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Estimating a model of excess demand for public housing

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The purpose of this paper is to develop and estimate a new equilibrium model of public housing that acknowledges the fact that the demand for public housing may exceed the available supply. We show that ignoring these supply side restrictions leads to an inconsistent estimator of household preferences. We estimate the parameters of the model based on a unique panel data set of low-income households in Pittsburgh. We find that public housing is an attractive option for seniors and exceedingly poor households headed by single mothers. We also find that for each family that leaves public housing there are, on average, 3.8 families that would like to move into the vacated unit. Simple logit demand models that ignore supply side restrictions cannot generate reasonable wait times and wait lists. Demolitions of existing units increase the degree of rationing and potentially result in welfare losses. An unintended consequence of demolitions is that they increase racial segregation in low-income housing communities.

KEYWORDS. Excess demand, rationing, search, equilibrium analysis, welfare analysis, enriched sampling, computational general equilibrium analysis.

JEL CLASSIFICATION. C33, C83, D45, D58, H72, R31.

1. INTRODUCTION

Providing adequate housing and shelter for low-income households is a stated policy goal of most administrations in the United States and Europe. One important policy, implemented by the Department of Housing and Urban Development (HUD), subsidizes the construction and maintenance of affordable housing communities in cities and metropolitan areas in the United States.[1] Low-income households are eligible for public housing assistance in the United States if their income is below a threshold that depends on household composition and region. Given the current standards for determining eligibility, there is typically a large number of households in each metropolitan area eligible for public housing.

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[1] Low-income housing programs in the United States grew out of the demand to address threats to public health and safety that resulted from low-cost, high-density housing neighborhoods for poor, mostly immigrant, families in the early twentieth century. Similar government institutions and programs exist in most European countries.

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The supply of public housing units is primarily determined by current and past political decisions that have allocated funding to local housing authorities. Since the rent charged for public housing is a fixed percentage of household income, there is no price mechanism to ensure that public housing markets clear. When the demand for public housing exceeds supply, there are long wait lists to get in. As a consequence, we cannot use standard demand models to estimate the preferences for public housing. The purpose of this paper is to develop and estimate a new model of demand for public housing that acknowledges the fact that the demand for public housing may exceed the available supply. We show that ignoring these supply side restrictions leads to inconsistent estimators of household preferences.

There are long wait lists for public housing in many metropolitan areas. While we can obtain some aggregate summary statistics that broadly measure the average wait time in these markets, these aggregate statistics are not sufficient to estimate a model that captures heterogeneity across households. Local housing authorities are not willing to disclose detailed microlevel data on wait lists. To our knowledge, there is no empirical research that uses household level, wait list data to study rationing in public housing markets. The key challenge is, therefore, to estimate a model that treats the wait list as latent.

We develop an equilibrium model that incorporates supply restrictions that arise from the administrative behavior of the local housing authority. A household can move into public housing if and only if the housing authority offers them a vacant apartment. The ability of the housing authority to offer apartments to eligible households is largely determined by voluntary exit decisions of households that currently live in housing communities. Exit from public housing is a stochastic event since it is partially determined by idiosyncratic preference and income shocks that are not observed by administrators. The housing authority's objective is to fill all vacant units. If the potential demand exceeds the available units at any point in time, the housing authority has to ration access to public housing.

Eligible households that have not been offered an apartment in an affordable housing community are placed, in our model, on a wait list. Each period, a fraction of households on the wait list will receive an offer to move into one of the apartments that has recently become available. If the total supply of public housing is fixed and vacancy rates are constant over time, the housing authority adjusts the offer probabilities in equilibrium so that the inflow into public housing equals the voluntary outflow. We define an equilibrium for our model and characterize its properties. We show that a unique equilibrium exists if there are no transfers between public housing communities. If transfers are possible, the equilibrium is also unique as long as the housing authority adopts an equal treatment policy.

We show how to identify and estimate the parameters of the model using data on observed choices, but unobserved wait lists. Since we do not observe the wait list, we do not know which households received offers to move into housing communities. We only observe those offers that were accepted and resulted in a move.[2] The basic insight

[2]This type of selection problem is also encountered in labor search and occupational choice models. For a discussion of identification and estimation of labor search model, see, among others, Eckstein and Wolpin

of our identification approach is that offer probabilities are endogenous and are constrained to satisfy equilibrium conditions. Hence, offer probabilities can be expressed as functions of the structural parameters of the housing choice models. Moreover, exit is purely voluntary and does not depend on offer probabilities. As a consequence, exit behavior is informative about the structural parameters of the utility function. Imposing the equilibrium conditions then establishes identification of the structural parameters of the model.

We estimate the model using a unique data set from the Housing Authority of the City of Pittsburgh (HACP).[3] We supplement these data with a sample of eligible low-income households in the Survey of Income and Program Participation, which allows us to follow eligible households outside of public housing.

We find that households that have income well below the poverty line and are headed by single mothers have strong preferences for public housing. African American households also have strong public housing preferences. The income coefficient shows that there are strong incentives for households to leave public housing as their income grows larger. These incentives are offset by the presence of significant moving costs that constrain potential relocations of households. We find that for each family that leaves public housing there are, on average, 3.8 families that would like to move into the vacated unit. For seniors, the rationing is more pronounced. For each senior who moves out of a housing community there are 23.2 senior households that would like to move in.

Finally, we conduct some counterfactual policy experiments. We evaluate a policy that considers the demolition of some of the existing public housing units. We find that the welfare costs of demolishing even the least desirable units are substantial. Displaced African American females are disproportionately disadvantaged, which raises some serious issues related to the distributional impact of these demolition programs. An unintended consequence is that the resulting equilibrium demographic distribution in the remaining public housing communities exhibits some increase in the proportions of female and African American residents, and thus an increase in segregation in these already highly segregated communities.

The remainder of the paper is organized as follows. Section 2 introduces our data set. Section 3 provides an equilibrium model that treats public housing as a differentiated product that is subject to rationing. Section 4 discusses identification and derives the maximum likelihood estimator for this model. The empirical results are presented in Section 5. Section 6 conducts some counterfactual policy analysis. We offer some conclusions in Section 7.

2. Institutional background and data

The U.S. Housing Act of 1937 formed the U.S. Public Housing Program that funds local governments in their ownership and management of buildings to house low-income

(1990) and Postel-Vinay and Robin (2002). Heckman and Honore (1990) discussed identification in the Roy model.

[3]Olsen, Davis, and Carrillo (2005) used restricted use data from HUD to study the impact of variations in local housing policies on household behavior.

residents at subsidized rents.[4] The U.S. government pursued an active policy of constructing public housing communities during the 1950's and 1960's. The Reagan administration significantly reduced financing for the construction of new housing projects during the 1980's to shift the focus to creating voucher programs. Since the early 1990's, HUD has given financial incentives under HOPE VI and related programs to tear down projects that are considered to be distressed.

New programs to encourage the construction of privately owned low-income housing emerged as construction of public housing ceased and demolition of public housing began. As detailed in Eriksen and Rosenthal (2010), the Low Income Housing Tax Credit (LIHTC) program was created in 1986 as part of the Tax Reform Act of 1986 as an alternative to public housing. They observed that "LIHTC has quickly overtaken all previous place-based subsidized rental programs to become the largest such program in the nation's history." They found, however, that this program has failed to result in new construction that serves the population served by public housing, largely due to crowd-out effects. As a consequence, there is not an adequate supply of affordable housing and there are long wait lists to get access to public housing in many U.S. cities.[5]

Currently, the U.S. Department of Housing and Urban Development funds the efforts of hundreds of city and county housing authorities in the United States. In Pennsylvania alone, there are 92 distinct housing authorities. In 2006, the estimated HUD budget for public housing was \$24.604 billion.[6] Within the public housing program, this funding supports administration, building maintenance, and even law enforcement.

The empirical analysis presented in this paper focuses on communities owned and managed by the Housing Authority of the City of Pittsburgh. In 2005, HUD provided the HACP with \$83.7 million in grants for public housing, housing vouchers, and other programs. In the same year, HACP received \$8.3 million from tenant payments. Only a small number of public housing communities were demolished during the course of our survey.[7] As a consequence, the supply of public housing was approximately fixed during our study period.

The public housing stock in the city of Pittsburgh is heterogeneous, including small houses converted into several apartment units, large high-rises, and large communities of low-rise housing that is spread continuously over several blocks and offers as many as 600 units. These communities are usually designated as either "family" communities or "senior" communities, where senior communities target households age 62 or older.

[4]Olsen (2001) provided a detailed description of the history and current practices of the various different U.S. Public Housing Programs.

[5]There is some evidence to suggest negative spillover effects (such as higher crime rates and lower educational achievement) associated with living in public housing (Oreopoulos (2003)). However, Jacob (2004), who considered the impact of demolitions in Chicago, found that there are very few positive effects associated with moving out of the projects using a variety of different outcomes.

[6]HUD (2007) provides details. Note that this figure does not include housing voucher programs, low-income community development programs, or other non-state-owned and managed housing programs.

[7]Much of the demolition was motivated by the argument that growing up in public housing might be negative for children, although this conjecture is controversial in the literature (Currie and Yelowitz (2000)). For an analysis of the impact of public housing demolitions in Chicago, see Jacob (2004).

There are 34 separate sites: 19 of these sites are family units, 11 are designated for seniors, and 4 have both senior- and family-designated units. There are 16 large communities that have more than 100 units, 8 that are medium sized, and 10 that are small, which have less than 40 units. Heterogeneity in public housing also arises due to differences in local amenities. The 34 public housing communities in the HACP are located across 19 of Pittsburgh's 32 wards and across 28 census tracts. These public housing communities also vary in terms of neighborhood amenities such as crime, school quality, property values, and demographic characteristics.[8]

The HACP data contain records of household entry, exit, and transfer from June 2001 to June 2006 within the 34 public housing communities actively used during this time period. The data set also includes annual updates of each of these households as well as any nonperiodic reports that update information about household composition or pre-rent income that is reported to the HACP. These records contain most of the information fields requested of all U.S. housing authorities including age, race, household composition (including age and relationship of family members and housemates), earnings, and income adjustment exclusions (including disability, medical, and childcare expenses). We also observe the monthly rent being charged to a particular household, the number of bedrooms of the housing unit, whether the community is targeted to seniors, and the address and unit number. There are 7,070 households observed at least once during this time period; there are 2,907 households that move in for the first time, 3,155 households that move out, and 1,244 that transfer from one public housing unit to another.

Table 1 summarizes key descriptive statistics for the full sample and for four subsamples that are differentiated by community type. Although some families live in senior housing and some seniors live in non-senior housing, age and family composition distributions are bimodal with respect to these two types of communities. In mixed communities, demographic variables look similar to a weighted average of senior and family communities; however, there are more cohabiting adults and a higher number of children in mixed housing than in family-only or senior-only housing. The mean age in senior housing is 31 years greater than the mean age in non-senior housing. The majority of households in both senior-only and family-only communities are female, but females are a much larger majority in family-only communities. African American households are a very high proportion of residents in family and mixed housing, while senior units have nearly equal proportions of African American and white households. Marriage rates are low: 2.20% in family housing and 3.93% in senior housing, and there are more

[8]There is much evidence that suggests that households make residential decisions based on neighborhood characteristics and local public goods. This evidence is based on estimated locational equilibrium models such as Epple and Sieg (1999), Epple, Romer, and Sieg (2001), Sieg et al. (2004), Calabrese et al. (2006), Ferreyra (2007), Walsh (2007), and Epple, Peress, and Sieg (2010). Bergstrom, Rubinfeld, and Shapiro (1982), Rubinfeld, Shapiro, and Roberts (1987), Nesheim (2001), Bajari and Kahn (2004), Bayer, McMillan, and Reuben (2004), Schmidheiny (2006), Bayer, Ferreira, and McMillan (2007), and Ferreira (2009) are examples of related empirical approaches that are based on more traditional discrete choice models or hedonic frameworks.

Table 1. Descriptive statistics of HACP demographics.

	All Units	Family Units	Mixed Units	Senior Units	2 Bedroom Apartments
Age	48.86	40.42	49.06	71.15	34.45
	(20.76)	(16.98)	(20.53)	(11.77)	(13.36)
Percent female	80.59	84.87	83.85	64.90	84.78
Percent married	2.66	2.20	2.65	3.93	1.43
Number of adults	1.16	1.17	1.21	1.06	1.06
	(0.44)	(0.45)	(0.50)	(0.23)	(0.24)
Number of children	0.95	1.00	1.59	0.00	0.76
	(1.36)	(1.22)	(1.71)	(0.00)	(0.75)
Percent with children	43.95	53.46	58.31	0.00	57.40
Percent Afr. Amer.	88.53	96.67	97.00	55.59	96.11
Annual income	9,082	8,516	9,714	9,784	6,305
	(7,776)	(8,957)	(6,968)	(4,602)	(6,771)

Note: Standard deviations are given in parentheses.

cohabiting adults in family housing than in senior housing.[9] There are fewer households in non-senior housing that have children than one might expect (about 53%).[10]

In the HACP data, we only observe households that have lived in public housing at some point during the sample period. Once households leave the housing communities, the HACP does not conduct any follow-up surveys. To learn about households that are eligible for public housing, but do not live in one of the housing communities, we turn to the 2001 Survey of Income and Program Participation (SIPP). The SIPP is a survey managed by the U.S. Census Bureau that interviews households every 4 months for 3 years. Each month, households are asked about their previous 4-month family composition, sources of income, and participation in government programs such as public housing and school lunch programs. We create a sample based on the SIPP that contains households eligible for housing aid.[11]

Table 2 provides some descriptive statistics for the SIPP sample used in this analysis and compares it to Census and HACP data. We find that low-income households that

[9]There is a strong incentive for families to not report the existence of a cohabiting adult or partner, as it would lead to an increase in rent if the cohabiting adult earns an income. As a result, the number of cohabiting adults as well as household income are surely larger than our estimates from the data.

[10]Our sample differs from other studies in that Pittsburgh public housing seems to house a higher percent of African American households, female-headed households, and households with children, but a much lower percent of married households. For example, Hungerford's 1996 sample from the 1986–1988 SIPP panel was 52% female, 23% African American, 32% married, and the mean number of children was 0.21 (Hungerford (1996)).

[11]The SIPP contains only 14 households that participated in public housing in Pittsburgh at some point during the sample period. There were 156 Pittsburgh households eligible for public housing in the first quarter. We constructed a subsample of the SIPP using the unweighted data of households eligible for public housing that live in metropolitan areas similar to Pittsburgh. Appendix B contains information on how the SIPP sample was constructed and compares characteristics of Pittsburgh with characteristics of the metropolitan areas selected in our SIPP subsample.

Table 2. Descriptive statistics of the SIPP subsample compared to Census and HACP.

	Census All	SIPP All	SIPP Private	SIPP Public	HACP Public
Age	50.83	52.70	52.72	52.19	48.86
Percent female	54.6%	59.94%	59.06%	76.56%	80.59%
Percent married	22.6%	30.79%	32.09%	6.25%	2.66%
Number of adults	1.450	1.274	1.284	1.094	1.160
Number of children	0.495	0.617	0.616	0.641	0.950
Percent with children	24.73%	30.32%	30.27%	31.25%	43.95%
Percent Afr. Amer.	32.64%	28.28%	27.05%	51.56%	88.53%
Annual income	$14,079	$18,979	$19,391	$11,184	$9,082

Note: "All" refers to all eligible households in the sample. "Private" refers to all eligible households in the sample in private housing. "Public" refers to all eligible households in the sample in public housing.

rent in the private market are, on average, more likely to be married, are less likely to be African American, and have substantially higher income than households in public housing. Comparing the SIPP with the HACP sample, we find that the SIPP sample is slightly older, average income is slightly higher, and children are fewer than in the HACP. Comparing the SIPP with the Census, the SIPP contains slightly older heads of household, more female heads of household, more married householders, households with more children, and fewer African American households. However, the differences between the SIPP sample and the Census sample of eligible households in Pittsburgh are relatively small.[12]

The 34 communities are classified into broad community types: family large (PH1), family medium (PH2), family small (PH3), mixed (PH4), senior large (PH5), and senior small (PH6). These six types of housing units are fairly homogenous, but seem to attract different types of households. Large, medium, and small low-rise non-senior communities primarily house families with children. Most senior-dominated communities include a significant percentage of non-senior adults without kids, ranging from 13 to 37%. Most family-only communities include some senior households, ranging from 0 to 20%, about a third of which are caring for children.

Table 3 shows the transition matrix for the HACP data. We find that locational choices are persistent since most households stay with their past choices. However, the off-diagonal elements of the transition matrix indicate that there is a fair amount of entry into and exit from public housing.[13] Moreover, there are a number of transitions within public housing communities. These transfer are largely voluntary and indicate that household differentiate among the heterogeneous community types.[14]

[12] See Appendix B.

[13] The HACP does not record the reason or the next destination of a household that moves out.

[14] In the SIPP sample, we observe 89 transitions from private to public housing and 98 transitions from public to private housing.

Table 3. Transition matrix.

	Private	PH1	PH2	PH3	PH4	PH5	PH6	Freq.
Private	0	677	144	24	300	59	191	1,395
PH1	855	16,264	16	2	75	7	10	17,229
PH2	233	16	5,371	3	17	8	7	5,655
PH3	44	2	29	1,438	1	0	2	1,516
PH4	572	16	8	1	12,156	5	9	12,767
PH5	105	1	0	0	1	2,017	29	2,153
PH6	302	0	0	1	47	37	8,129	8,516

Note: Rows indicate choices in $t-1$ and columns indicate choices in t. Freq. indicates row frequencies.

3. An equilibrium model of public housing

3.1 *The baseline model*

We consider a model with a continuum of low-income households. Each household is eligible for housing aid and can thus, in principle, live in one of the available public housing communities or rent an apartment in the private market. Denote the outside private market option with 0. Let J be the number of different housing communities that are available in the public housing program. Let $d_{jt} \in \{0, 1\}$ denote an indicator variable that equals 1 if the household chooses alternative j at time t and 0 otherwise.[15] Let the vector $d_t = (d_{0t}, \dots, d_{Jt})$ characterize choices of a household at t. Since the alternatives are mutually exclusive, we have

$$\sum_{j=0}^{J} d_{jt} = 1. \tag{1}$$

In our baseline model, we do not allow households to move or transfer between units in different public housing communities.[16]

Households differ along a number of characteristics x_t such as income, age, number of kids, number of adults, gender of household head, marital status, and race. We treat these characteristics as exogenous. While it is not difficult to endogenize income or family status from a conceptual perspective, it significantly increases the difficulty of computing equilibria.[17]

Household preferences are subject to idiosyncratic shocks denoted by $\varepsilon_{j,t}$. We assume that these shocks are continuous random variables with support over the real line. Moreover, in the baseline model they are independent and identically distributed (i.i.d.) across observations and time.[18]

[15] In our application, we use quarterly data.

[16] We relax this assumption in Section 3.2 and consider an extended version of the model with transfers between units in different communities.

[17] We do not observe labor supply or job market participation in the HACP data; this is a limitation of our data set. See Jacob and Ludwig (2012) for analysis of the impact of housing vouchers on income.

[18] One concern with this assumption is that it is plausible that households may have an overall preference for the public versus the private sector. One way to address this concern is to use a nested logit specification

Households face relocation costs if they decide to move. Thus lagged choices, denoted by d_{t-1}, are relevant state variables.

Households have preferences defined over all potential elements in the choice set. We model household preferences using a standard random utility specification.

ASSUMPTION 1. *Let $u(d_t, x_t, d_{t-1}, \varepsilon_t)$ denote the household utility function. We assume that the utility function is additively separable in observed and unobserved state variables, and thus allows the representation*

$$u(d_t, x_t, d_{t-1}, \varepsilon_t) = \sum_{j=0}^{J} d_{jt}\big[u_j(x_t, d_{t-1}) + \varepsilon_{jt}\big]. \tag{2}$$

This specification implicitly treats public housing as a differentiated product.

A key feature of our model is that all potential choices may not be available to a household at any given point of time. A household that is currently renting in the private market may not have access to public housing, even if the household meets all eligibility criteria.[19] We, therefore, need to formalize the fact that access to public housing is restricted by a local housing authority.

ASSUMPTION 2. *The public housing authority does not evict any households that have lost eligibility.*

This assumption is motivated by policies that are typically used by many local housing authorities. It implies that exit from public housing is purely voluntary. To characterize the voluntary outflow, let P_{jt} denote the fraction of eligible households living in community j at the beginning of period t. The outflow from public housing community j to the private sector, OF_{j0t}, is defined as

$$\mathrm{OF}_{j0t} = P_{jt} \int \Pr\big(u_0(x_t, d_{t-1}) + \varepsilon_{0t} \geq u_j(x_t, d_{t-1}) + \varepsilon_{jt}\big) f(x_t | d_{jt-1} = 1)\, dx_t, \tag{3}$$

where $f(x_t | d_{jt-1} = 1)$ denotes the conditional density function of households with characteristics x_t that live in j at the beginning of period t. As a consequence, the housing authority faces a stream of housing units that become available at each point of time. The authority needs to assign these units to new renters. To model this decision process, we need to model the potential demand for public housing.

Let P_{0t} denote the fraction of eligible households that are renting in the private market at the beginning of period t. We make the following assumption.

ASSUMPTION 3. *All eligible households that are renting in the private market are placed on a wait list for public housing.*

to capture correlation in unobserved preferences among public housing communities. We, therefore, also explore this specification as part of our robustness analysis.

[19] In practice, all eligible households are typically assigned to a wait list. A household will only receive an offer to move into public housing if it is on top of the wait list.

We offer four observations regarding this assumption. First, signing up for the wait list is, for all practical purposes, costless in practice.[20] Second, it is easy to relax the assumption and allow for systematic differences between households on the wait list and eligible households that have not signed up on the wait list. When we discuss the rationing implications, we relax this assumption and consider a case in which a demand signal triggers households to sign up on the wait list. Third, the assumption can be justified by empirical constraints. We do not observe the characteristics of all households on the wait list and neither does the housing authority. We also do not observe the priority ranking of households on the wait list. Assumption 3 implies that the households that have top priority on the wait list do not systematically differ from the eligible population.[21] Finally, it is also straightforward to assume that the housing authority has multiple wait lists for households with different family sizes.[22]

Next consider the potential demand for public housing. The probability that a household that is currently living in the private sector prefers j at time t is

$$\Pr(d_{jt} = 1 | x_t, d_{0t-1} = 1) = \Pr\bigl(u_j(x_t, d_{t-1}) + \varepsilon_{jt} \geq u_0(x_t, d_{t-1}) + \varepsilon_{0t}\bigr). \tag{4}$$

Let $f(x_t | d_{0t-1} = 1)$ denote the conditional density function of households with characteristics x_t that currently rent in the private market, are eligible for public housing, and, thus, have been assigned to a wait list. The potential demand for community j is then characterized by the fraction of households on the wait list that prefer j at time t:

$$F_{0jt} = P_{0t} \int \Pr(d_{jt} = 1 | x_t, d_{0t-1} = 1) f(x_t | d_{0t-1} = 1)\, dx_t. \tag{5}$$

The most interesting case arises if demand exceeds supply. We therefore make the following assumption.

Assumption 4. (a) *The potential demand exceeds the voluntary outflow for each community at each point of time*. (b) *The authority offers the available units to households on the wait list that have the highest priority*. (c) *The housing authority continues to offer units until all available vacant units have been filled with eligible households*.

Assumption 4(a) and (b) are not necessary to obtain a well defined equilibrium, but they hold empirically in almost all large markets in the United States. Assumption 4(a) implies that the housing authority cannot meet the full demand. Instead, it can only offer public housing to a fraction of the households that are eligible. Assumption 4(b) implies that housing authority follows a first-in–first-out policy. Assumptions 2–4 imply that there is a fraction of households, denoted by Π_{0jt}, that will receive offers to move

[20] Of course, it does not matter that all eligible households sign up as long as there are no systematic differences between eligible households and households on the wait list.

[21] As a consequence, we can solve and estimate the model without observing the conditional distribution of households on the wait list.

[22] We discuss these issues when we estimate the model in Section 5.

into housing community j at time t. The total inflow to public housing is then given by

$$\mathrm{IF}_{jt} = \Pi_{0jt} F_{0jt}. \tag{6}$$

We also need to impose an assumption on the supply of public housing and the vacancy rates.

Assumption 5. *The supply of public housing is constant in each housing community at each point of time.*

We can relax this assumption and allow for exogenous changes in the supply of public housing due to new construction or demolitions. We discuss these issues in detail when we quantify the impact of demolitions in Section 6.

Assumption 5 then implies that the outflow must equal the inflow for each housing community at each point of time in equilibrium[23]:

$$\mathrm{IF}_{jt} = \mathrm{OF}_{jt}. \tag{7}$$

To close the model and define an equilibrium, we need to make an assumption about initial conditions.

Assumption 6. *We take the initial distribution of households at the beginning of period* 1, *which is fully characterized, the vector of probabilities* P_1, *and conditional densities* $f(x_1|d_{i,0} = 1)$ *as exogenously determined.*

Given P_t and $f(x_t|d_{i,t-1} = 1)$, the conditional choice probabilities $\Pr(d_{jt} = 1|x_t, d_{it-1} = 1)$ then uniquely determine the unconditional choice probabilities P_{t+1} and the conditional distribution functions $f(x_{t+1}|d_{i,t} = 1)$ that characterize the composition of households for each element in the choice set at the beginning of the next period. Since households are myopic, we can define an equilibrium for each point of time t. The sequence of one-period equilibria is linked by the law of motion that characterizes the composition of the public housing communities over time.[24]

An equilibrium for period t for the baseline model can, therefore, be defined as follows.

Definition 1. Given an initial distribution of households at the beginning of period t, denoted by P_t, and given $f(x_t|d_{j,t-1})$, an equilibrium of this model consists of a vector of probabilities $\Pi_{01t}, \dots, \Pi_{0Jt}$ such that the following statements hold.

- The housing authority offers a fraction Π_{0jt} of all households on the wait list the opportunity to move into community j.

[23] The assumption of a constant housing stock is common in many theoretical papers that study housing market equilibrium in urban metropolitan areas. See, for example, Nechyba (1997a, 1997b), Nechyba (2003), Bayer and Timmins (2005), and Ferreyra (2007).

[24] We are abstracting here from households that enter or leave the local economy. It is straightforward to account for that.

- Households maximize utility subject to the effective choice set.
- For each housing community, the inflow of households equals the outflow of households for each housing community as required by equation (7).

We have the following result that characterizes the existence and uniqueness of equilibrium.

Proposition 1. *If the potential inflow exceeds the voluntary outflow for each community, then there exists a unique housing market equilibrium with rationing.*

Proof. For each time period, equation (7) implies that equilibrium is defined by a linear system of equations with J market clearing conditions and J unknown offer probabilities. The equilibrium offer probabilities are then ratios of the potential demand given by the right hand side of equation (6) and the outflow given by equation (3). We assume that the potential demand exceeds at each point of time the voluntary outflow. As a consequence, the offer probabilities are all strictly less than 1. □

3.2 *An extended model with transfers*

We generalize our model and allow for transfers between public housing units. Transfers imply that the demand for public housing must be modified since households may have additional options. The probability that a households that lives in community i at the beginning of the period prefers to move to community j at time t is

$$\begin{aligned} &\Pr(d_{jt} = 1 | x_t, d_{it-1} = 1) \\ &\quad = \Pr\big(u_j(x_t, d_{t-1}) + \varepsilon_{jt} \geq \max\big[u_i(x_t, d_{t-1}) + \varepsilon_{it}, u_0(x_t, d_{t-1}) + \varepsilon_{0t}\big]\big). \end{aligned} \tag{8}$$

Note that households only compare options that are in the effective choice set, that is, that are available to them. As before, the potential demand is then characterized by the fraction of households living in community i that prefer j at time t:

$$F_{ijt} = P_{it} \int \Pr(d_{jt} = 1 | x_t, d_{it-1} = 1) f(x_t | d_{it-1} = 1)\, dx_t. \tag{9}$$

In contrast to entry into public housing and exit, there is no stated policy for transfers between public housing units. Nevertheless, we observe a fair number of transfers in practice. A useful modeling approach is then to mimic our assumptions imposed on the (external) wait list to generate a well defined transfer policy. Suppose that the housing authority also has an internal mechanism that determines transfer offers. In that case, a fraction of households that are currently living in i are offered the opportunity to transfer to community j.

Assumption 7. *The probability of obtaining an offer to move into housing community j while living in public housing i is given by Π_{ijt}. Households get, at most, one offer at each point of time.*

The total realized demand (or inflow) from community i to community j at time t is, therefore, $\Pi_{ijt}F_{ijt}$. Summing over all current housing choices other than j gives the total inflow into housing community j:

$$\mathrm{IF}_{jt} = \sum_{i=0, i\neq j}^{J} \Pi_{ijt}F_{ijt}. \tag{10}$$

Similarly we can modify the equation that characterizes the total voluntary outflow from community j,

$$\mathrm{OF}_{jt} = \mathrm{OF}_{j0t} + \sum_{i=1, i\neq j}^{J} \Pi_{jit}F_{jit}, \tag{11}$$

where the outflow to the private sector, OF_{j0t}, is defined as

$$\begin{aligned}\mathrm{OF}_{j0t} = {} & P_{jt}\Pi_{jjt}\int \Pr\bigl(u_0(x_t, d_{t-1}) + \varepsilon_{0t} \geq u_j(x_t, d_{t-1}) + \varepsilon_{jt}\bigr) f(x_t|d_{jt-1} = 1)\, dx_t \\ & + P_{jt}\sum_{k=1, k\neq j}^{K} \Pi_{jkt}\int \Pr\bigl(u_0(x_t, d_{t-1}) + \varepsilon_{0t} \\ & \geq \max\bigl[u_j(x_t, d_{t-1}) + \varepsilon_{jt}, u_k(x_t, d_{t-1}) + \varepsilon_{kt}\bigr]\bigr) \\ & \times f(x_t|d_{jt-1} = 1)\, dx_t.\end{aligned} \tag{12}$$

In the extended model, we have J^2 offer probabilities and J market clearing conditions. Moreover, the system of equations that defines equilibrium is linear in the offer probabilities. An equilibrium for the economy exists if the linear system of market clearing equations has a solution. These solutions (generically) exist, but are not unique, since the number of equations is smaller than the number of unknowns.[25]

The potential for multiplicity in equilibrium arises because we have not sufficiently restricted the ability of the housing authority to allow households to transfer between different units. There are many transfer policies that are consistent with equilibrium in the public housing market. The market clearing conditions alone do not uniquely determine the offer probabilities. To obtain a unique solution to this system of equations, we need to impose additional assumptions. It is plausible that the housing authority does not discriminate based on current residence and uses the same odds ratio for insiders and outsiders. We therefore make the following assumption.

Assumption 8. *The fraction of households that receive an offer to transfer between units in different communities does not depend on current residence*:

$$\Pi_{ijt} = \Pi_{jt}, \quad i = 1, \dots, J. \tag{13}$$

[25] See, for example, the discussion in Strang (1988).

The odds ratios are the same for households inside and outside of public housing:

$$\Pi_{0jt} = R_{0t}\Pi_{jt}. \tag{14}$$

Note that this assumption is plausible since housing authorities are not allowed to discriminate based on income, race, and gender. As a consequence, it is hard to believe that they could discriminate based on residency. The parameter R_{0t} measures the relative degree of preferential treatment that is given to outsiders. In practice, R_{0t} should be much greater than 1. As a consequence, households on the wait list get preferential treatment over households that are already in public housing. Substituting Assumption 7 into the definition of equilibrium, we obtain

$$R_{0t}\Pi_{jt}F_{0jt} + \sum_{i \neq j} \Pi_{jt}F_{ijt} = \mathrm{OF}_{j0t} + \sum_{i \neq j} \Pi_{it}F_{jit}, \tag{15}$$

which is a system of J equations in $J + 1$ unknowns. Thus the equilibrium conditions define the offer probabilities up to the factor R_{0t}. We have therefore shown the following result.

Proposition 2. *For each value of R_{0t}, there exists a unique housing market equilibrium with rationing.*

In summary, we have developed an equilibrium model of public housing that generates rationing and excess demand in equilibrium. The model also explains transfers between heterogeneous housing communities. One key simplifying assumption of the model is that we treat households as myopic. If households are forward looking, they need to forecast if and when they are offered units in public housing. As a consequence, the value functions and the demand for public housing depend on expectations about future offer probabilities. The equilibrium can no longer be characterized by a sequence of one-period equilibria. As a consequence, the equilibrium is much more difficult to characterize and to compute.

4. Identification and estimation

We estimate the model using two different samples. The first sample is a choice-based sample that is provided by a local authority. This sample tracks households as long as they stay in public housing. The second sample is a random sample of households that are eligible for housing aid. In this section, we introduce a parametrization of our model. We then derive the conditional choice probabilities and develop our maximum likelihood estimator (MLE). We then discuss the role that equilibrium conditions play in establishing identification of the model. Finally, we show that our approach works in a Monte Carlo study when the data generating process is known.

4.1 *A parametrization*

We assume that the utility associated with community j is given by

$$u_{jt} = \gamma_j + \beta \ln(y_{jt}) + \delta x_t + \mathrm{mc}\, 1\{d_t \neq d_{t-1}\} + \varepsilon_{jt}, \quad j = 1, \dots, J. \tag{16}$$

The utility of the outside option is normalized to be equal to the expression

$$u_{0t} = \ln(y_{0t}) + \mathrm{mc}\, 1\{d_t \neq d_{t-1}\} + \varepsilon_{0t}. \tag{17}$$

In the preceding equations, y_{jt} denotes household net income, mc is a moving cost parameter, and γ_j is a community specific fixed effect.[26] Households that live in public housing typically pay 30% of their income in rent. As a consequence, net income is choice specific due to the implicit tax. As income increases, living outside of public housing should become more attractive. We would, therefore, expect that $\beta < 1$. The community specific fixed effects capture observed and unobserved differences among the public housing communities. The specification also accounts for (psychic) moving costs. Idiosyncratic shocks account for factors that are not observed by the econometrician. Following McFadden (1973), we assume that the ε's are i.i.d. type I extreme value distributed.

4.2 *Conditional choice probabilities*

Our main data set is from a local housing authority and follows households as long as they are in public housing. This is, therefore, a choice-based sample since we only observe households that have chosen to live in one of the housing communities at time t. A household that lived in community j at the end of the last time period, has potentially three options. First, the household moves back to the private housing market. Second, the household moves to a different housing community. Third, the household stays in its current community j. Given the distributional assumptions on the idiosyncratic shocks, the probability of moving to the private sector is then

$$\begin{aligned}
&\Pr\{d_{0t} = 1 | d_{jt-1} = 1, x_t\} \\
&\quad = \sum_{k=1, k\neq j}^{J} \Pi_{jkt} \frac{\exp(u_0(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))} \\
&\qquad + \Pi_{jjt} \frac{\exp(u_0(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))}.
\end{aligned} \tag{18}$$

The probability of moving from community j to community k is given by

$$\Pr\{d_{kt} = 1 | d_{jt-1} = 1, x_t\} = \Pi_{jkt} \frac{\exp(u_k(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))} \tag{19}$$

[26]We are implicitly imposing the budget constraint by using net income in the utility function.

and the probability of staying in community j is given by

$$\begin{aligned}
&\Pr\{d_{jt} = 1 | d_{jt-1} = 1, x_t\} \\
&\quad = \sum_{k=1, k \neq j}^{J} \Pi_{jkt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t)) + \exp(u_k(x_t))} \\
&\qquad + \Pi_{jjt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))}.
\end{aligned} \tag{20}$$

Finally, we also observe new entrants into public housing. The probability of observing a new household in community j is

$$\Pr\{d_{jt} = 1 | d_{0t-1} = 1, x_t\} = \Pi_{0jt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))}. \tag{21}$$

The conditional choice probabilities for the choice-based sample are thus defined by equations (18), (19), (20), and (21).

Our second sample is a random sample of low-income households that tracks households both inside and outside of public housing. In contrast to the choice-based sample, this sample does not allow us to identify the exact housing community in which a household lives. As a consequence, we only observe a coarser version of the choice set in the random sample. For households that are currently not living in public housing, we have two possible outcomes: (i) the household stays in private housing or (ii) the household moves to a public housing unit.

The probability of moving to any of the J public housing communities is given by

$$\Pr\{d_{0t} = 0 | d_{0t-1} = 1, x_t\} = \sum_{j=1}^{J} \Pi_{0jt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))}. \tag{22}$$

Note that (22) is obtained by summing the probabilities in (21) over all possible choices. Similarly, the probability of staying in private housing is defined as

$$\Pr\{d_{0t} = 1 | d_{0t-1} = 1, x_t\} = 1 - \sum_{j=1}^{J} \Pi_{0jt} \frac{\exp(u_j(x_t))}{\exp(u_0(x_t)) + \exp(u_j(x_t))}. \tag{23}$$

Note that we do not observe whether the household obtained an offer and we also do not observe to which housing unit it moved if it decided to move.

Next consider a household that currently lives in public housing. Again there are two possible outcomes: (i) the household moves back to private housing or (ii) the household stays in public housing. Consider the first case, in which the household moves back to private housing. Now, in the random sample, we do not observe in which unit the household lives. However, we can compute relative frequencies based on the choice-based sample that assign probabilities to each community type. Let us denote these probabilities by $\Pr\{d_{jt-1} = 1 | d_{0t-1} = 0, x_t\}$. The choice probability conditional on living in community j is given by equation (18). Summing over all J housing units and properly weighting each conditional choice probability implies that the probability of moving out of public housing is then

$$\begin{aligned}&\Pr\{d_{0t}=1|d_{0t-1}=0,x_t\}\\&\quad=\sum_{j=1}^{J}\Pr\{d_{0t}=1|d_{jt-1}=1,x_t\}\Pr\{d_{jt-1}=1|d_{0t-1}=0,x_t\}.\end{aligned}\tag{24}$$

Next consider the case in which a household stays in public housing. We cannot distinguish between the case in which a household stays in the same community or moves to a different housing community within public housing. Thus, conditional on living in community j, the probability of staying in public housing is the sum of the probabilities in equations (19) and (20), that is, the probability of staying, conditional on living in j at the end of the previous period, is

$$\begin{aligned}\Pr\{d_{0t}=0|d_{jt-1}=1,x_t\}&=\Pr\{d_{jt}=1|d_{jt-1}=1,x_t\}\\&\quad+\sum_{k=1,k\neq j}^{J}\Pr\{d_{kt}=1|d_{jt-1}=1,x_t\}.\end{aligned}\tag{25}$$

Summing over all J housing units and properly weighting each conditional choice probability implies that the probability of staying in public housing is then

$$\begin{aligned}&\Pr\{d_{0t}=0|d_{0t-1}=0,x_t\}\\&\quad=\sum_{j=1}^{J}\Pr\{d_{0t}=0|d_{jt-1}=1,x_t\}\Pr\{d_{jt-1}=1|d_{0t-1}=0,x_t\}.\end{aligned}\tag{26}$$

The conditional choice probabilities for the random sample are thus defined by equations (22), (23), (24), and (26).

4.3 *The likelihood function under enriched sampling*

To compute the likelihood function, we need to take into account the fact that we use a random and a choice-based sample in estimation. This sampling scheme is also called enriched sampling as discussed in detail by Cosslett (1978, 1981).[27] Let us denote the corresponding sample sizes with N_1 and N_2. Similarly, let T_1 and T_2 denote the length of the two panels. Observations are assumed to be independent across samples, ruling out sampling the same household in both data sets. The joint likelihood function of observing the two samples is thus the product of the two likelihood functions,

$$L=L_1L_2.\tag{27}$$

[27] Notice that our sampling scheme satisfies Assumptions 9 and 10 in Cosslett (1981), which guarantees a sufficient overlap in the relevant choice sets between the two samples.

The likelihood associated with the random sample L_1 is given by

$$L_1 = \prod_{i=1}^{N_1} \prod_{t=1}^{T_1} l_{1nt}, \tag{28}$$

where l_{1nt} is given by

$$\begin{aligned} l_{1nt} &= \left[\Pr\{d_{0nt} = 0 | d_{0nt-1}, x_{nt}\}\right]^{1-d_{0nt}} \\ &\quad \times \left[\Pr\{d_{0nt} = 1 | d_{0nt-1}, x_{nt}, \}\right]^{d_{0nt}} f(x_{nt}, d_{nt-1}). \end{aligned} \tag{29}$$

The likelihood for the choice-based sample L_2 is defined as

$$L_2 = \prod_{i=1}^{N_2} \prod_{t=1}^{T_2} \frac{\Pr\{d_{jnt} = 1 | d_{nt-1}, x_{nt}\} f(x_{nt}, d_{nt-1})}{\tilde{Q}_t(J)}, \tag{30}$$

where

$$\tilde{Q}_t(J) = \sum_{j=1}^{J} Q_t(j). \tag{31}$$

The term $Q_t(j)$ is the unconditional probability that choice j is chosen that is defined as

$$\begin{aligned} Q_t(j) &= \sum_{j=1}^{J} \int \Pr\{d_{jnt} = 1 | d_{t-1}, x_t\} f(x_t, d_{t-1})\, dx_t\, d_{t-1} \\ &= \sum_{j=1}^{J} \int \sum_{i=0}^{J} \Pr\{d_{jnt} = 1 | d_{it-1} = 1, x_t\} f(x_t | d_{it-1} = 1) \Pr\{d_{it-1} = 1\}\, dx_t. \end{aligned} \tag{32}$$

We assume that $f(x_t, d_{t-1}, \theta)$ is known up to a finite vector of parameters θ and we treat the $Q_t(j)$ as unknown. We then define our enriched sampled maximum likelihood estimator (ESMLE) as the argument that maximizes equation (27).[28]

4.4 *Imposing the equilibrium constraints*

One problem associated with the likelihood estimator above is that the offer probabilities are not separately identified from the choice specific intercepts. To obtain identification, we use the equilibrium conditions and express the endogenous offer probabilities as functions of the structural parameters of the choice model. To illustrate the basic

[28] If the $Q_t(j)$'s are known, we can define a constrained enriched sampled maximum likelihood estimator (CESMLE) as the argument that maximizes equation (27) subject to the J constraints in equation (32). Finally, one could follow Cosslett (1978, 1981) and treat $f(x_t, d_{t-1})$ as unknown and then define a pseudo MLE by concentrating out the weights that characterize the empirical likelihood of the data. These estimators extend the standard choice-based estimators discussed in Manski and Lerman (1977).

ideas, consider first the model without transfers. In that model the structural parameters of the utility function are identified from the exit behavior of households. The conditional exit probability does not depend on the probability of getting an offer to move into public housing. Unattractive housing units will have higher exit rates and lower potential demand than attractive housing communities. Given the voluntary exit rates and the potential demand for moving into public housing, the offer probabilities are then uniquely determined by the equilibrium conditions. Solving this linear system of equations, we can express the offer probabilities as functions of the voluntary outflow and the potential demand, which only depend on the structural parameters of the utility function. Imposing the equilibrium conditions thus resolves the key identification problem encountered in the model without transfers.

In the model with transfers, the sequential identification argument breaks down since exit probabilities depend on unobserved transfer probabilities. Nevertheless, we can still express the offer probabilities as functions of the structural parameters of the utility function. If a community is attractive, voluntary outflows will be low and potential demand will be high. As a consequence, offer probabilities are low. Similarly, if the community is unattractive, voluntary outflows and transfers will be high and the potential inflow will be low. As a consequence, offer probabilities need to be sufficiently large to meet the equilibrium condition. Thus a similar logic for identification applies in the extended model that accounts for transfers.

To provide some additional insights into our approach to identification, we conducted a Monte Carlo study.[29] We find that our estimator works well under random and enriched sampling. The absolute errors are small and approximately centered around zero. Generally, we find that the estimates for the fixed effects are slightly biased upward and the coefficients on income are slightly biased downward in samples with 2,000 observations. Larger samples help reduce the estimation bias. Imposing the equilibrium conditions works well and establishes identification. The estimates of the offer probabilities that are implied by the equilibrium conditions are accurate.

5. Empirical results

We implemented our estimator for a number of different model specifications.[30] Table 4 reports the parameter estimates and estimated standard errors for four models that capture the essence of our modeling approach. In column I, we estimate the model with transfers using the full sample.[31] We are thus implicitly assuming that the housing

[29] Details are reported in Appendix A.

[30] In all models, we use the empirical demographic distributions to estimate $f(x_{nt}, d_{nt-1})$. Race (African American, white) and age (senior, non-senior) are modeled as a multivariate distribution; sex is a binomial conditional on race–age; number of children is a multinomial conditional on sex and race–age; income is a truncated normal based on number of children, sex, and race–age. We fit a logit model to estimate $\Pr\{d_{jt-1} = 1 | d_{0t-1} = 0, x_t\}$, which is needed in equations (24), (25), and (26) for the SIPP likelihood. We calibrate R_0 based on the observed ratios of mobility for households inside and outside of public housing.

[31] We also estimated a version of the model that only used households in the SIPP that live in Pittsburgh. Using the smaller Pittsburgh subsample largely affects the precision of the estimates, but not the magnitude of the point estimates. See Appendix B for details.

Table 4. Parameter estimates.

	I		II		III		IV	
	Full Sample		2-BR Subsample		2-BR Subsample		Senior Subsample	
Income	0.329	(0.028)	0.280	(0.084)	0.166	(0.084)	0.395	(0.084)
Moving cost	−3.186	(0.017)	−4.282	(0.065)	−4.694	(0.064)	−2.605	(0.958)
Afr. Amer. and non-senior	1.222	(0.071)	0.822	(0.178)	1.394	(0.165)		
White and senior	0.209	(0.113)						
Afr. Amer. and senior	1.000	(0.101)					−2.261	(0.792)
Children	−0.315	(0.123)						
Female	0.053	(0.061)	0.253	(0.205)	0.986	(0.190)		
Female and senior	−0.174	(0.094)					0.065	(0.064)
Female with children	0.426	(0.130)						
PH1 × children					0.000	(0.289)		
PH2 × children					0.000	(0.324)		
PH3 × children					−0.900	(0.574)		
PH6 × Afr. Amer.							4.040	(0.808)
	Community Fixed Effects		Community Fixed Effects		Community Fixed Effects		Community Fixed Effects	
log likelihood	−688,796		−123,144		−123,111		−128,899	

Note: Estimated standard errors are given in parentheses.

authority has only one wait list. This estimator controls for differences in income, race, age, family status, and number of children. In column II, we estimate the model for the subsample of households that are eligible for two-bedroom non-senior apartment units. In column III, we consider the same subsample and add interactions between number of children and the fixed effects. Finally, column IV estimates a model for seniors only. The last three specifications models thus explicitly acknowledge the fact that there are separate wait lists for different family and apartment sizes.

We find that African Americans have stronger preferences for public housing than whites. This result is largely driven by the fact that African American households are overrepresented in public housing in Pittsburgh. We also find that age has an impact.[32] Male seniors have stronger preferences for public housing than female seniors. Females with children also have stronger preferences for public housing than other households. In contrast, fathers or married couples with children have lower valuations for public housing than those without children.

The income coefficient shows that there are strong incentives for households to leave public housing as income increases. This finding is consistent with the fact that there are only a few higher income household in our sample that live in public housing. There are only 52 households in our sample that, at some time during the study,

[32]The HACP does not record the reason a household vacates an apartment, so we might misclassify a death as an event where the household moves to private housing. If most exits from senior public housing are the results of death, we may be underestimating the fixed effects of senior housing.

exceeded the income eligibility limit of approximately $45,000.[33] We also estimate community specific fixed effects, which are not reported in Table 4. Our findings suggest that smaller communities are, in general, more desirable than larger communities.

We also find that there are significant moving costs that constrain potential relocations of households. One concern with the independence assumption is that nonpersistent preference shocks may be responsible for the high estimate of moving costs. Recall that these costs are identified in our model of lagged choices. As a consequence, we can also view these estimates as reflecting habit persistence. An alternative modeling approach would be to directly model persistence in unobserved preference shocks. We did not implement this approach, but we would expect to find similar results. It is well known that it is difficult to distinguish between habit formation and persistence in preference shocks in short panel data sets.

We argued above that incorporating the supply side restrictions is essential to obtain a consistent estimator for the underlying parameters of the model. To illustrate this important insight, we compare the estimates of our model with those obtained from a simpler logit model that ignores the supply side restrictions. (Table 10 in Appendix C reports the full set of estimates.) We find important differences between that model and our model. According to the estimates of the simple logit model, households view public housing communities as a relatively unattractive option. Our model estimates tell a different story. The estimated fixed effects associated with public housing communities are positive and much larger than those associated with private housing. Public housing is, therefore, an attractive option for low-income households. However, households do not live in public housing due to the strong supply restrictions. There is only a small probability of obtaining an offer to move into public housing. The estimate of the moving costs is even larger in the logit that ignores supply restrictions than in that for our baseline model. The simple logit model predicts that households are "locked into" public housing and do not leave public housing due to very high moving costs. Our model also creates some lock-in effect due to high moving costs, but public housing is still an attractive option for households with very low incomes.

A concern with the model specification is that the logit specification does not capture the correlation in unobserved preferences among public housing communities. We, therefore, also explored nested logit specifications. (Details of the nested logit are provided in a supplementary file available on the journal website, http://qeconomics.org/supp/148/supplement.pdf.) Using different optimization algorithms (including a simplex method with simulated annealing, a gradient-based approach, and a grid search over possible values for the correlation coefficient and the moving cost), we do not find that the likelihood function increases with any estimate of nonzero correlation. Therefore, we find that the nested logit model does not improve the fit of the model. Formal tests suggest that the simple logit model is appropriate.

Next we analyze the goodness of fit of our model. One measure of goodness of fit is to compare the residency distribution predicted by the model to the actual residency distribution observed in the sample. We find that the predictions that are based on our

[33] Note that this limit depends on year and size of household.

TABLE 5. Actual versus estimated composition of communities.

		Private	PH1	PH2	PH3	PH4	PH5	PH6
% Afr. Amer.	Observed	0.24	0.98	0.94	0.90	0.97	0.56	0.55
	Estimated	0.26	0.95	0.92	0.90	0.95	0.51	0.56
% Female	Observed	0.67/0.53	0.85/0.88	0.89/0.75	0.93/1.00	0.84/0.67	0.63/0.53	0.66/0.68
	Estimated	0.67/0.53	0.82/0.67	0.87/0.71	0.93/0.83	0.84/0.64	0.57/0.48	0.67/0.66
% Have kids	Observed	0.46/0.24	0.55/0.64	0.62/0.43	0.62/0.38	0.58/0.1	0/0	0/0
	Estimated	0.42/0.24	0.49/0.28	0.57/0.36	0.60/0.37	0.59/0.19	0.06/0.02	0.05/0.02
Income	Observed	19.3/21.0	8.4/7.2	12.3/12.9	14.1/10.3	9.9/11.3	9.1/8.5	9.3/9.8
	Estimated	19.3/21.5	8.5/6.2	12.3/8.1	12.6/7.5	9.9/8.1	8.3/8.0	9.4/9.9

Note: Composition shown by race African American/white.

preferred model are accurate. Our model thus matches the unconditional distributions of households among choices well. A more challenging exercise is to predict the composition of the housing communities using our model. We focus on the composition by gender and by family status conditional on race. The results are summarized in Table 5. The findings are by and large encouraging. Our model explains the demographic compositions of all communities well.

We compare the observed mobility with the mobility generated under the model. With the model parameters from our preferred model, the predicted number of move-ins during this whole sample is 1,796. The actual number is 1,581. The predicted move-outs is 2,273 (the actual is 2,106). Finally, the predicted number of transfers is 374 compared to 349 observed in the data.[34]

6. Policy analysis

To share some additional insights into the effects of supply side restrictions in the market for public housing, we consider demolishing some of the least attractive public housing units. We analyze how demolitions affect the equilibrium, analyze the composition of housing communities, and compute standard welfare measures. We consider demolishing communities with a large number of units. These communities have been the target of demolitions in many cities. Our estimates confirm that they have the lowest fixed effect parameter and are thus the least attractive of all communities.

We consider the demolition of public housing community 1 during the third period of a 12-quarter study. We use the estimates based on our preferred model in column II of Table 4. It is well known that these types of discrete choice models do not yield closed form solutions for compensating variations. We, therefore, follow McFadden (1989, 1995) and adopt a simulation-based approach. An additional complication in our model is that we not only need to simulate draws from distributions of the error terms, but also from the equilibrium offer probabilities. To initialize, the demographic characteristics in the first quarter are the same as those observed in the data. For families

[34]Some periods in the HACP data were eliminated. Only quarters that overlap with the SIPP data were included in the estimation.

of varying demographic characteristics, we compute the median compensating variation for an evicted household earning \$12,000 per year. We find that the estimates range from \$11,656 for a white male with kids to \$116,010 for an African American female with kids. White households require lower compensation to leave public housing than African American households. Overall, the estimates suggest that there may be significant welfare losses associated with demolishing existing units.[35] The policy experiment shows a decline in overall welfare for low-income African Americans. However, for some low-income households earning more than \$12,000 a year, there is a small welfare gain.

Compared to the baseline equilibrium, offer probabilities immediately decrease after the eviction because many evicted tenants wish to move back into public housing. Offer probabilities decrease 2.6% for medium communities, 12% for small family communities, 6.3% for mixed family and senior communities, and 16% for mostly senior communities. Over time, the composition of the remaining public housing communities changes. The public housing communities experience an increase of 3% in African American households and a 12% decrease in non-African American households; there is a 1.3% increase in female-headed households and 2.2% increase in households with children. Average income in the public housing communities decreases 2%. The demolitions of public housing, therefore, lead to an increase in racial and socioeconomic segregation.

To better understand the mechanism that drives these estimates, it is useful to provide a more complete characterization of the rationing process that results in equilibrium. Simulating the estimated model, we predict an estimated mean wait time of 12 months. In the HACP data, the mean wait time is 22 months with a mode of 14. We believe our model generates plausible estimates of the wait time. There are large outliers in the HACP wait time data that may contain measurement error. Based on the parameter estimates of our preferred model in column I, we estimate the fraction of the population that would like to move into public housing if it were possible. This fraction varies by quarter due to quarterly differences in income and demographic heterogeneity. Table 6 shows the percentage of households willing to move for the twelfth quarter (a quarter in the middle of the study).

Comparing the fraction of households willing to move into a housing community with the number of available units in that community, we find that this ratio is equal to 3.77 for community 1, which is the least attractive community. For the other three family communities, this ratio ranges between 7.10 and 72.71. For senior communities, this ratio is equal to 37.79 for communities with a small number of units and 18.17 for communities with a large number of units. If we restrict our attention to the subsample of households that are eligible for two-bedroom apartments, the demand–supply ratios are 2.65, 3.90, 15.88, and 4.64 for the four types of housing communities. The fraction of households willing to move into a public housing unit largely depends on the community specific fixed effects and thus reflects the attractiveness of the housing community. However, it also depends on the characteristics of eligible households. Older households

[35]It should be pointed out that the magnitude of the welfare estimates depends on the estimates of the "moving cost" parameter.

Table 6. Percent of households in community i who would accept an offer to move to j.

	Would Move to						
Current Residence:	Private	PH1	PH2	PH3	PH4	PH5	PH6
Private		0.006	0.012	0.009	0.008	0.009	0.012
PH1	0.080		0.067	0.054	0.044	0.055	0.071
PH2	0.063	0.020		0.029	0.023	0.029	0.039
PH3	0.075	0.023	0.043		0.028	0.035	0.045
PH4	0.077	0.031	0.056	0.045		0.046	0.059
PH5	0.102	0.022	0.041	0.032	0.026		0.043
PH6	0.085	0.019	0.034	0.027	0.022	0.028	

and extremely poor households are more willing to move from the private sector to public housing communities. These households suffer the highest welfare costs from policies that restrict the supply.

It is also interesting to compare the costs of public housing programs to voucher programs. In 1996, the U.S. Congress passed legislation requiring housing authorities to replace, that is, demolish, public housing structures if the expected cost of maintaining the structure for the next 20 years exceeded the expected cost of offering housing vouchers to the residents for the next 20 years. As a result of this law, it is predictable that for the years covered in our panel analysis, the cost of providing housing to those in public housing in Pittsburgh was lower than the cost of providing them with housing vouchers. Although exact cost measures are not available, in 2006 the HACP spent roughly $11,375 per year per housing voucher household and $8,900 per year per public housing household (HACP (2007)).

There are other important differences between voucher and public housing programs. One fact that is often overlooked is that more seniors and disabled persons are served by the public housing program than the voucher program; this fact may be a result of historical reasons or of the fact that disadvantaged populations find that public housing offers more convenient facilities than a typical apartment in the private housing market. There is some evidence that voucher households make different choices than households in public housing. Geyer (2012) analyzed a unique data set of voucher recipients in Pittsburgh Geyer (2012). She found that voucher recipients in Pittsburgh live in neighborhoods that have lower crime rates and better schools than the neighborhoods of public housing residents, suggesting that at least with respect to neighborhood quality, vouchers offer an improvement over public housing.[36]

[36]Research on the education, employment, and health outcomes of the voucher program in comparison to public housing offers additional valuable insights. For example, in studying public housing demolitions in Chicago, Jacob (2004) found that children in households offered a housing voucher did not fare better or worse than their peers who remained in public housing. In the Moving to Opportunities study, Katz, Kling, and Liebman (2007) found moving to lower poverty neighborhoods improved physical and mental health, but produced mixed outcomes for children's behavior and had little impact on employment outcomes.

7. Conclusions

We have developed a new method that can be used to estimate a demand model for public housing that captures key supply restrictions. Our empirical analysis of the Pittsburgh metropolitan area shows that public housing is an attractive option for seniors and exceedingly poor households headed by single mothers. Simple logit demand models that ignore supply side restrictions generate very different results. As a consequence, simpler models cannot explain the persistent existence of long wait lists in many U.S. cities. In contrast, our model generates low offer probabilities and long wait times.[37]

Our estimates and welfare analysis indicate that some low-income households strongly prefer public housing over private housing. Moreover, operating expenses appear to be lower for public housing than the voucher program in Pittsburgh. However, a complete cost–benefit analysis of public housing needs to be augmented by estimates of land purchases and construction costs, and to capture the potential spillover effects of public housing on a variety of outcomes such as human capital accumulation, earnings, and criminal behavior. More research and better data are needed to conduct such a comprehensive benefit–cost analysis of public housing.

The framework presented in this paper can be extended in a number of fruitful directions. In our model, households maximize current period utility. It is possible to model the dynamic decision problem faced by forward looking households. The value function that corresponds to this problem depends on current and future offer probabilities. We can still define demand as before and obtain a dynamic equilibrium with forward looking households. Characterizing the equilibrium of this model and estimating its parameters is, however, more challenging since the market clearing conditions are nonlinear in the offer probabilities.

It is possible to estimate even richer versions of the model discussed here. We have abstracted from unobserved heterogeneity in tastes for public housing. It is possible that there is stigma associated with living in public housing. Moffitt (1983) showed that stigma plays a role in explaining participation in other welfare programs. We can extend our framework and allow for unobserved heterogeneity in tastes for public housing. Such heterogeneity would provide an alternative explanation for the differential flow rates into and out of public housing. Some households may obtain a sufficiently strong negative utility from public housing that they effectively are never interested in the public sector. Other households might be less affected by stigma and are willing to choose public housing when they receive a sufficiently strong idiosyncratic shock. However, we can still define the equilibrium for this modified model. If the offer probabilities can be expressed as functions of the structural demand parameters, our approach for identification and estimation is valid and can be used to estimate richer specifications of the demand side.

[37]Excess demand can also occur in private housing markets due to other forms of regulation. Glaeser and Luttmer (2003) studied the misallocations that arise in private housing markets due to rent control.

Table 7. The 95% confidence intervals of estimation error.

Name	Variable	Random Sample	Enriched Sample
Fixed effect PH1	γ_1	$[-0.887,\ 1.763]$	$[-0.947,\ 1.763]$
Fixed effect PH2	γ_2	$[-0.8142,\ 1.585]$	$[-1.010,\ 1.585]$
Fixed effect PH3	γ_3	$[-0.806,\ 1.744]$	$[-0.850,\ 1.744]$
Beta	β	$[-0.191,\ 0.079]$	$[-0.191,\ 0.082]$
Offer prob PH1	π_1	$[-0.021,\ 0.019]$	$[-0.020,\ 0.019]$
Offer prob PH2	π_2	$[-0.043,\ 0.050]$	$[-0.046,\ 0.055]$
Offer prob PH3	π_3	$[-0.013,\ 0.010]$	$[-0.013,\ 0.010]$

Appendix A: A Monte Carlo study

Since our estimation procedure is nonstandard, we conducted a number of Monte Carlo studies to study the properties of the estimators when the true data generating process is known. In Table 7, we report the results for one specification that we tested.[38]

In our Monte Carlo study there is only one observed household characteristic (income). We assume that $f(x_t, d_{t-1})$ is log normally distributed with known mean and variance. We consider a model with three public housing communities with $\gamma_1 = 7.6$, $\gamma_2 = 7.0$, and $\gamma_3 = 0.4$. We set the coefficient of income $\beta = 0.4$. We assign 30% of the population to private housing, and 24, 28, and 18% to the three housing communities. This implies that in equilibrium, the offer probabilities are $\pi_1 = 0.11$, $\pi_2 = 0.24$, and $\pi_3 = 0.05$.

We consider the properties of the estimator above under two sampling designs: random sampling and enriched sampling. For each parameter vector, 100 model simulations and estimations are completed, each with sample size 2,000. Starting values are initially chosen from a uniform distribution between $(0, 1)$ for β and between $[0, 12]$ for the fixed effects, but any starting values that would lead to unreasonable offer probabilities (probabilities greater than 40%) are rejected. Table 7 summarizes the performance of the model and reports 95% confidence for the absolute error of parameter estimate and the implied offer probabilities.

In general, we find that our estimator works well under both random and enriched sampling. The absolute errors are small and approximately centered around zero. Generally, we find that the estimates for the fixed effects are slightly biased upward and the coefficients on income are slightly biased downward in samples with 2,000 observations. In general, larger samples help reduce the estimation bias. Imposing the equilibrium conditions seems to work well, and the estimates of the offer probabilities that are implied by the structural parameters of the model are accurate.

Appendix B: The extended SIPP sample

In addition to the Pittsburgh sample, we also constructed a larger sample, adding data from 13 metropolitan areas that have ratios of public housing units per household simi-

[38] More results for different parametrizations, sample sizes, and sampling schemes are available on request from the authors.

Table 8. Urban areas included in the sample.

City	Eligible for Public Housing	Median Income	Unemployment Rate	Minority	Fair Market Rent 2001
Pittsburgh	0.0546	37,467	4.4%	10%	476
Columbus	0.0384	44,782	2.7%	19%	471
Allentown	0.0375	43,098	4.2%	10%	511
Albany	0.0373	43,250	3.4%	10%	494
Dayton	0.0372	41,550	4.5%	18%	389
Buffalo	0.0339	38,488	5.3%	16%	453
Scranton	0.0607	34,161	5.6%	3%	408
St. Louis	0.0169	44,437	3.5%	22%	429
Madison	0.0124	49,223	1.7%	11%	559
Detroit	0.0159	49,160	3.9%	27%	598
Cleveland	0.0291	42,215	4.2%	21%	555
Cincinnati	0.0109	44,914	3.5%	15%	416
Philadelphia	0.0266	47,528	4.1%	27%	657
Milwaukee	0.0193	46,132	3.1%	22%	504

lar to Pittsburgh. Table 8 provides some summary statistics of these metropolitan statistical areas (MSA's).

Table 8 reports the MSA's ratio of public housing units to households eligible for public housing. We also show the 1999 MSA median income, 1999 unemployment rate, and the HUD-determined 2001 fair market rent for a one-bedroom unit.[39] Table 8 shows that Pittsburgh is representative of many other large urban areas in the Northeast and Midwest that face similar challenges in providing affordable housing for low-income households.

A more formal sensitivity test is to compare estimates of the model obtained using the full HACP and SIPP samples to the estimates of the model obtained using full HACP sample and only the SIPP samples from Pittsburgh. The results are reported in Table 9. We find that the point estimates are very similar. Not surprisingly, the estimated standard errors are larger when the SIPP sample is reduced to include only observations in Pittsburgh. We conclude that our estimates are not seriously driven by the composition of our "control" sample.

Appendix C: Ignoring the supply side restrictions

Table 10 reports the estimates obtained from a logit demand model that ignores the supply side restrictions and compares to our baseline estimates.

[39] The number of public housing units is taken from the HUD 1998 Picture of Subsidized Housing. Percent minority and median incomes are from the 2000 Census. Unemployment is from The Real Estate Center at Texas A&M University. Fair Market Rents are published on the HUD website.

Table 9. Parameter estimates.

	Full Sample		Pitt SIPP Only	
Income	0.329	(0.028)	0.327	(0.038)
Moving cost	−3.186	(0.017)	−3.203	(0.036)
Afr. Amer. and nonsenior	1.222	(0.071)	1.221	(0.107)
White and senior	0.209	(0.113)	0.209	(0.152)
Afr. Amer. and senior	1.000	(0.101)	1.001	(0.138)
Children	−0.315	(0.123)	−0.317	(0.185)
Female	0.053	(0.061)	0.054	(0.090)
Female and senior	−0.174	(0.094)	−0.174	(0.121)
Female with children	0.426	(0.130)	0.424	(0.195)
	Community Fixed Effects		Community Fixed Effects	

Note: Estimated standard errors are given in parentheses.

Table 10. Comparison with simple logit.

	Our Model		Simple Logit	
Income	0.329	(0.028)	0.436	(0.018)
Moving cost	−3.186	(0.017)	−5.512	(0.015)
Afr. Amer. and nonsenior	1.222	(0.071)	1.775	(0.034)
White and senior	0.209	(0.113)	0.612	(0.043)
Afr. Amer. and senior	1.000	(0.101)	1.161	(0.042)
Children	−0.315	(0.123)	−0.766	(0.083)
Female	0.053	(0.061)	0.195	(0.035)
Female and senior	−0.174	(0.094)	−0.136	(0.043)
Female with children	0.426	(0.130)	0.479	(0.086)
PH1	4.217	(0.254)	−0.636	(0.158)
PH2	4.848	(0.261)	−0.895	(0.165)
PH3	4.604	(0.277)	−1.672	(0.170)
PH4	4.394	(0.260)	0.101	(0.163)
PH5	4.626	(0.263)	−1.538	(0.158)
PH6	4.907	(0.258)	−0.837	(0.154)
log likelihood	−688,796		−653,936	

Note: Estimated standard errors are given in parentheses.

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