Supplement to "Estimating matching games with transfers"

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APPENDIX B: ESTIMATION AND INFERENCE FOR THE CAR PARTS EMPIRICAL WORK

I first describe point estimation and then inference. I sample 10,000 inequalities for each of 29 matching markets given by component categories. For the 30th component category, I use the maximum number of inequalities, 8272, for that market. To create one of the 10,000 inequalities for a market, I randomly sample two car parts from different suppliers and include the resulting matching maximum score inequality where the two car parts are exchanged between the suppliers.

For point estimation, I use the numerical optimization routine differential evolution in Mathematica. For differential evolution, I use a population of 250 points and a maximum of 15,000 iterations. The numerical optimization is run 20 times with different initial populations of 250 points. For the specification in Table 1, all 20 runs found the same objective function value and the same point estimates, up to two significant digits. Therefore, it might be reasonable to consider the model to be point identified in the limit.

In the maximum score objective function, an inequality is satisfied if the left side plus the constant +0.0000000001 exceeds the right side. This small perturbation to the sum of profits on the left side ensures that inequalities such as $0 > 0$ are counted as being satisfied consistently, rather than inconsistently because of some numerical-approximation error resulting in, say, $2.0 \times 10^{-15} > 1.0 \times 10^{-15}$.

I use the inference procedure of Romano and Shaikh (2008), which is valid under both set identification and point identification. Let Θ_0 be the identified set for the parameter θ. The authors provide a subsampling procedure to construct a confidence region $\mathscr{C}$ called, by those authors, a "confidence region for the identifiable parameters that is uniformly consistent in level," under the conditions of their Theorem 3.3. This definition of the properties of a confidence region is equation (3) of their paper. Under easier

to verify conditions (namely that a limiting distribution exist for the objective function), their procedure produces "confidence regions for identifiable parameters that are point-wise consistent in level"—or their equation (2). Computationally, I implement equations (12) and (10) in their paper.

I have 30 component categories. I treat these component categories as separate markets for the industry-specific reasons discussed in the main text. I perform asymptotics in the number of suppliers in the market, as this is an explicitly two-sided market where a supplier cannot also be an assembler, as in Example 2. Subsampling requires a choice of subsample size. Unfortunately, the literature has not provided a data driven method to pick this tuning parameter. I use 33% of the suppliers from each of the 30 component categories. Given each subsampled set of suppliers, that subsample uses only matching maximum score inequalities where both suppliers whose valuation functions are in the inequality are in the subsample. I use 500 subsamples; in experiments, results are robust when using more subsamples.

In the car parts empirical work, the maximum score or maximum rank correlation objective function is, for $D = 30$ component categories,

$$Q(\theta) = \binom{N}{2}^{-1} \sum_{d=1}^{D} \sum_{i_1=1}^{N_d-1} \sum_{i_2=i_1+1}^{N_d} \sum_{g \in G_{i_1,i_2}^d} 1[Z'_{g,d}\theta \geq 0],$$

where G_{i_1,i_2}^d is the set of included matching maximum score inequalities where i_1 and i_2 exchange one car part each. Romano and Shaikh (2008) is written where one minimizes a function and where the population objective function's minimum value is 0. In maximum score, one maximizes the objective function value and the value of the population objective function can only be estimated using the finite sample objective function. Following the suggestion at the beginning of their Section 3.2, I work with the objective function $-Q(\theta) - (-\max_\theta Q(\theta))$.

I define the "sample size" for the entire procedure to be $N = \max_{d \in D} N_d$. This choice is arbitrary and does not impact the reported confidence regions. I implement the procedure of Romano and Shaikh (2008) using the rate of convergence for the objective function of $\sqrt{N}$, following the results of Sherman (1993) for the case of point identified maximum rank correlation estimators.

The confidence regions reported in tables are projections onto the axes of the confidence region $\mathscr{C}$ from Romano and Shaikh (2008). Computationally, the lower bound for the confidence region for a scalar parameter θ_1 is found by minimizing the parameter θ_1 subject to the constraint that the entire parameter vector θ is in $\mathscr{C}$. Likewise, the upper bound for the confidence region θ_1 is found by maximizing the parameter θ_1 subject to the entire parameter vector being in $\mathscr{C}$.

APPENDIX C: ALTERNATIVE WEIGHTING SCHEMES FOR SPECIALIZATION MEASURES

This appendix discusses versions of the estimates in Table 1 where the included HHI-specialization measures use different weighting schemes, many involving data on car sales for car models in Europe and the United States. These alternative weighting

schemes result in worse statistical fit than the estimates in Table 1, in the sense that fewer inequalities are satisfied. For this reason, I report the estimates in this appendix instead of the main text.

In terms of sales, car models primarily sold in Europe are matched to European sales from Western Europe and car models primarily from North America are matched to sales from the United States. Note that all weighting schemes affect the HHI-specialization measures. I do not explore weighting the maximum score inequalities in the objective function, although that could be pursued.

Let w_c be a weight for a particular car part c. The example parent-group supplier HHI measure in (13) becomes

$$X_{\text{PG}}(j^s, \Psi) = \left(\frac{\text{sum Chrysler weights in } \Psi}{\text{sum all weights in } \Psi} \right)^2 + \left(\frac{\text{sum Ford weights in } \Psi}{\text{sum all weights in } \Psi} \right)^2$$
$$+ \left(\frac{\text{sum GM weights in } \Psi}{\text{sum all parts in } \Psi} \right)^2.$$

Analogous schemes are used for other supplier HHI-specialization measures and for assembler specialization at the parent-group level. The example brand assembler HHI measure in (14) becomes

$$X_{\text{brand}}(j^b, \Phi) = \bar{w}_{\text{Opel}} \sum_{i \in s(\Phi, \text{Opel})} \left(\frac{\text{sum weights sold by supplier } i \text{ to Opel in } \Phi}{\text{sum weights for Opel in } \Phi} \right)^2$$
$$+ \bar{w}_{\text{Chevy}} \sum_{i \in s(\Phi, \text{Chevy})} \left(\frac{\text{sum weights sold by supplier } i \text{ to Chevy in } \Phi}{\text{sum weights for Chevy in } \Phi} \right)^2,$$

where, for example,

$$\bar{w}_{\text{Chevy}} = \frac{\text{sum weights for Chevy in GM}}{\text{sum weights in GM}}.$$

Note that the brand weights like $\bar{w}_{\text{Chevy}}$ are not recomputed for counterfactual trades Φ, as indeed Chevrolet and other brands still produce the same car models with the same need for car parts in the counterfactuals in matching maximum score inequalities.

Table 4 reports the estimates with weights. There are three specifications, each with different weights. Note that the parameters multiply different explanatory variables across the three specifications, so the parameters have slightly different interpretations. However, the means and standard deviations of the weighted HHI measures are qualitatively similar to those reported in Table 1 for the baseline specification.

The first weighting scheme in Table 4 does not use sales data; it sets

$$w_c = \frac{1}{\#\text{car parts for model in market } d},$$

This scheme gives less weight to a car part on a car model with more parts in a component category so as to equalize the contribution of car models to HHI calculations. The

TABLE 4. Specialization by Suppliers and Assemblers with Different Weighting Schemes

Weights→	1 Divided by #Parts		Sales		Sales Divided by # Parts	
HHI Measure	Point Estimate	95% CI	Point Estimate	95% CI	Point Estimate	95% CI
			Suppliers			
Parent Group	+1	Supercon.	+1	Supercon.	+1	Supercon.
Continent	0.982	(0.405, 4.66)	0.733	(0.582, 1.29)	0.717	(0.298, 3.12)
Brand	0.988	(−0.154, 8.52)	0.967	(0.158, 3.19)	0.894	(−0.52, 6.06)
Model	0.0806	(−8.79, 2.52)	−1.97	(−4.14, −1.31)	−1.88	(−6.18, −0.374)
			Assemblers			
Parent Group	0.474	(−0.0902, 4.70)	≈ 0	(−0.0318, 0.0654)	−0.0448	(−0.36, 0.0333)
Brand	−0.380	(−3.77, 1.79)	≈ 0	(−0.0435, 0.556)	0.0462	(−0.212, 1.2)
Model	202	(100, +∞)	+∞	(+∞, +∞)	+∞	(+∞, +∞)
# Inequalities	298, 272		298, 272		298, 272	
% Satisfied	76.3%		74.4%		74.3%	

Note: The parameter on parent group specialization is fixed at +1. Estimating it with a smaller number of inequalities always finds the point estimate of +1, instead of −1. The estimate of a parameter that can take only two values is superconsistent, so I do not report a confidence interval. See Appendix B for details on estimation and inference.

second weighting scheme uses the weights

$$w_c = \text{sales of car model},$$

The third weighting scheme combines the previous two, as in

$$w_c = \frac{\text{sales of car model}}{\#\text{car parts for model in market } d}.$$

Compared to Table 1, Table 4 shows less of a role for supplier model specialization and much more of a role for assembler model HHI specialization. In all three of the specifications, the lower bound for the confidence region for supplier HHI model specification is negative and large in magnitude relative to the +1 normalization for supplier parent-group specialization. The point estimate is negative in two of the three specifications. In all three specifications, the upper bound of the confidence region for assembler model specialization is unbounded. The point estimate for assembler model specialization is unbounded in two specifications and still quite large, at 202, for the other specification.

Also compared to Table 1, there is now statistical uncertainty about the sign of supplier brand specialization, although the point estimate is about the same as for parent-group specialization, which is normalized to +1. Similarly to Table 1, 0 is still in the confidence regions for assembler parent-group and brand specialization.

Overall, I emphasize Table 1 in the main text because of its higher statistical fit.

References

Romano, J. P. and A. M. Shaikh (2008), "Inference for identifiable parameters in partially identified econometric models." *Journal of Statistical Planning and Inference*, 138 (9), 2786–2807. [1, 2]

Sherman, R. P. (1993), "The limiting distribution of the maximum rank correlation estimation." *Econometrica*, 61 (1), 123–137. [2]