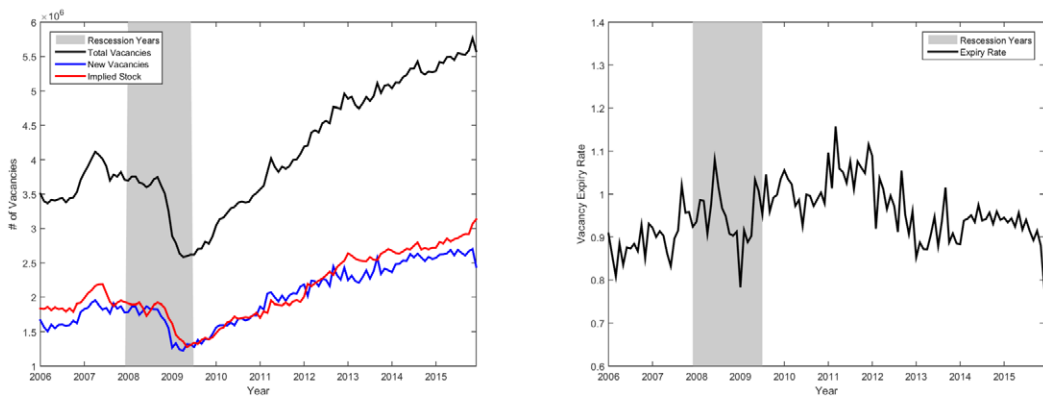


FIGURE A.1. Vacancy series.

We restrict the attention to the decade January 2006 to December 2016. Changing the time horizon does little to change the mean monthly expiration date which is computed as 0.95, implying that vacancies last a little longer than a month. The series are presented in Figure A.1. The first panel shows the raw series of total and new vacancies as well as the implied number of vacancies in the stock at that point in time. The second panel shows the implied expiry rate of vacancies over the period.



## A.5 Wage competition

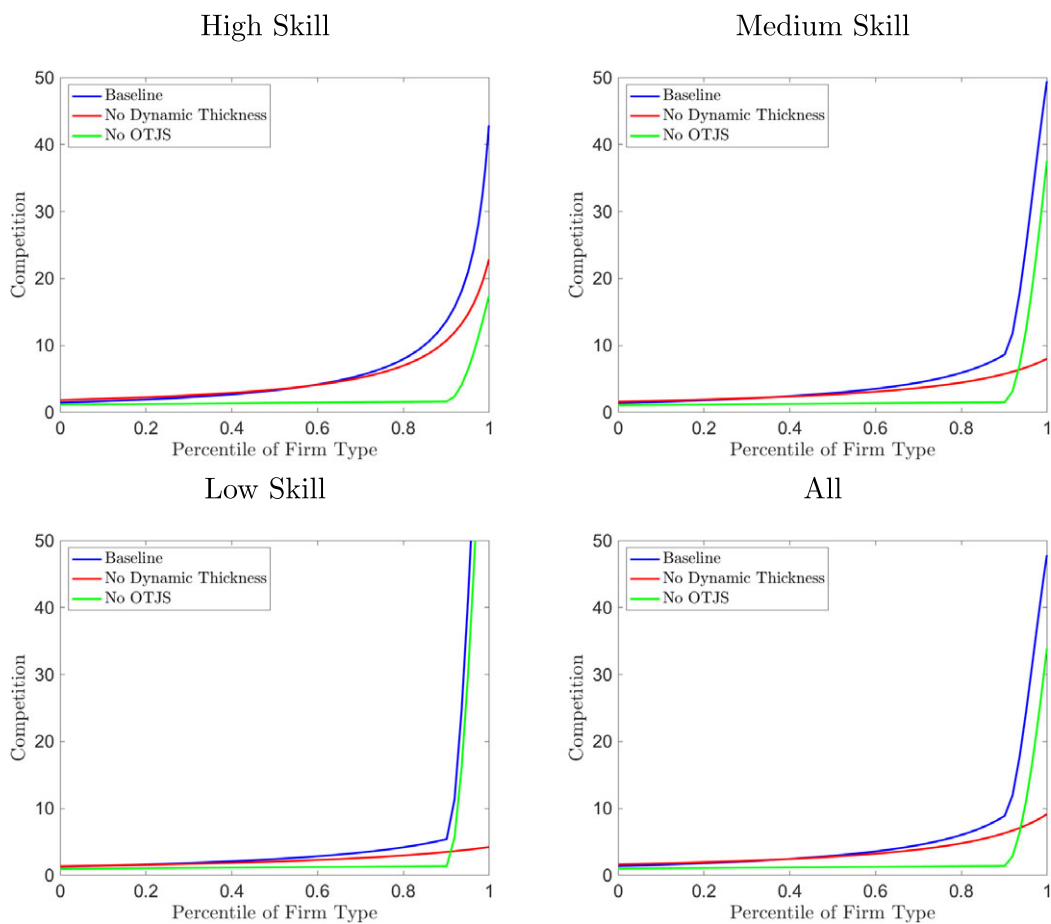


FIGURE A.2. Wage competition. *Note:* This figure plots the degree of competition in each model, as defined by  $\ell'(F)/\ell(F)$ .

## A.6 Proportion of wages driven by the retention motive

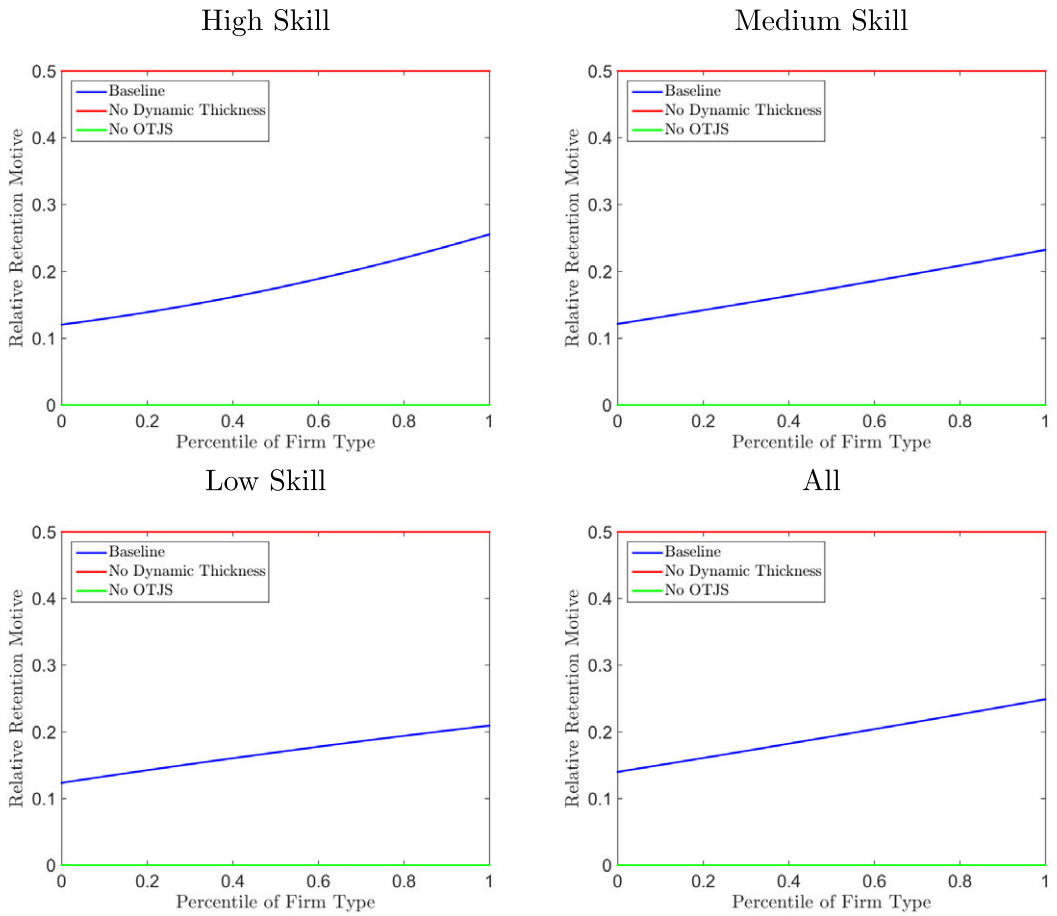


FIGURE A.3. Proportion of wages driven by the retention motive. *Note:* The relative retention motive is bounded on  $[0, 1]$  and defined as  $r(F)/(r(F) + h(F))$ .

## APPENDIX S: SUPPLEMENTARY APPENDIX

This Supplementary Appendix contains additional information for the paper by [Bradley and Gottfries \(2021\)](#). All references to sections and equations refer to this main paper.

S.1 *Sample selection*

We make a special effort to ensure that the variables circumscribing the samples are consistent across surveys. That is, the following filters are passed through each survey:

- (i) The attention is restricted to a sample of only male workers. The sex of a worker is defined in the SIPP by the variable *esex* and *pesex* in the CPS.
- (ii) We use the full-time window in the 1996 SIPP, including early observations based on recall of previous employment. This corresponds to observations from December 1995 until February 2000, inclusive. The identical window is used in the CPS.
- (iii) Motivated by differential mobility rates by age (see Appendix S.2), the attention is restricted to only workers between 25 and 45, where age is defined as a respondent's age as of last birthday in the variable *tage* in the SIPP and age as of the end of the survey week in the CPS by the variable *peage*. Note that this will introduce negligible differences across samples when a respondent's birthday occurs in a CPS surveying week.
- (iv) Skill groups are defined by the variables *eeducate* in the SIPP and *peeduca* in the CPS. The two variables are defined identically with one exception. The CPS variable differentiates between having a “*diploma or certificate from a voc, tech, trade, or bus school beyond high school*” and having an “*associate degree in college—occupational/vocational program*” while the SIPP variable agglomerates the two. We treat these two groupings as college educated and include them as high-skill workers. All other groupings are noncontroversial.

## S.2 Transition rates by age

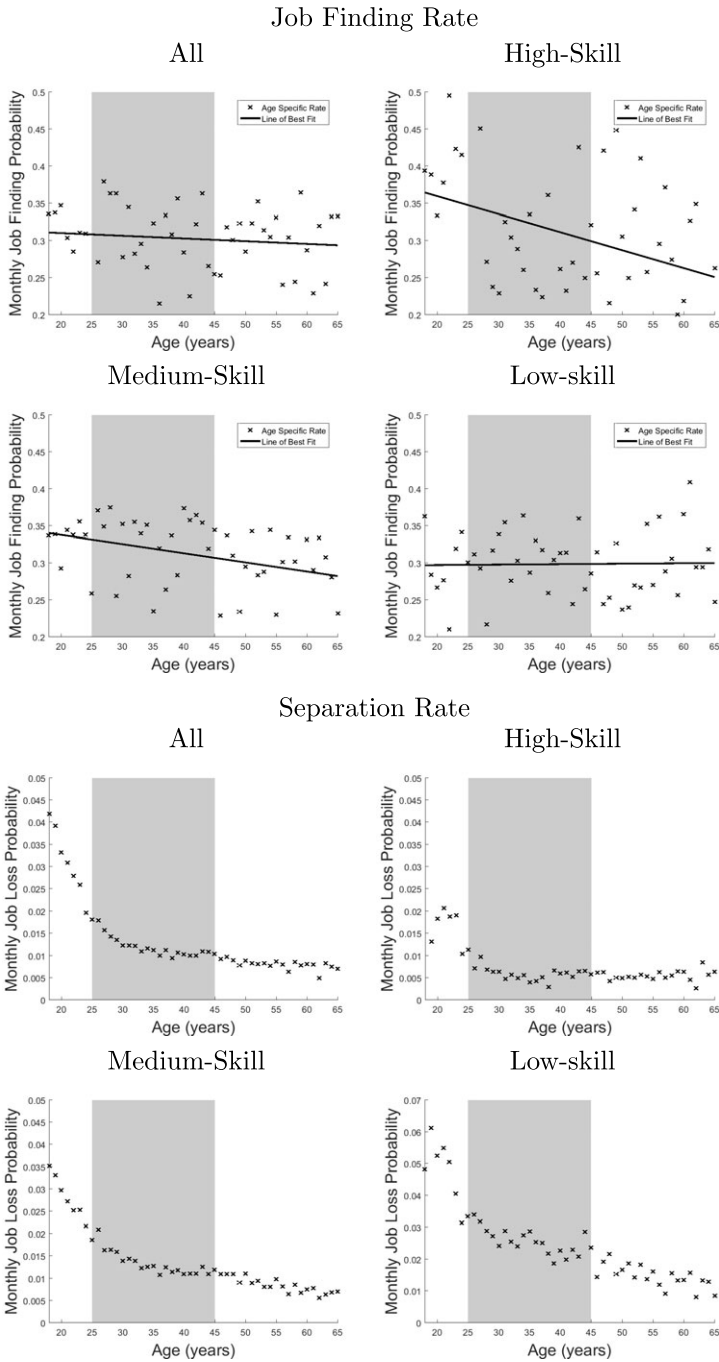


FIGURE S.1. Transition Rates by Age. *Note:* The x's represent the appropriate monthly transition probability for a male of that age. The shaded region represents the specific age we will focus on in our analysis. Data come from the CPS, 1996–1999 inclusive.

## S.3 Fit of the wage distribution

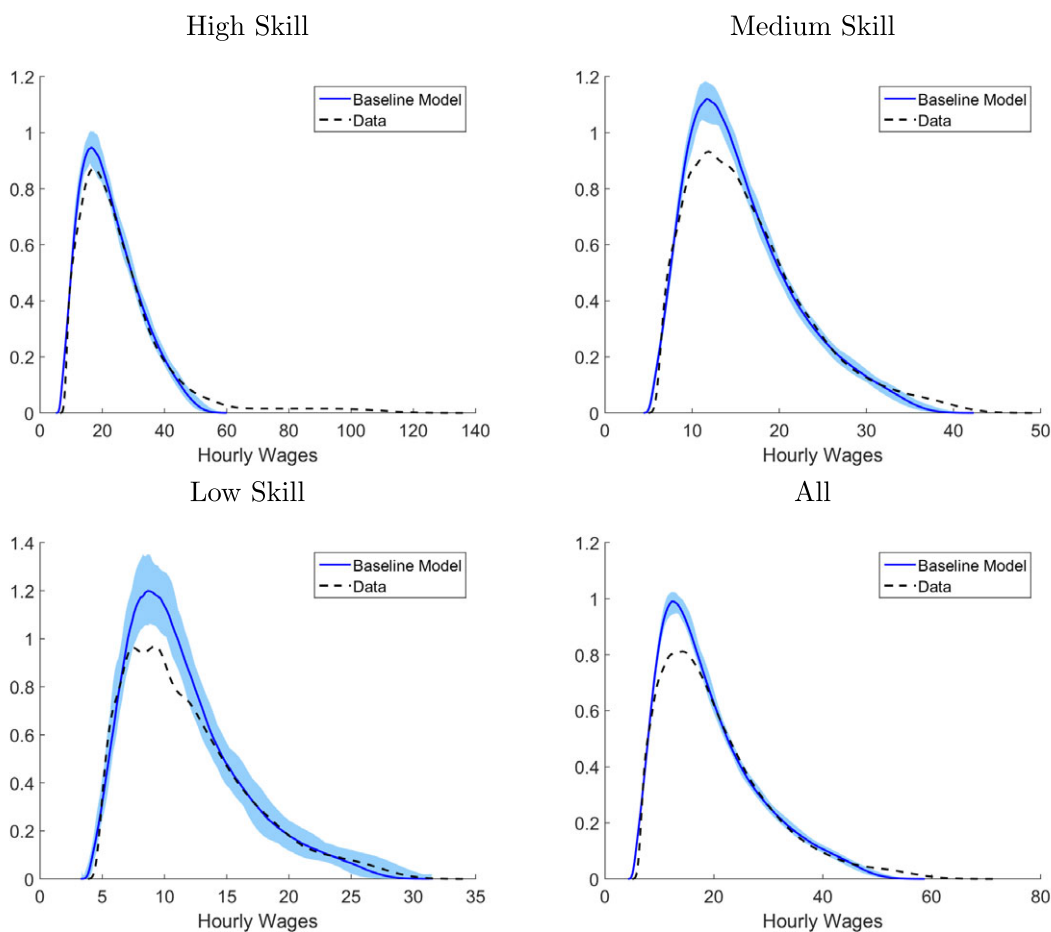


FIGURE S.2. Fit of the wage distributions. *Note:* Distributions are kernel density plots of the simulated and empirical data. The shaded areas represent 99% confidence intervals based on a repeated resimulation of the model.

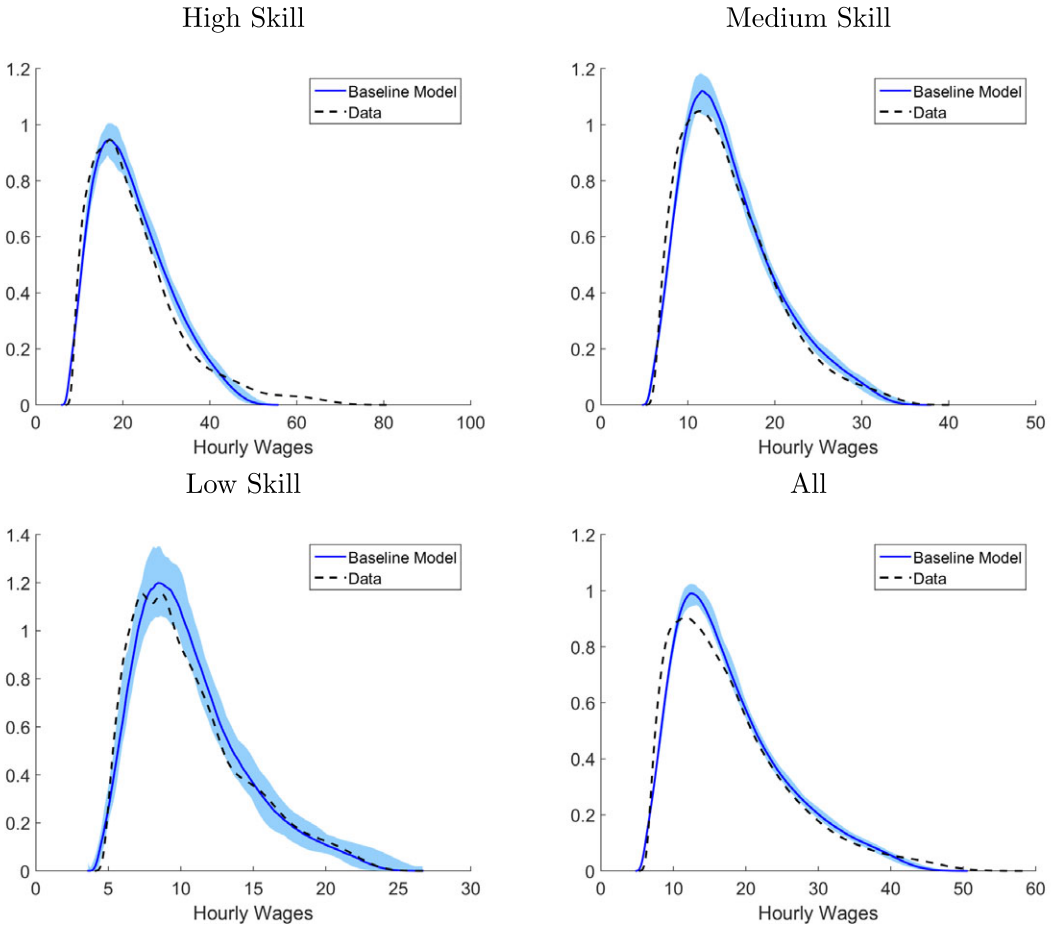


FIGURE S.3. Fit of the distribution of mean wages. *Note:* Distributions are kernel density plots of the simulated and empirical data. The shaded areas represent 99% confidence intervals based on a repeated resimulation of the model.

### S.4 Robustness and discussion for the expiry of prospects $\nu$

In this section, we assess the quantitative robustness of the model to the specific parameterizations of  $\nu$  and  $\gamma_s$ , that is, the rate that opportunities expire and the frequency to which the unemployed and employed apply to jobs. The only parameter calibrated outside of estimation is the frequency in which job opportunities/vacancies expire,  $\nu$ . Clearly, all estimates are conditional on the specific value of this calibration. Here, we demonstrate that calibrating  $\nu$  across a broad range of values found in the literature and re-estimating the model does not change our results in a quantitatively meaningful way. The rates at which workers in state  $s$  apply to jobs,  $\gamma_s$ , by contrast, are well identified. However, it is a parameter that could potentially be endogenized, workers optimally decide when to apply. To do this, one would have to take a stance on what is in a worker's information set. For example, are they aware of the number of opportunities afforded to them or form expectations, conditional on employment status and tenure? We show, in a partial equilibrium setting, that across skill-groups and employment states, little gains can be made by allowing the choice to apply to their set of opportunities. If a worker could access the offers at a small cost, that is, the  $\gamma_s$  are partially endogenous, a small cost would be sufficient prevent the worker from exercising this option. Thus, we interpret this as implying that our model, with an exogenous process for  $\gamma_s$ , is a good approximation for a setting in which applications are being made endogenously.

*Calibration of  $\nu$ .* By computing the average vacancy duration in Section A.4,  $\nu$  is calibrated as 0.95. However, there is a fairly broad range of *sensible* calibrations; see Table S.1. To assess the implication of the value of  $\nu$  we reestimate the first step of the model varying  $\nu$  from a half to one and a half, implying a vacancy duration between 3 weeks and 2 months. The productivity parameters are then reestimated ensuring the level of frictional wage dispersion remains fixed. This is analogous to the way in which the models with no dynamic thickness or no on the job search are estimated. For brevity, this exercise is conducted only on the unstratified sample.

TABLE S.1. Estimates of mean vacancy duration in the U.S.

Paper	Data Source	Time Period	Mean Duration	Implied $\nu$
Brenic and Norris (2012)	Monster.com	2004–2006	44 days	0.68
Davis and Samaniego de la Parra (2017)	JOLTS	2012–2016	41.9	0.72
Davis and Samaniego de la Parra (2017) + Crane, Davis, Faberman, and Haltiwanger (2016)*	JOLTS + SCE	2012–2016	58.1 days	0.52
Davis, Faberman, and Haltiwanger (2013) <sup>×</sup>	DHI-DFH	2001–2018	30.5	1.02
Marinescu and Wolthoff (2020)	CareerBuilder.com	2011	15.7 days	1.91
This paper	HWOL	2005–2018	28.5 days	0.95

*Note:* \* The 16.2 days from the addition of Crane et al. (2016) represents the additional lapsed time associated with the “start lag.” The time taken after a vacancy is filled and when the job commences.

<sup>×</sup> This moment is taken from the mean of the “DHI-DFH Mean Vacancy Duration Measure,” based on a time series for the U.S. from January 2001 until April 2018. Data are taken from DHI Group, Inc., DHI-DFH Mean Vacancy Duration Measure [DHIDFHMVDM], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/DHIDFHMVDM>. Note, this is measured in working days, so to compute calendar days durations are multiplied by 7/5.

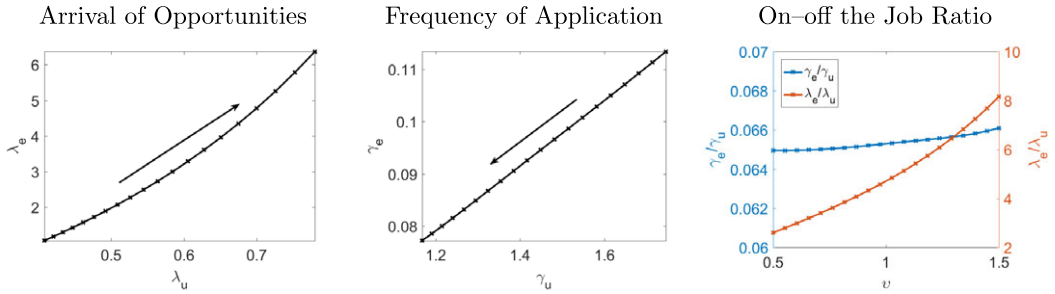


FIGURE S.4. Transition parameters-varying upsilon. *Notes:* The connected ‘x’s represent individual estimates for a given  $\nu$ . The large arrows in the first two panels show the direction in which  $\nu$  is increasing, values range from 0.5 to 1.5.

Figure S.4 shows how the transitional parameters change varying the parameter  $\nu$ . As the rate at which vacancies expire increases, in order to match the transitional moments,  $\lambda_u$  and  $\lambda_e$ , the rate opportunities arrive increase. However, since workers now lose their opportunities at a higher rate, differences between the short and long term unemployed reduce. To maintain the same job finding rates by duration of unemployment, the estimates of  $\lambda_e$  increase at a faster rate than  $\lambda_u$ . Changes to  $\gamma_u$  and  $\gamma_e$ , the frequency to which unemployed and employed workers access the market are, by comparison, small. This is because the majority of workers, particularly the employed and recently unemployed, have some opportunities. Further, the ratio of  $\gamma_e$  to  $\gamma_u$  remains fairly stable for any  $\nu$  whereas implied differences in the arrival rate of opportunities across employment state vary enormously with  $\nu$ .

Rather than the parameter values, what is perhaps more important is how the implications of the model vary with  $\nu$ . In particular, in determining the primary cause of unemployment and the ability of the model to replicate wage dispersion. In Section 3.4, the unemployment rate is computed assuming all workers have labor market opportunities. Since as  $\nu$  increases the estimate of  $\gamma_u$  decreases, so fewer unemployed apply for jobs, thus this hypothetical unemployment rate increases with  $\nu$ . However, since changes in  $\gamma_u$  are relatively small, so are changes to the unemployment decomposition. Across the entire span of  $\nu$  both a lack of opportunity and simply not accessing the market are quantitatively important. The latter varies from explaining 38% of total unemployment, when  $\nu$  is a half, to 56% when  $\nu$  is one and a half.

Turning to wage dispersion, as is made apparent in the main body of the text, the canonical job ladder model struggles to generate the level of frictional wage dispersion seen in the data without negative replacement rates. Figure S.5 documents how the implied replacement rate changes with  $\nu$ , fixing the same level of frictional wage dispersion. Over the span of calibrated  $\nu$  the replacement rate varies from a low of 15% to a high of 30%. Although the implied value of home production clearly depends on the specific value of  $\nu$ , the model can also generate positive replacement ratios consistent with the macro labor literature for the empirically relevant range of values of  $\nu$ . Given the level of frictional wage dispersion, the two components that determine the replacement rate are the search option and the insurance option. These two effects move in



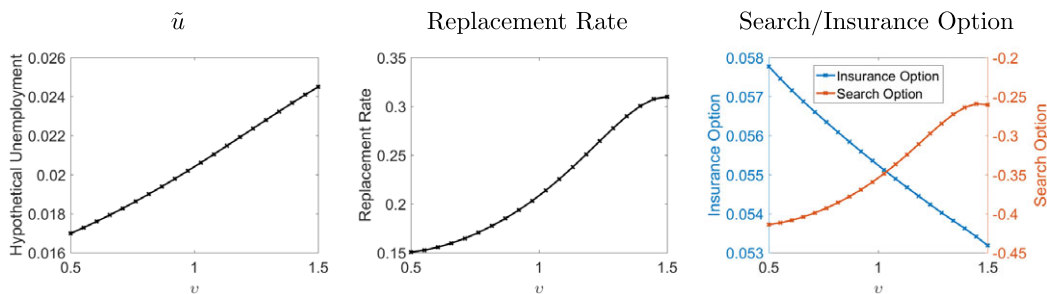


FIGURE S.5. Replacement rate-varying upsilon.

opposite directions in response to a change in the expiration of opportunities; again see Figure S.5. The search option increases, decreasing in absolute terms. Since  $\lambda_e$  is growing relative to  $\lambda_u$  as  $v$  increases, there is an increase in the returns to taking a job for a given wage, and less value in remaining unemployed. The insurance option, the benefits of returning to unemployment in a better position, declines with  $v$ . Since opportunities disappear more quickly, the short and long term unemployed are in similar positions after a shorter lapse of time. Since the search option is an order of magnitude larger than the insurance option, our estimates of the replacement rate increase with larger values of  $v$ .

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