

Communication with multiple senders: An experiment

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We implement multi-sender cheap talk in the laboratory. While full-information transmission is not theoretically feasible in the standard one-sender–one-dimension model, in this setting with more senders and dimensions, full revelation is generically a robust equilibrium outcome. Our experimental results indicate that fully revealing outcomes are selected in particular settings, but that partial-information transmission is the norm. We uncover a number of behavioral patterns: On the one hand, senders follow exaggeration strategies, qualitatively similar to those predicted by a special case for the fully revealing equilibrium. Receivers, on the other hand, follow differing heuristics depending on the senders' biases, which are not always sequentially rational. When full revelation is observed it can be explained as the intersection of the receiver heuristics with the equilibrium response.

KEYWORDS. Information transmission, cheap talk, multiple senders, full revelation.

JEL CLASSIFICATION. C72, C92, D83, D84.

1. INTRODUCTION

In many strategic situations decision-makers must rely on experts' advice to make informed choices. A tension arises when the decision-maker's preference over final outcomes is misaligned with the experts'. A theoretical literature on strategic information transmission, starting from Crawford and Sobel (1982), shows that with just a single informed, expert strategic motives can substantially reduce information transfer. With multiple experts a new challenge for the decision-maker emerges, resolving the

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Our thanks to the following people: Marco Battaglini, Gary Charness, John Duffy, Matthew Embrey, Ignacio Esponda, Rod Garratt, Daniel Martin, Rebecca Morton, Jack Ochs, Ryan Oprea, Charles Plott, Luca Rigotti, Matan Tsur, Isabel Trevino, Lise Vesterlund, Joseph Wang, Leeat Yariv, Sevgi Yuksel, and seminar participants at Caltech, Carnegie Mellon, CeSifo, Econometric Society, ESA, Princeton IAS, Montreal, NC State, NYU, Pittsburgh, Rutgers, Ilades, Stanford, and SWET. Research support from the Center for Experimental Social Science at NYU is gratefully acknowledged. Special thanks to Guillaume Fréchet for great advice on the design and implementation. We have also benefited from the feedback of many others through the publication process.

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DOI: 10.3982/QE500

potentially conflicting advice. However, multiple experts also creates an opportunity, as decision-makers may be able to fully extract all the available information from experts by comparing their advice. Through a series of laboratory experiments, we examine whether such fully revealing outcomes are selected, and how human subjects respond to conflicting advice. While the underlying tensions we investigate are present in the field—judicial proceedings, congressional testimony, consumers making large purchases, health care, etc.—a controlled experiment allows us to isolate the strategic tensions, with data and exogenous variation not commonly available outside the laboratory.

Battaglini (2002) constructs a fully revealing equilibrium with multiple senders over a multi-issue policy, and our experiment will examine this particular equilibrium. The setup is as follows: A decision-maker (DM) needs to make a choice on a multi-issue policy, but she is uncertain over the best choice. However, she consults with a number of experts, who are perfectly informed on the ideal policy, but are known to be biased. Experts independently provide their recommendations to her, and the DM then makes a policy choice. Battaglini shows by construction that an equilibrium generically exists where the DM combines the experts' recommendations and infers her best policy.

Consider the simplest case for the equilibrium construction: two experts provide guidance for a two-issue policy, with preferences as illustrated by Case 0 in Figure 1. Here the realized state of the world, representing the DM's ideal choice for the two-issue policy, is given by the solid black point, and the ideal points for the experts are given by the two white points. In the Case 0 environment, expert X is known to have an ideal policy coincident with the DM on issue 1, but biased upward on issue 2. Conversely, expert Y 's ideal policy on issue 2 is coincident with the decision-maker, but he is biased to the right on issue 1. While the DM does not know the realized state, she does know the relative position of each expert's ideal policy in relation to her own—the experts' biases. In the fully revealing equilibrium, the DM combines recommendations from the two experts, following expert X on issue 1 and expert Y on issue 2. Given the DM's reaction to their recommendations, each expert only has influence over the final choice on a single issue and, conditional on the other expert truthfully revealing, it is a best response for each to truthfully reveal themselves.

Battaglini (2002) shows that the intuitive construction in Case 0 generalizes. For a simple illustration, consider experts' relative ideal policies in Case 1, labeled with dark-

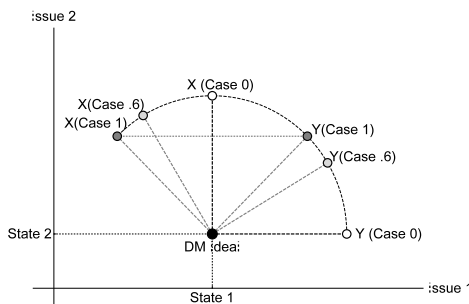


FIGURE 1. Experimental treatments.

gray points in Figure 1. This environment is simply a rotation of the Case 0 coordinate system, and the equilibrium construction can be similarly rotated. In fact, the equilibrium construction works whenever we can find a coordinate system that mirrors the alignment over issues in Case 0, reconstructing the equilibrium in the new basis. However, while the general case and Case 0 share a common structure in different coordinate systems, the equilibrium intuition viewed within the original coordinate system is quite different. The construction in Case 1 requires the DM to combine and compare recommendations from *both* experts in *both* canonical issues, while in contrast, Case 0 requires no inference across issues.

Our experiment implements the two-expert–two-issue game in the laboratory using a modified state space (a pair of circles), where our treatment variable is the experts' bias directions. Achieving full revelation in our environment is certainly challenging, as the decision-maker must select the one correct point out of 129,600 possibilities, which are equally likely *ex ante*. But in the treatment that matches Case 0, our data are qualitatively close to a fully revealing outcome, with just under a quarter of DM choices exactly coincident with the true state, and many more close by. Moreover, the component strategies for a majority of subjects in both the expert and DM roles form a mutual best response.

However, we also document conditions under which full revelation is not achieved. Rotating the experts' biases, as presented in Case 1 and the intermediate Case .6, as in Figure 1, the match to full revelation disappears. The observed failure in these cases does not seem to be driven by equilibrium selection. Subjects in experts' roles have a consistent pattern of response across all treatments. The large majority of experts can be classified as providing recommendations that add an *exaggeration* to the true state in the direction of their own self-interest. Refining the equilibrium set by restricting experts to linear exaggeration strategies, we show that the only equilibria consistent with experts' exaggeration patterns are fully revealing. Given experts' behavior, a sequentially rational DM will use strategies similar to those predicted by the fully revealing equilibrium. Yet, DM choices are only consistent with the equilibrium response in Case 0, and the amount of information extracted across treatments is stochastically ordered by the rotation—with greater extraction in Case 0 than Case .6 and in Case .6 than Case 1.

Further analysis of DMs' response to recommendations provides an explanation for the failure: subjects treat the canonical issues independently, as if responding in two unrelated single-issue problems. Where the equilibrium (and sequentially rational) response in Case .6 and Case 1 makes substantial use of across-issue information, our estimations show subjects ignore it completely. In contrast, the receiver response within issue is fairly sophisticated, where subjects show a significant reaction to experts' biases, with behavior clustered around the unidimensional best response. Subjects react strategically to the incentives, demonstrating they understand the environment and tensions within issue; their failure is in not understanding the connections between the distinct choice dimensions.

A positive description of receivers' response is as follows: In issues where the two experts' ideal points lie on opposite sides of the DM's ideal, final choices are a weighted-average of the recommendations, with greater weight placed on experts with smaller biases within the issue. In issues where the expert's ideal points lie on the same side of

the DM's, a common response is to identify the recommendation that seemed to be the least exaggerated, and then shade a fixed amount from it. In fact, the success for the fully revealing prediction in Case 0 can be explained in terms of these within-issue heuristics: all the weight on one sender and shading nothing from the minimum.

The failures in the rotated environments and success in Case 0 suggest room for policy interventions to select fully revealing outcomes through framing. Whenever it is possible for the policy-maker to control the terms/framing of the debate, inducing a Case-0-like environment might lead to gains in information transmission. Moreover, because the behavioral responses in Case 0 form a mutual best response for the experts and DM, full revelation is likely to be a robust long-run outcome.

1.1 Literature

Our paper is part of a larger experimental literature examining cheap talk and strategic information transmission. This literature starts with a number of studies examining the one-sender–one-dimension Crawford and Sobel (1982) environment.¹ Though experimental data follow the broad comparative statics over the best-case equilibrium predictions (partial revelation of information, decreasing in the sender's bias magnitude), subjects overcommunicate relative to equilibrium, with excessive truth-telling and naive response by receivers. Our paper instead studies the case with two senders within a two-dimensional environment, where a fully revealing equilibrium exists. Our focus is on how receivers combine senders' recommendations and whether fully revealing outcomes are selected.

Recent experimental papers have begun to expand from the one-sender–one-receiver setting to include additional senders and receivers.² Closest to our paper is the independent work of Lai, Lim, and Wang (2015). Their focus is the ideas in Ambrus and Takahashi (2008), who show that Battaglini-type fully revealing equilibria may not exist when the state space is bounded, as sender messages can lead to out-of-equilibrium inferences. In a baseline treatment similar to the Case 0 setting above,

¹Prominent examples are Dickhaut, McCabe, and Mukherji (1995), Blume, DeJong, Neumann, and Savin (2002), Cai and Wang (2006), Kawagoe and Takizawa (2009), Wang, Spezio, and Camerer (2010). Outside of the standard Crawford and Sobel framework, Gneezy (2005) documents a strong behavioral aversion to sending deceptive messages in a simple sender–receiver game, making comparisons to behavior in a dictator game with similar payoffs (see also Ottaviani and Squintani (2006), Kartik (2009), Kartik, Ottaviani, and Squintani (2007), for theoretical treatments). Chung and Harbaugh (2014) experimentally examine persuasive cheap talk where senders have state-independent preferences (for theory, cf. Chakraborty and Harbaugh (2010), finding broad support for the theoretic predictions (see also Lafky and Wilson (2015), who vary a conditional transfer in a related one-sender environment).

²Battaglini and Makarov (2014) experimentally examine the tensions when those providing information must cater to multiple audiences. While observing excessive honesty in some senders, they do find that when facing multiple audiences senders react strategically. An additional sender in one dimension (with some additional uncertainty on the preferences) is analyzed in Minozzi and Woon (2015), in a companion paper to the present one (Vespa and Wilson (2016)), and in Wilson (2014), while Evdokimov and Garfagnini (2014) examine communication/centralization between elements of the same firm. Plott and Lewellyn (2015) study an environment with two biased senders and two issues, where the eventual decision is voted on by a committee of five. The preferences of the committee members differ and each member votes after receiving the public advice from the informed experts.

Lai, Lim, and Wang find evidence consistent with full revelation. Their main treatments manipulate the set of available states and messages, thereby changing the possibility for senders to force out-of-equilibrium inferences (messages implying an impossible state). They find significantly lower revelation where these out-of-equilibrium inferences are possible. In contrast, our paper examines environments where out-of-equilibrium inferences cannot be forced. Our concentration is on examining whether or not the same fully revealing equilibrium is selected, and the extent to which receivers incorporate the available information as the frame shifts.

The paper is structured as follows: Section 2 outlines the underlying theory; Section 3 presents our experimental design; the analysis is carried out in Section 4; finally, Section 5 discusses the results, after which we conclude.

2. THEORY

In this section we first describe the general environment and then briefly describe the fully revealing equilibrium constructed in Battaglini (2002) when the state space is a subset of \mathbb{R}^n .³ Second, we introduce our experimental environment, which alters the state space over which the theory is constructed. Finally, we illustrate a version of the Battaglini construction, and show how the main behavioral intuition—uninformed parties take a weighted average of the informed parties' recommendations on each issue, but then add a penalizing term based on the degree of their divergence in other issues—is common to both \mathbb{R}^n and our own experimental environment.

2.1 Setup and Battaglini (2002) construction

The game has three players: two senders, X and Y , and a decision-maker/receiver Z . Nature chooses a state of the world $\theta \in \Theta$ according to a commonly known distribution G , and the realization is perfectly observed by X and Y ; however the decision-maker Z is uninformed. The state space Θ is two dimensional, where each dimension $j = 1, 2$ represents an issue over which a decision must be made. Each sender decides on a message to send to the receiver, $\mathbf{x} \in \mathcal{M}_X$, $\mathbf{y} \in \mathcal{M}_Y$. The decision-maker observes the message pair (\mathbf{x}, \mathbf{y}) and selects a final decision $\mathbf{z} \in \Theta$. Preferences for each player $i \in \{X, Y, Z\}$ are defined through a state-dependent ideal point, $\theta + \delta^i$, with preferences over the final decision \mathbf{z} represented by the utility function $u_i(\mathbf{z}, \theta; \delta^i) = -\|\theta + \delta^i - \mathbf{z}\|$, so that preferences are strictly decreasing in the Euclidean distance between the final decision \mathbf{z} and the ideal point. The receiver's ideal point is normalized to be the state, so $\delta^Z = \mathbf{0}$, while the senders' relative ideal points are defined by two common-knowledge vectors, $\delta^X, \delta^Y \in \Theta$, the *biases* for senders X and Y . The timing of the game is as follows: (i) the state of the world θ is drawn and observed by the senders; (ii) X and Y simultaneously send messages \mathbf{x} and \mathbf{y} , respectively; (iii) Z observes (\mathbf{x}, \mathbf{y}) and selects \mathbf{z} , after which payoffs are realized.

³Readers interested in the fuller development of the theoretical information transmission literature are referred to the following papers and references therein: Crawford and Sobel (1982), Gilligan and Krehbiel (1989), Krishna and Morgan (2001), Battaglini (2002), Levy and Razin (2007), Ambrus and Takahashi (2008).

Strategies for the two senders are functions $\xi_X : \Theta \rightarrow \Delta\mathcal{M}_X$ and $\xi_Y : \Theta \rightarrow \Delta\mathcal{M}_Y$, from states to probability distributions over messages, while a strategy for the receiver is a function $\zeta : \mathcal{M}_X \times \mathcal{M}_Y \rightarrow \Delta\Theta$, taking observed message pairs into a probability distribution over final decisions. The belief $\mu_Z(\theta|\mathbf{x}, \mathbf{y})$ for the decision-maker reports a posterior distribution over Θ for each pair of messages in $\mathcal{M}_X \times \mathcal{M}_Y$. A *fully revealing equilibrium* (FRE) is a perfect Bayesian equilibrium: a strategy triple $\{\xi_X^*, \xi_Y^*, \zeta^*\}$ and a conditional belief μ_Z^* updated according to Bayes' rule, which for all $\theta \in \Theta$ satisfies $\zeta^*(\xi_X^*(\theta), \xi_Y^*(\theta)) = \theta$ with probability 1.

Battaglini examines an n -dimensional version of the above model where the state space is \mathbb{R}^n and the senders' utility functions are quasi-concave at the point $\mathbf{z} = \theta$. He constructs a fully revealing equilibrium using two supporting hyperplanes with slopes γ^X and γ^Y , which strongly separate the upper-contour sets at full revelation. So for sender i , the set $\Theta_i(\theta) := \{\mathbf{z} | u_i(\mathbf{z}, \theta) \geq u_i(\theta, \theta)\}$ is separated by the hyperplane going through the point θ with slope γ^i . The condition for the existence of an FRE in this construction is that the hyperplanes γ^X and γ^Y (or lower-dimensional components, thereof) form a basis for Θ . The construction has each sender provide a message $\xi_i(\theta)$, revealing the hyperplane through the true state that separates the other sender's upper-contour set $\Theta_j(\theta)$, $j \neq i$. The message sent is therefore synonymous with sending the set-valued message

$$\xi_i(\theta) = \{\mathbf{z} | \exists \kappa \in \mathbb{R} \text{ such that } \mathbf{z} = \kappa \cdot \gamma^i + \theta\}$$

with certainty.

The final choice by the receiver is simply the unique intersecting point between the two hyperplane messages, so $\zeta^*(\mathbf{x}, \mathbf{y}) = \mathbf{x} \cap \mathbf{y}$. Given the receiver's strategy and sender j 's revelation that the state lies on the hyperplane $\xi_j(\theta)$, sender i is constrained to deviations resulting in a final choice somewhere on the hyperplane revealed by j . But, by construction, $\xi_j(\theta)$ separates the set of points that i prefers to full revelation. Revealing that the state lies on the hyperplane $\xi_i(\theta)$ is therefore a best response, leading to the final choice $\zeta^*(\xi_X(\theta), \xi_Y(\theta)) = \xi_X(\theta) \cap \xi_Y(\theta) = \theta$, and an FRE.

2.2 Exaggeration equilibria

In the Battaglini construction of the FRE, messages are hyperplanes; however, in our experimental environment, we will force senders' messages to be specific points in the state space, so that $\mathcal{M}_X = \mathcal{M}_Y = \Theta$. Given that our message space is the same as our state space, we will refer to each message as a *recommendation*. We will now present an alternative version of the Battaglini construction using this message space, which we will refer to as an exaggeration equilibrium. Though the mathematical underpinnings of our exaggeration construction are not substantially different from the geometric construction in Battaglini (2002), one advantage of our approach is in providing a framework to test broad patterns in sender response within our data, and a decomposition through which to understand the FRE prediction for receiver's response and to analyze the observed behavior.

Senders' recommendation strategies can always be decomposed to

$$\xi_X(\boldsymbol{\theta}) = \boldsymbol{\theta} + \tilde{\mathbf{x}}(\boldsymbol{\theta}),$$

$$\xi_Y(\boldsymbol{\theta}) = \boldsymbol{\theta} + \tilde{\mathbf{y}}(\boldsymbol{\theta}),$$

where $\tilde{\mathbf{x}}(\boldsymbol{\theta})$ and $\tilde{\mathbf{y}}(\boldsymbol{\theta})$ are exaggerations—random vectors added to the true state by the senders X and Y , respectively. We introduce two simple restrictions on the exaggerations senders use in equilibrium, which refine the equilibrium set substantially. Our restrictions allow for a linear family of recommendation strategies used by the senders, where our experimental data will allow us to test the empirical validity of these restrictions. By restricting on-path behavior of senders, we show that the sequentially rational response for receivers is also linear, which allows for direct econometric testing. Moreover, the best response for receivers to a generic linear strategy for the two senders has an intuitive decomposition: take a weighted average of the recommendation within the issue and then modify this point based on the divergence between the recommendations in other issues.

SENDER RESTRICTION A (State Independence). *Exaggerations $\tilde{\mathbf{x}}(\boldsymbol{\theta})$ and $\tilde{\mathbf{y}}(\boldsymbol{\theta})$ are independent of the state $\boldsymbol{\theta}$.*

SENDER RESTRICTION B (Linear Exaggerations). *The equilibrium exaggerations vary only in a unidimensional subspace.*

Our first restriction is a form of symmetry, asking for similarly chosen exaggeration components across all realized states $\boldsymbol{\theta} \in \boldsymbol{\Theta}$. The second restriction provides a linear structure for the exaggerations, and constrains our focus to equilibria where the exaggeration component acts in a specific vector direction. Exaggerations satisfying the two restrictions can be written

$$\xi_X(\boldsymbol{\theta}) = \boldsymbol{\theta} + \kappa_X \cdot \boldsymbol{\gamma}^X, \tag{1}$$

$$\xi_Y(\boldsymbol{\theta}) = \boldsymbol{\theta} + \kappa_Y \cdot \boldsymbol{\gamma}^Y, \tag{2}$$

where $\boldsymbol{\gamma}^X$ and $\boldsymbol{\gamma}^Y$ are unit length vectors indicating the direction each sender exaggerates within, and where κ_X and κ_Y are the exaggeration magnitudes—scalars drawn independently from distributions F_X and F_Y .

Let senders' exaggeration directions be given by $\boldsymbol{\Gamma} := [\boldsymbol{\gamma}^X \quad \boldsymbol{\gamma}^Y]$. Though the receiver does not observe the point $\boldsymbol{\theta}$, the information contained in the recommendation pair (\mathbf{x}, \mathbf{y}) and the knowledge that senders have exaggerated along linearly independent directions is enough to infer the state $\boldsymbol{\theta}$ with probability 1. The proposition below provides a characterization of the linear sequentially rational response when $\boldsymbol{\Gamma}$ has full rank.

PROPOSITION 1. *When senders exaggerate in linearly independent directions given by $\boldsymbol{\Gamma}$ (for any exaggeration distributions F_X, F_Y), the sequentially rational receiver response is*

$$\zeta_{\boldsymbol{\Gamma}}(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} \alpha \cdot x_1 + (1 - \alpha) \cdot y_1 + \beta_1 \cdot (y_2 - x_2) \\ (1 - \alpha) \cdot x_2 + \alpha \cdot y_2 + \beta_2 \cdot (y_1 - x_1) \end{pmatrix}$$

for scalars α , β_1 , and β_2 defined by

$$\begin{bmatrix} \alpha & -\beta_1 \\ -\beta_2 & 1 - \alpha \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \boldsymbol{\gamma}_Y \end{bmatrix} \boldsymbol{\Gamma}^{-1}.$$

See the [Appendix](#) for the proof.

The characterization—which we will use to motivate our estimation procedure for receiver’s strategies—tells us that to infer the state in any issue, the receiver linearly combines information from both senders in both dimensions. *Within* each issue j the receiver considers the identifiable difference between the two recommendations $\nabla_j(\mathbf{x}, \mathbf{y}) = y_j - x_j$. This initial point is then modified by an amount proportional to the divergence in recommendations in the other issue $k \neq j$, the difference $\nabla_k(\mathbf{x}, \mathbf{y}) = y_k - x_k$, where the degree of this modification is weighted by the parameter β_j .

The receiver response $\zeta_{\boldsymbol{\Gamma}}(\mathbf{x}, \mathbf{y})$ is defined for any full rank exaggeration basis $\boldsymbol{\Gamma}$, but in a fully revealing equilibrium we need to make sure that senders’ cannot benefit from any deviation at any realized state. This leads us to the following result.

EQUILIBRIUM RESTRICTION C. *Given linearly independent bias vectors (so $\boldsymbol{\Delta} := [\boldsymbol{\delta}^X \ \boldsymbol{\delta}^Y]$ has full rank) the only equilibria possible that satisfy Restrictions A and B with linearly independent exaggeration directions has the exaggeration basis $\boldsymbol{\Gamma}^*(\boldsymbol{\Delta}) := [\boldsymbol{\gamma}^X(\boldsymbol{\Delta}) \ \boldsymbol{\gamma}^Y(\boldsymbol{\Delta})]$, with exaggeration directions $\boldsymbol{\gamma}^X(\boldsymbol{\Delta}) \perp \boldsymbol{\delta}^Y$ and $\boldsymbol{\gamma}^Y(\boldsymbol{\Delta}) \perp \boldsymbol{\delta}^X$.⁴*

This restriction comes from the receiver’s sequentially rational response and sender optimality. The receiver’s strategy translates each sender’s recommendation into an exaggeration coordinate via the $\boldsymbol{\Gamma}^{-1}$ transformation, extracting the $\boldsymbol{\gamma}^Y$ coordinate from sender X and the $\boldsymbol{\gamma}^X$ coordinate from sender Y . If sender Y follows the exaggeration strategy in (2), and the receiver responds through $\zeta_{\boldsymbol{\Gamma}^*(\boldsymbol{\Delta})}(\mathbf{x}, \mathbf{y})$, every possible recommendation from X leads to a final choice on the $\boldsymbol{\gamma}^Y$ hyperplane through $\boldsymbol{\theta}$. What remains is to check which points on this hyperplane maximize X ’s outcome.

For any recommendation \mathbf{x} within a neighborhood of $\boldsymbol{\theta}$, if X sends a recommendation with exaggeration in the $\boldsymbol{\gamma}^X$ direction only, then the final outcome is $\boldsymbol{\theta}$, so X is indifferent over every exaggeration in the $\boldsymbol{\gamma}^X$ direction. With our assumed preferences (negative of the Euclidean distance from the ideal point), the set of final choices strictly preferred to $\boldsymbol{\theta}$ by X is given by all points within a distance of $\|\boldsymbol{\delta}^X\|$ to X ’s ideal point $\boldsymbol{\theta} + \boldsymbol{\delta}^X$. Given this upper-contour set, there is a *unique* locally supporting hyperplane at the point $\boldsymbol{\theta}$, which is orthogonal to the $\boldsymbol{\delta}^X$ direction. If the exaggeration direction $\boldsymbol{\gamma}^Y \not\perp \boldsymbol{\delta}^X$, X would be able to profitably deviate by sending the recommendation $\mathbf{x} = \boldsymbol{\theta} + \kappa_X \cdot \boldsymbol{\gamma}^X + \varepsilon \cdot \boldsymbol{\gamma}^Y$ for some $\varepsilon \in \mathbb{R}$. A similar argument for Y implies that $\boldsymbol{\gamma}^X \perp \boldsymbol{\delta}^Y$. Given the sender restrictions and that the two exaggeration directions are linearly independent, the sender optimality conditions indicate that the only FREs possible are those with the exaggeration basis $\boldsymbol{\Gamma}^*(\boldsymbol{\Delta})$.⁵

⁴In [Battaglini \(2002\)](#) the preferences are quasi-concave, so the hyperplane $\boldsymbol{\gamma}^Y$ through the point $\boldsymbol{\theta}$ strongly separates the convex upper-contour set $\{\mathbf{z} \in \boldsymbol{\Theta} \mid u_X(\mathbf{z}, \boldsymbol{\theta}) \geq u_X(\boldsymbol{\theta}, \boldsymbol{\theta})\}$.

⁵If the exaggeration directions are not linearly independent and each sender exaggerates in a common $\boldsymbol{\gamma}$ direction, the sequentially rational on-path response is $\zeta(\mathbf{x}, \mathbf{y}) = \mathbf{x} - \mathbb{E}(\kappa_X \mid \nabla(\kappa_X \cdot \boldsymbol{\gamma}, \kappa_Y \cdot \boldsymbol{\gamma}) = \nabla(\mathbf{x}, \mathbf{y})) \cdot \boldsymbol{\gamma}$.

2.3 Toroidal state space

While the FRE constructed above is over \mathbb{R}^2 with the standard topology, our experiment will instead use what is referred to as a Clifford torus—a space with a circular topology.⁶ This choice will have some theoretical advantages when testing for full revelation in a laboratory setting that we will discuss next in the design section. Moreover, as we now show, the intuition for the FRE’s mechanics is the same in both \mathbb{R}^2 and our toroidal space.

Our experiment frames the state space Θ through two separate circles, where a point on the circumference of each circle is uniformly and randomly chosen. The state space is therefore a surface in $\mathbb{R}^2 \times \mathbb{R}^2$, where the two-dimensional realized state $\theta = (\theta^1, \theta^2)$ is indicated through a location on the two circles. The points on each circle are defined by the map $\mathcal{C} : [0^\circ, 360^\circ)^2 \rightarrow \mathbb{R}^2 \times \mathbb{R}^2$ given by

$$\mathcal{C}(\theta) := \left(\begin{pmatrix} \sin \theta_1 \\ \cos \theta_1 \end{pmatrix}, \begin{pmatrix} \sin \theta_2 \\ \cos \theta_2 \end{pmatrix} \right),$$

where to mirror the experimental interface, all angular measurements will be given in degrees. Because $\mathcal{C}(\cdot)$ is a bijection, rather than work with the range (the positions on each circle’s circumference), we will instead focus our analysis on the domain of \mathcal{C} , so $\Theta = [0^\circ, 360^\circ)^2$, and the realized state θ is a two-dimensional random vector, with each component uniformly distributed between 0° and 360° .

The presentation on two circles is illustrated in the left panel of Figure 2. Senders perfectly observe the position $\mathcal{C}(\theta)$, and choose a position on the circumference of each of the two circles to represent their recommendations, $\mathcal{C}(\mathbf{x})$ and $\mathcal{C}(\mathbf{y})$ (for clarity, the

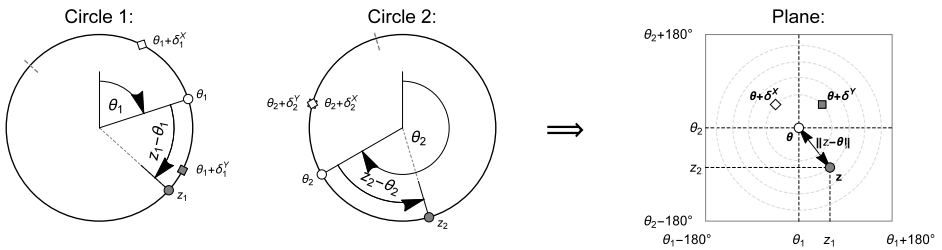


FIGURE 2. Toroidal state space: coordinate on two circles to flat plane. *Note:* The example corresponds to Case 1 in Figure 1. The realized state is indicated with θ_i for each circle $i = 1, 2$. In circle 1 senders are biased in opposite directions, so that θ_1 lies in between $\theta_1 + \delta_1^X$ and $\theta_1 + \delta_1^Y$. In circle 2 senders are biased in the same direction and with equal magnitude so that $\theta_2 + \delta_2^X$ and $\theta_2 + \delta_2^Y$ overlap. The term z_i indicates the choice of the receiver.

Full revelation without full rank for Γ requires that the supports of κ_X, κ_Y be such that $(\kappa_X | \kappa_Y - \kappa_X)$ is degenerate for all possible $\kappa_Y - \kappa_X$ realizations.

⁶We are not aware of theory papers examining cheap talk in toroidal state spaces, though Filipovich (2008) characterizes one-sender cheap talk on a single circle. However, our interest in this topology is methodological. We view this state space as a simple test tube for the FRE, one that replicates the main tensions and equilibrium form for the exaggation equilibrium outlined above.

recommendations are not shown on the figure). The receiver observes the recommendations $\mathcal{C}(\mathbf{x})$ and $\mathcal{C}(\mathbf{y})$, and then makes her own choice, the location $\mathcal{C}(\mathbf{z})$. Preferences for all agents are defined through an ideal point on each circle $\mathcal{C}(\boldsymbol{\theta} + \boldsymbol{\delta}^i)$, where each final choice \mathbf{z} given the state $\boldsymbol{\theta}$ is ranked by agent i via

$$u_i(\mathbf{z}, \boldsymbol{\theta}; \boldsymbol{\delta}^i) = -\sqrt{\phi_d(z_1, \theta_1 + \delta_1^i)^2 + \phi_d(z_2, \theta_2 + \delta_2^i)^2},$$

where $\phi_d(a, b)$ is the angular length of the shortest arc (clockwise positive, counterclockwise negative) connecting two angles a and b .⁷ So, in the figure, the receiver ranks the indicated decision $\mathcal{C}(\mathbf{z})$ via

$$u_Z(\mathbf{z}, \boldsymbol{\theta}; \mathbf{0}) = -\sqrt{(\theta_1 - z_1)^2 + (\theta_2 - z_2)^2}.$$

The game on the two circles is locally identical (for all states/choices in $[\theta_1 - 180^\circ, \theta_1 + 180^\circ] \times [\theta_2 - 180^\circ, \theta_2 + 180^\circ]$) to a model with $\boldsymbol{\theta}, \mathbf{x}, \mathbf{y}$, and \mathbf{z} in \mathbb{R}^2 , where agents' preferences are the negative of the Euclidean distance, $-\|\boldsymbol{\theta} + \boldsymbol{\delta}^i - \mathbf{z}\|$. The right panel of Figure 2 illustrates this equivalence, where the angles on the two circles are represented as points on the plane. The length of the shortest arcs on each circle are by construction less than 180° for all points in the illustrated plane, where the presentation is as if we cut the two circles at the two gray dashed lines diametrically opposite the state and folded them out to form the two axes of the plane.⁸

Where the torus differs from \mathbb{R}^2 for the multi-sender game is over the measurements of differences between any two points, in particular, identifying the difference between the recommendations \mathbf{x} and \mathbf{y} . For \mathbb{R}^2 the difference in dimension j is $\nabla_j(\mathbf{x}, \mathbf{y}) = y_j - x_j$. However for circles this difference will be an angular length for one of *two* possible arcs—either clockwise from x_j to y_j or counterclockwise from x_j to y_j —where the receiver must make an inference over which one is the correct measurement. For simplicity, and to fix ideas, we can think of the case where we always select the shortest arc connecting x_j and y_j (which would be the correct inference when the exaggerations are not large). The angular difference between the recommendation vectors \mathbf{x} and \mathbf{y} along the minor arc would therefore be given by $\nabla_j(\mathbf{x}, \mathbf{y}) = \phi_d(x_j, y_j)$.⁹ As long as the inferred vector difference $\nabla(\mathbf{x}, \mathbf{y})$ is equal to the actual exaggeration difference

⁷So more formally, for two angles $a, b \in [0^\circ, 360^\circ)$, we define $\phi_d(a, b) := \text{mod}(b - a + 180) - 180$.

⁸Our main treatments use the frame with two circles presented on the left panel of Figure 2. We also conduct robustness treatments with the frame on the right panel and results are provided in the Supplement, available as a file on the journal website, <http://qeconomics.org/supp/500/supplement.pdf>.

⁹If the exaggerations are opposed in sign on a particular issue j (for example, where $\kappa_X \gamma_j^X \leq 0 \leq \kappa_Y \gamma_j^Y$ for all $\kappa_i \in \text{supp } F_i$) and the exaggeration magnitudes were believed to satisfy $|\kappa_i \gamma_j^i| \in [0, 180^\circ)$, one could instead focus on a specific arc from the two (the clockwise arc from x_j to y_j). The inferred recommendation difference would therefore be $\nabla_j(\mathbf{x}, \mathbf{y}) = \text{mod} \in [0, 360^\circ)$ and could therefore be any measurement from 0° to 360° . For issues where the biases are in the same direction (for example, where $0 \leq \kappa_X \gamma_j^X, \kappa_Y \gamma_j^Y$) and where $|\kappa_i \gamma_j^i| \in [0, 180^\circ)$, one would still focus on the minor arc as the difference $\kappa_Y \gamma_j^Y - \kappa_X \gamma_j^X \in [-180^\circ, 180^\circ)$ always picks out the minor arc. More generally, the theory will work whenever the support of each exaggeration component $\kappa_i \cdot \gamma_j^i$ is assumed to be no wider than 180° , so that the support of the observed differences is within the 360° resolution of the circle.

$\xi_Y(\boldsymbol{\theta}) - \xi_X(\boldsymbol{\theta}) = \kappa_Y \cdot \boldsymbol{\gamma}^Y - \kappa_X \cdot \boldsymbol{\gamma}^X$, then the sequentially rational response of the receiver is the similar to that in Proposition 1:

$$\boldsymbol{\zeta}(\mathbf{x}, \mathbf{y}; \alpha, \beta_1, \beta_2) = \begin{pmatrix} \mathbf{x} + (1 - \alpha) \cdot \nabla_1(\mathbf{x}, \mathbf{y}) + \beta_1 \cdot \nabla_2(\mathbf{x}, \mathbf{y}) \\ \mathbf{x} + \alpha \cdot \nabla_2(\mathbf{x}, \mathbf{y}) + \beta_2 \cdot \nabla_1(\mathbf{x}, \mathbf{y}) \end{pmatrix}$$

for the same parameters α , β_1 , and β_2 as functions of Γ .

Following the intuition for the FRE in \mathbb{R}^2 , if the receiver uses a linear rule, it must be that senders cannot make a small deviation from their linear exaggeration strategy to improve their expected outcome. Because the toroidal environment's preferences are identical to those in \mathbb{R}^2 in a neighborhood of the fully revealing decision, $\mathbf{z} = \boldsymbol{\theta}$, it must be that any FRE with linearly independent exaggeration directions satisfies Equilibrium Restriction C. However, beyond this local condition, we must also check a global condition that large enough exaggerations cannot force the receiver to make an incorrect inference over the difference $\nabla(\mathbf{x}, \mathbf{y})$, and through this derive a benefit to the sender. A sufficient condition for the existence of an equilibrium satisfying this global condition is given in the following proposition.

PROPOSITION 2. *If the biases for each sender $i \in \{X, Y\}$ satisfy $\|\boldsymbol{\delta}^i\| \leq \sqrt{5} \cdot 45^\circ$, an FRE exists on the toroidal state space satisfying Sender Restrictions A and B.*

See the Supplement for the proof.

The sequentially rational receiver response in the FRE is exactly that characterized above for \mathbb{R}^2 , where the parameter weights are given by

$$\begin{bmatrix} \alpha^* & -\beta_1^* \\ -\beta_2^* & 1 - \alpha^* \end{bmatrix} := [\mathbf{0} \quad \boldsymbol{\gamma}^Y(\boldsymbol{\Delta})] \boldsymbol{\Gamma}^*(\boldsymbol{\Delta})^{-1},$$

with the only added restriction being that the biases are not too large in absolute size.

The intuition for the receiver's response on the two circles is given in Figure 3. Senders X and Y have realized recommendations \mathbf{x} and \mathbf{y} , illustrated as white diamonds and gray squares. For circle 1, the inferred difference is the length of the clockwise arc between x_1 and y_1 , while for circle 2, it is the clockwise arc from x_2 to y_2 . The initial within-dimension response is to choose the angles illustrated by the gray circles, the weighted average of the sender's recommendations: $x_1 + (1 - \alpha) \cdot \nabla_1(\mathbf{x}, \mathbf{y})$ and $x_2 + \alpha \cdot \nabla_2(\mathbf{x}, \mathbf{y})$. These initial points are then modified using the across-issue differences. Assuming that both β_1 and β_2 are negative—so that both $\beta_j \cdot \nabla_k(\mathbf{x}, \mathbf{y})$ terms are counterclockwise—the diagram illustrates the modification on each circle based on the across-issue difference $\nabla_k(\mathbf{x}, \mathbf{y})$.

3. EXPERIMENTAL DESIGN

3.1 Experiment frame

State space Our multidimensional experiment will use the toroidal environment to examine whether or not the Battaglini FRE is selected. As noted above, the strategic tensions and FRE in both the torus and \mathbb{R}^2 are very similar. Why then have we chosen to

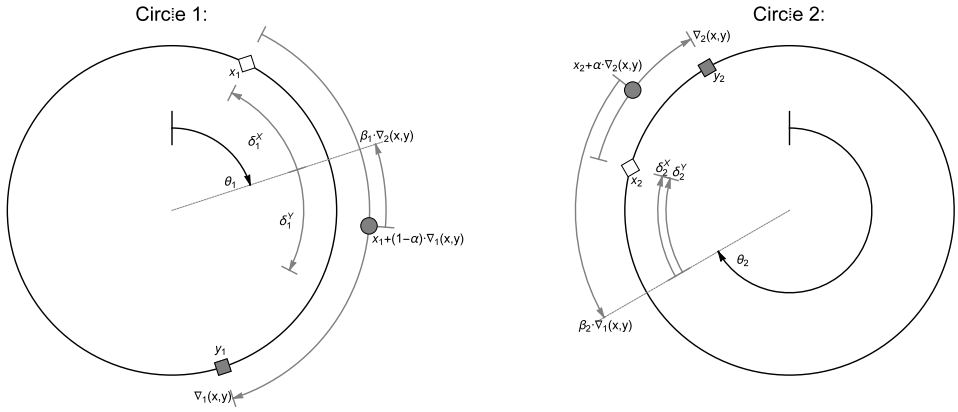


FIGURE 3. Example inference on the circles. *Note:* The example corresponds to Case 1 in Figure 1. The realized state and messages are indicated with θ_i , x_i , and y_i for each circle $i = 1, 2$, respectively. The within-dimension response is illustrated with gray circles. For each dimension the figure also illustrates the adjustment using across-issue differences.

use the torus? Our reasons for the choice are both methodological and theoretical. To select a random state θ in the laboratory we need to inform subjects about the distribution it is drawn from. If we did not inform them, subjects may think that there is something meaningful to learn about the distribution, and this could add unnecessary noise to our data. The feasibility of full revelation could be tested over \mathbb{R}^2 if we used a distribution with full support, such as a bivariate normal. While the equilibrium construction does not depend on the specific distribution G , it is not unreasonable to expect that nonuniform distributions might have an effect on subjects' behavior. Specifically, we worry about tensions between how likely a state was *ex ante* and what the advice they receive from senders indicates *ex post*.

Using a uniform distribution for the state introduces a new challenge. A uniformly distributed state requires a compact support, such as the unit square, $[0, 1] \times [0, 1] \subset \mathbb{R}^2$. However, economic theory (see [Ambrus and Takahashi \(2008\)](#), for more details) shows that FRE may not exist in such state spaces. Specific pairs of recommendations can lead to out-of-equilibrium inferences, indicating that the state implied by the two recommendations is outside of the state space—geometrically, one sender reveals a line going through the state and the deviating sender reveals a line with an intersecting point outside the unit square. Whenever this happens, consistent receivers will conclude they are out of equilibrium and take some alternative action. Depending on the off-the-path actions, senders might benefit in certain states from unilaterally forcing the out-of-equilibrium inference. We want to give the FRE its best chance, and our topological shift removes this possibility. Because of the circularity of our environment, recommendation pairs leading to an inference outside of the state space, say $(370^\circ, -30^\circ)'$, will within the torus indicate a point to the receiver that is still within it and on path; here $(10^\circ, 340^\circ)'$.

Laboratory environment The realized angles for the state and all recommendations/final decisions are represented graphically to subjects as compass headings on two cir-

cles, where 360° is the origin (*North*) on each circles. We discretize the circles, so that the state realization θ in our experiments is a vector of two independently distributed integers θ_1 and θ_2 , drawn uniformly and independently from $\Theta_1 = \Theta_2 = \{1^\circ, 2^\circ, \dots, 360^\circ\}$ at the start of each round. In each of the first 15 rounds of the experimental sessions, three subjects are randomly and anonymously matched together in a group. Two of the matched subjects take on the role of senders, X and Y , though within the experiment we use the neutral labels *blue* and *red* player (with the receiver called *green*).¹⁰

Each sender perfectly observes the state of the world θ , represented as a green point on the edge of each circle. The senders then choose recommendations on each circle, angular vectors \mathbf{x} and \mathbf{y} , which are sent to the receiver. Given the recommendations, the receiver chooses a point on each circle for the group—the angular vector \mathbf{z} , which determines the round payoffs. Preferences are induced by providing each of the three roles with an ideal angular location in Θ . For sender X this is parametrized by an angular bias δ^X , so that X has an ideal point δ_1^X clockwise from θ_1 on issue 1 and δ_2^X clockwise from θ_2 on issue 2. Similarly, sender Y has an angular bias δ^Y , inducing the ideal point $\theta + \delta^Y$. Final payments for all subjects/roles were derived using the angular distance of the chosen point \mathbf{z} from the subject's ideal point. The exact monetary payment from a session is given by the sum of *two* randomly selected rounds, where the payment for the selected round is given by

$$\max \left\{ \$5, \$20 - \$8 \frac{\sqrt{(\text{degrees from ideal}_1)^2 + (\text{degrees from ideal}_2)^2}}{45^\circ} \right\}.$$

That is, all subjects in all roles receive \$20 if the chosen point \mathbf{z} exactly matches their ideal location. Otherwise, the minimal angular distance from the ideal point (in either the clockwise or counterclockwise directions) in each circle is used to construct a cost. For every 45° distance from their ideal point, subjects lose \$8, until a \$5 floor is reached.¹¹ Subjects are given both the above payoff formula and a table indicating their payoff for every 15° difference from their optimal point in each issue, while the interface provides additional calculation aids.

To choose recommendations, sender subjects simply click within each of the two circles (labeled circle A and circle B in the experiment) and their choice is rendered as a radial angle from the circle's center. Each sender's screen graphically illustrates his/her own most preferred point (blue/red radial angles), that of the other sender (red/blue angles), and the receiver (θ , indicated by green points on the circles' circumference). While senders make their decisions, receivers view a screen graphically illustrating the relative location of the best points (the bias vectors δ^X and δ^Y). After each sender has confirmed

¹⁰The instruction script read to subjects and slide shows shown to them are available in the Technical Supplement file on the journal website, http://qeconomics.org/supp/500/code_and_data.zip, while screen shots of the interface are provided in Supplement Section D. Additionally, interested readers can download the z -tree code (see Fischbacher (2007)) for both the session and the interactive instructions subjects followed alongside the instructions on their own screens.

¹¹The flatness in the preferences far from the true state does not affect the theoretical existence result, which requires senders to have a convex upper-contour set at the fully revealing receiver decision $\mathbf{z} = \theta$ and that the biases are not too large.

his/her chosen recommendation, the receiver final-decision stage begins. Receivers are provided with the two recommendations (\mathbf{x} and \mathbf{y} , illustrated as blue and red radial angles or purple if coincident) alongside the senders' relative ideal locations (δ^X and δ^Y). The receiver chooses the group action $\mathbf{z} = (z_1, z_2)'$ by clicking on the circular interface.

To review, group $g = \{i_X, i_Y, i_Z\}$ in round t is randomly formed from the session participants and the state θ_{gt} is uniformly drawn from Θ . The state is perfectly observed by the two senders, X and Y , who chose recommendations \mathbf{x}_{gt} and \mathbf{y}_{gt} within Θ in the *message selection* stage. The message pair $(\mathbf{x}_{gt}, \mathbf{y}_{gt})$ is observed by the receiver, who makes the group choice $\mathbf{z}_{gt} \in \Theta$ in the *choice selection* stage. Finally, at the end of the round, subjects are given the feedback $(\theta_{gt}, \mathbf{x}_{gt}, \mathbf{y}_{gt}, \mathbf{z}_{gt})$, along with the round payoffs (π_X, π_Y, π_Z) .

Each experimental session was divided into two parts. In the first part, subjects played 5 consecutive rounds in each role (X , Y , and Z) for a total of 15 rounds. In the second part, every subject takes on the role of the receiver Z , and is given the recommendations from groups in rounds 11–15 that they were not a member of. The second part has each subject acting as a receiver in rounds 16–20, removing potential other-regarding payment concerns, as the players who sent those recommendations are not compensated for outcomes in the second part. Final payment in the experiment was made on two randomly selected rounds from the entire session, so the maximum payment was \$40 and the minimum was \$10.

3.2 Treatments

We utilize a *between-subject* design with the treatment variable $\Delta = [\delta^X \ \delta^Y]$ fixed in each session, matching Figure 1 in the [Introduction](#). The experimental treatments are more precisely summarized here in Table 1.¹² The goal of the design is to diagnose whether fully revealing equilibria are selected when theory indicates their existence, and to examine the robustness of this selection to a simple coordinate-system transformation. This paper will focus on three treatments for Δ where the biases are orthogonal and have symmetric magnitudes—so Δ always has full rank and $\|\delta^X\| = \|\delta^Y\|$ —where an FRE with our exaggeration restrictions exists. Up to rounding error, the three treatments are simple rotations of the same coordinate system. The FRE represents the upper bound for receivers; however, in all three treatments another equilibrium exists—the uninformative babbling outcome. Babbling leads to the lower-bound outcome for receivers, attaining their individually rational payoff.¹³ A companion paper ([Vespa and Wilson \(2016\)](#)) provides details on behavior in sessions where Δ does not have full rank (and Battaglini-type equilibria do not exist), complementing the present work.

We refer to each of our *revelation* treatments by the label $R(\tan \psi)$, where ψ is a counterclockwise rotation of the coordinate system and $\tan \psi$ indicates the ratio of sender's

¹²However, we do experimentally vary the positive direction for biases (clockwise or counterclockwise) and dimensions (circle a as issue b), while making sure to keep the relative orientations of the senders' biases fixed.

¹³Within the toroidal state, space babbling equilibria can satisfy Sender Requirement A (uniform exaggerations from 1 to 360 across both issues); however, they necessarily violate Sender Requirement B, which guarantees revelation within at least a unidimensional subspace of Θ .

TABLE 1. Treatments.

Treatment	Biases		Within α^*	Across β^*	Payoff Babble/FRE	
	δ^X	δ^Y			Senders	Receiver
R(0), P(0)	$\begin{pmatrix} 0^\circ \\ 60^\circ \end{pmatrix}$	$\begin{pmatrix} 60^\circ \\ 0^\circ \end{pmatrix}$	1	0	\$5.86/\$9.33	\$5.86/\$20.00
R(.6)	$\begin{pmatrix} -30^\circ \\ 50^\circ \end{pmatrix}$	$\begin{pmatrix} 50^\circ \\ 30^\circ \end{pmatrix}$	$\frac{25}{34}$	$-\frac{15}{34}$	\$5.86/\$9.63	\$5.86/\$20.00
R(1), P(1), E(1)	$\begin{pmatrix} -45^\circ \\ 45^\circ \end{pmatrix}$	$\begin{pmatrix} 45^\circ \\ 45^\circ \end{pmatrix}$	$\frac{1}{2}$	$-\frac{1}{2}$	\$5.86/\$8.68	\$5.86/\$20.00

bias magnitudes on each issue. In each treatment the bias vector magnitude for each sender is approximately 60° , with small differences across the rotations to obtain round numbers for the experiment. Senders' bias directions are summarized by the rotation angle ψ : sender X has a bias in the direction $(-\sin \psi, \cos \psi)'$ and sender Y has a bias in the direction $(\cos \psi, \sin \psi)'$. Because the bias directions are orthogonal, senders' equilibrium exaggerations are predicted to be in the directions of their own biases, so $\Gamma^*(\Delta) = \Delta$, and the receiver's equilibrium response is

$$\zeta^*(\mathbf{x}, \mathbf{y}) = \mathbf{x} + \begin{bmatrix} 1 - \alpha^* & \beta^* \\ \beta^* & \alpha^* \end{bmatrix} (\mathbf{y} - \mathbf{x}), \quad (3)$$

where the across-issue modifiers are $\beta_1^* = \beta_2^* = -\frac{1}{2} \sin 2\psi =: \beta^*$ and the within-issue weight is $\alpha^* = \cos^2 \psi \equiv \frac{1}{2} + \frac{1}{2} \cos 2\psi$.¹⁴

Table 1 indicates the FRE parameters— α^* and β^* , the within- and across-issue components in the equilibrium response $\zeta^*(\mathbf{x}, \mathbf{y})$ —as well as the predicted payoffs to the senders and receiver under babbling/full revelation. The three R(.) treatments are the following.

R(0). The senders are each aligned with the receiver on a particular issue, but misaligned by 60° in the other. Within issue, the equilibrium weight parameter is given by $\alpha^* = 1$ and there is no across-issue modification (so $\beta^* = 0$). The clear asymmetry in bias magnitudes within issue fully resolves the problem, and the receiver response is the intuitive $\zeta^*(\mathbf{x}, \mathbf{y}) = (x_1, y_2)'$. The response $\zeta^*(\mathbf{x}, \mathbf{y})$ is to compose the two sources, following the recommendation of the unbiased sender in each circular issue. The receiver puts full weight on the unbiased sender within the issue and ignores across-issue information.

R(1). The fully rotated treatment has biases symmetrically opposed on one issue (-45° and $+45^\circ$) and symmetrically aligned on the other (both $+45^\circ$; so $\tan \psi = \frac{45}{45} = 1$), and vector magnitude 63.6° . Ex ante the two senders are symmetrically biased within each issue. In issue 1 they are diametrically opposed; in issue 2 they are perfectly aligned. This symmetry has to be broken using the *realized* recommendations, \mathbf{x} and \mathbf{y} . This treatment's equilibrium has a within-issue simple averaging response with $\alpha^* = \frac{1}{2}$, but

¹⁴Henceforth, we will for simplicity use $\mathbf{y} - \mathbf{x}$ to refer to the angular difference $\nabla(\mathbf{x}, \mathbf{y})$.

the across-issue modifier reaches an extreme, with $\beta^* = -\frac{1}{2}$. Because the two senders have an identical bias in issue 2, the sign and magnitude of the observable difference ($y_2 - x_2$) is informative on which of the two senders provided the larger exaggeration, and by how much. Starting from the recommendation midpoint in issue 1, the equilibrium response $\zeta_1^*(\mathbf{x}, \mathbf{y}) = \frac{1}{2}x_1 + \frac{1}{2}y_1 - \frac{1}{2} \cdot (y_2 - x_2)$ breaks symmetry by making a final choice in issue 1 closer to the sender who has exaggerated less in issue 2. A similar intuition matches the equilibrium response in issue 2, where the across-issue modification is given by $\frac{1}{2}(y_1 - x_1)$. The recommendation difference in issue 1 is informative about the total exaggeration magnitude. So the equilibrium response $\zeta_2^*(\mathbf{x}, \mathbf{y}) = \frac{1}{2}x_2 + \frac{1}{2}y_2 - \frac{1}{2} \cdot (y_1 - x_1)$ is equivalent to removing the inferred average exaggeration from the average issue 2 recommendation—the larger the separation in issue 1 recommendations, the greater the receiver's shading from the issue 2 midpoint.

R(6). The partially rotated treatment has senders with opposed biases of -30° and $+50^\circ$ in one issue, while in the other issue the senders have same-signed biases of $+50^\circ$ and $+30^\circ$, respectively (so $\tan \psi = \frac{30}{50} = 0.6$). The vector magnitudes of the senders' biases are the same—here 58.3° —but in each specific issue, one sender has a smaller bias. The receiver response $\zeta^*(\mathbf{x}, \mathbf{y})$ is therefore in between that for R(0) and R(1). There is some initial asymmetry within each issue, which shows up as the receiver placing greater weight on the less biased sender within issue, with $\alpha^* = \frac{25}{34} \approx 0.74$. Across issue there is still a lot to learn from the realized differences, and $\beta^* = -\frac{15}{34} \approx -0.44$.

The design examines the simplest permutations of the special case R(0), rotations of the coordinate system. Given the experimental failures we will document in the simplest transformations (rotations), we see little reason to favor the FRE in more general transformations (rotations and shears).

In addition to these three treatments, in the Supplement we describe results from three further treatments conducted to examine robustness. These extension treatments are strategically identical to either the R(0) or R(1) environments. Two treatments modify the graphical interface to examine presentation effects. Instead of subjects making separate decisions on each circular issue, subjects in these treatments make choices jointly, as if locating a point on the plane depicted in Figure 2. The final robustness treatment extends the second part of the experiment to provide subjects with further experience in the receiver role. For the first 15 rounds the treatment is identical to R(1); however, after round 15, all subjects face a sequence of 15 rounds as a receiver instead of 5. In these final rounds the state and recommendation data come from previous R(1) sessions. We refer to the *extended* second-part treatment as E(1). Instructions and additional details for these extensions are also included in the Supplement, where the results indicate no substantial deviations from the R(·) treatments.

4. RESULTS

In this section we present results using data from sessions conducted at NYU's Center for Experimental Social Science with a total of 249 subjects recruited from the student population. The section is organized as follows: After briefly summarizing the main

economic findings, Section 4.1 describes the response by senders. Section 4.2 then describes the receiver choices and provides our main empirical findings. This is followed by Section 4.3, which provides a positive description of the receiver's behavior in this environment.

Before we document the observed behavior in the experiment in greater detail, we first outline the main *economic* findings. We present final outcomes through an efficiency measure that captures the distance between the receiver's final choice and the true state:

$$Y = \frac{\text{babbling distance} - \text{observed distance}}{\text{babbling distance} - \text{fully revealing distance}}.$$

Values close to 100 percent reflect full revelation, with zero observed distance from the true state, while values close to 0 percent reflect no information transfer over the prior, the babbling outcome.

In the baseline, R(0), we find very high efficiency levels. Looking at the last five rounds, the average efficiency is 77 percent, and 58 of the 240 observations are at the upper boundary, with receivers making choices with zero distance from the true state—exactly choosing the true state from the 129,600 possible locations. In contrast, despite minimal *strategic* differences as we rotate the coordinate system, the efficiency drops to 56 percent in R(.6) and 39 percent in R(1), while just a single round across the 665 observations in these two treatments is coincident with the true state.

What makes the R(0) treatment special? Why does the efficiency drop as we rotate the coordinate system? We will show that across treatments, subjects' behavior in the sender roles is qualitatively close to the fully revealing exaggeration strategies presented in the theory section—so much so that the receiver empirical best response is not starkly different from the equilibrium response. There are certainly deviations from the sender restrictions, and behavior becomes noisier and more heterogeneous as the coordinate system is rotated. However, noise in the senders' response accounts for at most half of the observed efficiency losses. We will show that most of the efficiency losses in the rotated treatments can be directly ascribed to a failure in sequential rationality by receivers. Though receivers do well at incorporating within-issue information, they systematically fail to incorporate across-issue information, approaching each issue in isolation. Due to this failure, receiver response in our rotated treatments is not consistent with any perfect Bayesian equilibrium. The goal of the results section will be to document and explain these findings.

Summary statistics for each of the R(·) treatments are provided within Table 2. Results are split into four parts: (i) the recommendation exaggerations $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ added to the true state by the senders X and Y , respectively, as coordinates on the plane; (ii) the final choice error $\tilde{\mathbf{z}} = \mathbf{z} - \boldsymbol{\theta}$ made by the receiver Z as a coordinate on the plane, again with units of angular degrees; (iii) the round profits for each of the three subjects (π_X , π_Y , and π_Z) given in dollars; and (iv) the final efficiency Y , as well as the fraction of subject rounds with perfect revelation, where $Y = 1$.¹⁵ For each variable, in each treatment, the

¹⁵All angular differences reported are the shortest arcs between the two points, so all measurements are clockwise differences in $[-180^\circ, +180^\circ)$.

TABLE 2. Summary statistics.

	R(0), 4 Sessions				R(.6), 4 Sessions				R(1), 3 Sessions [†]			
	Avg.	σ^B	σ^W	N	Avg.	σ^B	σ^W	N	Avg.	σ^B	σ^W	N
<i>Exaggerations</i>												
$\bar{\mathbf{x}} = \mathbf{x} - \boldsymbol{\theta}$	$\begin{pmatrix} 0.2 \\ 47.8 \end{pmatrix}$	$\begin{pmatrix} 15.6 \\ 30.8 \end{pmatrix}$	$\begin{pmatrix} 24.5 \\ 44.7 \end{pmatrix}$	300	$\begin{pmatrix} -29.1 \\ 49.8 \end{pmatrix}$	$\begin{pmatrix} 23.9 \\ 31.5 \end{pmatrix}$	$\begin{pmatrix} 31.7 \\ 29.6 \end{pmatrix}$	300	$\begin{pmatrix} -61.1 \\ 53.1 \end{pmatrix}$	$\begin{pmatrix} 38.1 \\ 34.7 \end{pmatrix}$	$\begin{pmatrix} 44.8 \\ 34.1 \end{pmatrix}$	345
$\tilde{\mathbf{y}} = \mathbf{y} - \boldsymbol{\theta}$	$\begin{pmatrix} 50.1 \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 32.6 \\ 11.7 \end{pmatrix}$	$\begin{pmatrix} 40.7 \\ 19.7 \end{pmatrix}$	300	$\begin{pmatrix} 50.3 \\ 28.9 \end{pmatrix}$	$\begin{pmatrix} 30.5 \\ 16.6 \end{pmatrix}$	$\begin{pmatrix} 33.1 \\ 28.1 \end{pmatrix}$	300	$\begin{pmatrix} 62.0 \\ 52.1 \end{pmatrix}$	$\begin{pmatrix} 36.0 \\ 30.0 \end{pmatrix}$	$\begin{pmatrix} 43.1 \\ 34.9 \end{pmatrix}$	345
<i>Choice error</i>												
$\bar{\mathbf{z}} = \mathbf{z} - \boldsymbol{\theta}$	$\begin{pmatrix} 9.5 \\ 9.7 \end{pmatrix}$	$\begin{pmatrix} 17.0 \\ 13.5 \end{pmatrix}$	$\begin{pmatrix} 35.7 \\ 34.5 \end{pmatrix}$	540	$\begin{pmatrix} 2.9 \\ 25.9 \end{pmatrix}$	$\begin{pmatrix} 12.9 \\ 22.3 \end{pmatrix}$	$\begin{pmatrix} 51.0 \\ 42.4 \end{pmatrix}$	540	$\begin{pmatrix} 2.5 \\ 18.7 \end{pmatrix}$	$\begin{pmatrix} 23.7 \\ 25.1 \end{pmatrix}$	$\begin{pmatrix} 78.0 \\ 55.0 \end{pmatrix}$	450
<i>Round payoffs</i>												
π_X	\$9.31	\$1.57	\$3.07	300	\$9.56	\$2.50	\$4.00	300	\$8.08	\$2.03	\$3.72	345
π_Y	\$9.24	\$1.36	\$3.09	300	\$9.23	\$2.08	\$3.87	300	\$8.11	\$2.19	\$3.70	345
π_Z	\$15.19	\$2.83	\$4.92	540	\$11.12	\$2.30	\$4.09	540	\$9.28	\$1.99	\$4.22	450
<i>Efficiency</i>												
Y	76.7%	14.8%	28.5%	540	57.3%	15.8%	28.2%	540	40.4%	18.6%	37.0%	450
$Y = 1$	23.0%	20.1%	39.4%	540	0.2%	1.3%	4.3%	540	0.0%	–	–	450

Note: Results are from fixed-effect panel estimates across subjects and session periods, with the relevant variables regressed on a constant term. The columns headed σ^B provide the standard deviation *between* subjects' average levels; columns headed σ^W provide the standard deviation *within* each subject. The results for one R(0) session and one R(.6) session only have data for the first period from periods 16–20 due to a coding error; for this reason we conducted an additional session for each of these two treatments. The total number of unique subjects in each session is 15. [†] Sender data for R(1) includes data from 24 subjects in the first part of two E(1) sessions.

table provides the average, as well as the within- and between-subject standard deviations obtained through a fixed-effects panel estimate, and the number of subject-round choices, N .

4.1 Senders and exaggeration strategies

Across our treatments the average exaggeration is close to the sender's bias, with the interpretation that recommendations are centered around the sender's ideal points. For instance, the average X subject in treatment R(0) sends a nearly unexaggerated recommendation in the unbiased issue 1, but a recommendation that adds 47.8° to the true state in issue 2, where their induced best point adds 60° to the true state. However, the table also indicates substantial variation both within and between subjects. We show that this variation is primarily in the directions predicted in the theory section. To illustrate this, Figure 4 displays all the exaggeration data in each treatment. The figure provides a scatter for the exaggerations \tilde{x} and \tilde{y} sent in every subject round in each treatment. Each point represents an exaggeration added to the true state—gray points for X , white for Y —where the area of each plotted point is proportional to the mass of observations at each exaggeration vector.¹⁶ Across the 129,600 possible realizations, there is substantial variation, but there are also clear patterns.

Exaggerations are clustered around the bias direction rays—qualitatively matching not only the linear restrictions for sender on-path behavior, but also matching the equilibrium direction. However, even though there are strong patterns conforming to these restrictions, there are also differences, noticeably so in the R(.6) and R(1) treatments. Though these differences will lead to efficiency losses, they do not substantially alter the receiver's optimal response.

Exaggeration strategies In the theory section we specified three restrictions on behavior that we now examine. The first, Sender Restriction A, asked that the exaggeration components \tilde{x} and \tilde{y} be independent of the realized state θ . To explore this restriction we take each subject-round recommendation (with subscript it) as the unit of observation and compute the correlation between the exaggerations, \tilde{x}_{ait} and \tilde{y}_{ait} , and state θ_{bit} across issues $a, b \in \{1, 2\}$.¹⁷ Computing the eight correlations in each treatment (four for each sender) we find that the highest absolute value for a correlation in any treatment is 0.036, and no relationships are significant. We conclude that there is no evidence for dependence between exaggerations and the state.

Sender Restriction B and Equilibrium Restriction C require that agents' exaggerations are linear and in the direction of the biases, respectively. Hence, one simple measure for B and C jointly is the fraction of exaggerations in the bias direction. In R(0), 69 percent of the exaggerations sent are *exactly* coincident with the equilibrium exaggeration direction, while the figures for R(.6) and R(1) are much smaller, at 13 percent and 10

¹⁶For R(1), we pool data from the 24 subjects who make choices as senders within the first 15 rounds of E(1). The first 15 rounds in E(1) have identical treatment to R(1), and the treatments differ only in the length of the second part.

¹⁷Because the random variable we calculate correlations for is defined on a circle, we use the circular correlation measure of Fisher and Lee (1983).

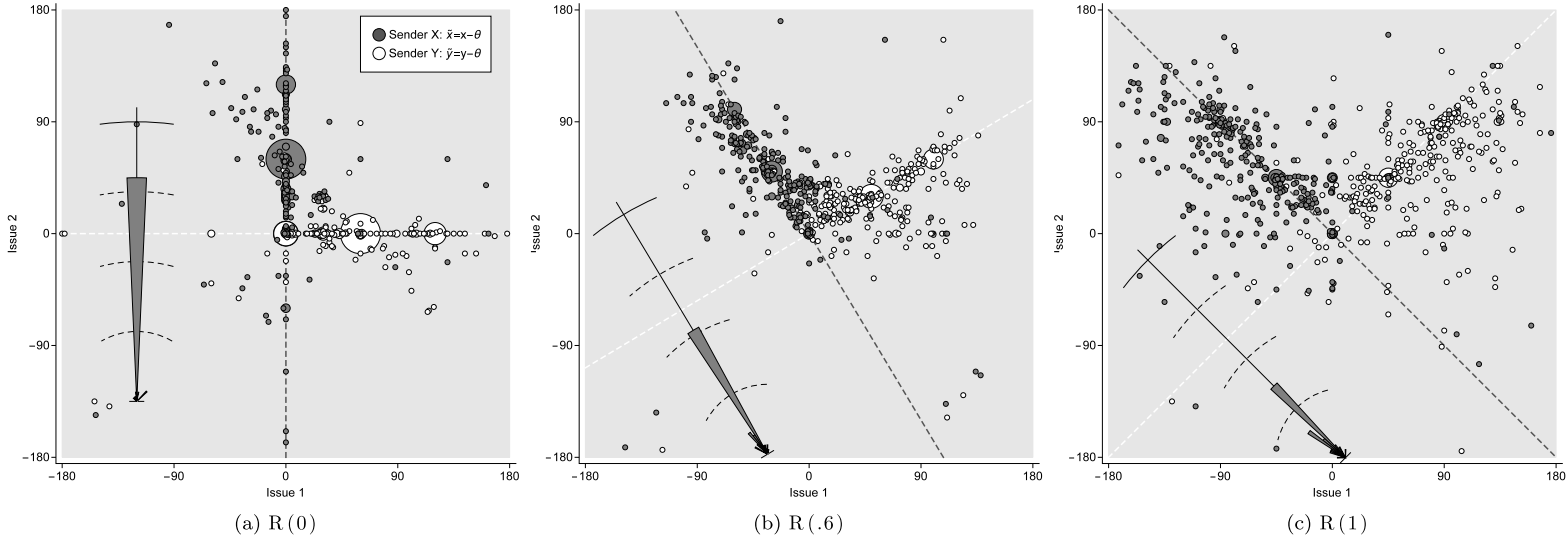


FIGURE 4. Sent exaggerations. *Note:* In figures (a)–(c), each circular point represents the difference between the message/recommendation sent by one of the sender subjects and the true state for periods 1–15. Dark gray points represent the message exaggerations \bar{x} sent by sender X , while white points represent the message exaggerations \bar{y} sent by sender Y . The dotted lines represent the Battaglini (2002) straightforward equilibrium strategies. Angular histograms within each scatter plot indicate subject-level exaggeration directions $\omega + \psi$ for sender X , binned into $\frac{\pi}{36}$ radian cones, with the scale aligned to the rotation direction ψ . Note that panel (c) contains sender exaggeration data pooled across $R(1)$ and $E(1)$.

percent, respectively. Allowing for some noise (a 10° band either side of the bias directions) these figures jump up to 82 percent adherence in R(0), 59 percent in R(.6), and 49 percent in R(1). While there is clearly a qualitative match with the assumptions, we also observe deviations that increase as we rotate the coordinate system. In the Supplement we provide a detailed analysis of Restrictions B and C, but here we provide only a brief summary of the main results:

(i) Sender Restriction A. There is no evidence for dependence between exaggerations and the realized state.

(ii) Sender Restriction B. Deviations from *linear exaggeration* account for approximately half of the observed efficiency losses, where the magnitude of these deviations increases as the coordinate system is rotated. However, the gains to receivers from relaxing Restriction B and responding with a nonlinear best response are small while the additional complexity is very large. We therefore treat deviations from the restriction as white noise.

(iii) Equilibrium Restriction C. The modal response for senders across treatments is to exaggerate in the direction of their bias. At the aggregate level, exaggeration directions are not significantly different from the bias direction in R(0) and R(.6), and though there is a significant difference in R(1), it is quantitatively small.

In the next section we analyze subjects' behavior as receivers. Given the sender results, our approach will be to examine sequential rationality under the assumption that Restrictions A and B hold, and then examine receivers' response in comparison to the Proposition 1 prediction.

4.2 Receiver behavior and full revelation

As described, outside of the R(0) case, the full-revelation equilibrium is not validated by the data from our experiments. Two strategically similar rotated treatments, R(.6) and R(1), have substantially lower information transmission. Below, we first characterize receiver outcomes in more detail across the three treatments. We then document a failure in sequential rationality in R(.6) and R(1), and outline why this failure occurs: that subjects do not use across-issue information. Given this failure, Section 4.3 then goes on to outline a descriptive model of how receivers process conflicting advice.

4.2.1 Final outcomes To measure the information extracted by receivers we use the distance of the receiver's final choice from the true state, $\|\tilde{\mathbf{z}}_{it}\|$. We focus on the last five rounds of the experimental sessions, rounds 16–20, because (i) receivers have had experience with all roles in the environment (10 rounds as senders and 5 rounds as a receiver); (ii) the last 5 rounds use sender/state data from rounds 11–15, but are paid as decision problems, isolating potential other-regarding confounds; and (iii) all subjects play the receiver role in the last 5 rounds, so we have a large number of receiver–subject

observations at a consistent point in the experimental session.¹⁸ Given the set of subjects in each treatment, \mathcal{I} , we construct the subject-level efficiency for each $i \in \mathcal{I}$:

$$Y_i = 1 - \frac{\frac{1}{5} \sum_{t=16}^{20} \|\tilde{\mathbf{z}}_{it}\|}{\mathbb{E}\|\boldsymbol{\theta}\|}.$$

Empirical cumulative distribution functions (CDFs) for Y_i over the set of subjects \mathcal{I} are presented for each R(·) treatment in Figure 5(a). Inspecting the figure, we see that not only is there a significant response in the mean efficiency (the circle labeling the CDF with the treatment label located at the average efficiency with the 95 percent confidence interval illustrated), but there is a clear first-order stochastic relationship between treatments:

$$\text{full revelation} \succ_1^{***} Y(\text{R}(0)) \succ_1^{**} Y(\text{R}(.6)) \succ_1^{**} Y(\text{R}(1)) \succ_1^{***} \text{babbling}.$$

Though none of our treatments attains the (degenerate) full-revelation upper bound, it is also clear that we have more information transmission than the lower bound, babbling.¹⁹ In addition to stochastic dominance, a one-sided test for a babbling null (by subject instead of jointly over the distribution) can be inferred from the figure. The babbling distribution's 95th percentile is indicated, and it is clear that we can reject babbling at this level for every subject in R(0). Similarly we can reject babbling for 93 percent of subjects in R(.6) and for 67 percent in R(1).

As we outlined in our discussion of sender response, deviations from Sender Restriction B increase as the coordinate system is rotated, and the additional noise could be the primary driver for the observed efficiency losses. To show this is not the case, we construct a second efficiency measure, which controls for losses due to noise in the observed recommendations, which we call *receiver efficiency*. Receiver efficiency measures the distance between the observed receiver decision \mathbf{z}_{it} and the Proposition 1 sequentially rational response. Given the best-fit exaggeration basis $\hat{\Gamma}$, we compute the counterfactual response $\hat{\zeta}_{it} = \zeta_{\hat{\Gamma}}(\mathbf{x}_{it}, \mathbf{y}_{it})$. The subject-level receiver efficiency is defined as

$$\hat{Y}_i = 1 - \frac{\frac{1}{5} \sum_{t=16}^{20} \|\mathbf{z}_{it} - \hat{\zeta}_{it}\|}{\mathbb{E}\|\boldsymbol{\theta}\|}.$$

¹⁸Our findings do not change if we used the rounds in part 1 when subjects acted as receivers.

¹⁹All of our distributions are significantly different from one another using a Kolmogorov–Smirnov test for nonequality of the distributions, at any conventional level. The reported significance levels for the first-order stochastic dominance relation \succ_1 , following Barrett and Donald (2003), use a bootstrap of size 1000 to calculate p -values. The observed stochastic ordering is significant at the following levels: ***1 percent; **5 percent; *10 percent. The order \succ_1 is complete and transitive over the five distributions at the 5 percent level, as $Y(\text{R}(0)) \succ_1^{**} Y(\text{R}(1))$ and all treatments are stochastically dominated by full revelation, and dominate babbling at the 1 percent level.

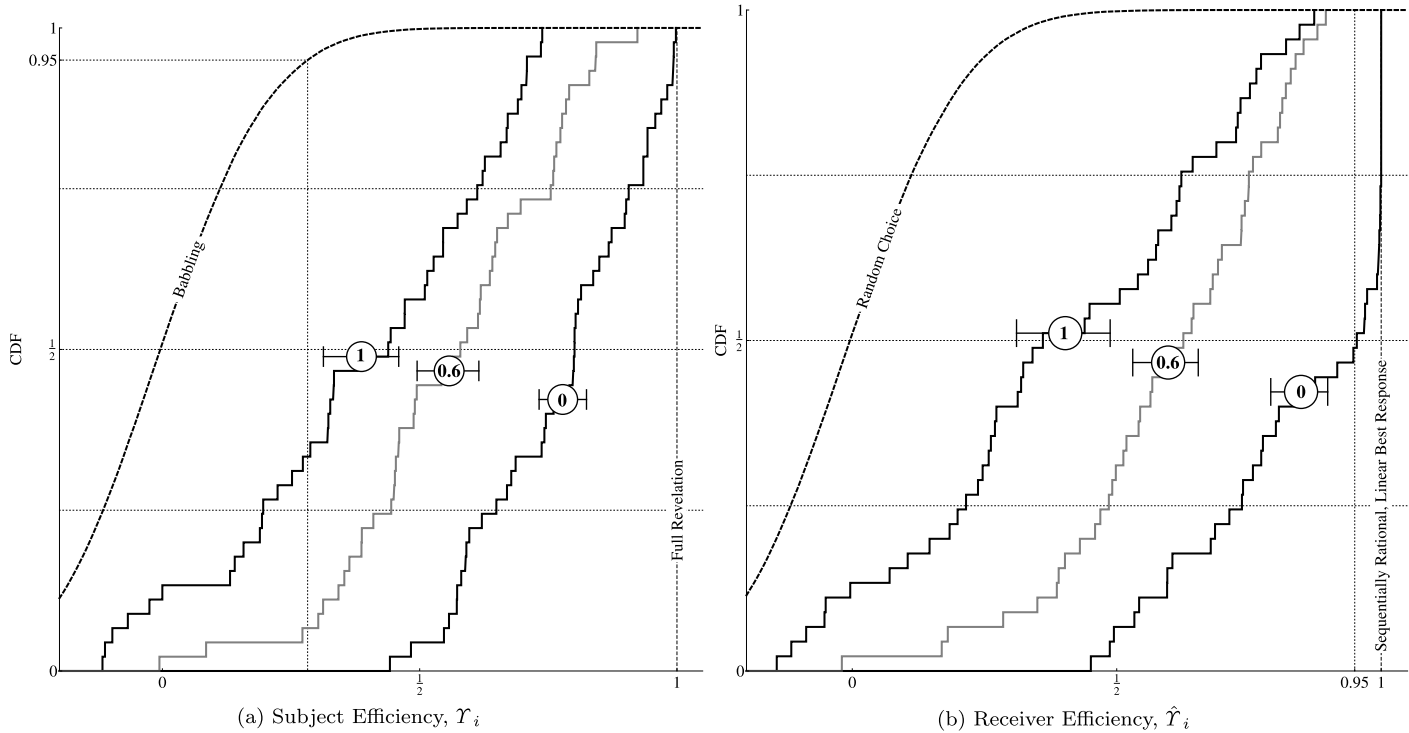


FIGURE 5. CDFs for subject's average decision efficiency. *Note:* Plot labels are located at the average efficiency level within the treatment; bars through the plot label represent the 95 percent confidence region for the distribution mean, extracted from a bootstrap. There is a reference CDF for Y_i under babbling because the babbling equilibrium distance is a random variable. The fully revealing equilibrium's efficiency is a degenerate random variable with a certain value of 1 in every subject round.

Receiver efficiency is the average distance between the subject's choice and the sequentially rational response, normalized by the expected distance under babbling (or equivalently here, the expected distance between $\hat{\zeta}_{it}$ and a uniform random choice). This is our measure for sequential rationality, where \hat{Y}_i controls for noise in the sender recommendations, as both \mathbf{z}_{it} and $\hat{\zeta}_{it}$ respond to the same (potentially noisy) recommendations $(\mathbf{x}_{it}, \mathbf{y}_{it})$. A subject with full receiver efficiency makes final choices exactly coincident with the sequentially rational response in all five final rounds, while a subject with zero receiver efficiency has the same average distance from $\hat{\zeta}_{it}$ as would a random uniform choice over Θ .

Empirical CDFs for \hat{Y}_i are provided by treatment in the Figure 5(b). Again, treatments differ significantly over the average receiver efficiency and are completely ordered by first-order stochastic dominance at the 5 percent level. The clearest difference in comparison to Figure 5(a) is the stronger indication that many subjects in R(0) follow the sequentially rational response. Approximately a quarter of subjects in R(0) *exactly* match the response $\zeta_{\hat{r}}(\mathbf{x}_{it}, \mathbf{y}_{it}) = (x_{1it}, y_{2it})'$ in *all* five rounds, while 56 percent have final choices with receiver efficiency greater than 95 percent—corresponding to an average choice within a 4.9° radius of $\hat{\zeta}_{it}$. Deviations by senders from Restriction B account for these subjects not having values for Y_i closer to the fully efficient level.

In contrast to R(0), in R(.6) and R(1) not one subject has a receiver efficiency above 95 percent, so subjects in these treatments are consistently far from the sequentially rational response. We conclude then that receivers in the rotated treatments are not following the $\zeta_{\hat{r}}(\mathbf{x}_{it}, \mathbf{y}_{it})$ response.²⁰

Having provided evidence of what they are not doing, we now turn to more constructive evidence for the strategies receivers do use. In particular, receivers do fairly well at incorporating the available information within each issue (the α terms). Crucially though, we find no evidence that subjects use *any* available information across issues (the β terms).

4.2.2 Receiver strategies To understand how subjects use the provided recommendations we estimate how receivers' final choices respond to the recommendations they received. In particular, we examine a generalization of the Proposition 1 response, where we estimate the values α_k , β_k , and η_k in the econometric identity

$$\begin{pmatrix} z_1^{it} \\ z_2^{it} \end{pmatrix} = \begin{bmatrix} \alpha_1 & -\beta_1 \\ -\beta_2 & 1 - \alpha_2 \end{bmatrix} \begin{pmatrix} x_1^{it} \\ x_2^{it} \end{pmatrix} + \begin{bmatrix} 1 - \alpha_1 & \beta_1 \\ \beta_2 & \alpha_2 \end{bmatrix} \begin{pmatrix} y_1^{it} \\ y_2^{it} \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1^{it} \\ \varepsilon_2^{it} \end{pmatrix}. \quad (4)$$

Given our orthogonal design, equilibrium strategies in treatment R($\tan \psi$) fit in to the above estimation equation as $\alpha_1^* = \alpha_2^* = \cos^2 \psi$ and $\beta_1^* = \beta_2^* = -\frac{1}{2} \sin 2\psi$, with zero offset ($\eta_1^* = \eta_2^* = 0$). Moreover, given state-independent exaggerations in independent directions given by Γ , the sequentially rational response $\zeta_{\Gamma}(\mathbf{x}, \mathbf{y})$ fits into the above family with $\alpha_1 = \alpha_2$ and $\eta_1 = \eta_2 = 0$.²¹

²⁰In fact, through a similar exercise we can also conclude that receivers in the rotated treatments follow neither a more sophisticated affine best response nor the unrestricted, nonparametric best response $\tilde{\zeta}(\mathbf{x}_{it}, \mathbf{y}_{it})$.

²¹Parameters for the best affine response to the distributions of senders' exaggerations are provided in the Supplement.

TABLE 3. Estimated receiver strategy.

	Estimated Aggregate Receiver Strategies					
	Within Issue		Across Issue		Offset	
	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\eta}_1$ [0.0]	$\hat{\eta}_2$ [0.0]
R(0)	0.87 [1.0] (0.04)	0.88 [1.0] (0.05)	0.00 [0.0] (0.02)	0.07 [0.0] (0.03)	2.6 (1.4)	-0.7 (3.1)
R(.6)	0.65 [0.74] (0.07)	0.63 [0.74] (0.07)	-0.11 [-0.44] (0.07)	0.02 [-0.44] (0.06)	2.3 (5.7)	-19.2 (6.3)
R(1)	0.38 [0.5] (0.08)	0.54 [0.5] (0.07)	0.02 [-0.5] (0.08)	-0.05 [-0.5] (0.03)	-2.7 (10.4)	-29.3 (5.3)

Note: Estimation results are from an iterated generalized method of means (GMM) regression enforcing the identifying constraints across the two equations (moment conditions are that all regressors are uncorrelated with the errors across both issues) using data from rounds 16–20. For each treatment there are 225 observations, with 3 sessions, and 45 subjects. Theoretical values are given in square brackets next to the estimate where applicable. Standard errors are given in parentheses below the estimate.

We estimate parameters in (4) using an iterated GMM approach, with data across all subjects in rounds 16–20, where the results are provided in Table 3. Examining the estimated coefficients (and comparing them to the equilibrium predictions in the square brackets), the most striking features are the following:

(i) The within-issue weights ($\hat{\alpha}_1$ and $\hat{\alpha}_2$) shift significantly by treatment in the directions of the equilibrium prediction. The only treatment where we can reject the equilibrium value at the 95 percent level is R(0), where the equilibrium coefficient is located at a boundary.

(ii) The across-issue terms ($\hat{\beta}_1$ and $\hat{\beta}_2$) are not significantly different from zero at the 95 percent level (with the exception of issue 2 in R(0) where equilibrium/best response does predict zero), but are significantly different from the equilibrium prediction in R(.6) and R(1).

(iii) The offset terms ($\hat{\eta}_1$ and $\hat{\eta}_2$) are only significantly different from zero when the biases of the senders have the same sign within the issue—issue 2 in R(.6) and R(1).

To briefly summarize the regression results, as these estimates constitute one of the paper's main findings, receivers *do* react to senders' biases within issue (the α_j terms). However, where the full-revelation strategies require a multidimensional response, subjects in our treatments do *not* exhibit *any* reaction to across-issue differences (the β_j terms).²²

²²A linear estimation similar to (4) with fewer free parameters conducted at the subject level mirrors the intuition from Figure 5(b). In R(0), 49 percent of subjects have choices with excellent fit to the model (an R^2 coefficient of 0.99 or greater), and have estimated parameters quantitatively close to the equilibrium point prediction. When we perform the same exercise in the R(.6) and R(1) treatments, the fraction of subjects with even adequate fit to the model (an R^2 coefficient of 0.8 or greater) is 13 and 20 percent, respectively. Of this subset, none has estimated parameters close to the equilibrium values.

4.3 Receiver reactions within each issue

We now focus on providing a positive characterization of subject's within-issue response. In each specific issue, senders have two general orientations: either their biases are *opposed* (each wants the receiver's choice to move in a different direction) or they are *aligned* (both want the receiver's choice to move in the same direction) with treatment R(0) representing a boundary case in between the two. We study these two cases in turn, and provide evidence for two distinct types of unidimensional response by the receivers: an averaging response $\zeta_{\text{Avg}}(\tilde{x}_j, \tilde{y}_j)$, and a response that identifies the minimal recommendation and shades a fixed amount from it $\zeta_{\text{Min}}(\tilde{x}_j, \tilde{y}_j)$.

Opposed issue The senders have opposed biases in issue 1 for the R(.6) and R(1) treatments, and in a limiting sense both issues in R(0). If both senders use linear exaggeration strategies in the bias direction, the received recommendations in issue 1 would be $x_1 = \theta_1 - \kappa_X \cdot |\delta_1^X|$ and $y_1 = \theta_1 + \kappa_Y \cdot |\delta_1^Y|$ for the exaggeration magnitudes κ_X and κ_Y . The exaggeration difference $y_1 - x_1$ is directly observable, and is equal to $\kappa_Y \cdot |\delta_1^Y| + \kappa_X \cdot |\delta_1^X|$. In general—that is, outside of the R(0) case where one bias is zero—there will be a continuum of (κ_X, κ_Y) pairs consistent with the difference $y_1 - x_1$, each leading to a different inference on the precise location of θ_1 .

The FRE response $\zeta^*(\mathbf{x}, \mathbf{y})$ uses the across-issue difference $y_2 - x_2$ to eliminate the degree of freedom on the location of θ_1 , pinning down unique values for both κ_X and κ_Y . As we have described, subjects do not use this across-issue difference. Instead, we will show that in opposed issues the majority of subjects act *as if* responding to a symmetric conjecture on the exaggeration magnitudes that pins down the response, that $\kappa_X = \kappa_Y$. Given this conjecture, the within-issue best response is a weighted average of the recommendations, so $z_1 = \alpha \cdot x_1 + (1 - \alpha) \cdot y_1$, with the weight parameter $\alpha = |\delta_1^Y| / (|\delta_1^Y| + |\delta_1^X|)$. In R(0) the biases are 0° and 60° for the two senders in each issue, so this type of response leads to a final choice equal to the recommendation sent by the unbiased sender in the issue (matching the equilibrium response). In R(.6) the biases in issue 1 are -30° and 50° , so a symmetric conjecture on the exaggeration magnitudes leads to $z_1 = \frac{5}{8} \cdot x_1 + \frac{3}{8} \cdot y_1$, while in R(1), the senders' biases in the first issue are -45° and $+45^\circ$, leading to the simple-average response, $z_1 = \frac{1}{2}x_1 + \frac{1}{2}y_1$.

For each subject $i \in \mathcal{I}$ we estimate the best-fitting parameter α_i in the following averaging response $\zeta_{\text{Avg}}(x_1, y_1; \alpha_i) = \alpha_i \cdot x_1 + (1 - \alpha_i) \cdot y_1$ in the opposed-sign issues.²³ Figure 6(a)–(c) provide histograms illustrating the distribution of the estimated $\hat{\alpha}_i$ parameters by treatment. The histogram illustrates the fraction of subjects with values with model fit greater than 99 percent (95 and 90 percent, respectively) with black (gray and light gray) bars.²⁴

²³That is, we estimate the parameter α_i that minimizes the objective $\sum_{t=16}^{20} |z_{1it} - \zeta_{\text{Avg}}(x_{1it}, y_{1it}; \alpha_i)|$ for each subject. Because data are circular, there are two potential midpoints in between x_1 and y_1 ; we always choose the midpoint to minimize the error $|z_{1it} - \zeta_{\text{Avg}}(x_{1it}, y_{1it})|$. For R(0) because both issues are opposed, we pool observations across the two issues using sender symmetry, and minimize the objective $\sum_{t=16}^{20} |z_{1it} - (\alpha_i \cdot x_{1it} + (1 - \alpha_i) \cdot y_{1it})| + |z_{2it} - (\alpha_i \cdot y_{2it} + (1 - \alpha_i) \cdot x_{2it})|$.

²⁴Using the residuals, $\varepsilon_{ijt}(\hat{\alpha}_i) = z_{ijt} - \zeta_{\text{Avg}}(x_{ijt}, y_{ijt}; \hat{\alpha}_i)$, we calculate subjects' goodness-of-fit via $R_i^2 := 1 - \widehat{\text{Var}}(\varepsilon_{ijt}(\hat{\alpha}_i)) / \text{Var}(\theta_j)$.

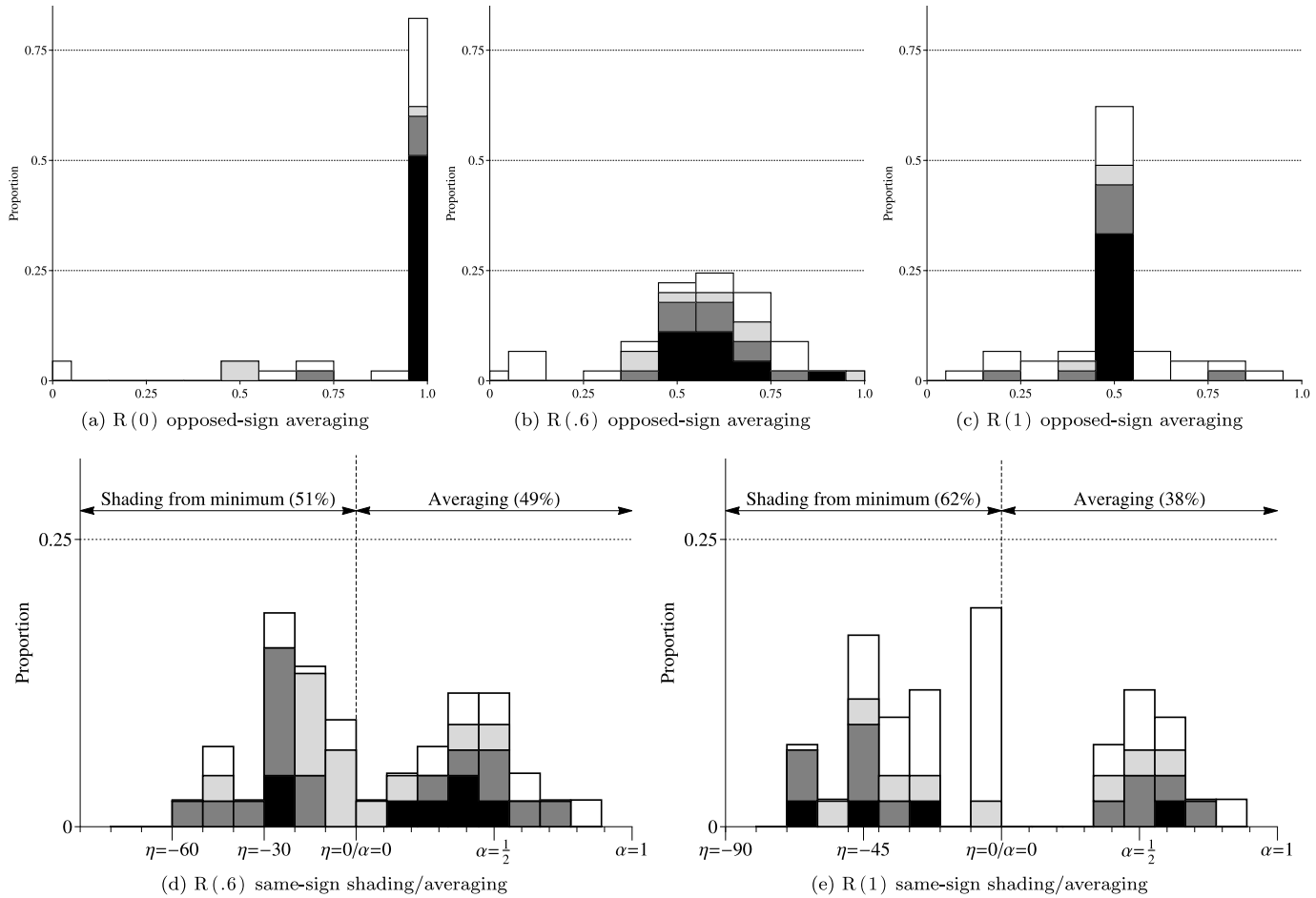


FIGURE 6. Subject level within-issue behavior models. *Note:* Shading represents the subjects assessed coefficient of determination R^2 relative to babbling within the issue. Shading represents R^2 greater than 0.99 (black); 0.95 (dark gray); 0.9 (light gray).

Inspecting the three opposed-issue histograms, the modal response clearly follows the bias-weighted average motivated above. The starkest pattern is within the R(0) treatment, where just over 50 percent of subjects have an estimated weight on the unbiased sender of 0.95–1.00, with a very strong fit to the model. Clearly, given that this averaging process mirrors the sequentially rational response, these are the same subjects with high receiver efficiencies in Figure 5. However, the remaining subjects' strategies seem to fall between those using a simple average (a form of *false equivalence*, given the known asymmetric sender biases in the issue) and those leaning more toward the less biased sender.²⁵ In R(.6) the modal responses is for weights in 0.55–0.65, placing a greater weight on the recommendation from the sender with the bias of -30° , and less weight on the sender with the $+50^\circ$ bias. Subjects are therefore again clustered around the bias-weighted average of 0.625. Finally, in R(1), we again see a large spike at the bias-weighted point, this time indicating subjects who use a simple averaging strategy, equally weighting the two recommendations. Given the responses of R(.6) and R(0), the mass of subjects in the 0.45–0.55 bin might best be thought of as the union of false-equivalence subjects, and those using a bias-weighted average.²⁶

Same-sign issue When the senders have opposed biases, the recommendation difference *within the issue* allows receivers to observe the sum of the exaggerations, but provides no information on the relative exaggeration of each sender. In contrast, when senders have same-signed biases, the converse holds: within the issue the recommendation speaks to the relative exaggerations of each sender, but there is no information on the absolute magnitude. The clearest case for this is issue 2 in R(1). Both senders have the same bias relative to the receiver, $\delta_2^X = \delta_2^Y = +45^\circ$. Given linear exaggerations in the bias direction, the observable recommendation difference is $y_2 - x_2 = (\kappa_Y - \kappa_X) \cdot \delta_2^X$. So the recommendation difference is informative on which sender exaggerated by more and by how much. But without the across-issue difference, the absolute values of κ_X and κ_Y are again not pinned down.

Examining receiver responses, subjects seem to use two main forms of response in same-signed issues: (i) identifying the sender who exaggerated the least and shading the recommended point by a constant amount η , so the response is $\zeta_{\text{Min}}(x_j, y_j; \eta) := \min\{x_j, y_j\} + \eta$, or (ii) averaging the two recommendations via $\zeta_{\text{Avg}}(x_j, y_j; \alpha)$. To distinguish between these two very different types of response we estimate both models for each subject, and then provide the model parameter $\hat{\eta}_i \in (-180, 0]$ for shading or $\hat{\alpha}_i \in (0, 1]$ for averaging that attains the best fit from the two.²⁷

²⁵Mann–Whitney tests indicate significant differences across all treatments for the median $\hat{\alpha}_i$, with R(0) larger than R(.6) larger than R(1), at the 5 percent level.

²⁶The results in R(0) and R(.6) also make clear the aggregate within-issue parameters assessed in Table 3. However, the subject-level results here control for a common mistake in the first issue in the R(1) treatment, choosing the wrong arc. When both senders exaggerate to a large degree, receivers sometimes make choices on the “wrong” arc, where x_{1it} is clockwise and y_{1it} is counterclockwise, the opposite directions from the biases. This happens in 28 percent of the R(1) data. This error leads to a choice very far from the true state, and as such creates a bias in the GMM estimates. Controlling for this mistake (looking at the weight parameter α_i on the *chosen* arc) the subject-level estimates more clearly indicate the bias-averaging strategy.

²⁷For $\zeta_{\text{Min}}(x_j, y_j; \eta)$ we use a circular variant of the min function: we find the least clockwise recommendation on the smallest arc connecting x_j and y_j . A parameter estimate $\hat{\eta}_i$ captures a fixed amount

The subject-level distribution for the two same-signed issues—issue 2 in R(6) and R(1)—are provided as histograms in Figure 6(e) and (f), with a nonlinear scale to account for the differing model supports. We again illustrate the model's goodness-of-fit through the histogram bin shading.

Subjects whose response as a receiver matches the shading from the minimum strategy represent 51 percent of subjects in R(6) and 62 percent of subjects in R(1). While the shading strategy represents the majority, a large minority acts as if averaging the recommendations. Interestingly, the subjects who are averaging do seem to react to the bias magnitudes, with the distribution closer to 0 in R(6) than in R(1). For the subjects shading from the minimum recommendation, the treatments R(6) and R(1) differ in the magnitude of the shading, with subjects in R(1) on average removing a larger amount.²⁸

A common strategy seems to be removing one multiple of the bias from the minimal recommendation, matching an as-if conjecture that the more honest player recommends his/her ideal point, so $\min\{\kappa_X, \kappa_Y\} = 1$. In R(6) the minimal sender is frequently the less biased X , with a bias of $+30^\circ$. The histogram makes clear that the larger mass of subjects (with good fit to the shading model) removes between 20° and 30° from the minimal recommendation. In contrast, the most common strategy estimated with a good fit in R(1) is to remove between 40° and 50° from the minimal recommendation.

Taken together the three treatments indicate the following general patterns to subjects' within-issue response.

(i) In opposed-sign issues, the vast majority of subjects act as if computing a bias-weighted average. A small minority exhibit false equivalence, and compute simple averages of the senders' recommendations.

(ii) The majority of subjects in same-signed issues act as if identifying the minimal recommendation, and removing a constant amount from it. The average amount removed responds to the treatment variable $\min\{|\delta_j^X|, |\delta_j^Y|\}$, the size of the least biased sender. A (somewhat large) minority of subjects use averaging strategies.

(iii) Subjects who do not adhere to one of our assessed within-issue models fare much worse than those who do.²⁹

The modal response by subjects is in fact somewhat close to a *within-issue* restricted best response. Consider the within-issue family of strategies that takes averages with

added to the minimal recommendation, which is found by minimizing $\sum_{t=16}^{20} |z_{2it} - \zeta_{\text{Min}}(x_{2t}, y_{2t}; \eta)|$ across $\eta \in (-180, 0]$. An estimate $\hat{\alpha}_i$, capturing a weighted averages response, is found by minimizing $\sum_{t=16}^{20} |z_{2it} - \zeta_{\text{Avg}}(x_j, y_j; \alpha)|$ across $\alpha \in (0, 1]$. We then compare each model through the fitted residual magnitudes, $\frac{1}{5} \sum_{t=16}^{20} |z_{2it} - \zeta_{\text{Min}}(x_{2t}, y_{2t}; \hat{\eta}_i)|$ and $\frac{1}{5} \sum_{t=16}^{20} |z_{2it} - \zeta_{\text{Avg}}(x_{2t}, y_{2t}; \hat{\alpha}_i)|$. The two models were chosen to match clear patterns while not overlapping (the minimum shading model makes choices counterclockwise from the smallest arc; the averaging model makes choice on the smallest arc).

²⁸This difference is not significant when looking at *all* shading-strategy subjects; however, it is significant at the 1 percent level for subjects with good fit, an $R_i^2 > 0.9$, using a Mann–Whitney test.

²⁹We break subjects into those with good fit ($R_i^2 > 0.95$) and those with poor fit ($R_i^2 < 0.95$) and examine subject efficiency Y_i . Subjects with good fit to the averaging strategy in the *opposed-sign* issue have efficiencies 20–25 greater than those with a poor fit. When we look at the *same-signed* issue we find a similar efficiency gain for subjects with good fit to a *shading from the minimum* response, in relation to those with poor fit to any response. Subjects with good fit to an *averaging response* in the same-signed issue have efficiencies 5 percent lower than those with *poor fit* to either model.

weight $\alpha \in [0, 1]$ on sender X in opposed issues and adds $\eta \in [-180, 0)$ to the minimal recommendation in same-signed issues. Within this family the best response parameters are $\alpha = 0.64$ and $\eta = -32^\circ$ in R(.6), and $\alpha = 0.5$ and $\eta = -37^\circ$ in R(1). Comparison with the modal response in Figure 6 makes clear that many subjects are close to this within-issue best response. The efficiency attainable with a within-issue response is relatively high, achieving 75 percent efficiency in R(.6) and 59 percent in R(1). In comparison, the efficiency attained by the multidimensional, sequentially rational response $\zeta_{\hat{r}}$ is 81 percent in R(.6) and 72 percent in R(1). However, though the within-issue response does well, outside of the R(0) special case, it cannot be part of an FRE, where the averaging strategies in particular give both senders an incentive to exaggerate further in those issues.³⁰

5. DISCUSSION

The data indicate clear patterns for both senders and receivers in a multidimensional, multi-sender cheap talk setting. A large majority of senders can be modeled as using a strategy that exaggerates in the direction of their own bias. In our experimental setting this behavior intersects with a component strategy in a fully revealing equilibrium. Receivers, however, do not use information in the optimal manner, and fail to adjust their choice in one dimension with information learned in another. Instead, their response treats each dimension independently. Though many receivers' within-dimension response shows strategic sophistication with a significant response to the biases, their behavior is not consistent with multidimensional sequential rationality. In our rotated environments final outcomes are far from full revelation, and we observe increased noise in sender response. However, in one case, R(0), the *specific* framing of the multidimensional problem makes the receiver best response entirely within issue. In this treatment, not only are the final outcomes qualitatively close to full revelation, but the component behavior for both senders and receivers forms a mutual best response.³¹

Our findings suggest several avenues for future research on behavioral models. The data illustrate a failure of sequential rationality. Despite calculating fairly sophisticated conditional expectations within each dimension, subjects completely fail to understand connections between choice dimensions. Because of this failure of sequential rationality, behavioral concepts may better organize the data than equilibrium predictions. One alternative would be to extend models such as Jehiel's (2005) analogy-based expectations, which attempt to modify equilibrium thinking to allow conditional expectations

³⁰Our companion paper (Vespa and Wilson (2016)) compares the results from R(1) to two unidimensional environments, where senders have the same opposed or same-signed bias in *both* issues. The results make clear that increased noise in R(1) mostly stems from increased exaggeration in any environment where receivers take averages.

³¹Except for R(0), the joint behavior of senders and receivers does not constitute an equilibrium. Given that receivers' behavior is not consistent with sequential rationality, senders could benefit from further exaggerating. The finding that senders are not using an empirical best response is common to experiments with one sender and one dimension (e.g., Cai and Wang (2006)). One approach in the literature uses disequilibrium concepts (level- k cognitive hierarchy) to better organize the data. Below we comment on the challenges that these approaches face to rationalize our data.

to be miscalculated. Our receivers are effectively failing to compute a conditional expectation: instead of assessing $\mathbb{E}(\theta_a | \mathbf{y} - \mathbf{x})$ in each dimension a (conditioning on the vector difference $\mathbf{y} - \mathbf{x}$), many subjects act as if calculating $\mathbb{E}(\theta_a | y_a - x_a)$.³²

Related cheap talk papers have modeled sender/receiver behavior (in one-sender–one-dimension settings) with level- k models, with agents best responding to simple conjectures about the other player's response.³³ However, for level- k models to explain behavior in sequential multi-agent environments such as ours, they must address how conjectures on others respond to observed play. With multiple senders and multiple issues, receivers in our environment obtain many observables, and many conjectures over senders' behavior can be falsified by comparing recommendations. Any level- k model of behavior must address how agents modify their conjectures in response to conflicting information. Our data offer some examples for how subjects respond. On the one hand, when subjects are confronted with information about senders' exaggerations within dimension, they readily adapt. Where senders' biases have opposed signs, receiver response mirrors that of an updated *symmetric* conjecture over exaggerations.³⁴ When senders' biases have the same sign, our results point to receivers understanding the realized asymmetry and shading from the identified minimal sender. On the other hand, our results suggest a stark failure to resolve conflicts in the conjectured response *across* dimensions. Receivers' behavior reveals an inconsistency in conjecture over each individual sender. For example, a receiver might react as if a specific sender is naively truthful in one dimension, but that same sender has exaggerated by multiples of his bias in another dimension—they are fully cursed over dimensions of choice. Any extensions of level- k to multi-sender–multidimension settings needs to address the extent to which conjectures are constrained and updated with respect to observables. Our evidence suggests that conjectures across dimensions are unresponsive, while our within-issue models of response suggest much greater receiver sophistication.

Our results suggest several directions for future experimental research on multidimensional cheap talk. An unsettling finding is that the behavior of senders and receivers in our rotated treatments is not a mutual best response. While we do not observe senders displaying quantitatively large deviations from bias-direction exaggerations (at the aggregate level), it is possible that greater experience may have an effect. One natural way to investigate this in the laboratory would be with computers taking the role

³²In this sense, one can think of extensions to the Jehiel (2005) model to allow analogy classes within subspaces of the problem rather than simply subsets, where subjects take unconditional expectations over the variable $y_b - x_b$, as if responding to $\mathbb{E}[\mathbb{E}(\theta_a | \mathbf{y} - \mathbf{x}) | y_a - x_a]$. Alternatively, a simple dimension-specific agent-form model can accommodate the findings. In this model, receivers simply believe that they are responding to recommendations from a different pair of senders in each dimension.

³³Cai and Wang (2006) find support for a level- k behavioral model in a Crawford–Sobel setting, while Wang, Spezio, and Camerer (2010) further analyze subject behavior with eye-tracking data, providing further evidence for level- k explanations (with further evidence in Kawagoe and Takizawa (2009)). A companion paper to this (Vespa and Wilson (2016)) provides evidence from the same toroidal interface with just one sender, uncovering very similar level- k behavior to the previous literature.

³⁴Indeed, even the subjects who are closest in spirit to a level-0 response have a symmetric conjecture. Subjects exhibiting false equivalence (both in opposed and same-sign issues) can be thought of as responding to a belief of truthful response for each sender with symmetric white noise in recommendations.

of receivers, following some predetermined strategy.³⁵ Two types of computer strategy would seem natural candidates: the equilibrium strategy $\zeta^*(\mathbf{x}, \mathbf{y})$, and the behaviorally focal within-issue strategy (averaging in opposed-sign issues; shading one multiple of the bias in same-signed issues). In particular, response to a computerized within-issue strategy would help understand the steady state behavior, whether outcomes converge to low-efficiency, babbling-like outcomes or whether some partially revealing equilibrium emerges. Sender behavior with computers programmed to use the equilibrium response $\zeta^*(\mathbf{x}, \mathbf{y})$ in an R(1)-like setting would help understand the degree to which the increased noise we observe from senders in R(1) is caused by the receiver's within-issue response. Additionally, experiments using computerized receivers would be useful for understanding collusion. For example, a common market interest rate used in contracts (the London Interbank Offered Rate) is set via a periodic survey of banks' borrowing rates, where the published rate is a trimmed mean. Ongoing investigations by various fiduciary authorities have alleged collusion on the part of the bankers being surveyed. Automated-receiver experiments might help understand the coordination requirements for successful collusion, and whether multidimensional reporting could inhibit it.

Another aspect of our experimental design that might be further explored is sender response. Our treatments do not address whether senders' behavior—exaggerating in the direction of their own bias—is an artifact of the orthogonal biases or whether this behavior holds more generally. Environments with linearly independent but nonorthogonal biases have differing equilibrium predictions, and would help provide evidence on how senders behave in reaction to generic competing voices. Similarly, varying other environmental variables would help provide greater insight into receiver response. How do subjects respond to senders with differing bias magnitudes? When the prior provides a focal choice, does this influence the response? Greater experimental variation here will help refine our understanding.

A related open question is the degree to which interested parties select experts with particular biases. Do receivers choose experts with large biases but in R(0)-like directions, and fully extract, or do they instead choose experts with smaller biases in R(1)-like directions? When we seek out second opinions, do we select those with diametrically opposed interest? Similarly, do interested parties (for instance, the defending/prosecuting sides in a jury trial) select experts to take advantage of the receiver's heuristics? Further exploration of these questions would seem fruitful.

Finally, while our results might lead us to be skeptical toward the FRE in generic situations, from a policy perspective, our results do suggest a behavioral avenue. The FRE's theoretical existence result is generic. In contrast, our experimental findings indicate success only in a special case. However, the full-revelation existence proof works by showing that the generic environment, through a change in multidimensional frame, is the special case. Behaviorally, we can use this result, but in the opposite direction. In situations where policy-makers can manipulate the frame—through mandating specific reporting metrics, instructing jurors on how to understand testimony, manipulating the

³⁵In addition to receiver automation, similar exercises with automated senders could help identify the conditions/instruction required for receivers to begin incorporating across-issue information.

message space, and so on—generic environments might be translated into an $R(0)$ -like setting.³⁶ In this sense, environments with low-information transmission, such as $R(.6)$ or $R(1)$, might be avoided by institutional designs aware of human behavior. By changing the frame, it might be possible to affect equilibrium selection.

6. CONCLUSION

We examine three cheap-talk environments with multiple senders providing recommendations to a receiver in multiple dimensions. In each environment, full revelation can be supported as an equilibrium outcome using a similar construction to that in Battaglini (2002), where our treatment variables are reframings—rotations of the coordinate system. Though senders' strategic response is qualitatively similar across the three environments, there are large differences in the amount of information extracted by receivers, and whether the observed strategies are consistent with an equilibrium outcome. In one frame, when each sender is perfectly aligned with the receiver only in one dimension, $R(0)$, the majority of receivers succeed in extracting information, and come close to matching the fully revealing equilibrium. The majority of subjects in all roles use strategies that form a fully revealing mutual best response.

However, when we reframe the environment, and the sequentially rational response has receivers combine information across dimensions, we document substantial failures. Our results suggest that receivers treat each choice dimension independently, as if solving two isolated one-dimension problems. Therefore, though there are particular frames under which fully revealing equilibria emerge, generically, their selection seems unlikely. However, the results do suggest a potential policy intervention: if it is possible to reframe an environment by bringing it closer to $R(0)$, potentially large gains in information transmission can result.

Our paper offers a simple, positive description of how receivers process conflicting advice: averaging opposed interest senders according to their biases, directly following unbiased senders, and shading from the minimum for senders with aligned interests. The results indicate that most subjects do not suffer from false equivalence, that they react to the magnitudes and directions of bias, and by and large follow sensible heuristics. Where they fail is in their treatment of multidimensionality, treating strategically connected dimensions as separate.

APPENDIX

PROOF OF PROPOSITION 1. The sequentially rational response for all recommendations (\mathbf{x}, \mathbf{y}) solves

$$\min_{\mathbf{z}} \mathbb{E}_{\theta, \kappa_X, \kappa_Y} \{ \|\mathbf{z} - \theta\| \mid \mathbf{x}, \mathbf{y} \},$$

³⁶Indeed, the results of Lai, Lim, and Wang (2015) show that message-space restrictions can increase information transfer.

given beliefs that senders follow the strategy (1) and (2). The receiver has four equations in four unknowns, θ_1 , θ_2 , κ_X , and κ_Y .³⁷

Because $\Gamma = [\gamma^X \ \gamma^Y]$ has full rank, the posterior belief over $\theta|\mathbf{x}, \mathbf{y}$ is degenerate. The sequentially rational response is

$$\xi_{\Gamma}(\mathbf{x}, \mathbf{y}) = \begin{bmatrix} \mathbf{0} & \gamma_Y \end{bmatrix} \Gamma^{-1} \mathbf{x} + \begin{bmatrix} \gamma_X & \mathbf{0} \end{bmatrix} \Gamma^{-1} \mathbf{y} \quad (\text{A.5})$$

for any inferred exaggeration levels $(-\hat{\kappa}_X, \hat{\kappa}_Y)' := \Gamma^{-1}(\mathbf{y} - \mathbf{x})$ in the support of F_X and F_Y , respectively. This receiver's response transforms each sender's recommendation into an exaggeration-direction coordinate through the transformation Γ^{-1} . Given the sender's strategies

$$\xi_X(\theta) = \theta + \Gamma \begin{pmatrix} \kappa^X \\ 0 \end{pmatrix} \quad \text{and} \quad \xi_Y(\theta) = \theta + \Gamma \begin{pmatrix} 0 \\ \kappa^Y \end{pmatrix},$$

the receiver's response selects the γ_Y coordinate from X , and the γ_X coordinate from Y , which results in the final choice $\xi_{\Gamma}(\xi_X(\theta), \xi_Y(\theta)) = \theta$, the maximal outcome for the receiver. The proposition characterization follows from $[\mathbf{0} \ \gamma_Y] \Gamma^{-1} + [\gamma_X \ \mathbf{0}] \Gamma^{-1} \equiv \mathbf{I}$, and the fact that

$$\text{tr} \left\{ \begin{bmatrix} \mathbf{0} & \gamma_Y \end{bmatrix} \Gamma^{-1} \right\} = \text{tr} \left\{ \Gamma \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Gamma^{-1} \right\} = 1$$

by circularity of the trace operator. □

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³⁷Whenever the support for the exaggerations means that the approximate position of θ relative to \mathbf{x} and \mathbf{y} can always be worked out—one sufficient condition being that $\frac{\kappa^i}{\max\{\delta_1^i, \delta_2^i\}} \in [\sigma^\circ, 180^\circ - \sigma^\circ]$ for both senders—then the analysis on the plane illustrated in Figure 2 is going to be equivalent to working on the two circles.

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Co-editor Rosa L. Matzkin handled this manuscript.

Submitted October, 2014. Final version accepted May, 2015.