

# Supplement to “Model averaging, asymptotic risk, and regressor groups”

(*Quantitative Economics*, Vol. 5, No. 3, November 2014, 495–530)

BRUCE E. HANSEN  
University of Wisconsin

## SIMULATION DETAILS

The simulation programs were written in R and run under Windows Vista. The programs are available on the journal website [http://qeconomics.org/supp/332/code\\_and\\_data.zip](http://qeconomics.org/supp/332/code_and_data.zip).

The results are presented graphically, with MSE displayed as a function of  $R^2$ . The value of  $R^2$  was varied on the 19-point grid  $\{0.00, 0.05, 0.10, 0.15, \dots, 0.90\}$ . For a fixed  $\alpha$  and  $R^2$ , the value of  $c$  was then determined as

$$c = \sqrt{\frac{R^2}{\sum_{j=1}^M j^{-2\alpha}(1 - R^2)}}.$$

Given  $c$ , we then set  $\beta_j = cj^{-\alpha}$  and

$$y_i = \beta_0 + \sum_{j=1}^M \beta_j x_{ji} + e_i$$

with  $\beta_0 = 0$ .

We varied  $\alpha \in \{0, 1, 2, 3\}$  and  $n \in \{50, 150, 400, 1000\}$ .

The default model (Model 1) set the errors  $e_i$  and regressors  $x_{ji}$  as i.i.d.  $N(0, 1)$  and set  $M = 12$ . The remaining models explored the deviations from these default settings.

We explored nonnormal errors, heteroskedastic errors, correlated regressors, and  $M = 24$ .

All models were designed so that the error is conditionally mean zero and has an unconditional variance of 1.

The results for Model 1 and Model 6 are calculated using 10,000 simulation replications. For Models 2–5, the calculations used 2000 simulation replications.

### 1. Model 1: Normal regression

- $e_i \sim N(0, 1)$

---

Bruce E. Hansen: [behansen@wisc.edu](mailto:behansen@wisc.edu)

Copyright © 2014 Bruce E. Hansen. Licensed under the Creative Commons Attribution-NonCommercial License 3.0. Available at <http://www.qeconomics.org>.

DOI: 10.3982/QE332

- uncorrelated regressors
- $M = 12$

## 2. Model 2: Nonnormal error

- $e_i \sim \frac{4}{5}N(-\frac{1}{3}, \frac{5}{9}) + \frac{1}{5}N(\frac{4}{3}, \frac{5}{9})$

## 3. Model 3: Heteroskedastic error

- $e_i \sim N(0, \frac{1}{2}(1 + x_{2i}^2))$

## 4. Model 4: Correlated regressors

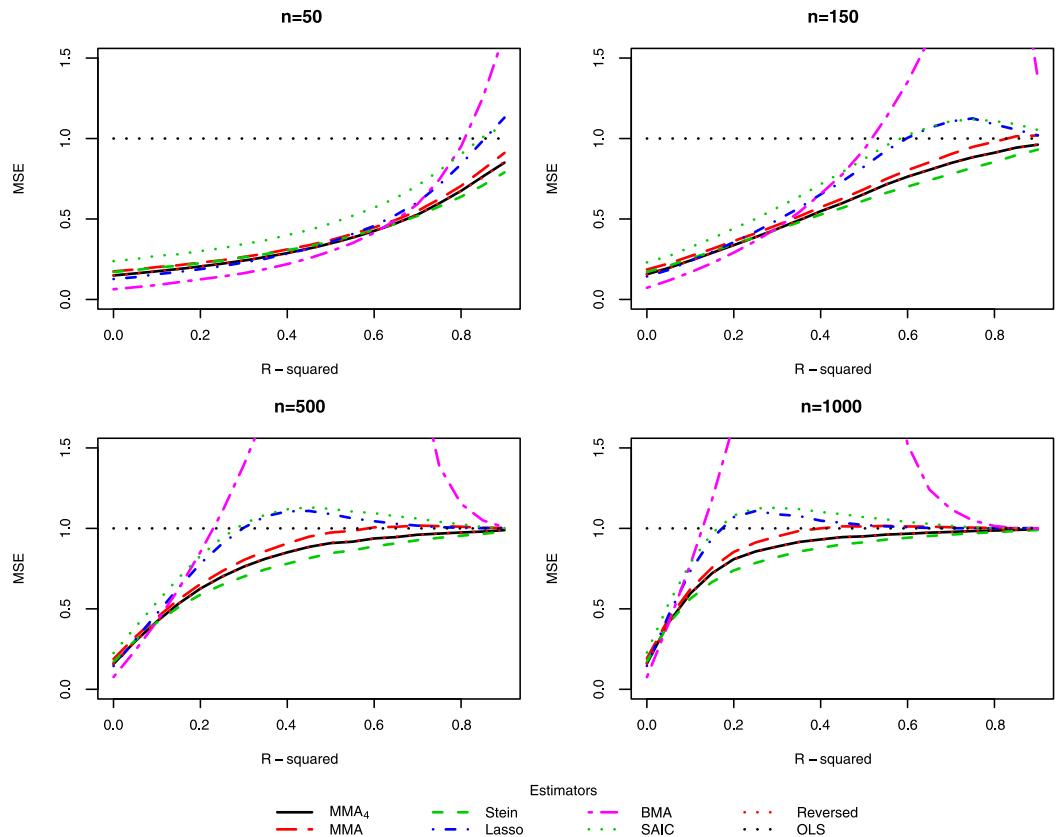
- $e_i \sim N(0, 1)$
- $E(x_{ji}^2) = 1, E(x_{ji}x_{ki}) = 0.5$  for  $j \neq k$

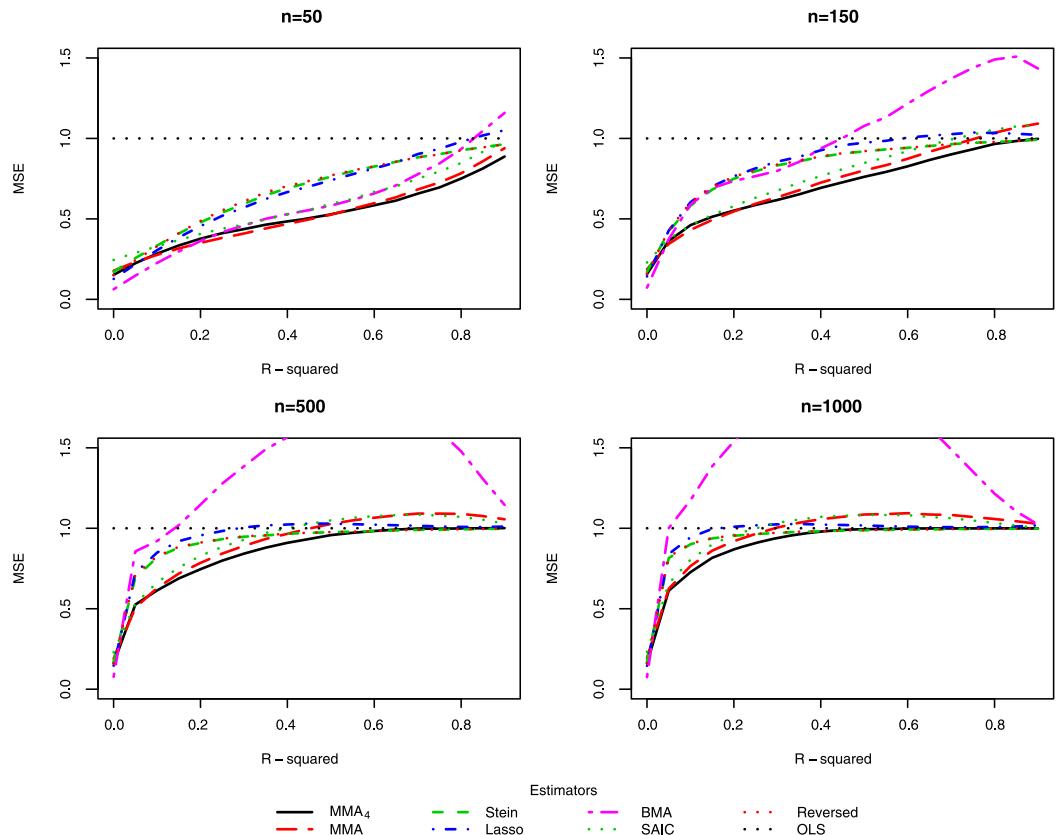
## 5. Model 5: Increased number of regressors

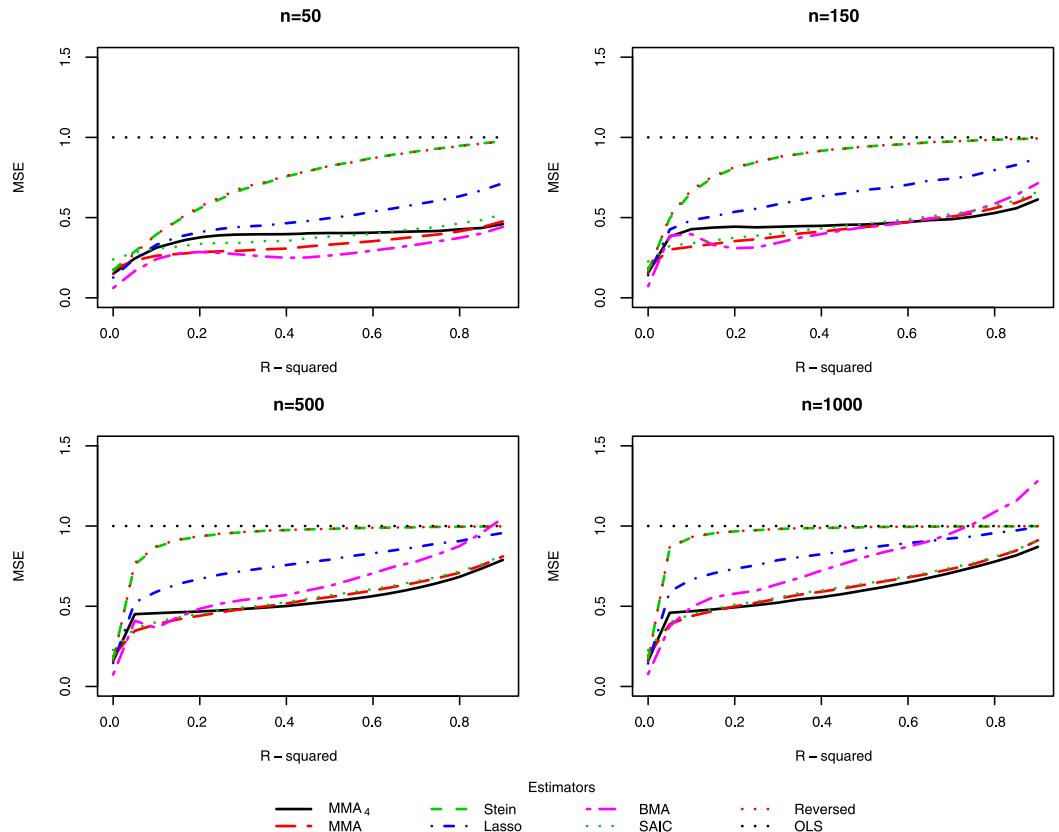
- $e_i \sim N(0, 1)$
- $M = 24$

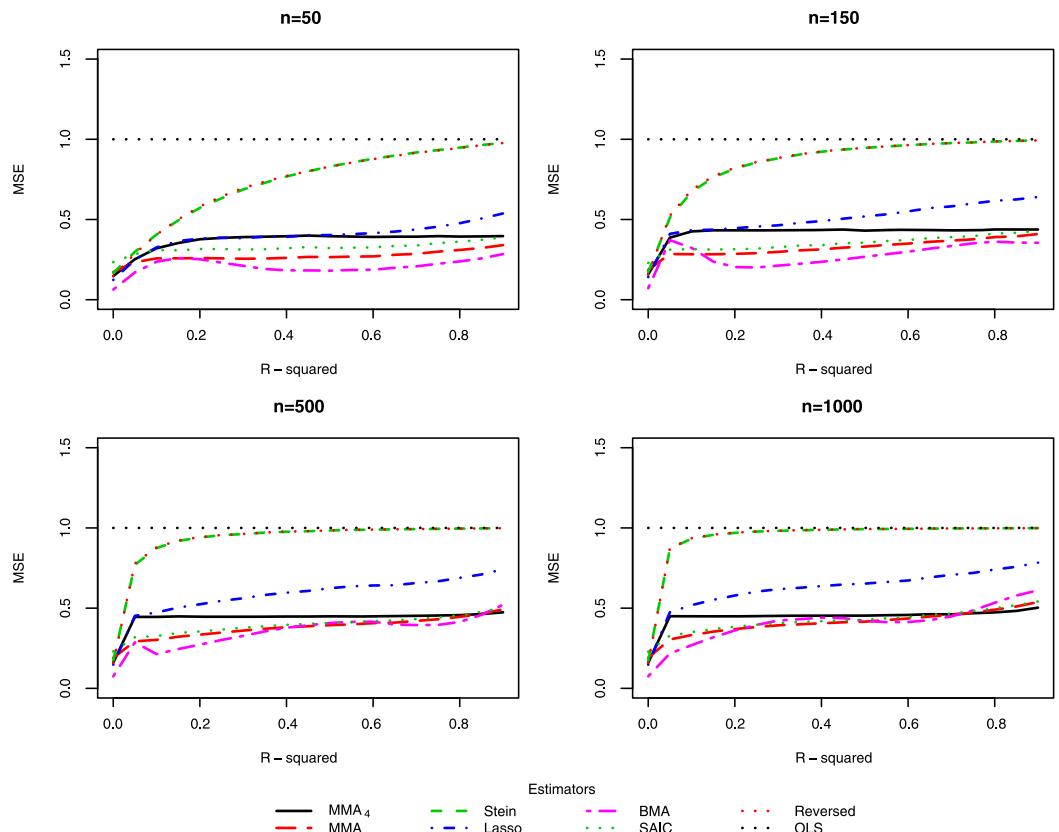
## 6. Model 6: Autoregression

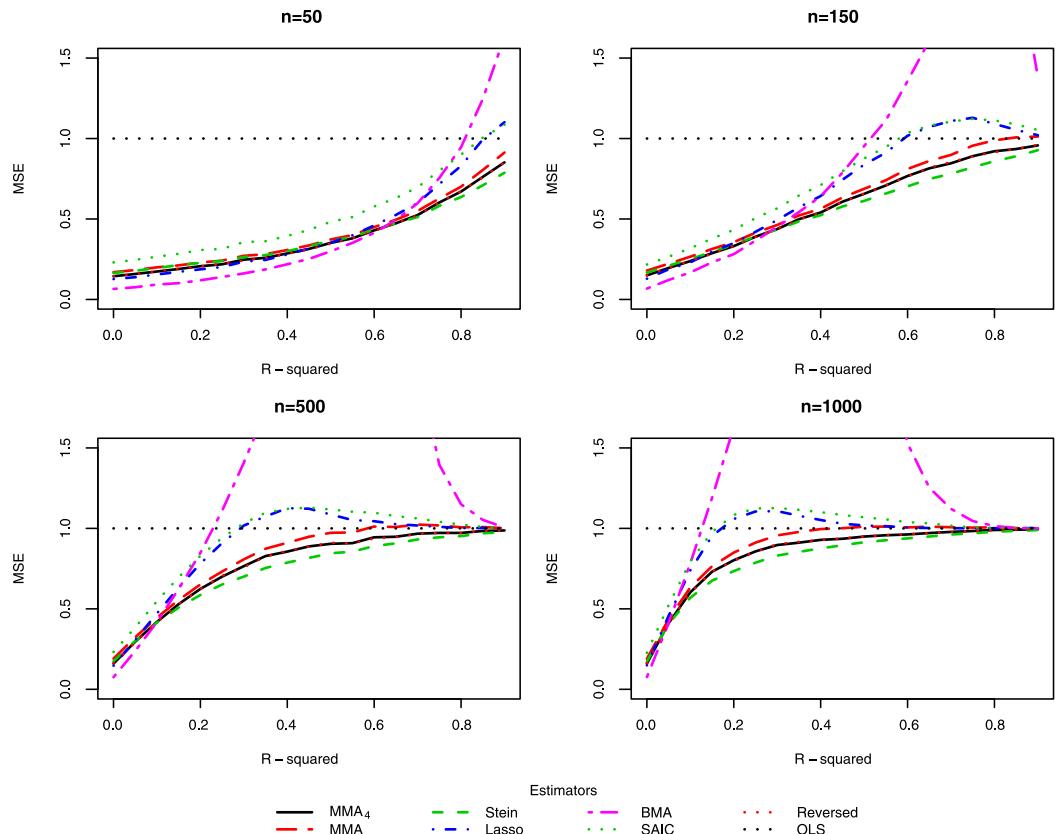
In the paper, the figures display the normalized MSE for the estimators MMA<sub>4</sub>, MMA, Stein, Lasso, and BMA. Here, we also display the normalized MSE for the estimator SAIC and the MMA<sub>4</sub> estimator with the regressors ordered in reverse (from smallest to largest coefficients) and labeled as “Reversed.”

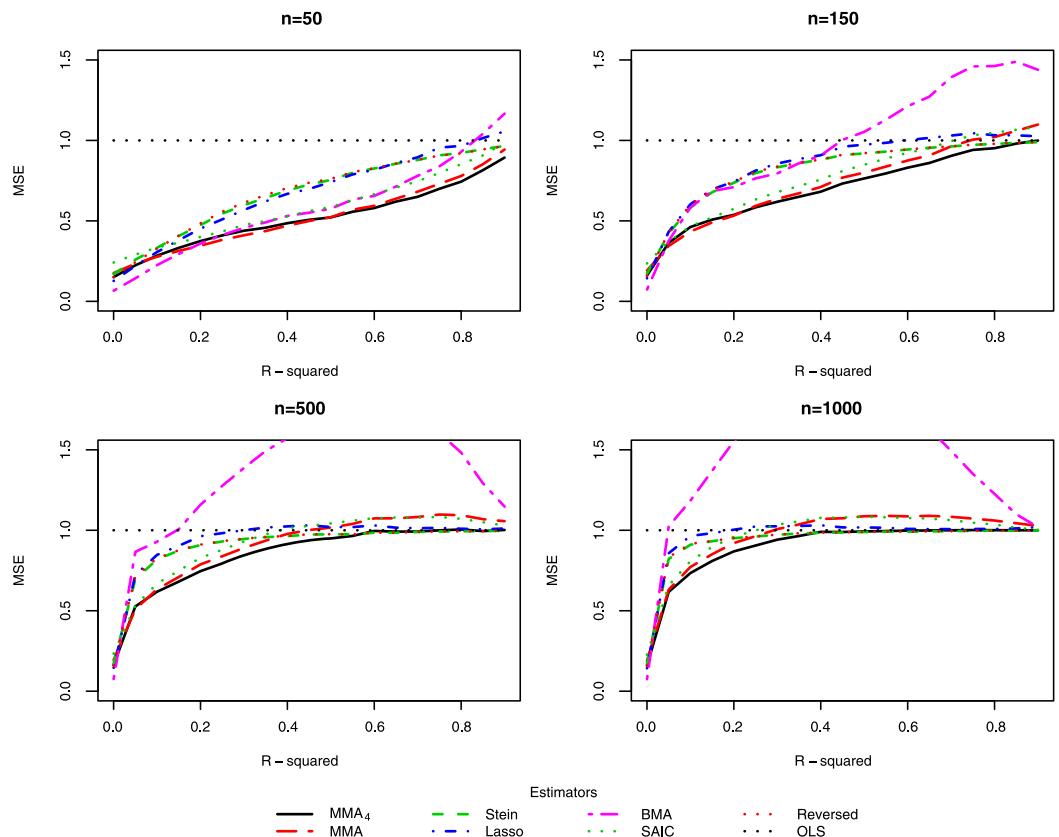
FIGURE 1. Model 1:  $\alpha = 0$ .

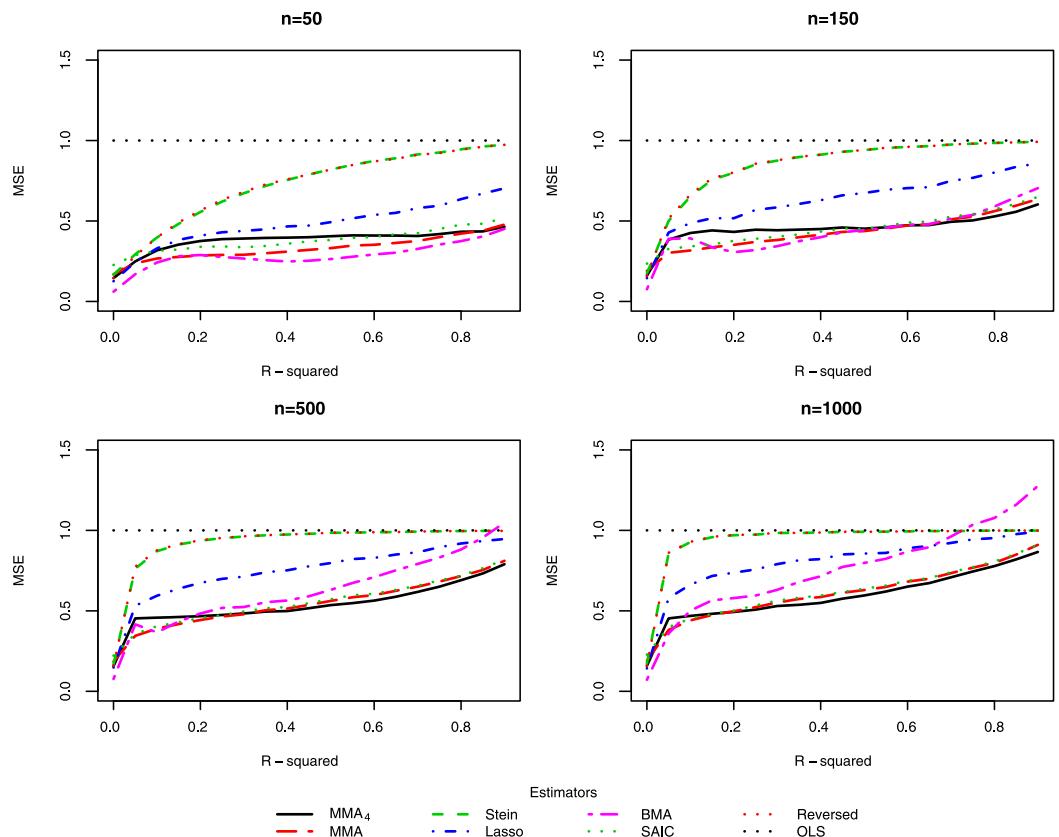
FIGURE 2. Model 1:  $\alpha = 1$ .

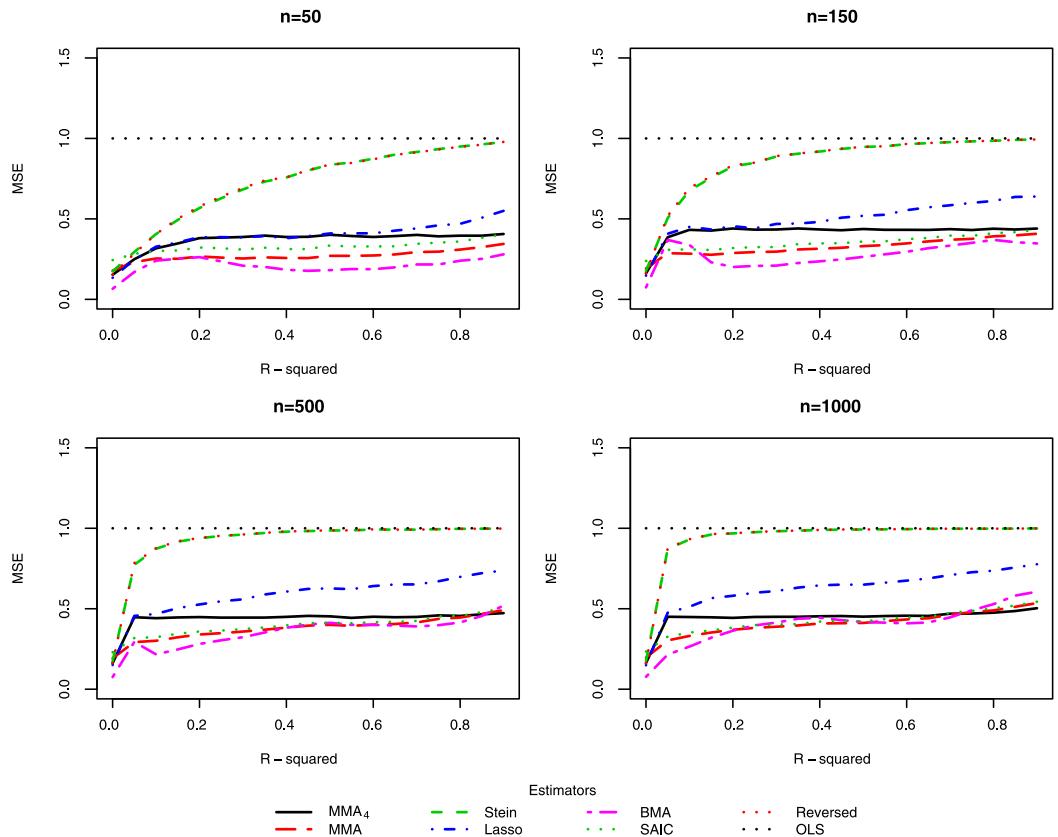
FIGURE 3. Model 1:  $\alpha = 2$ .

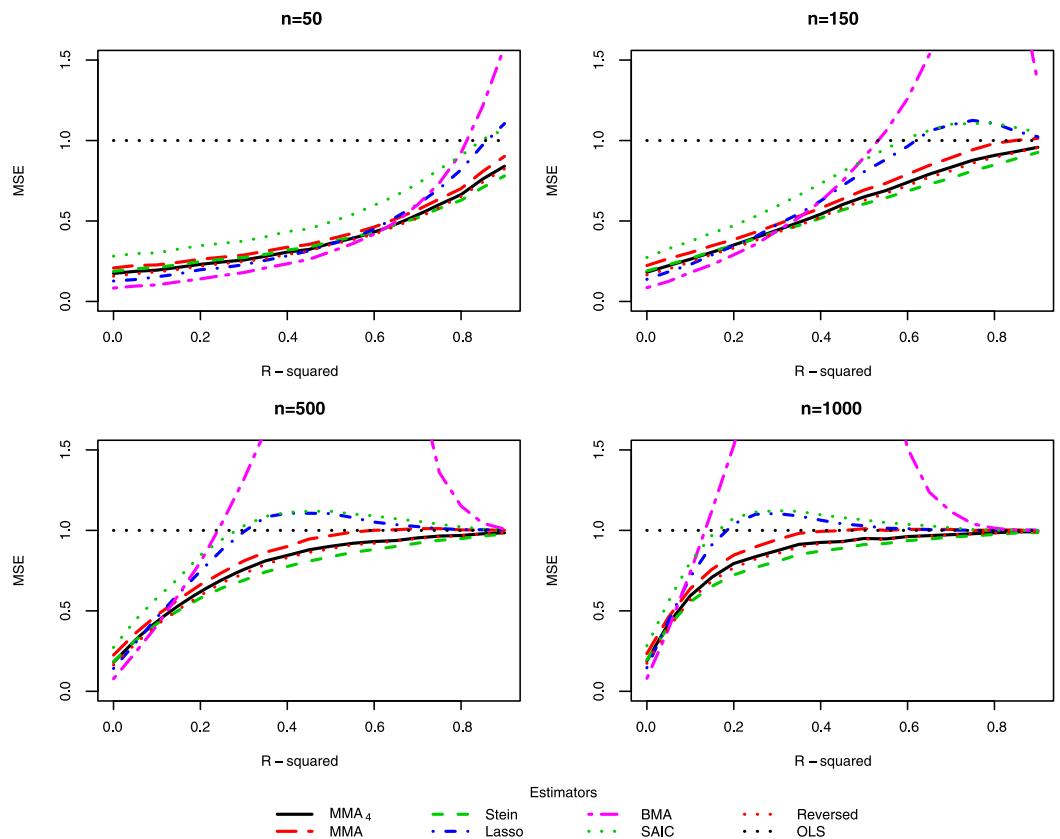
FIGURE 4. Model 1:  $\alpha = 3$ .

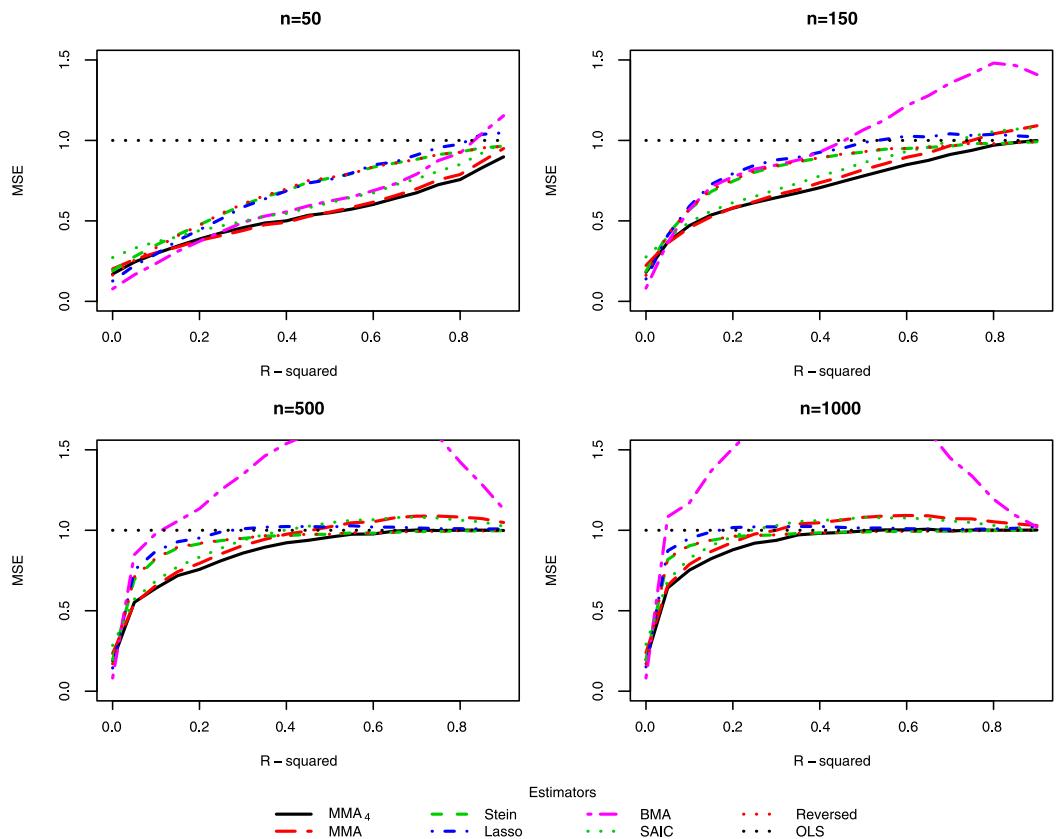
FIGURE 5. Model 2:  $\alpha = 0$ .

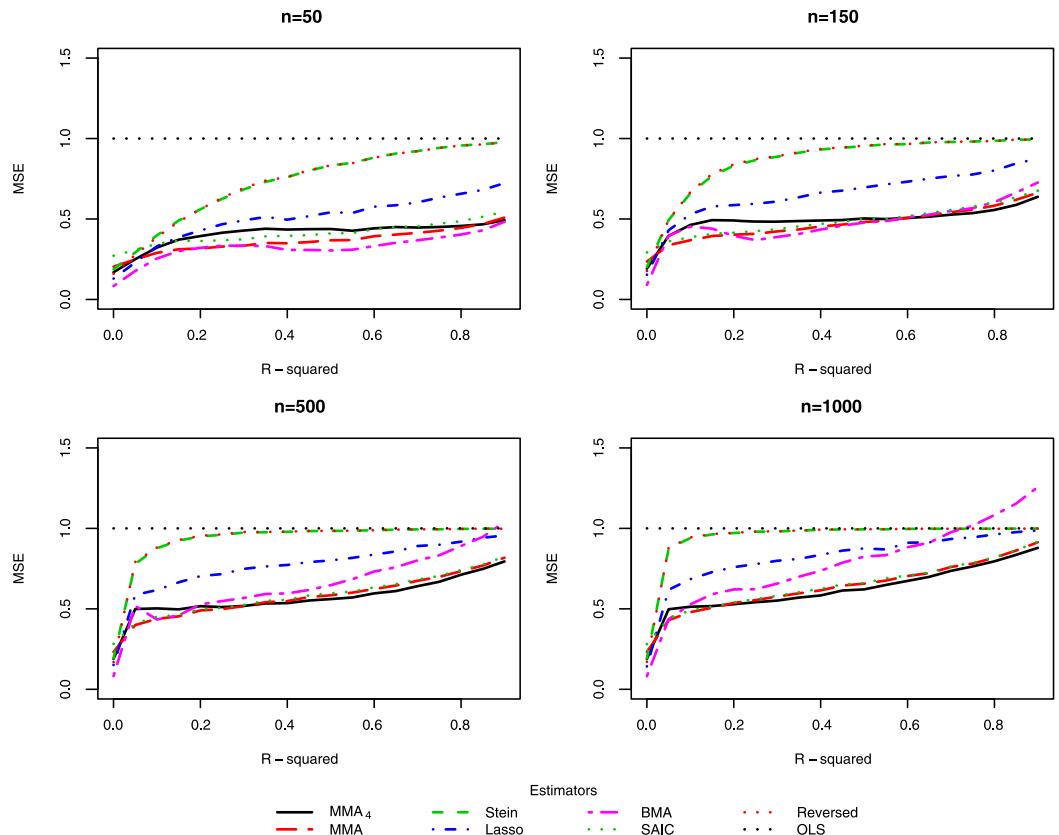
FIGURE 6. Model 2:  $\alpha = 1$ .

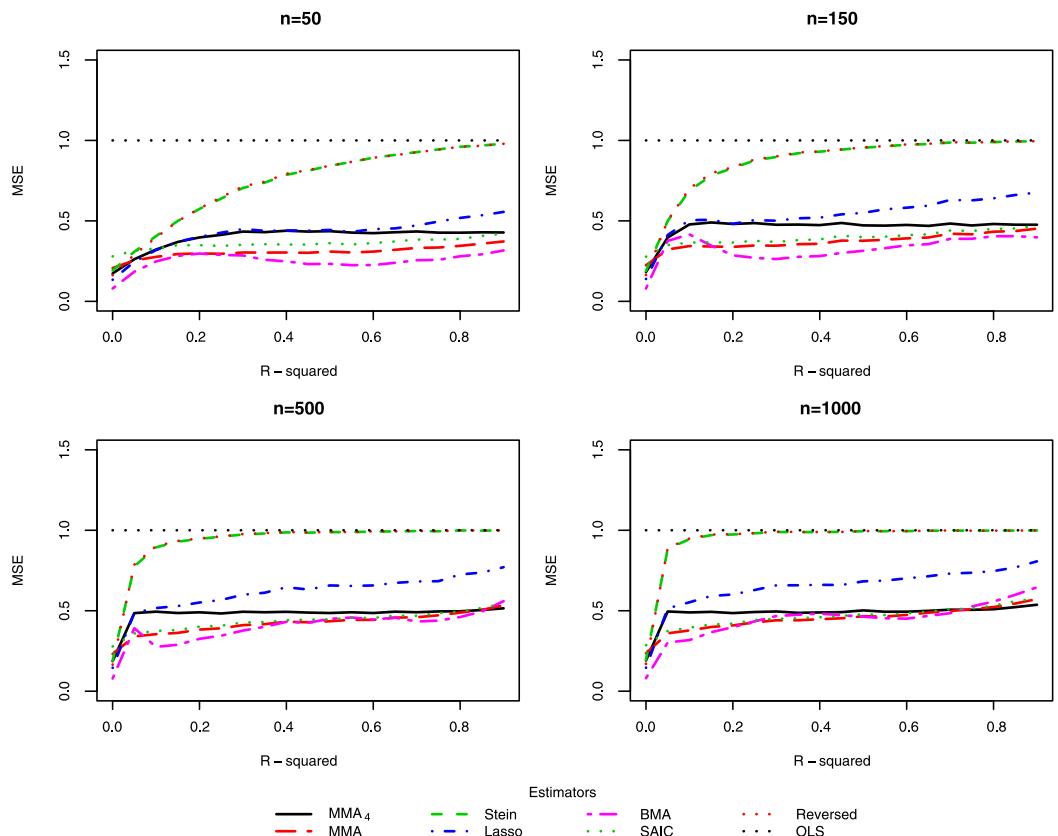
FIGURE 7. Model 2:  $\alpha = 2$ .

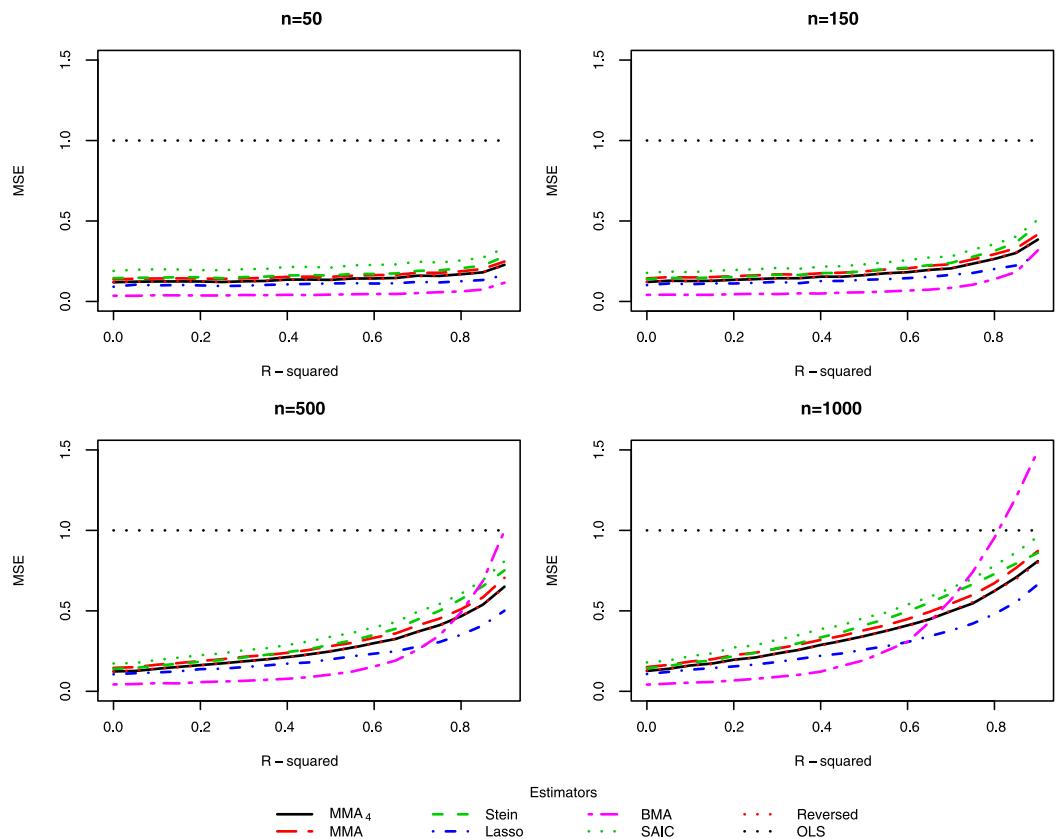
FIGURE 8. Model 2:  $\alpha = 3$ .

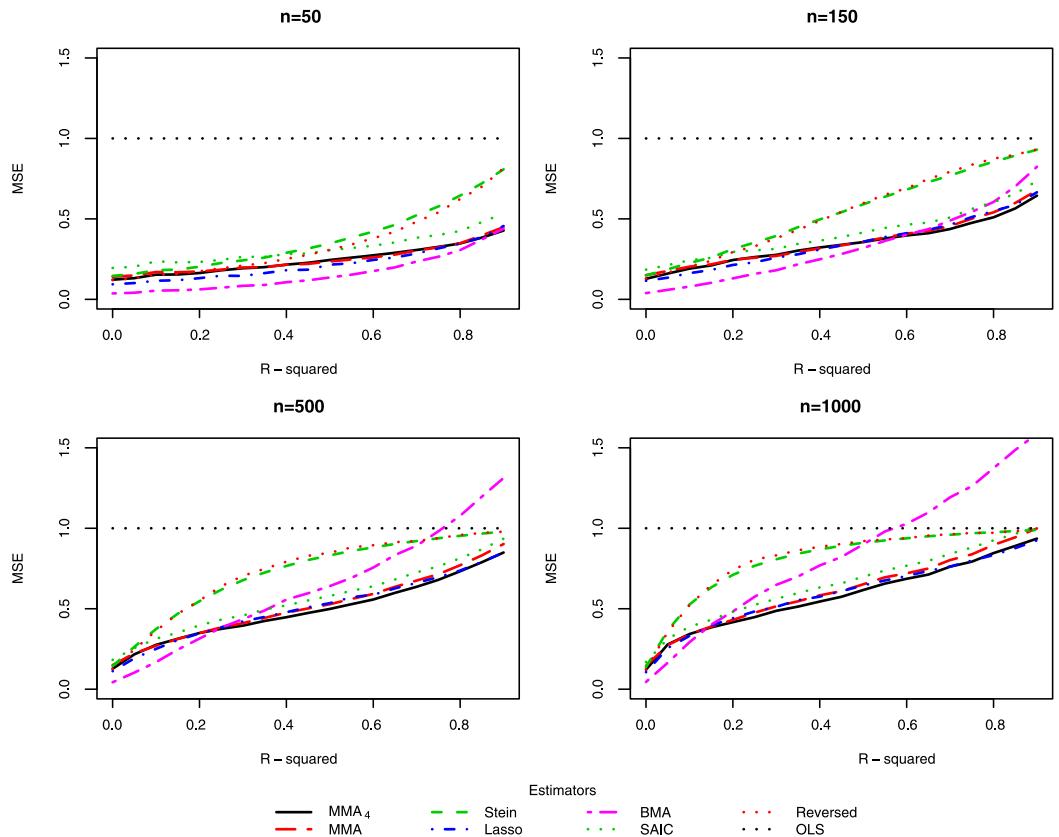
FIGURE 9. Model 3:  $\alpha = 0$ .

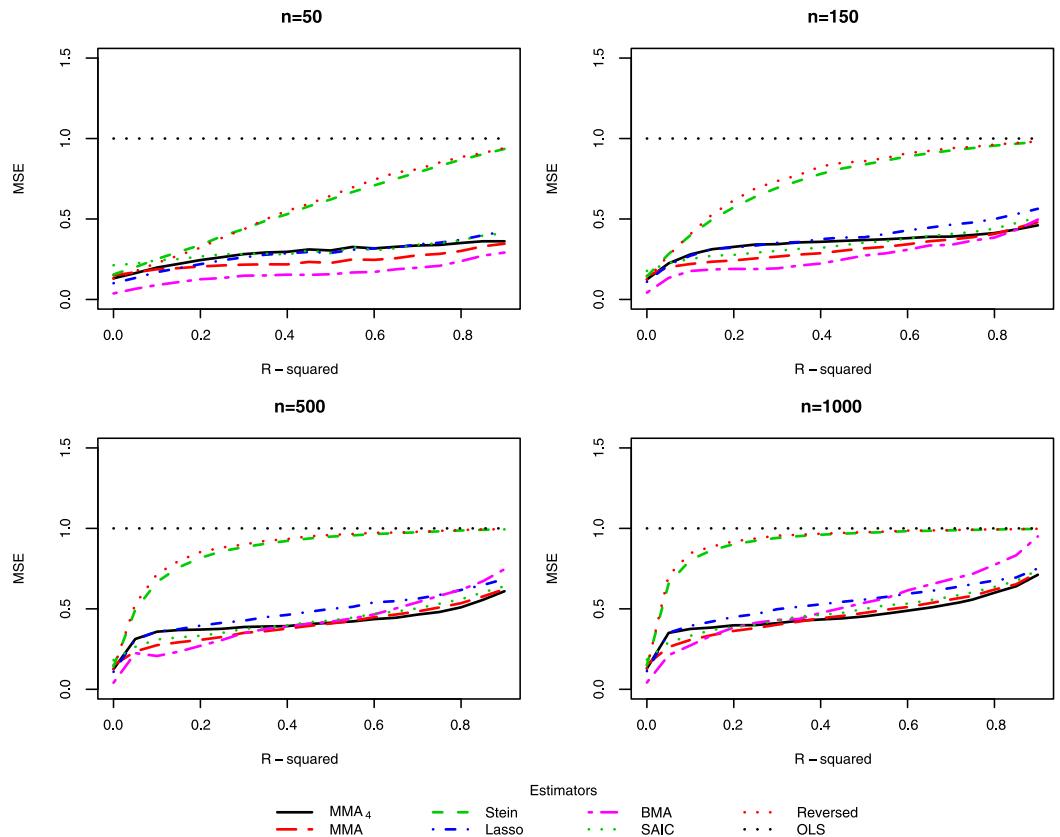
FIGURE 10. Model 3:  $\alpha = 1$ .

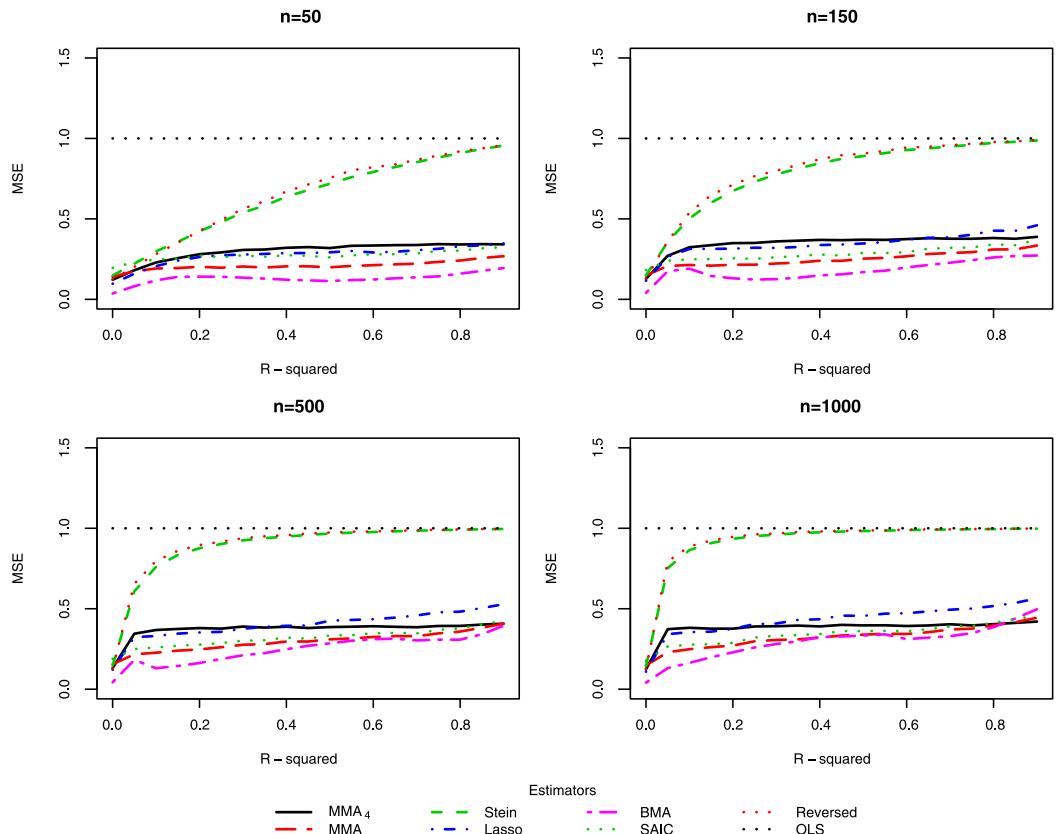
FIGURE 11. Model 3:  $\alpha = 2$ .

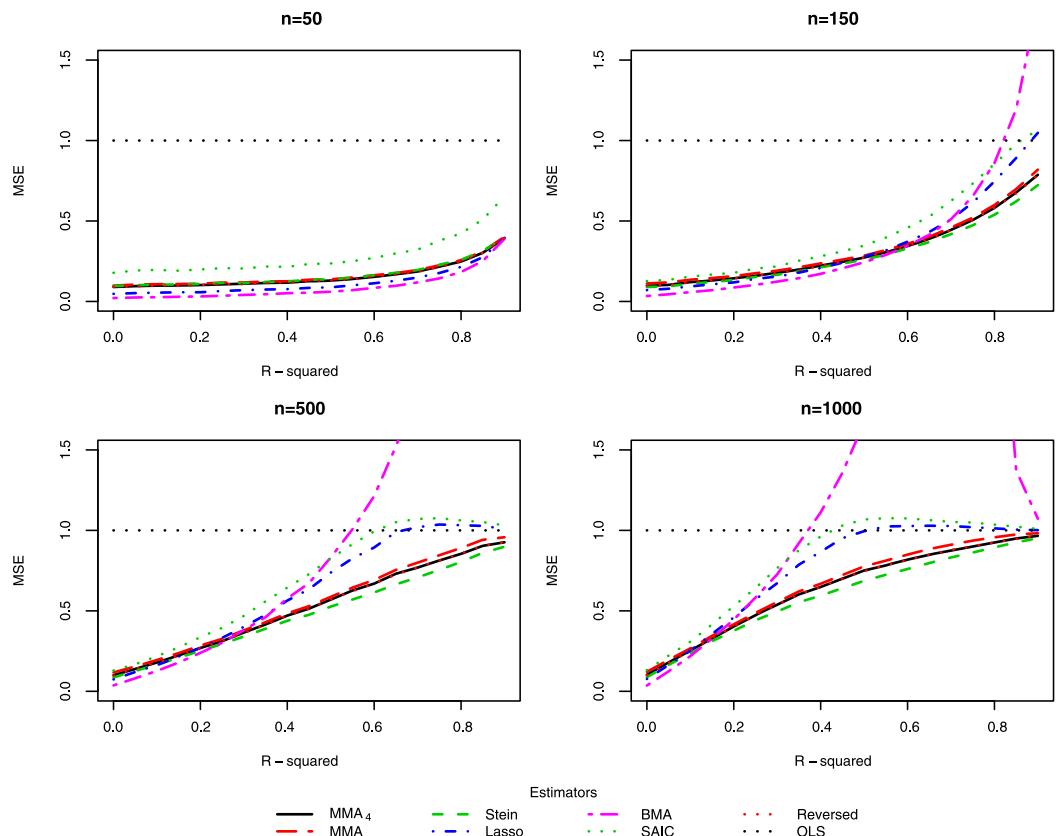
FIGURE 12. Model 3:  $\alpha = 3$ .

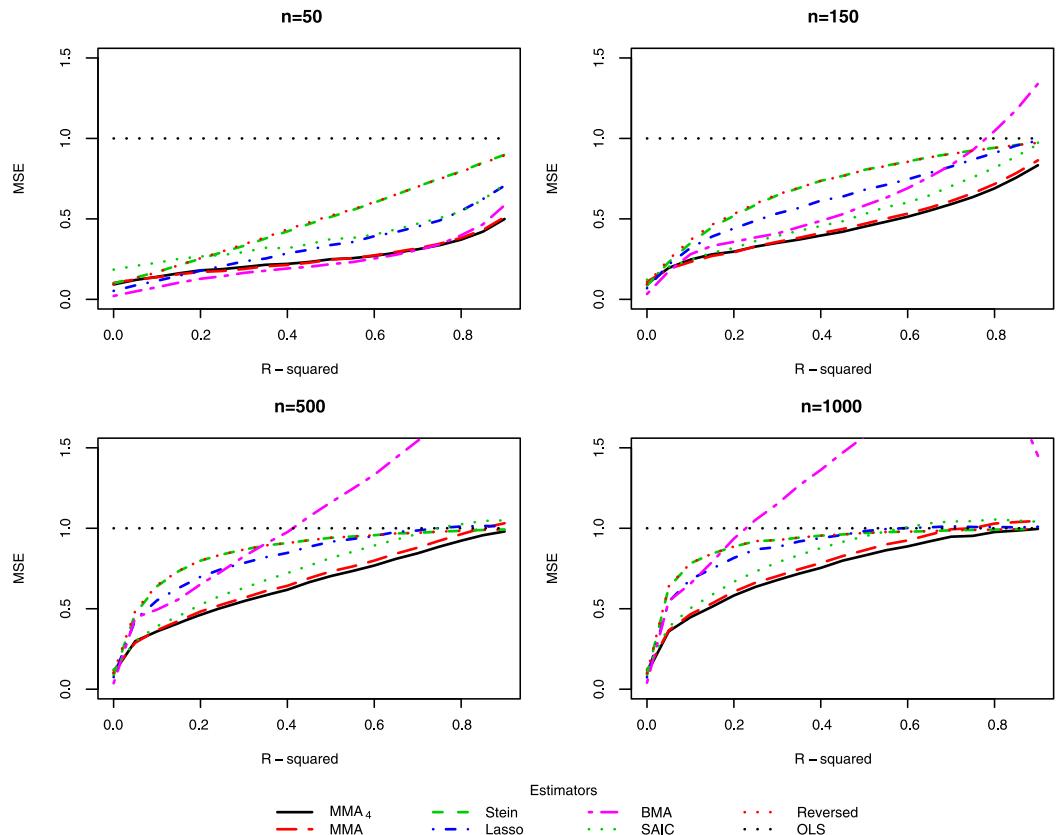
FIGURE 13. Model 4:  $\alpha = 0$ .

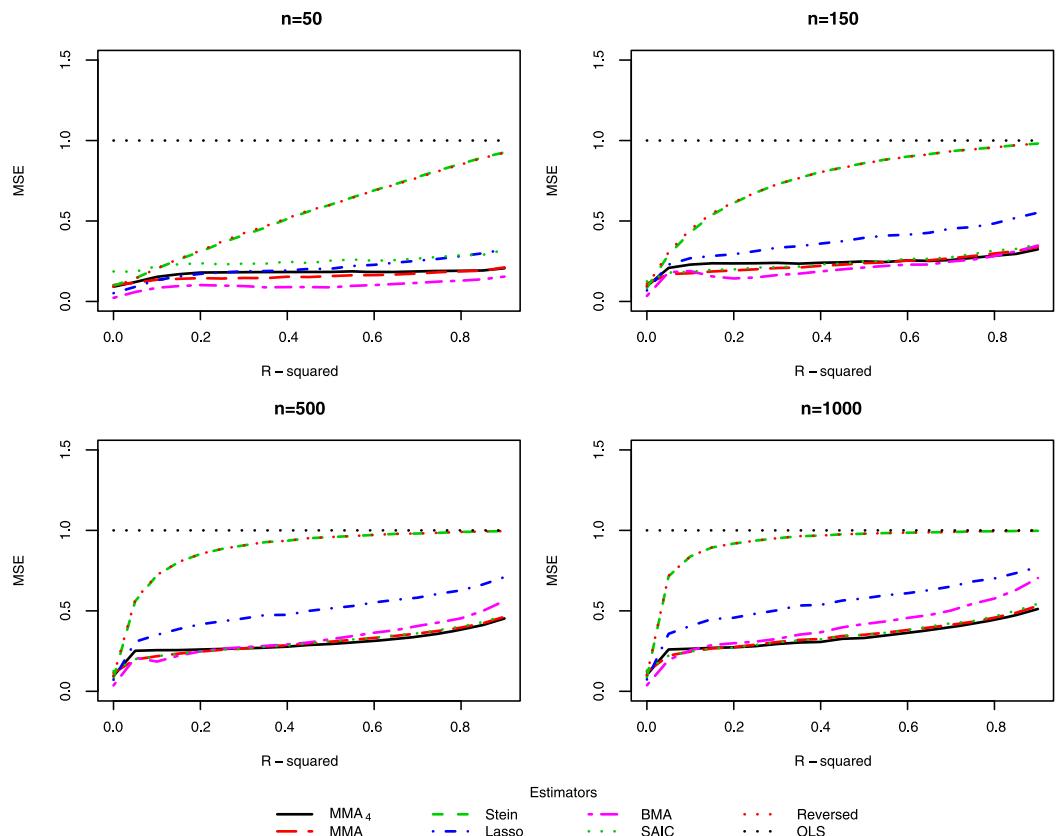
FIGURE 14. Model 4:  $\alpha = 1$ .

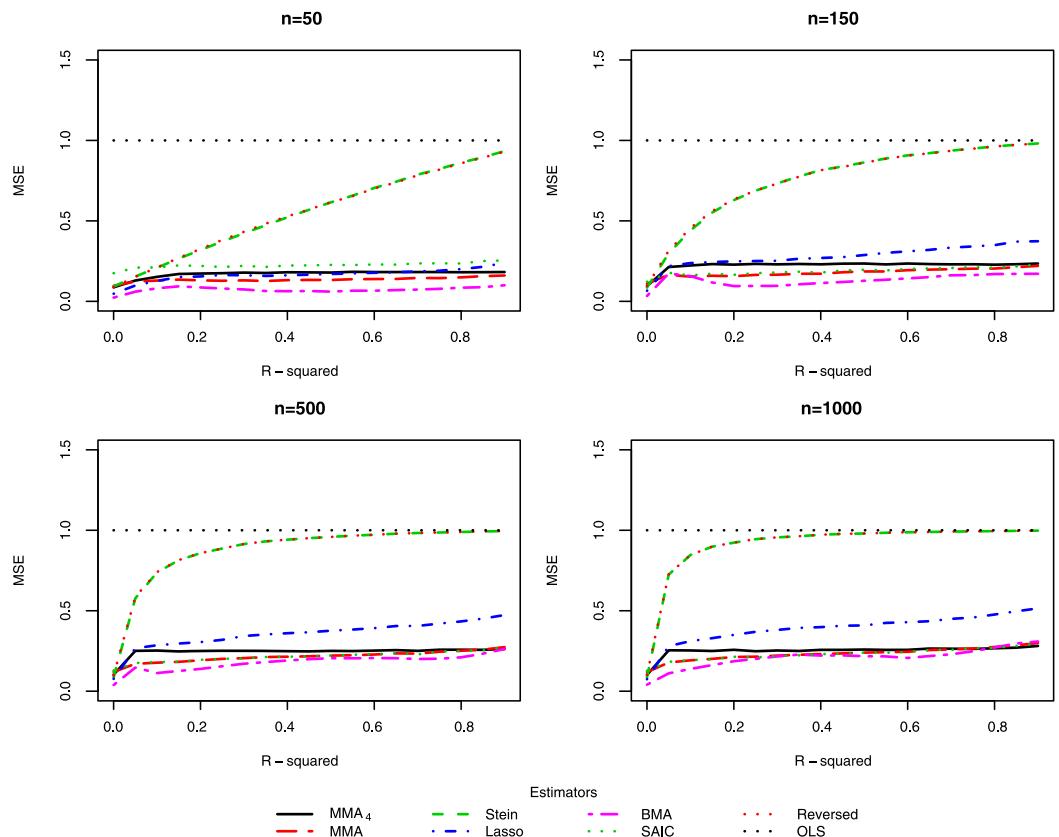
FIGURE 15. Model 4:  $\alpha = 2$ .

FIGURE 16. Model 4:  $\alpha = 3$ .

FIGURE 17. Model 5:  $\alpha = 0$ .

FIGURE 18. Model 5:  $\alpha = 1$ .

FIGURE 19. Model 5:  $\alpha = 2$ .

FIGURE 20. Model 5:  $\alpha = 3$ .

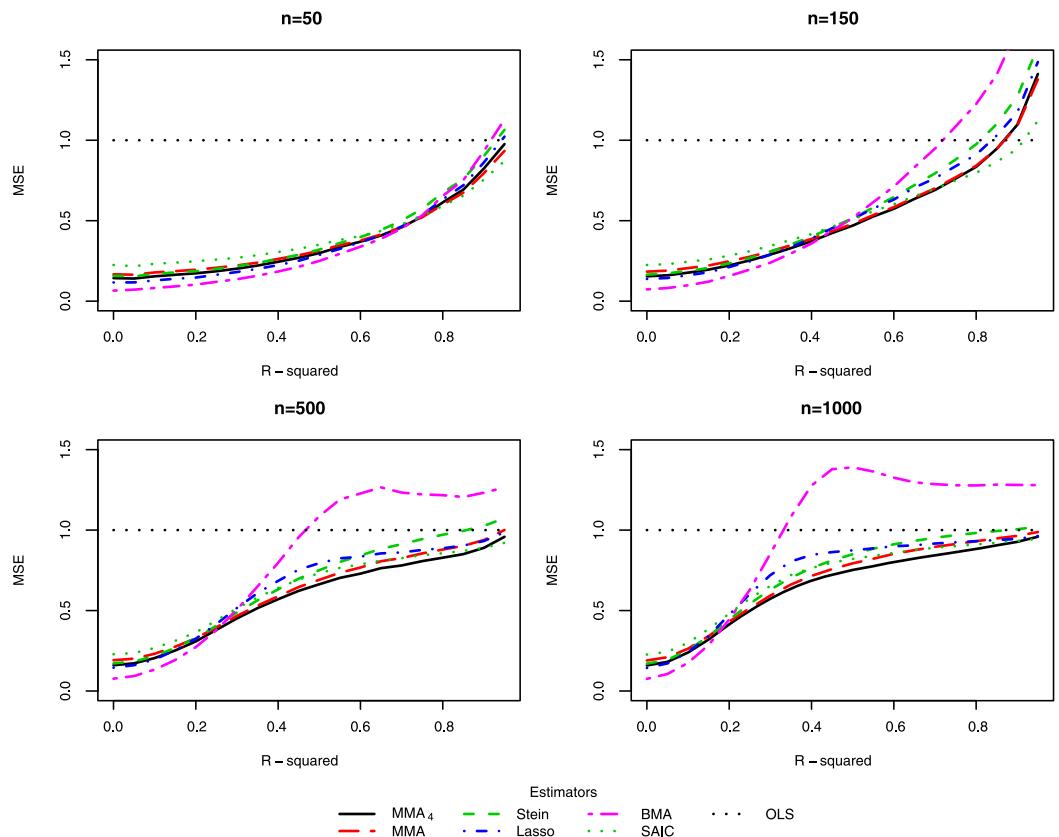


FIGURE 21. Model 6: Autoregression.

Submitted November, 2012. Final version accepted August, 2013.