

Supplement to “Revealing a preference for mixtures: An experimental study of risk”

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APPENDIX A: EXPERIMENTAL INSTRUCTIONS

Each instruction page was read out loud and subjects were instructed not to click ahead. The full set of instructions appears below. After the first set of instructions, there was a short series of training examples. Static versions of the training tasks appear in Figures 13(a)–14(b). After the training tasks, payments were explained using each subject’s final training exercise. An example is depicted in Figure 15. Important points were emphasized as “things to remember.” Part I is for the convex choice tasks and part II is for the repeated discrete choice tasks. After all tasks from part I were completed, a second set of instructions was given for part II. Once the task that pays was determined, a subject could examine it to observe their choices from the task. An example of this page appears in Figure 16.

[Instructions:] PLEASE READ CAREFULLY AND DO NOT PRESS NEXT UNTIL INSTRUCTED TO DO SO.

This is a study about your own preferences. There are no right or wrong answers. This study has two parts. Part I has 79 tasks and Part II has 6 tasks. Once you have finished, we will pick a task at random as the **Decision-that-counts**. Since all decisions are equally likely to be chosen, you should approach each task as if it is the **Decision-that-counts**. Part II will be explained once you complete Part I.

For Part I, your objective in each of the 79 tasks is to pick the **Chance** that you like the most. Every task has a changing visual aid to assist with picking your preferred **Chance**.

In every task, you must choose among the different **Chances** of receiving three prizes. The three prizes are \$2, \$10, and \$30. Each **Chance** will assign different chances to the three prizes. To determine your preferred chance, and all possibilities, you will have to move a slider. For all tasks, picking 100 will always give you the largest chance of the middle prize of \$10. As you move the slider toward 0, the chance of both \$30 and \$2 will increase. The largest chance of \$10 and how the other two chances change, as you move the slider, will be different for different tasks.

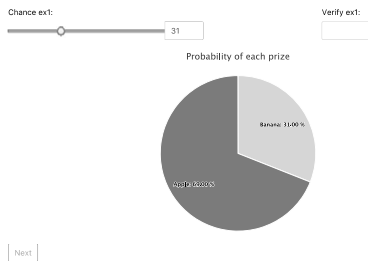
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Example 1

Please select your preferred **Chance**. You can do so, by moving the slider around. This example shows you how your chances could change if the extremes were two sure prizes. Once you have selected your preferred chance, you can proceed by pressing next at the bottom. Note, this Task **does not count!**

Available prizes are an Apple and a Banana.

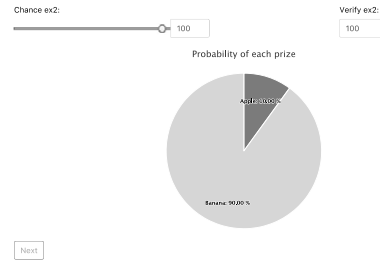


(a) Example 1

Example 2

Please select your preferred **Chance**. This example shows you what could happen when the two extremes are not sure prizes. Note, this Task **does not count!**

Available prizes are an Apple and a Banana.



(b) Example 2

FIGURE 13.

Most **Chances** might involve some risk. For example, a **Chance** could be a 25 in 100 chance of \$30, a 50 in 100 chance of \$10, and a 25 in 100 chance of \$2 while another **Chance** could be a 100 in 100 chance, or a sure prize, of \$10. To aid with your choice, there will be a changing display for every possible **Chance**. Therefore, for any **Chance** you will always be able to see the chance of receiving each of the prizes.

The next few pages contains 4 examples to familiarize you with “how this works.” The examples have prizes that are different from the main tasks. Take your time and make sure you understand “how it works.” We will not begin until everybody completes these examples and payments are explained. After the examples, there will be a detailed explanation of how payments will be determined.

Important: You must move the slider around and then verify your answer next to it. If the chosen **Chance** on the slider does not match the **Verified Chance** next to it, or if you do not move the slider around, you will not be allowed to proceed to the next task. Once you have picked a **Chance** for a given task and verified it, you will no longer be able to change it.

Very Important: For each task, the slider is a tool to help you decide the choice you like the best. Therefore, it is in your best interest to move it around to help you determine which **Chance** you like better.

[Examples 1–4]

See Figures 13–14.

[Earning Money]

See Figure 15.

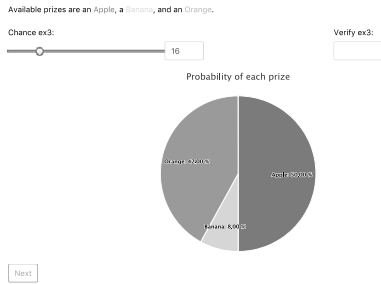
[Things to Remember:]

PLEASE READ CAREFULLY AND DO NOT PRESS NEXT UNTIL INSTRUCTED TO DO SO.

- You will complete 79 tasks for Part I. Part II has 6 tasks and will be explained after Part I.
- Different **Chances** will determine a different chance for each prize. All you have to do is pick the **Chance** you like the best.
- There is no right or wrong answer for any of these questions. We are interested in studying your preferences.
- Once all of your decisions have been made, we will choose one task and one decision as the decision-that-counts and will implement your preferred **Chance**.

Example 3

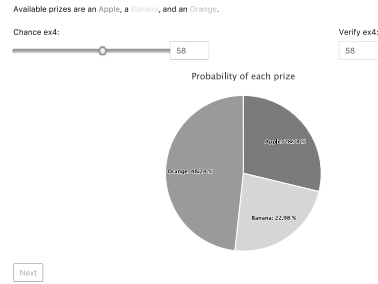
Please select your preferred **Chance**. This example has three possible prizes; however, the chance of apple does not change. Note, this Task **does not count**!



(a) Example 3

Example 4

Please select your preferred **Chance**. This example has three possible prizes. This task will be used to show you how your earnings will be determined. Note, this Task **does not count**!



(b) Example 4

FIGURE 14.

- Every decision is equally likely to be the **Decision-that-counts**. So, it is in your interest to treat each **Chance** as if it could be the one that determines your payoffs.

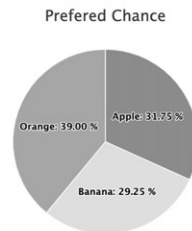
Earning Money:

PLEASE READ CAREFULLY AND DO NOT PRESS NEXT UNTIL INSTRUCTED TO DO SO.

First, we will roll two ten-sided dice to determine the **Decision-that-counts**. One for the ten's digit and another for the one's digit. Any number up to 85, the number of tasks, will count. Therefore any ten's die that is more than eight will have to be rerolled. A double zero will count as a hundred so this will also trigger a re-roll. In both cases, both dice will be rerolled.

Second, we will roll four ten-sided dice to determine your outcome. For example, lets say Example 4 was chosen as the **Decision that counts**. Then your preferred **Chance** gave you a 31.75 in 100 chance of an Apple, a 29.25 in 100 chance of a **Banana** and a 39.0 in 100 chance of an **Orange**. Then we would compute your earnings as follows:

1. We assign higher numbers to higher monetary outcomes. For this example assume Apples < Bananas < Oranges.
2. First die corresponds to the ten's digit, second die corresponds to the one's digit, third die is the first decimal and fourth die is the second decimal.
3. We split the chance of the potential outcomes according to your previously selected **Chance** of 25:



4. Therefore:
 - You get an Apple for any roll between 00.01 to 31.75, a 31.75 in 100 chance.
 - You get a Banana for any roll between 31.76 to 61.0, a 29.25 in 100 chance.
 - You get an Orange for any roll between 61.01 to 99.99, a 38.99 in 100 chance. You also get it if you roll 00.00, which gives the missing 00.01 in 100 chance.

Next

FIGURE 15. How to earn money.

- For each task, you must move the slider and verify your preferred **Chance**. Failure to move the slider or not match it will prevent you from moving to the next task.
- Once you have selected your preferred **Chance** and verified it, you will not be able to change it.
- The slider is a tool to help you determine your preferred **Chance**. Therefore, it is in your best interest to use it to evaluate all potential alternatives.

[Part I: Tasks 1–79] Occurs here

PLEASE READ CAREFULLY AND DO NOT PRESS NEXT UNTIL INSTRUCTED TO DO SO.

In this part of the study, you are asked again to choose your preferred **Chance**, just like you did in Part I. The difference between this part and Part I is that in this part you will see the same question three times, one after the other. That is, you will be asked one question; once you click the Next button, you will be asked the same question again; and once you click the Next button, the same question will appear for the third time. Once you click the Next button, then a new question will appear, which will also be asked three times. There are a total of 2 questions, each asked three times, for a total of 6 tasks.

Another difference is that you will only be able to choose between two **Chances**, 0 or 100. Please remember that there is no right or wrong answer. Remember you must move the slider around and then verify your answer next to it. If the chosen **Chance** on the slider does not match the **Verified Chance**, or if you do not move the slider around, you will not be allowed to proceed to the next task. Once you have picked a **Chance** for a given task and verified it, you will no longer be able to change it.

[Part II: Tasks 80–85] Occurs here: see Figure 16.

APPENDIX B: PAY ONE DECISION INCENTIVES

We pay one decision following the intuition of Azrieli, Chambers, and Healy (2018, 2019) who build on the work of Karni and Safra (1987) and Segal (1990). We discuss how our payment scheme maps into these papers after describing how payment uncertainty is resolved. Uncertainty is resolved using physical randomization devices. Each task could be chosen for payment with equal probability. We reminded subjects of this during each task. Before starting the discrete choice tasks, subjects were informed they would face the same binary choice three times in

The Decision-that-Counts is Task 15

Your choice for the **Decision-that-counts** appears below. We will now proceed to select volunteers so we can determine your payment. This screen is here to remind you about your choices.

Your preferred **Chance** was a 19.8 in 100 chance of \$30.00, a 17.0 in 100 chance of \$10.00 and a 63.2 in 100 chance of \$2.00.

You preferred **Chance** was: 34.

Your selected chances are:

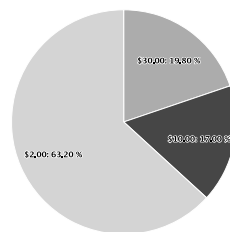


FIGURE 16. Decision-that-counts.

a row. They were also told there were two different binary choices for a total of six tasks. After the decisions from the convex and discrete choice tasks were completed, a ten-sided dice was rolled twice, once for the tens digit and another for the ones digit, to determine the task that counts. Any number above 85 was rerolled. Subjects entered the chosen task into the computer terminal and were reminded about their choice for that task and the resulting distribution over payoffs from that choice.

To determine payoffs for the chosen task, four ten-sided dice were rolled. So a roll of 5, 6, 7, and 9 would give 56.79%. The chosen distribution induced at most three intervals $[0, x)$, $[x, y)$, and $[y, 1]$ for the low, middle, and high prize, respectively. When a binary task was chosen for payment or the convex choice was at a corner, only two intervals applied to the relevant prizes. The interval that contained the number determined which outcome was obtained. To induce greater variation in outcomes, four sets of four dice were rolled. Each set of four dice applied to at most six subjects.

Finally, we relate the resolution of lotteries from the experiment to the pay one scheme discussion in [Azrieli, Chambers, and Healy \(2018\)](#). Here, we assume the die roll to select a task that pays induces a compound lottery over the objective lotteries used in the experiment. Thus, the result of the die roll can be viewed as a state using the notation of [Azrieli, Chambers, and Healy \(2018\)](#). We assume that the individual satisfies first-order stochastic dominance over the prizes in this state space. Finally, we assume that the individual is able to internally randomize according to their most preferred distribution following the interpretation of [Machina \(1985\)](#). Thus, we assume each “choice” in the repeated discrete choice tasks is an independent draw from an individual’s most preferred distribution. This last assumption is implicit in [Agranov and Ortoleva \(2017\)](#).

APPENDIX C: REVEALED PREFERENCE RESULTS

Here, we provide a revealed preference characterization for a utility model of risk preferences that respects first-order stochastic dominance for lotteries with three monetary prizes. The results are similar to those of [Varian \(1982\)](#), [Forges and Minelli \(2009\)](#), [Heufer \(2013\)](#), and [Cerrei-Vioglio, Dillenberger, and Ortoleva \(2018\)](#). We consider risk preferences over lotteries with three monetary prizes that are ordered $x_L < x_M < x_H$. The probability that the prizes x_L , x_M , and x_H occur are denoted p_L , p_M , and p_H , respectively. Moreover, let $\Delta = \{p \in \mathbb{R}_+^3 \mid p_L + p_M + p_H = 1\}$ be the probability simplex for three prize lotteries.

We consider budget sets generated by convex combinations of a numeraire lottery p^N and extreme lottery p^E . Here, the budgets are given by

$$B(p^N, p^E) = \left\{ p = (p_L, p_M, p_H) \in \mathbb{R}_+^3 \mid \begin{array}{l} p_H - r(p^N, p^E)p_L \leq p_H^N - r(p^N, p^E)p_L^N \\ | p_L + p_M + p_H = 1 \end{array} \right\},$$

where $r(p^N, p^E) = \frac{p_H^E - p_H^N}{p_L^E - p_L^N}$ is the relative price of the numeraire lottery described in the main text. Recall, we consider $r(p^N, p^E) \in \mathbb{R}_{++}$ since we are only considering budget sets that do not have lotteries ordered by FOSD.

Let $\{p^t, B(p^{N,t}, p^{E,t})\}_{t=1}^T$ be a data set generated by an individual choosing the lottery p^t from the budget $B(p^{N,t}, p^{E,t})$ where $p^{N,t}$ is the numeraire lottery from the t th task and $p^{E,t}$ is the extreme lottery from the t th task. We will often abbreviate the price so that $r^t = r(p^{N,t}, p^{E,t})$. Our main focus is to find conditions that determine when the choices an individual makes can be described by a utility function that satisfies first-order stochastic dominance. Recall that the lottery

$p' = (p'_L, p'_M, p'_H)$ first-order stochastically dominates the lottery $p = (p_L, p_M, p_H)$ when $p'_H \geq p_H$ and $p'_H + p'_M \geq p_H + p_M$ with at least one inequality strict.

DEFINITION 1. The data set $\{p^t, B(p^{N,t}, p^{E,t})\}_{t=1}^T$ is first-order stochastic dominance rationalized by a utility function when there exists a utility function $V : \Delta \rightarrow \mathbb{R}$ such that:

1. For all $t \in \{1, \dots, T\}$,

$$p^t \in \operatorname{argmax}_{p \in B(p^{N,t}, p^{E,t})} V(p);$$

2. For all p' that first-order stochastically dominate p ,

$$V(p') > V(p).$$

Now we define revealed preference relations for the lotteries above. We say that a lottery p^t is *directly revealed preferred* to the lottery p^s , denoted $p^t R p^s$, if $p_H^s - r^t p_L^s \leq p_H^{N,t} - r^t p_L^{N,t}$. We are able to use the terms for the numeraire lottery to define a revealed preference relation when looking for a first-order stochastic dominance utility rationalization since the lottery chosen from budget t must satisfy

$$p_H^t - r^t p_L^t = p_H^{N,t} - r^t p_L^{N,t}.$$

To see this, suppose by contradiction that

$$p_H^t - r^t p_L^t < p_H^{N,t} - r^t p_L^{N,t}.$$

In this case, one can find a lottery p' that first-order stochastically dominates p^t and is still feasible in $B(p^{N,t}, p^{E,t})$. For example, let $p'_H > p_H^t$, $p'_H + p'_L = p_H^t + p_L^t$, and

$$p'_H - r^t p'_L = p_H^{N,t} - r^t p_L^{N,t}.$$

This lottery first-order stochastically dominates p^t , which contradicts the data coming from a utility function that satisfies first-order stochastic dominance. Next, we say a lottery p^t is *directly strictly revealed preferred* to the lottery p^s , denoted $p^t P p^s$, if $p_H^s - r^t p_L^s < p_H^{N,t} - r^t p_L^{N,t}$. We say that lottery p^t is *revealed preferred* to the lottery p^s , denoted $p^t R^* p^s$, if there exists a sequence $\{p^m\}_{m=1}^M$ with $M \geq 2$ such that $p^1 = p^t$ and $p^M = p^s$ such that $p^m R p^{m+1}$ for all $m = 1, \dots, M-1$. Similarly, we say that the lottery p^t is *strictly revealed preferred* to the lottery p^s , denoted $p^t P^* p^s$, if there exists a sequence $\{p^m\}_{m=1}^M$ with $M \geq 2$ such that $p^1 = p^t$ and $p^M = p^s$ such that $p^m R p^{m+1}$ for all $m = 1, \dots, M-1$ where for some $m \in \{1, \dots, M-1\}$ there is a strict preference revelation so that $p^m P p^{m+1}$.

The revealed preference relation generated from the data set is *acyclic* when

$$p^t R^* p^s \text{ implies } \text{not } p^s P p^t.$$

We mention that $\text{not } p^s P p^t$ is the same as $p'_H - r^s p'_L \geq p_H^s - r^s p_L^s$. Acyclicity of a revealed preference relation is similar to the generalized axiom of revealed preference from Varian (1983). In particular, acyclicity requires that a researcher cannot find a cycle of lotteries such that a lottery is strictly revealed preferred to itself. We say there is a cycle when there exists a sequence $\{t_m\}_{m=1}^M$ with $M \geq 2$ where $t_m \in \{1, \dots, T\}$ such that $p^{t_m} R p^{t_{m+1}}$ for all $m = 1, \dots, M-1$ and $p^{t_M} P p^{t_1}$.

PROPOSITION 1. The following are equivalent for the data set $\{p^t, B(p^{N,t}, p^{E,t})\}_{t=1}^T$ when all $r^t > 0$:

1. *The data set is first-order stochastic dominance rationalized;*
2. *The revealed preference relations generated from R and P satisfy acyclicity;*
3. *For all $t \in \{1, \dots, T\}$, there exist numbers $u^t \in \mathbb{R}$ and $\lambda^t \in \mathbb{R}_{++}$ such that for all $s, t \in \{1, \dots, T\}$*

$$u^s \leq u^t + \lambda^t (p_H^s - p_H^{N,t} - r^t (p_L^s - p_L^{N,t}));$$

4. *The data set is first-order stochastic dominance rationalized with a continuous and concave utility function.*

PROOF. (1) \implies (2) Suppose by contradiction that the data set is FOSD rationalized and the revealed preference relation has a cycle for observations $\{t_m\}_{m=1}^M$ where $t_m \in \{1, \dots, T\}$ so that

$$\text{for } m = 1, \dots, M-1 \quad p^{t_m} R p^{t_{m+1}} \quad \text{and} \quad p^{t_M} P p^{t_1}.$$

It follows that

$$V(p^{t_1}) \geq V(p^{t_2}) \geq \dots \geq V(p^{t_M}) > V(p^{t_1}),$$

where the strict inequality holds since $p^{t_M} P p^{t_1}$. However, this contradicts the existence of a well-defined utility function.

(2) \implies (3) This follows from the arguments in [Fostel, Scarf, and Todd \(2004\)](#), [Forges and Minelli \(2009\)](#), or [Chambers and Echenique \(2016\)](#).

(3) \implies (4) We construct a utility function from the numbers u^t and λ^t that rationalize the data. In particular, consider the function

$$\tilde{V}(p) = \min_{t \in \{1, \dots, T\}} \{u^t + \lambda^t (p_H - p_H^{N,t} - r^t (p_L - p_L^{N,t}))\}.$$

It follows immediately that $\tilde{V} : \Delta^2 \rightarrow \mathbb{R}$ is continuous and concave since it is the minimum of finitely many affine functions. It remains to show the function satisfies first-order stochastic dominance and satisfies the optimality properties.

First, we show that the function satisfies first-order stochastic dominance. To see this, note that if p' first-order stochastically dominates p , then $p'_H \geq p_H$ and $p'_H + p'_M \geq p_H + p_M$ with one inequality strict. Let $t' \in \{1, \dots, T\}$ be chosen so

$$\tilde{V}(p') = u^{t'} + \lambda^{t'} (p_H - p_H^{N,t'} - r^{t'} (p_L - p_L^{N,t'})).$$

We now show that first-order stochastic dominance holds. It follows that

$$\begin{aligned} \tilde{V}(p') &= u^{t'} + \lambda^{t'} (p'_H - p_H^{N,t'} - r^{t'} (p'_L - p_L^{N,t'})) \\ &> u^{t'} + \lambda^{t'} (p_H - p_H^{N,t'} - r^{t'} (p_L - p_L^{N,t'})) \\ &\geq \min_{t \in \{1, \dots, T\}} \{u^t + \lambda^t (p_H - p_H^{N,t} - r^t (p_L - p_L^{N,t}))\} \\ &= \tilde{V}(p), \end{aligned}$$

where the strict inequality follows since either $p'_H > p_H$ or $p'_H + p'_M > p_H + p_M$. When $p'_H > p_H$, the strict inequality holds since $\lambda^{t'} > 0$, $r^{t'} > 0$, and $0 \leq p'_L \leq p_L$. When $p'_H + p'_M > p_H + p_M$, this implies that $p'_L < p_L$ and the strict inequality holds since $\lambda^{t'} > 0$, $r^{t'} > 0$, and $0 \leq p_H \leq p'_H$.

Finally, we show that the lotteries p^s are optimal for the given budgets. Note that for any lottery $p \in B(p^{N,s}, p^{E,s})$ that

$$\begin{aligned}\tilde{V}(p) &= \min_{t \in \{1, \dots, T\}} \{u^t + \lambda^t (p_H - p_H^{N,t} - r^t (p_L - p_L^{N,t}))\} \\ &\leq u^s + \lambda^s (p_H - p_H^{N,s} - r^s (p_L - p_L^{N,s})) \\ &\leq u^s \\ &= \tilde{V}(p^s),\end{aligned}$$

where the second inequality follows since $p_H - r^s p_L \leq p_H^{N,s} - r^s p_L^{N,s}$ and $\tilde{V}(p^s) = u^s$ by the construction of the inequalities in (2).

(4) \implies (1) is immediate. \square

This proof follows from standard arguments in revealed preference. This is also related to work by Nishimura, Ok, and Quah (2017) who give conditions for when there is a preference relation that preserves a partial order such as first-order stochastic dominance. Relative to the more general revealed preference analysis from Nishimura, Ok, and Quah (2017), Heufer (2013), and Cerreia-Vioglio, Dillenberger, and Ortoleva (2018), we are able to get a continuous and concave utility representation because we focus on finite dimensional lotteries and the budget sets we use here are convex.

In order to measure violations of a first-order stochastic dominance rationalization, we use the Houtman–Maks index (HMI). The HMI is proposed in Houtman and Maks (1985) and suggests using the largest number of choices that can be rationalized as a measure of closeness. Here, we find the largest number of choices that admit a first-order stochastic dominance rationalization. We interpret a higher HMI to mean an individual is “closer” to a first-order stochastic dominance rationalization since more data is consistent with this type of preference relation. While finding the HMI is a difficult problem in general, Demuyneck and Rehbeck (2021) shows that a mixed integer linear programming can quickly find the largest number of choices that can be rationalized for general budget sets. We present how to compute the HMI in the following proposition.

PROPOSITION 2. *The HMI for the data set $\{p^t, B(p^{N,t}, p^{E,t})\}_{t=1}^T$ is computed by solving*

$$\begin{aligned}\text{HMI} &= \max_{\substack{A_t \in \{0,1\} \\ U_{t,v} \in \{0,1\} \\ u_t \in [0,1]}} \sum_{t=1}^T A_t, \\ \text{s.t. } & u_t - u_v \leq -\varepsilon + 2U_{t,v} \quad \forall t, v \in \{1, \dots, T\}, \\ & U_{t,v} - 1 \leq u_t - u_v \quad \forall t, v \in \{1, \dots, T\}, \\ & -(p_H^v - r^t p_L^v - p_H^{N,t} + r^t p_L^{N,t}) \leq -\delta + \beta(U_{t,v} + (1 - A_t)) \quad \forall t, v \in \{1, \dots, T\}, \\ & \beta(U_{v,t} + A_t - 2) \leq (p_H^v - r^t p_L^v - p_H^{N,t} + r^t p_L^{N,t}) \quad \forall t, v \in \{1, \dots, T\},\end{aligned}$$

where $0 < \varepsilon < \frac{1}{T}$, $\beta > \max_{t,v \in \{1, \dots, T\}} \{|p_H^v - r^t p_L^v - p_H^{N,t} + r^t p_L^{N,t}|\} + \alpha$, and $0 < \delta < \alpha$ where $\alpha = \min\{1, \min_{t,v} \{p_H^v - p_H^{N,t} - r^t (p_L^v - p_L^{N,t}) \mid p_H^v - p_H^{N,t} - r^t (p_L^v - p_L^{N,t}) > 0\}\}$.

This proposition follows from Demuyneck and Rehbeck (2021, Corollary 7). We compute the measure using the mixed integer linear solving from Optimization, LLC Gurobi (2021) through Matlab. The HMI for each individual is given in Table 4.

TABLE 4. Houtman–Maks index for all subjects.

(i)ndividual	HMI	i	HMI	i	HMI	i	HMI	i	HMI
1	70	31	76	61	71	91	62	121	74
2	74	32	79	62	60	92	68	122	68
3	76	33	72	63	79	93	79	123	79
4	76	34	73	64	54	94	79	124	73
5	78	35	68	65	76	95	75	125	78
6	76	36	66	66	58	96	65	126	72
7	75	37	65	67	74	97	74	127	77
8	63	38	77	68	73	98	63	128	78
9	75	39	70	69	78	99	75	129	69
10	74	40	75	70	72	100	69	130	75
11	79	41	76	71	60	101	61	131	70
12	75	42	72	72	77	102	77	132	76
13	77	43	63	73	73	103	77	133	78
14	73	44	79	74	75	104	79	134	70
15	68	45	74	75	76	105	79	135	72
16	71	46	72	76	72	106	72	136	77
17	71	47	77	77	65	107	74	137	76
18	78	48	76	78	70	108	79	138	67
19	63	49	71	79	74	109	69	139	71
20	52	50	71	80	68	110	65	140	75
21	74	51	65	81	70	111	78	141	78
22	69	52	71	82	76	112	70	142	73
23	72	53	79	83	71	113	68	143	68
24	76	54	72	84	76	114	77	144	79
25	71	55	75	85	78	115	72		
26	78	56	79	86	71	116	78		
27	71	57	66	87	75	117	67		
28	79	58	60	88	75	118	74		
29	77	59	72	89	75	119	64		
30	75	60	74	90	60	120	72		

C.1 Benchmark behavior

Following the discussion in the main text, we compare the number of violations in the data from subjects to benchmarks of alternative behavior following the suggestion of Bronars (1987). In particular, we generate simulated choices p^t from $B(p^{N,t}, p^{E,t})$ by choosing an α^t that generates choices following equation (3.1). Following Andreoni, Feldman, and Sprenger (2017), we also consider sampling over the distribution of choices in the population. Thus, let $\alpha^{i,t}$ be the choice of α by the i th individual in the t th task from the experiment. We generate simulated benchmarks the following four ways:

1. Let α^t be distributed $U[45, 55]$.
2. Let α^t be distributed $U[0, 100]$.
3. Let α^t be distributed uniformly over $\{\alpha^{i,t}\}_{i,t}$.
4. Let α^t be distributed uniformly over $\{\alpha^{i,t}\}_j$.

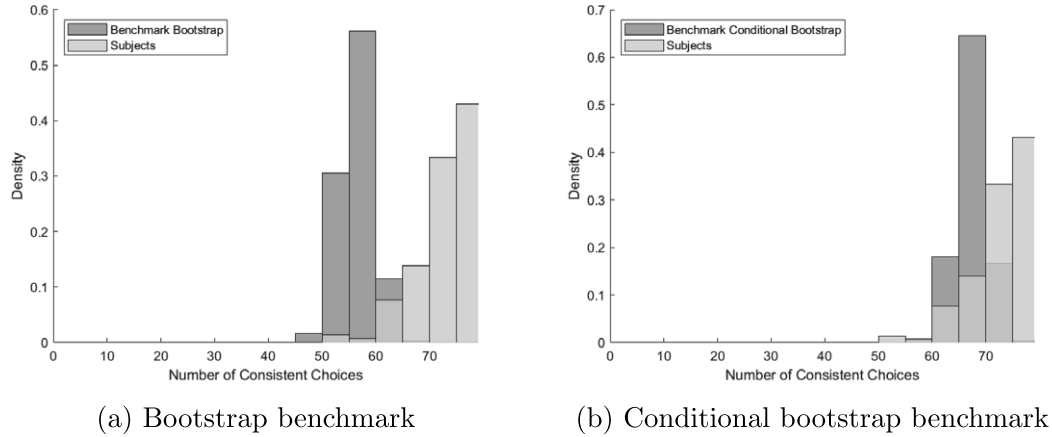


FIGURE 17. HMI probability density of subjects and benchmark behavior.

The first randomization scheme is our benchmark demand effects setting where an individual “trembles” around the initialized slide rule. The second randomization scheme is an implementation of a Bronars (1987) uniform random scheme and is our benchmark of noise. The third and fourth benchmarks follow Andreoni, Feldman, and Sprenger (2017) who suggest comparing behavior of individuals relative to randomness that is likely to be generated by individuals. We call the third randomization scheme a bootstrap benchmark since it bootstraps from all choices. Similarly, we call the fourth randomization scheme a conditional bootstrap benchmark since it bootstraps from choices conditional on a budget. We plot the distribution of simulated HMI along with the distribution of subject HMI for the third and fourth randomization schemes in Figure 17.

We compare subject behavior to the bootstrap benchmark in Figure 17(a). We find that 137/144 (95.1%) subjects are “closer” to a well-defined preference than 95% of the simulated bootstrap benchmark behavior according to the HMI. Moreover, we reject the null that the distribution of subject HMI and the bootstrap benchmark are the same according to the Wilcoxon rank sum test ($p < 0.0001$) and according to the Kolmogorov–Smirnov test ($p < 0.0001$). Comparing subject behavior to the conditional bootstrap benchmark in Figure 17(b), we find that 103/144 (71.5%) subjects are “closer” to a well-defined preference than 95% of the simulated conditional bootstrap benchmark behavior according to the HMI. Moreover, we reject the null that the distribution of subject HMI and conditional bootstrap benchmark are the same according to the Wilcoxon rank sum test ($p < 0.0001$) and according to the Kolmogorov–Smirnov test ($p < 0.0001$). In both cases, a majority of individuals outperform the benchmark. Thus, we conclude that the interface is able to elicit individual preference information.

APPENDIX D: ROBUSTNESS OF MIXING BEHAVIOR

We strengthen the definition of mixing to show that individuals are not making choices that are close to the boundaries of the budget sets. In particular, we define 2-mixing when $\alpha \in \{2, \dots, 98\}$ and 5-mixing when $\alpha \in \{5, \dots, 95\}$. Our results remain qualitatively unchanged. Table 5 and Table 6 replicate Table 2 in the main text for the stronger mixing definitions. The behavior is qualitatively similar to Table 2.

TABLE 5. Percentage of choices at each numeraire lottery (2-mix).

	N1	N2	N3	N4	N5	N6	N7	N8	Total
2-Mix	32%	44%	50%	50%	45%	42%	30%	18%	44%
Numeraire	52%	35%	28%	26%	26%	25%	30%	31%	32%
Extreme	16%	20%	21%	24%	29%	33%	40%	51%	24%
Obs	1296	2736	2304	1872	1440	1008	576	144	11,376

We also check whether individuals only mix close to some expected utility choices. In particular, let α_{EU}^t be the choices of an expected utility maximizer in task t . We call someone an EU-1 maximizer when there exists an expected utility where $|\alpha^t - \alpha_{\text{EU}}^t| \leq 1$ for all budgets that are not indifferent. We call someone an EU-4 maximizer when there exists some value of expected utility where $|\alpha^t - \alpha_{\text{EU}}^t| \leq 4$ for all budgets that are not indifferent. Recall that we have only one individual who can be described by expected utility. We find this same individual is the only subject to satisfy EU-1 or EU-4.

APPENDIX E: ALL COMPARATIVE STATICS OF MIXING AND LOG RELATIVE PRICE

In this section, we present how mixing behavior changes with respect to the relative price of the numeraire lottery for choices from budgets with numeraire lotteries N1, N3, N4, N5, N6, and N7. These graphs are all displayed in Figure 18. The graphs all share the qualitatively similar feature of mixing increasing with the price of the numeraire up to some point and then decreasing. We also find that demand for the numeraire decreases as the relative price of the numeraire increases. Lastly, we find that demand for the extreme lottery increases as the relative price of the numeraire increases.

APPENDIX F: AGGREGATE DEMANDS

This section presents the average demand curves for each numeraire lottery. In particular, Figure 19 gives the average demand curves for numeraire lotteries N1, N3, N4, N5, N6, and N7. We see from these figures that regardless of which numeraire lottery we examine, average demand has a hyperbolic shape. We also give regression results for the specification

$$\alpha_{i,t}^{N_j} = \beta_j + \eta_j \log(r^t),$$

where $\alpha_{i,t}^{N_j}$ is the α chosen for task t from the N_j numeraire and $\log(r^t)$ is the log price at the t th trial. Note we allow a separate intercept and slope of demand for each numeraire lottery. The

TABLE 6. Percentage of choices at each numeraire lottery (5-mix).

	N1	N2	N3	N4	N5	N6	N7	N8	Total
5-Mix	31%	43%	49%	48%	43%	41%	29%	17%	42%
Numeraire	53%	37%	30%	28%	28%	26%	31%	32%	33%
Extreme	16%	21%	22%	24%	29%	33%	40%	51%	24%
Obs	1296	2736	2304	1872	1440	1008	576	144	11,376

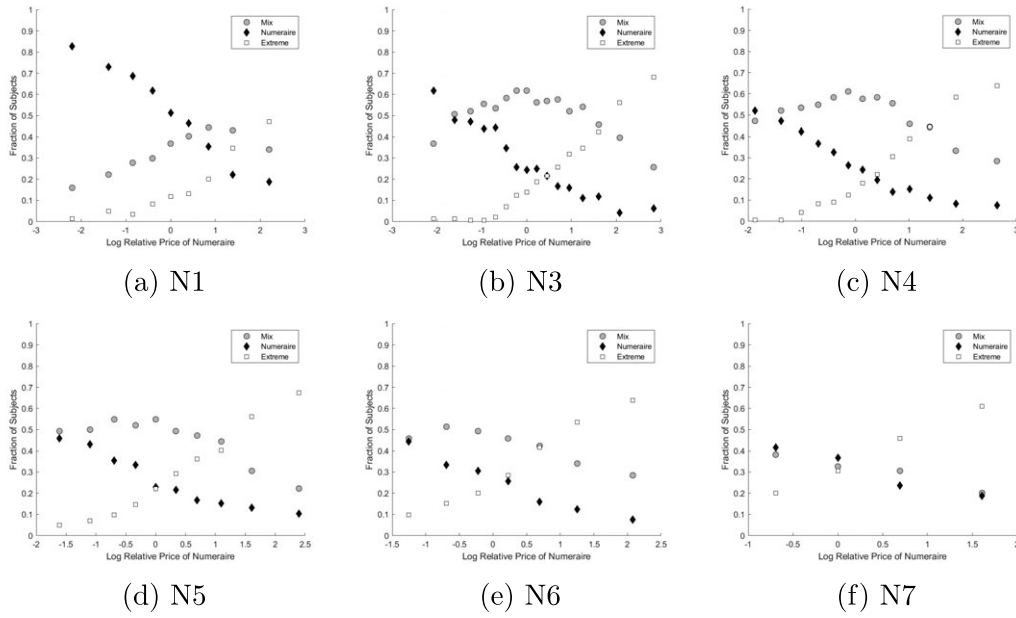


FIGURE 18. Log prices versus mixing behavior for remaining numeraire lotteries.

regression results are presented in Table 7. Here, we find that the aggregate demand as measured by the intercept decreases as the numeraire lottery “increases” with respect to FOSD. Moreover, we find that the slope on log relative prices is relatively consistent. One exception is for the nu-

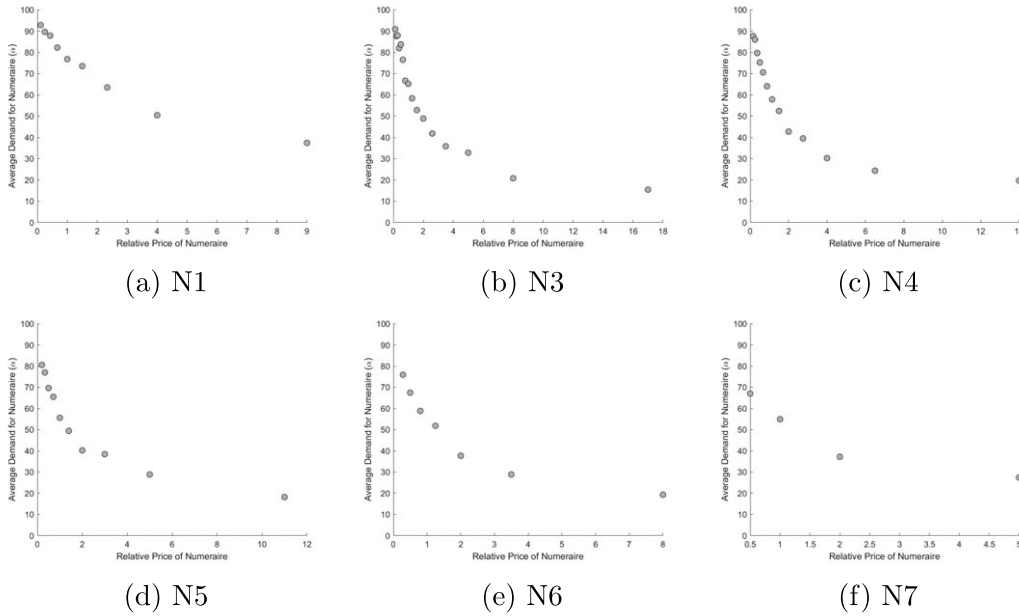


FIGURE 19. Average demands.

TABLE 7. Regression results of numeraire demand on log relative prices.

	N1	N2	N3	N4	N5	N6	N7
Intercept	72.67 (0.94)	61.62 (0.57)	62.38 (0.67)	59.73 (0.78)	56.35 (0.97)	53.88 (1.21)	53.81 (1.92)
$\log(r)$	-13.11 (0.72)	-17.48 (0.39)	-17.92 (0.50)	-17.58 (0.61)	-16.71 (0.80)	-17.94 (1.10)	-17.79 (2.04)
R^2	0.20	0.43	0.36	0.31	0.23	0.21	0.12
Adjusted R^2	0.20	0.43	0.36	0.31	0.23	0.21	0.12
Num Obs	1296	2736	2304	1872	1440	1008	576

Note: Regression results for all numeraire lottery demand for the linear-log specification, standard errors are in parentheses beneath each coefficient estimate.

numeraire lottery N1 where demand is high for low prices and the slope is much lower. This means that individuals are less elastic with respect to choosing lotteries that have extreme outcomes when the numeraire is low. This suggests that when individuals face a bad benchmark lottery with respect to FOSD, they may be less willing to choose an extreme lottery even when an extreme lottery has an attractive price (aka a low probability of getting the low prize). We note that the coefficients are significant for standard p-values.

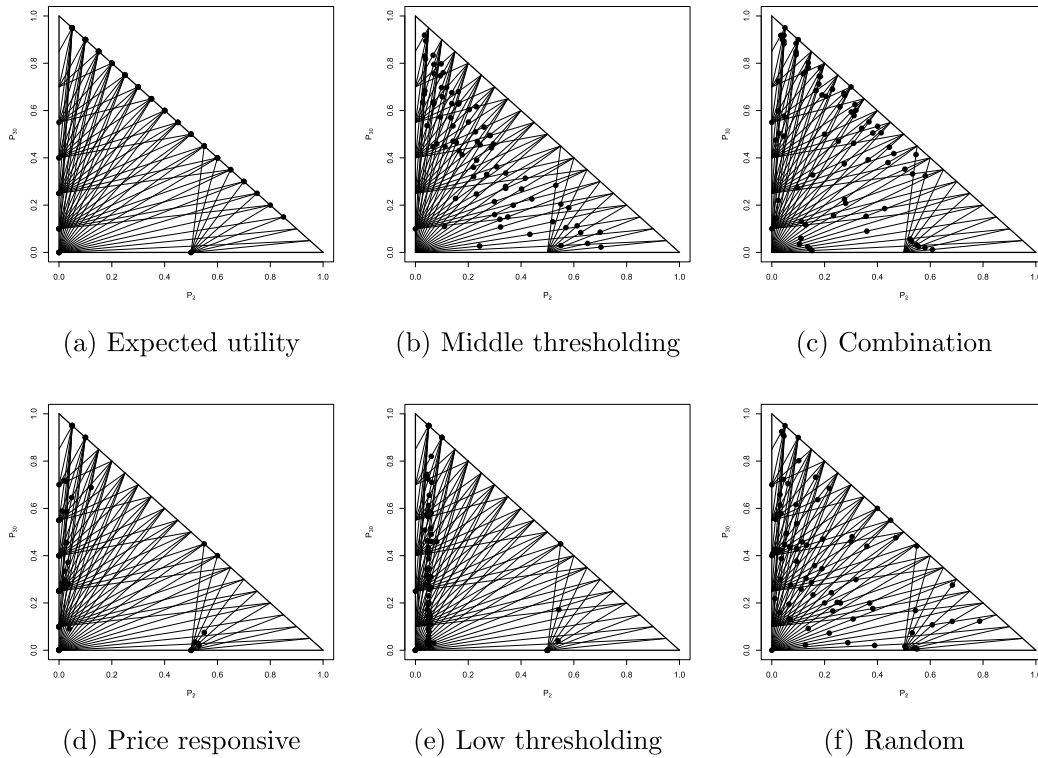


FIGURE 20. Example types of individual choice behavior for all endowments.

APPENDIX G: EXAMPLE TYPES: ALL BUDGET SETS

In Figure 20, we present all choices made for the individuals we used as examples of heterogeneous behavior in Section 5.2. We also present the behavior of all individuals from all budgets in Figure 21. In particular, each row in Figure 21 references a subject number at the start of the row and proceeds in order.

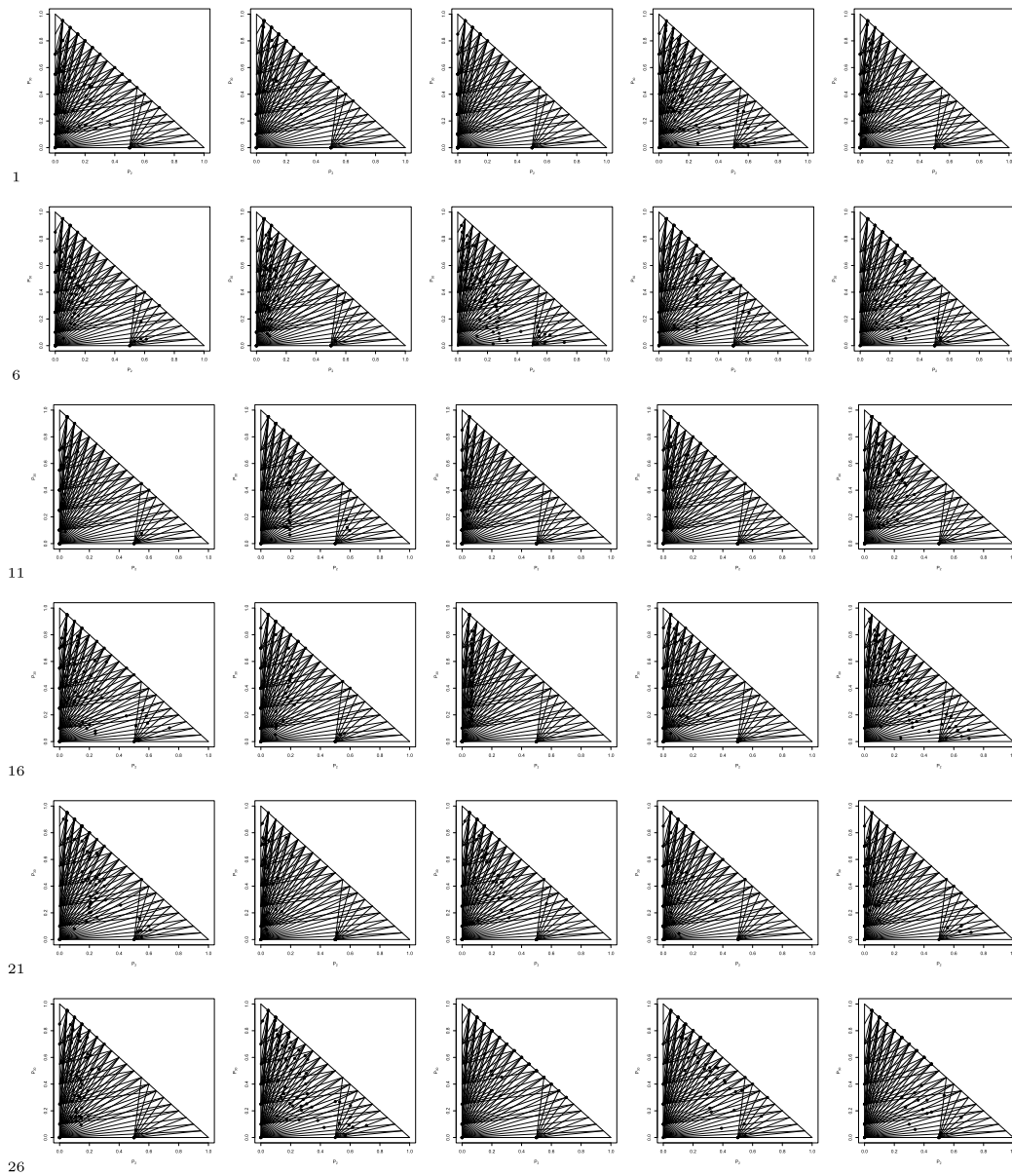


FIGURE 21. All choices from all subjects.

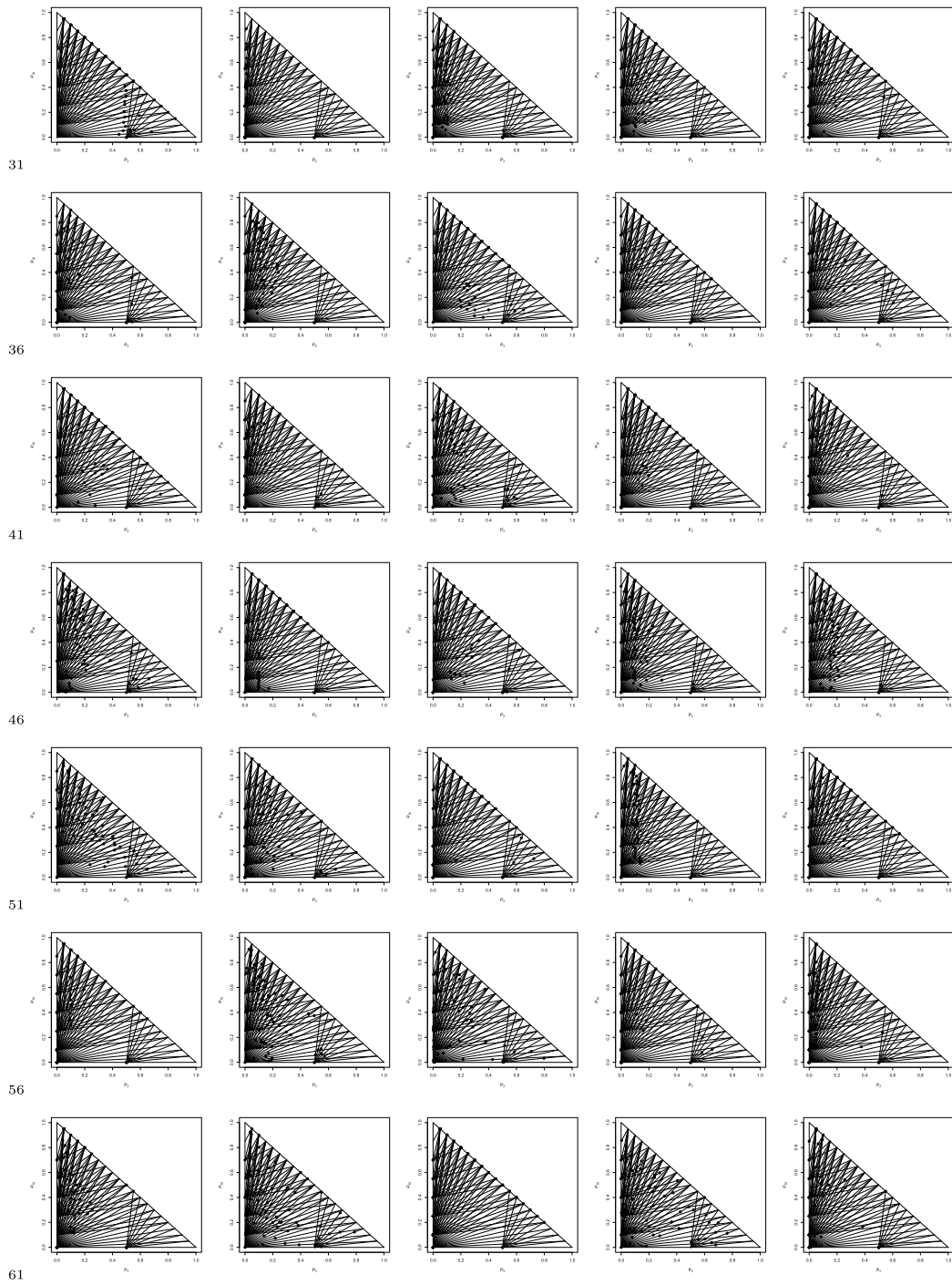
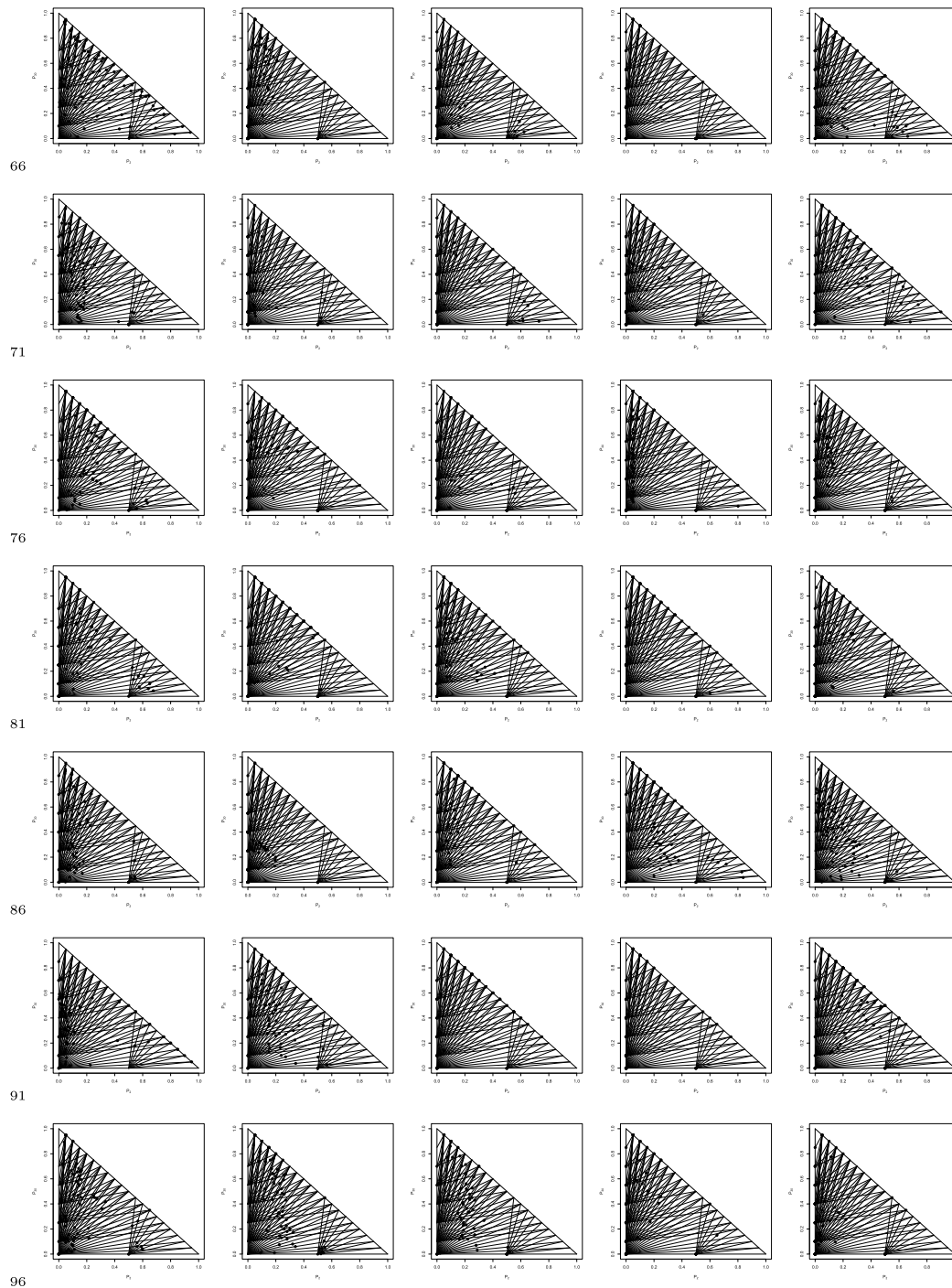


FIGURE 21. *Continued.*

FIGURE 21. *Continued.*

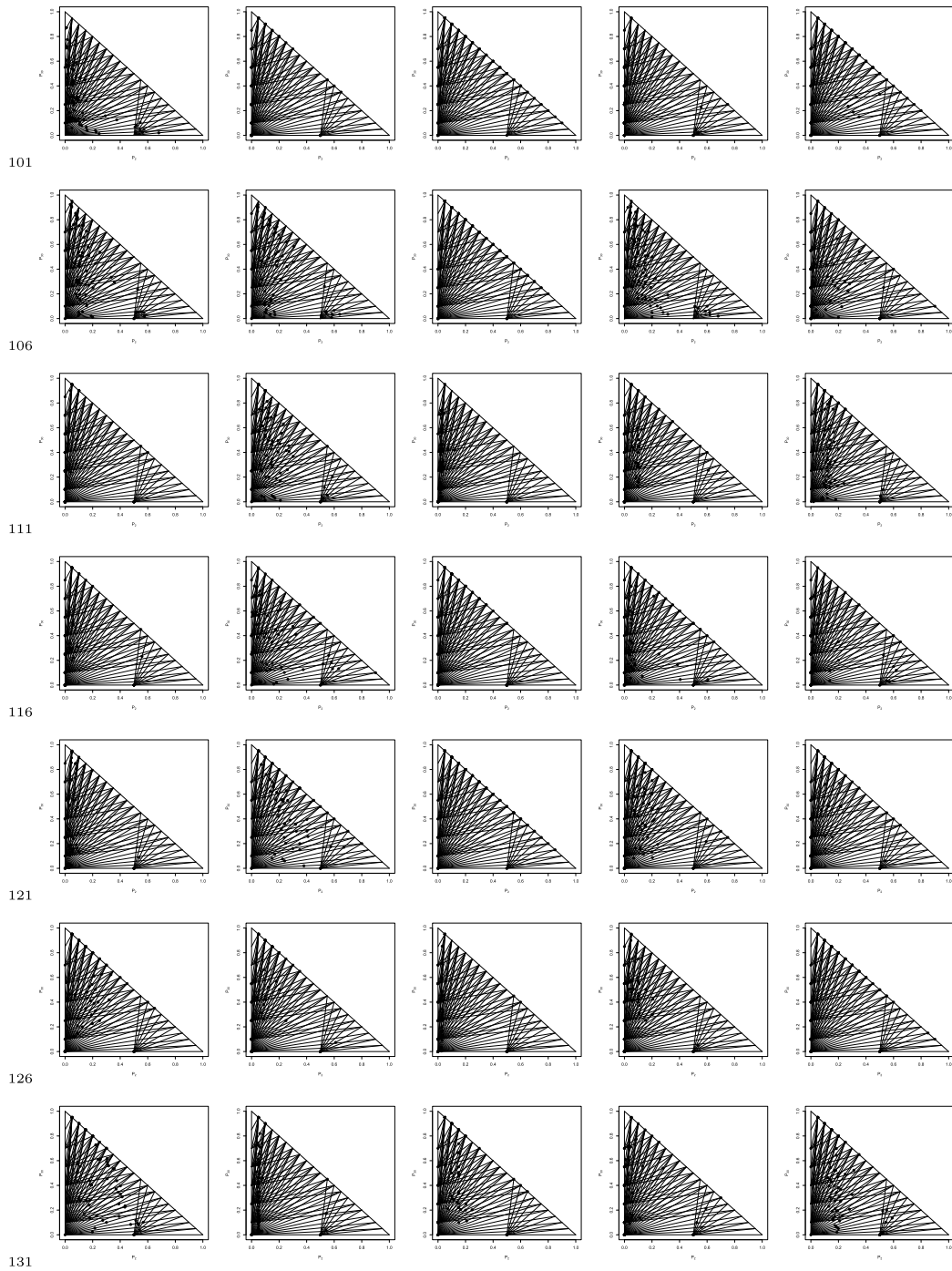
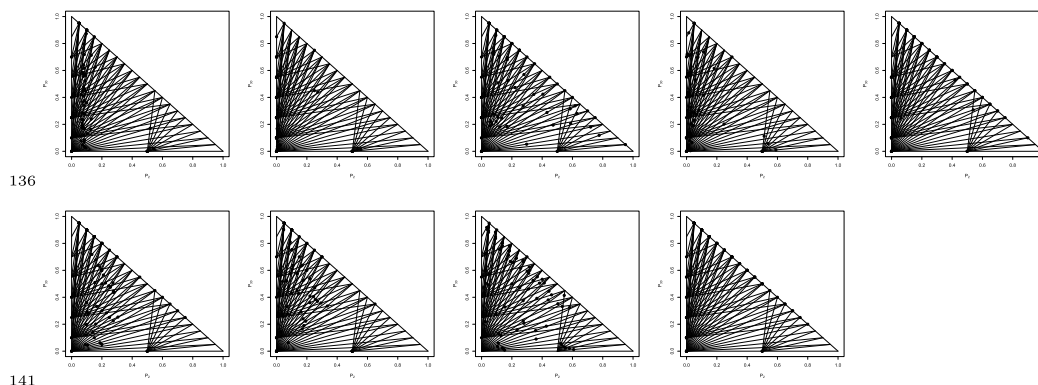


FIGURE 21. *Continued.*

FIGURE 21. *Continued.*

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