Supplementary Material

Supplement to "Does affirmative action lead to mismatch? A new test and evidence"

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Appendix B: Details about the implementation of the nonparametric estimation in Section 6

We propose an empirical strategy that consists of the following steps:

Step 1. Invoking Kotlarski's (1967) theorem, we separately recover the marginal distributions of X_C, X_U, and X_S from the observed joint distribution of (W_U, W_S).

Step 2. We draw random samples of $\{X_{Ci}, X_{Ui}, X_{Si}\}$ from the marginal distributions of X_C, X_U, and X_S recovered in Step 1.

Step 3. We obtain samples of $\{W_{Ui}, W_{Si}\}$ from the random samples of $\{X_{Ci}, X_{Ui}, X_{Si}\}$ generated in Step 2 and then recover a sample of Y_i conditional on $\{W_{Ui}, W_{Si}\}$ using multiple imputation methods.[29]

Step 4. We run regressions of Y on X_C, X_U, and X_S using the pseudo-sample $\{Y_i, X_{Ci}, X_{Ui}, X_{Si}\}$ simulated above to estimate γ_C, γ_U, and γ_S, and to perform variance decomposition.

We now provide more details about each of the steps, beginning with recovering the marginal distributions of X_C, X_U, and X_S. Let

$$\Psi(t_1, t_2) = \mathrm{E}\exp(it_1 W_U + it_2 W_S) \tag{B1}$$

[29]See Rubin (1987) for an extensive description of this methodology.

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denote the characteristics function for the observed joint random vector (W_U, W_S) and let

$$\begin{aligned}\Psi_1(t_1, t_2) &\equiv \frac{\partial \Psi(t_1, t_2)}{\partial t_1} \\ &= \mathrm{E}[iW_U \exp(it_1 W_U + it_2 W_S)]\end{aligned} \tag{B2}$$

denote the derivative of $\Psi(\cdot, \cdot)$ with respect to its first argument. Then the Kotlarski theorem shows that the characteristic functions for random variables X_C, X_U, and X_C are, respectively, given by

$$\begin{aligned}\Psi_{X_C}(t) &= \exp\left(\int_0^t \frac{\Psi_1(0, t_2)}{\Psi(0, t_2)}\, dt_2\right), \\ \Psi_{X_U}(t) &= \frac{\Psi(t, 0)}{\Psi_{X_C}(t)}, \\ \Psi_{X_S}(t) &= \frac{\Psi(0, t)}{\Psi_{X_C}(t)}.\end{aligned}$$

Finally the characteristic functions of these three random variables uniquely determines the probability density function via an inversion formula. Let f_{X_C}, f_{X_U}, and f_{X_S}, respectively, denote the marginal probability density function for random variables X_C, X_U, and X_S. Following the inversion formula described in Horowitz (1998, p. 104), we have

$$f_{X_K}(x_K) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp(-itx_K)\Psi_{X_K}(t)\, dt \quad \text{for } K \in \{C, U, S\}.$$

We are now in a position to describe the somewhat standard estimation procedure needed to carry out Step 1.[30] The key is to estimate $\Psi(\cdot, \cdot)$ and $\Psi_1(\cdot, \cdot)$ by their sample analogs: given a sample $\{W_U^j, W_S^j\}_{j=1}^n$,

$$\begin{aligned}\widehat{\Psi(t_1, t_2)} &= \frac{1}{n} \sum_{j=1}^n \exp(it_1 W_U^j + it_2 W_S^j), \\ \widehat{\Psi_1(t_1, t_2)} &= \frac{1}{n} \sum_{j=1}^n iW_U^j \exp(it_1 W_U^j + it_2 W_S^j).\end{aligned}$$

The characteristic functions $\Psi_{X_K}(t)$ for $K \in \{C, U, S\}$ can in turn be estimated by replacing $\Psi(\cdot, \cdot)$ and $\Psi_1(\cdot, \cdot)$ with their estimates above. Applying Kotlarski's decomposition to $\{W_U, W_S\}$ allows to generate data on $\{X_{Ci}, X_{Ui}, X_{Si}\}$ and, therefore, $\{W_{Ui}, W_{Si}\}$ (Steps 2 and 3) by simply drawing from the marginal distributions.

The next step, Step 4, is to obtain a sample of grades (i.e., Y_i) conditional on W_{Ui} and W_{Si} by multiple imputation. Here we follow Rubin (1987). The basic steps of Rubin multiple imputation are as follows:

[30] See Krasnokutskaya (2011) for similar estimation procedure. Horowitz (1998, Chapter 4) described some useful suggestions for issues related to smoothing.

(i) Calculate $V = (W'W)^{-1}$, $\widehat{\beta} = VW'Y$, and $\widehat{Y} = W'\widehat{\beta}$, where $W = \{W_U, W_S\}$.

(ii) Draw a random g from χ^2 distribution with degree of freedom $n_{\text{obs}} - r$.

(iii) Calculate $\sigma_*^2 = (Y - \widehat{Y})'(Y - \widehat{Y})/g$.

(iv) Draw an r-dimensional Normal random vector $D \sim N(0, I_r)$, where I_r is the identity matrix of dimension r.

(v) Calculate $\widehat{\beta}_* = \widehat{\beta} + \sigma V^{1/2} D$, where $V^{1/2}$ is the triangular square root of V obtained by the Cholesky decomposition.

(vi) Calculate predicted values $\widehat{Y}_i = W_i' \widehat{\beta}_*$.

(vii) For each missing value, find the respondent whose $\widehat{Y}$ is closest to $\widehat{Y}_i$ and take Y of this respondent as the imputed value (predictive mean matching).[31]

We then regress the generated outcomes on the generated regressors.

References

Horowitz, J. (1998), *Semiparametric Methods in Econometrics*. Springer. [2]

Krasnokutskaya, E. (2011), "Identification and estimation in highway procurement auctions under unobserved auction heterogeneity." *Review of Economic Studies* (forthcoming). [2]

Rubin, D. B. (1987), *Multiple Imputation for Nonresponse in Surveys*. Wiley, New York. [1, 2]

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[31] To test for robustness of the results, we also implemented a nonparametric approach to recover Y_i. Basically, we draw a sample of Z_i conditional on $\{W_{Ui}, W_{Si}\}$ from the observed conditional distribution $G(Y|W_U, W_S)$, which was obtained using the Epanechnikov kernel ($K(u) = \frac{3}{4}(1 - u^2)1_{(|u| \le 1)}$). The smoothing parameter was selected by following a refined plug-in method, which tries to find the bandwidth that minimizes the mean integrated square error. Results obtained using this strategy did not differ significantly from those using the multiple imputation technique.