Supplement to "Inequality and dynamics of earnings and disposable income in Denmark 1987–2016"

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This document contains supplementary material for LINK.

APPENDIX OA-1: ALTERNATIVE DEFINITIONS OF KELLEY'S SKEWNESS

We check how different definitions of Kelley's skewness impact the results. Instead of considering the standard p90/p10 spread, we look at p95/p5 and p99/p1 for both earnings and disposable income:

$$\text{p95/p5}: \frac{(\text{p95–p50}) - (\text{p50–p5})}{\text{p95–p5}},$$
$$\text{p99/p1}: \frac{(\text{p99–p50}) - (\text{p50–p1})}{\text{p99–p1}}.$$

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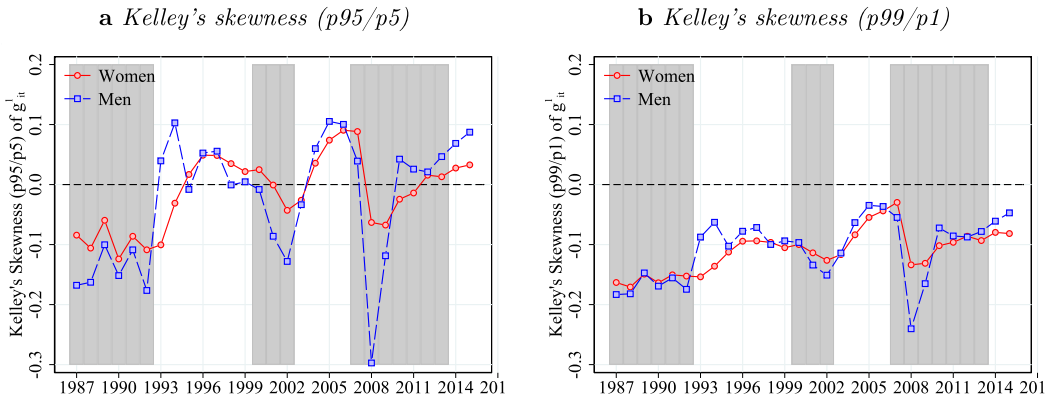**a** *Kelley's skewness (p95/p5)*   **b** *Kelley's skewness (p99/p1)*

FIGURE 28. Alternative Kelley's skewness of 1-year residual log earnings growth. *Notes*: Using residual one-year earnings changes, the figure plots the following variables against time: (a) Kelley's skewness calculated as $\frac{(p95-p50)-(p50-p5)}{p95-p5}$, (b) Kelley's skewness calculated as $\frac{(p99-p50)-(p50-p1)}{p99-p1}$. The shaded areas indicate recessionary periods with GDP growth below 2%.
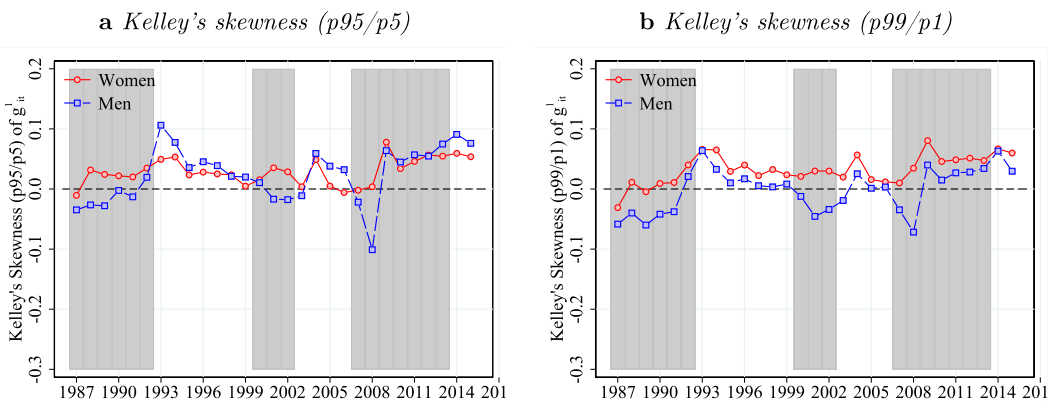
a *Kelley's skewness (p95/p5)*   **b** *Kelley's skewness (p99/p1)*

FIGURE 29. Alternative Kelley's skewness of 1-year residual log disposable income growth. *Notes*: Using residual 1-year disposable income changes, the figure plots the following variables against time: (a) Kelley's skewness calculated as $\frac{(p95-p50)-(p50-p5)}{p95-p5}$, (b) Kelley's skewness calculated as $\frac{(p99-p50)-(p50-p1)}{p99-p1}$. The shaded areas indicate recessionary periods with GDP growth below 2%.

OA-2.1 *How dispersion of residual log income growth correlates with log growth rate of GDP by permanent income rank*
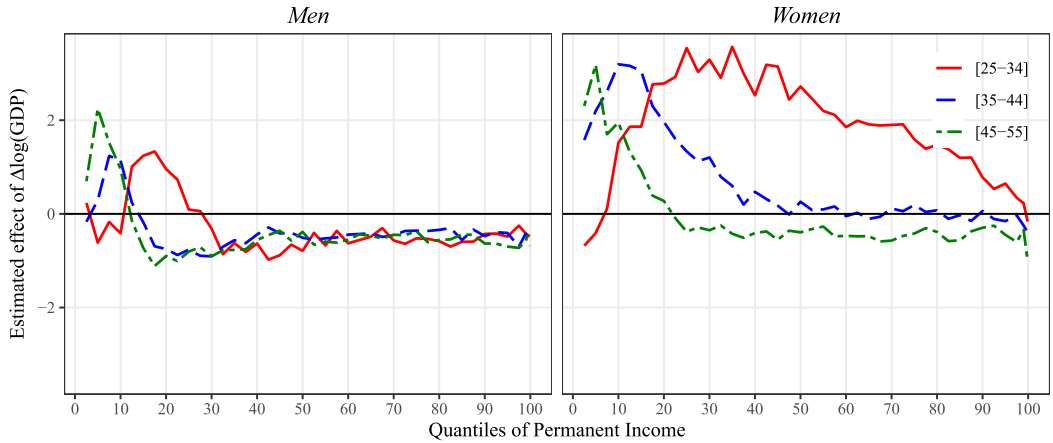

FIGURE 30. Dispersion, residual log earnings growth. *Notes*: The figure presents estimates of the slope parameter from a regression of the dispersion of residual log earnings growth on log growth rate of GDP and a linear time trend (cf. equation (1)), estimated by selected percentiles in the distribution of earnings for men and women and by age groups. Dispersion is calculated as p90–p10.
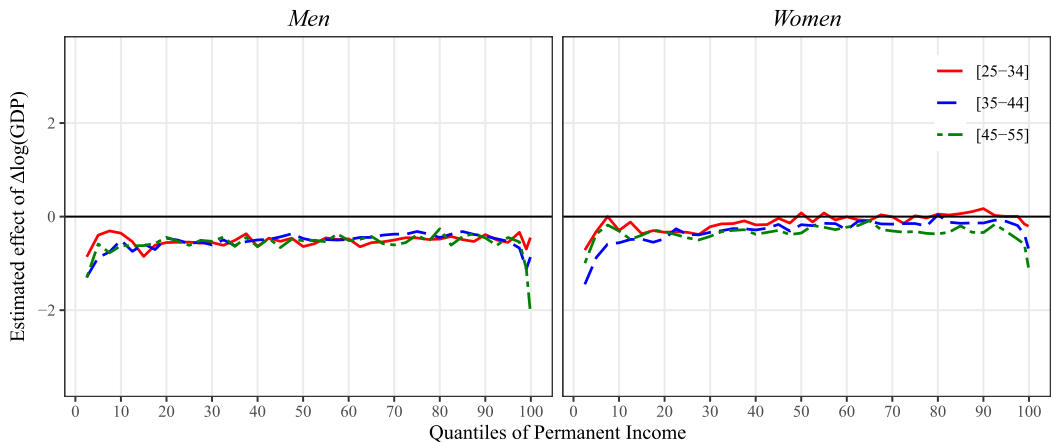

FIGURE 31. Dispersion, residual log disposable income growth. *Notes*: The figure presents estimates of the slope parameter from a regression of the dispersion of residual log disposable income growth on log growth rate of GDP and a linear time trend (cf. equation (1)), estimated by selected percentiles in the distribution of disposable income for men and women and by age groups. Dispersion is calculated as p90–p10.

OA-2.2 *How skewness of residual log income growth correlates with log growth rate of GDP by permanent income rank*
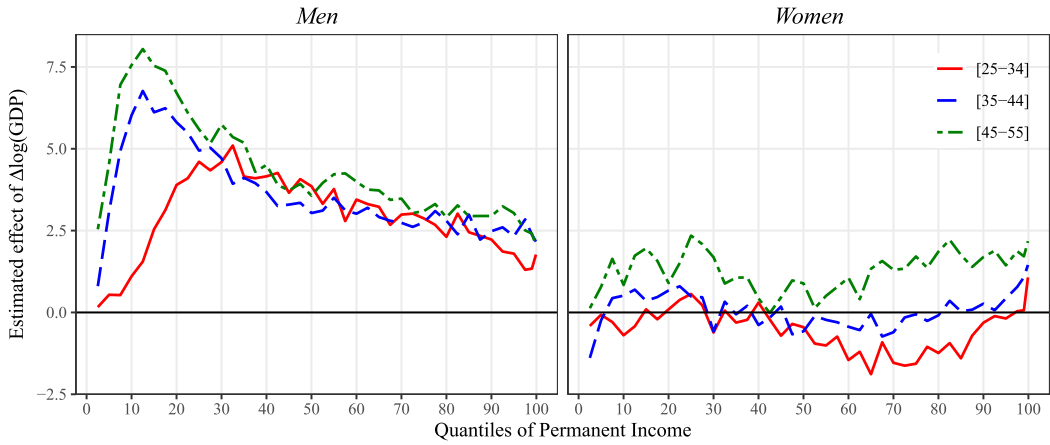

FIGURE 32. Skewness, residual log earnings growth rate. *Notes*: The figure presents estimates of the slope parameter from a regression of skewness of the distribution of residual log earnings growth rates on log growth rate of GDP and a linear time trend (cf. equation (1)), estimated by selected percentiles in the distribution of earnings for men and women and by age groups. Kelley's skewness is calculated as $\frac{(p90-p50)-(p50-p10)}{p90-p10}$.
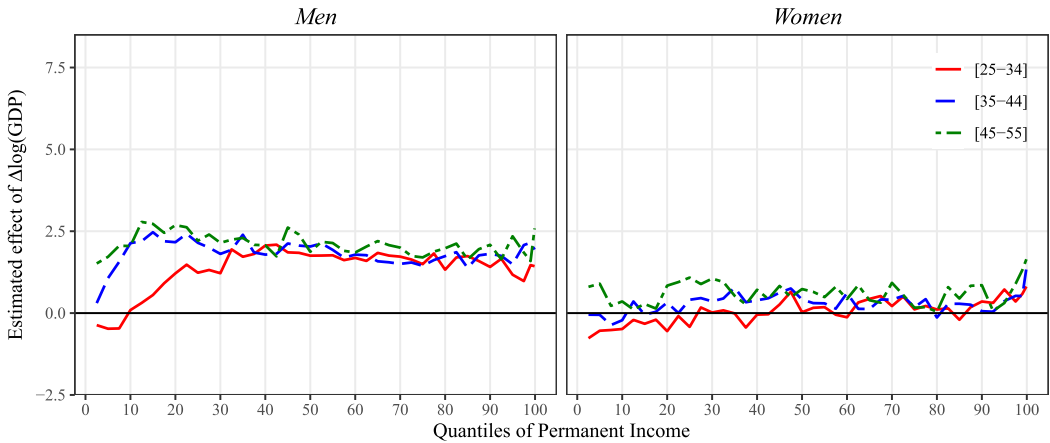

FIGURE 33. Skewness, residual log disposable income growth rate. *Notes*: The figure presents estimates of the slope parameter from a regression of skewness of the distribution of residual log disposable income growth rates on log growth rate of GDP and a linear time trend (cf. equation (1)), estimated by selected percentiles in the distribution of disposable income for men and women and by age groups. Kelley's skewness is calculated as $\frac{(p90-p50)-(p50-p10)}{p90-p10}$.

OA-2.3 *How kurtosis of residual log income growth correlates with log growth rate of GDP by permanent income rank*

FIGURE 34. Kurtosis, residual log earnings growth rate. *Notes*: The figure presents estimates of the slope parameter from a regression of kurtosis of the distribution of residual log earnings growth rates on log growth rate of GDP and a linear time trend (cf. equation (1)), estimated by selected percentiles in the distribution of earnings for men and women and by age groups. Excess Crow–Siddiqui kurtosis is calculated as $\frac{\text{p97.5–p2.5}}{\text{p75–p25}} - 2.91$.
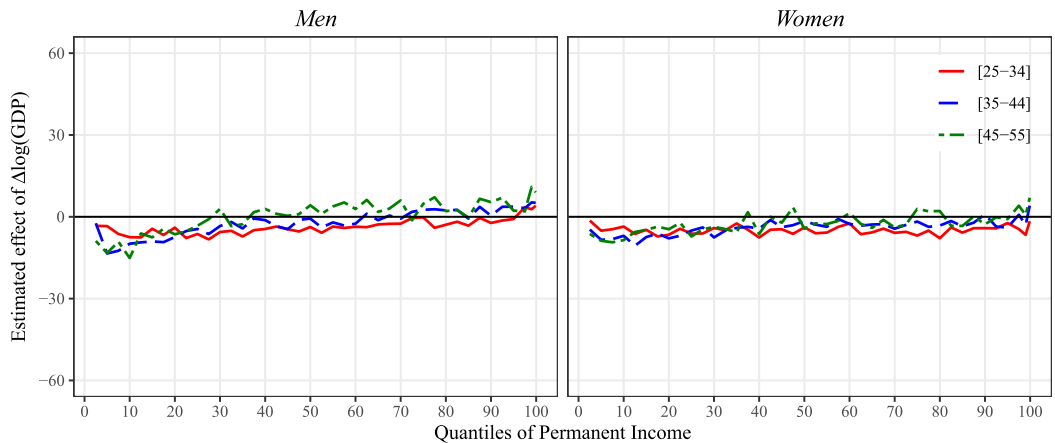

FIGURE 35. Kurtosis, residual log disposable income growth rate. *Notes*: The figure presents estimates of the slope parameter from a regression of kurtosis of the distribution of residual log disposable income growth rates on log growth rate of GDP and a linear time trend (cf. equation (1)), estimated by selected percentiles in the distribution of disposable income for men and women and by age groups. Excess Crow–Siddiqui kurtosis is calculated as $\frac{\text{p97.5–p2.5}}{\text{p75–p25}} - 2.91$.

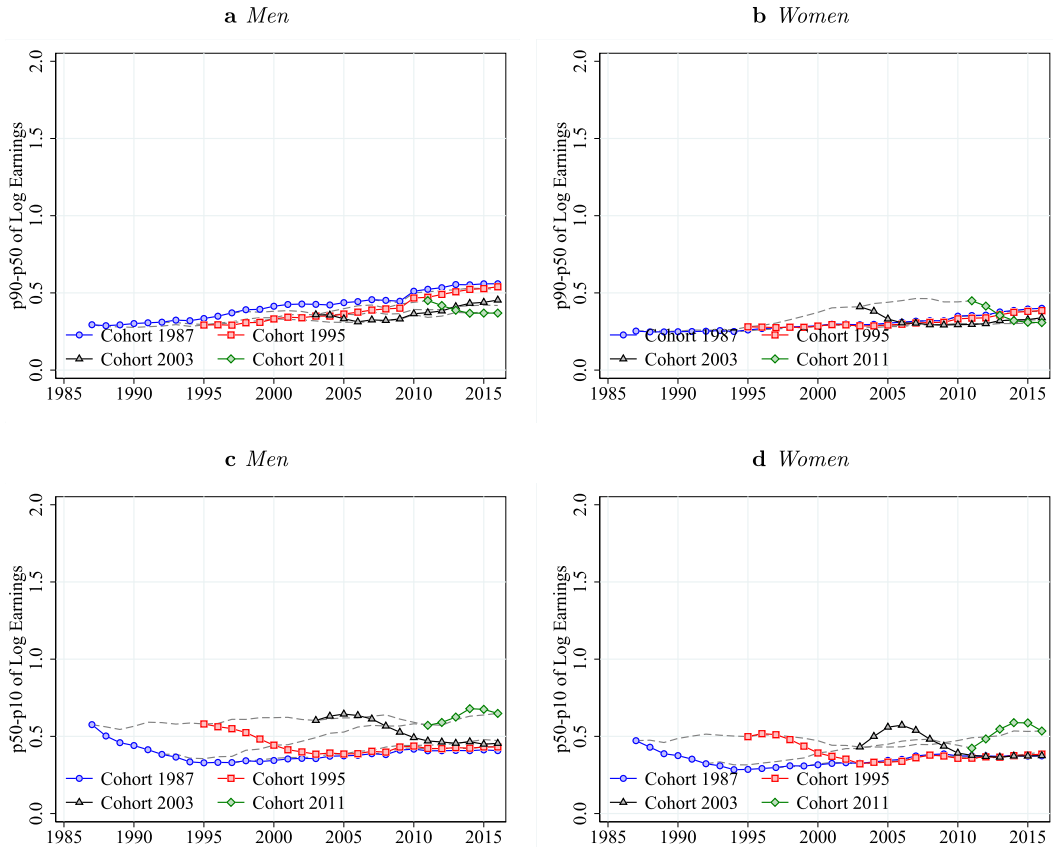APPENDIX OA-3: LIFE-CYCLE INEQUALITY OVER COHORTS, DISPOSABLE INCOME

FIGURE 36. Life-cycle inequality over cohorts, disposable income. *Notes*: The figure plots life–cycle profiles of dispersion of raw log disposable income. CS sample. The following variables are plotted against time over the life cycle for four selected cohorts (indexed by age 25): (a) Men: p50–p10, (b) Women: p50–p10, (c) Men: p90–p50, (d) Women: p90–p50.

APPENDIX OA-4: RESULTS BASED ON GROSS INCOME

Note: In 1994, multiple transfers changed status from tax-exempt income to taxable income, with a subsequent rise in level. This was done to ease comparability between earned income and transfers. In the figures below, this shows up as a spike in 1993 (since g_{it}^1 is forward looking).

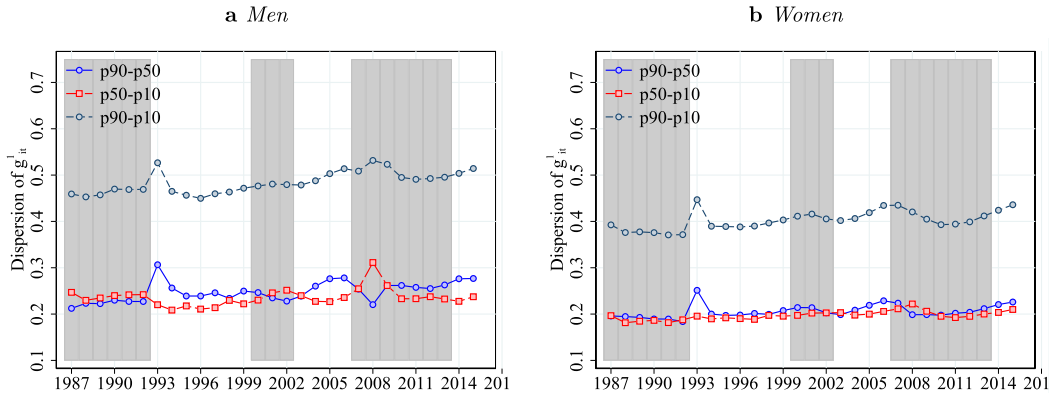**a** *Men*   **b** *Women*

FIGURE 37. Dispersion of distribution of 1-year residual log gross income growth rates. *Notes*: Dispersion of distribution of 1-year residual log gross income growth rates, $g^1_{it} = \varepsilon_{it+1} - \varepsilon_{it}$. LS sample. The figure plots the following variables against time: p90–p50 (blue), p50–p10 (red), and p90–p10 (grey) for men in panel a, and for women in panel b. The shaded areas indicate recessionary periods with GDP growth below 2%.
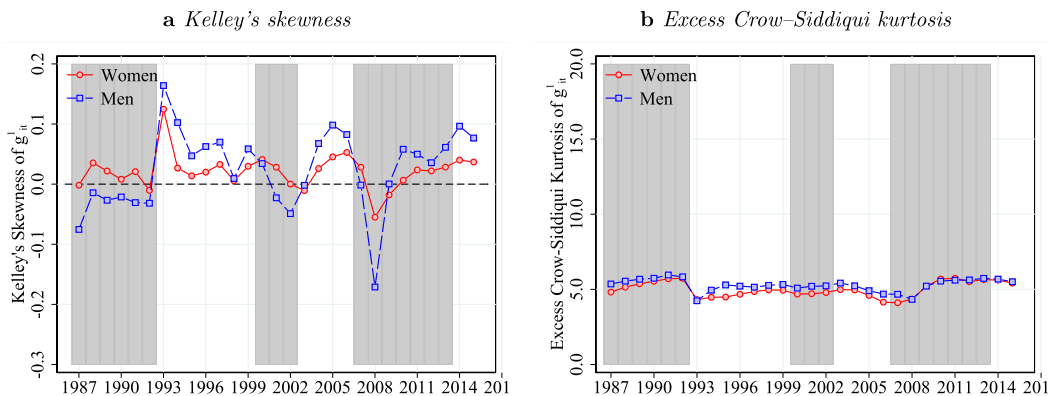

a *Kelley's skewness*   **b** *Excess Crow–Siddiqui kurtosis*

FIGURE 38. Skewness and kurtosis of distribution of 1-year residual log gross income growth rates. *Notes*: Skewness and kurtosis of distribution of 1-year residual log gross income growth rates, $g^1_{it} = \varepsilon_{it+1} - \varepsilon_{it}$. LS sample. The figure plots the following variables against time: (a) Men and Women: Kelley's skewness calculated as $\frac{(\text{p90–p50}) - (\text{p50–p10})}{\text{p90–p10}}$, (b) Men and Women: Excess Crow–Siddiqui kurtosis calculated as $\frac{\text{p97.5–p2.5}}{\text{p75–p25}} - 2.91$ where the first term is the Crow–Siddiqui measure of kurtosis and 2.91, corresponds to the value of this measure for a normal distribution. The shaded areas indicate recessionary periods with GDP growth below 2%.
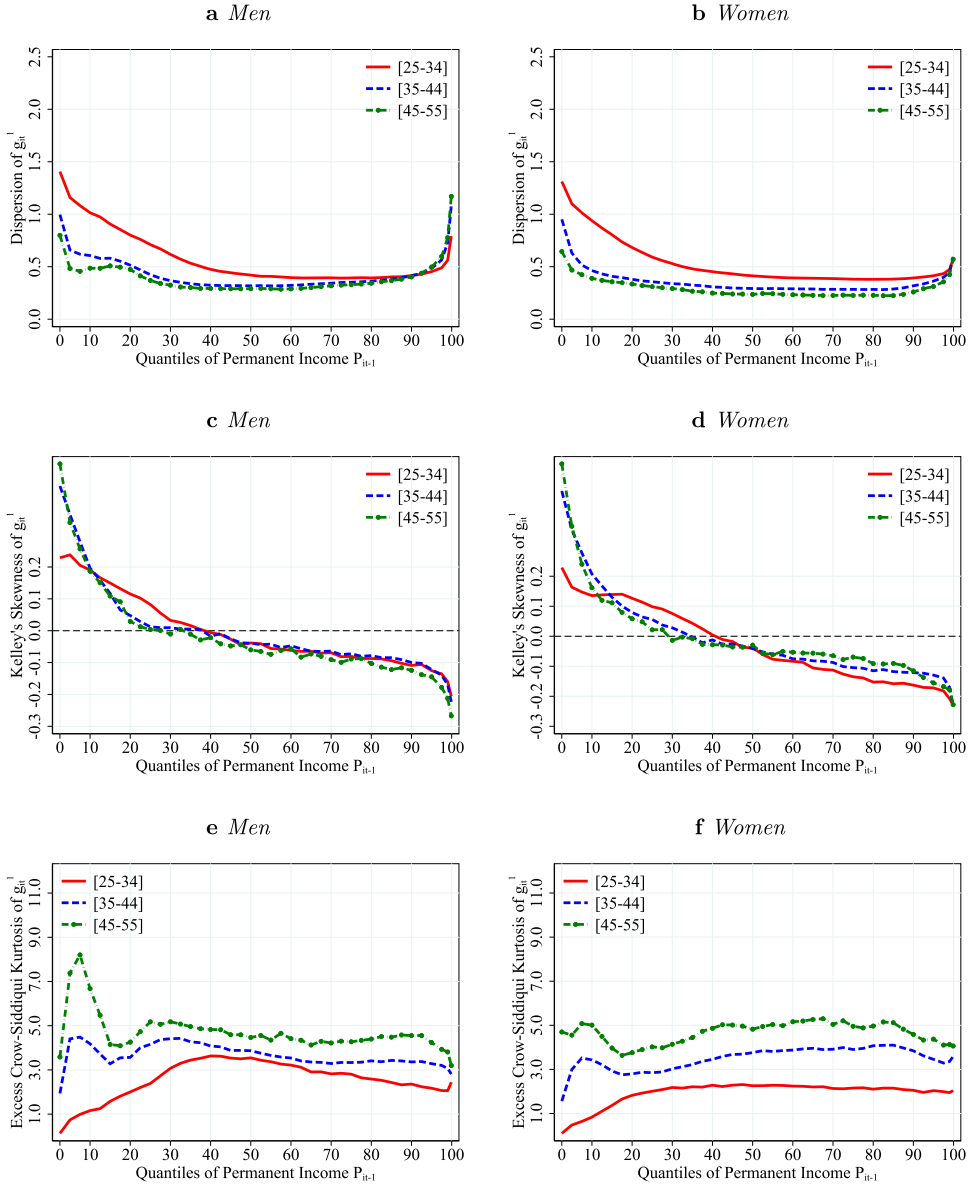
FIGURE 39. Dispersion, skewness and kurtosis of distribution of 1-year residual log gross income growth rates. *Notes*: Dispersion, skewness, and kurtosis of 1-year residual log gross income growth, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. H sample in the period 1997–2016. Permanent Income is based on three years of income, $t-4$, $t-3$, $t-2$. The figure plots the following variables against permanent income quantile groups for the 3 age groups (denoted by color): (a) Men: p90–p10, (b) Women: p90–p10, (c) Men: Kelley's skewness, (d) Women: Kelley's skewness, (e) Men: Excess Crow–Siddiqui kurtosis, (f) Women: Excess Crow–Siddiqui kurtosis. Kelley's skewness is calculated as $\frac{(\text{p90–p50})-(\text{p50–p10})}{\text{p90–p10}}$. Excess Crow–Siddiqui kurtosis is calculated as $\frac{\text{p97.5–p2.5}}{\text{p75–p25}} - 2.91$, where the first term is the Crow–Siddiqui measure of kurtosis, and 2.91 corresponds to the value of this measure for a normal distribution.
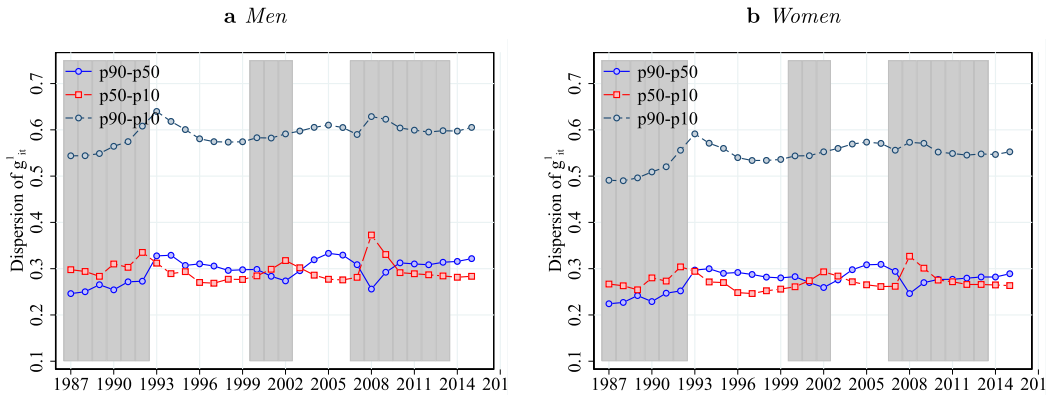
APPENDIX OA-5: RESULTS BASED ON HOUSEHOLD EARNINGS

FIGURE 40. Dispersion of distribution of 1-year residual log earnings growth rates, households. *Notes*: Dispersion of 1-year residual household log earnings growth rates, $g^1_{it} = \varepsilon_{it+1} - \varepsilon_{it}$. LS sample. The figure plots the following variables against time: p90–p50 (blue), p50–p10 (red), and p90–p10 (grey) for men in panel a, and for women in panel b. The shaded areas indicate recessionary periods with GDP growth below 2%.
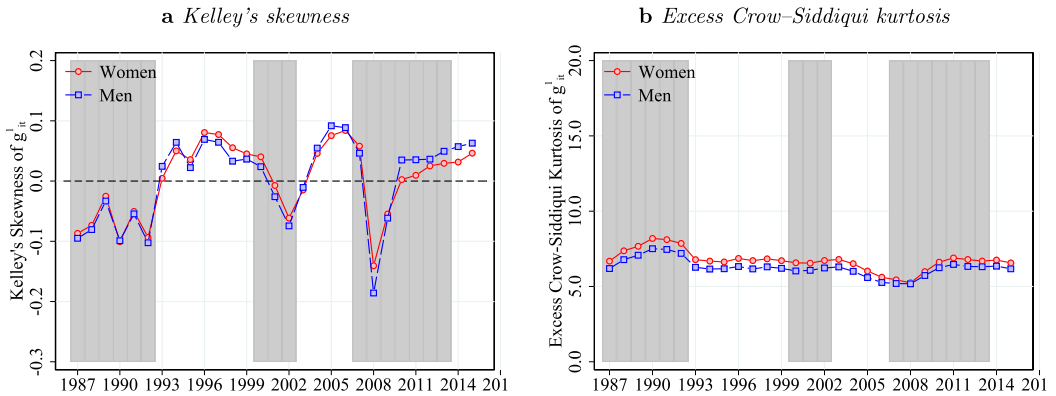

FIGURE 41. Skewness and kurtosis of distribution of 1-year residual log earnings growth rates, households. *Notes*: Skewness and kurtosis of distribution of 1-year residual household log earnings growth rates, $g^1_{it} = \varepsilon_{it+1} - \varepsilon_{it}$. LS sample. The figure plots the following variables against time: (a) Men and Women: Kelley's skewness calculated as $\frac{(p90-p50)-(p50-p10)}{p90-p10}$, (b) Men and Women: Excess Crow–Siddiqui kurtosis calculated as $\frac{p97.5-p2.5}{p75-p25} - 2.91$ where the first term is the Crow–Siddiqui measure of kurtosis and 2.91, corresponds to the value of this measure for a normal distribution. The shaded areas indicate recessionary periods with GDP growth below 2%.
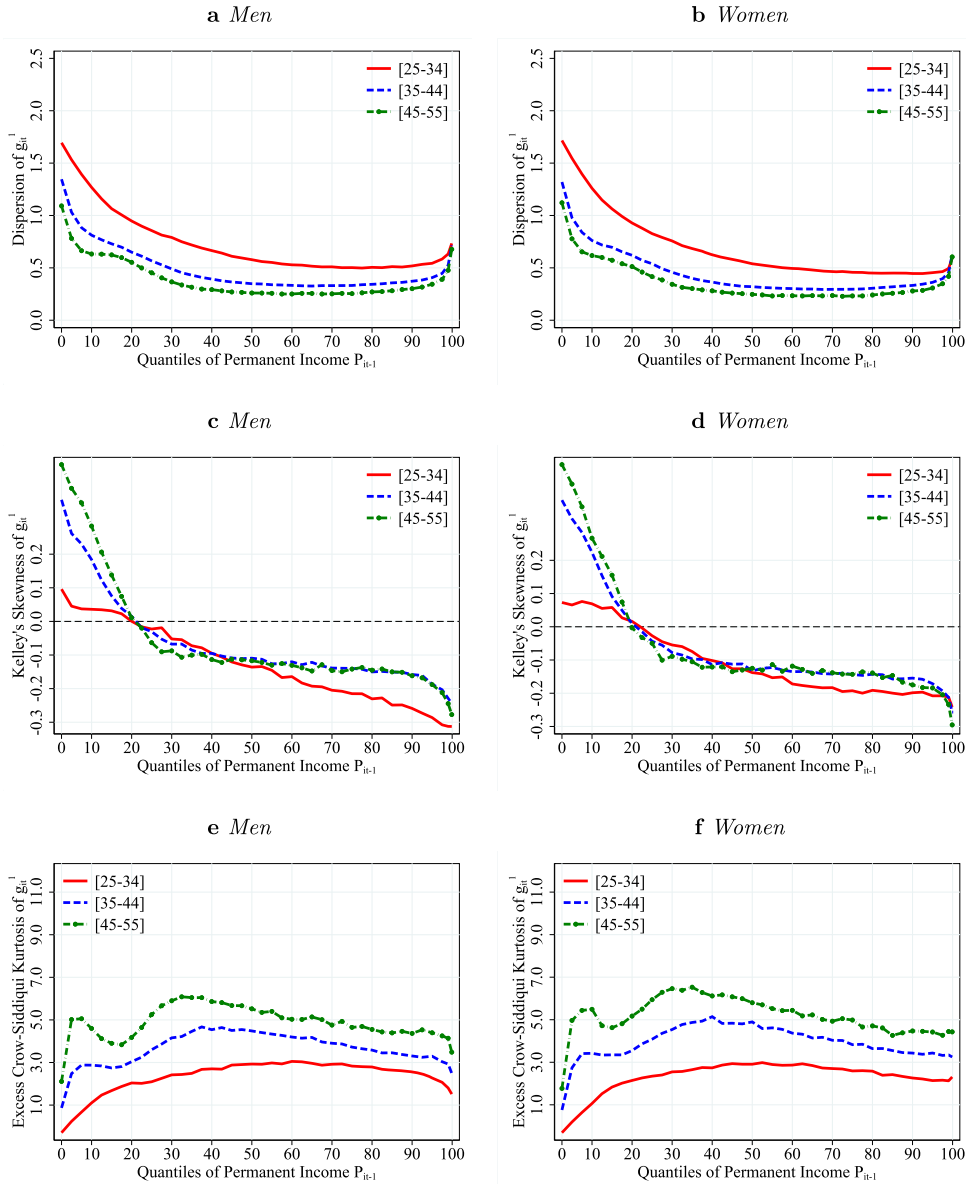
FIGURE 42. Dispersion, skewness and kurtosis of the distribution of 1-year residual log earnings growth rates, households. *Notes*: Dispersion, skewness, and kurtosis of distribution of 1-year residual household log earnings growth rates, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. H sample in the period 1997–2016. Permanent Income is based on three years of income, $t-4$, $t-3$, $t-2$. The figure plots the following variables against permanent income quantile groups for the 3 age groups (denoted by color): (a) Men: p90–p10, (b) Women: p90–p10, (c) Men: Kelley's skewness, (d) Women: Kelley's skewness, (e) Men: Excess Crow–Siddiqui kurtosis, (f) Women: Excess Crow–Siddiqui kurtosis. Kelley's skewness is calculated as $\frac{(p90-p50)-(p50-p10)}{p90-p10}$. Excess Crow–Siddiqui kurtosis is calculated as $\frac{p97.5-p2.5}{p75-p25} - 2.91$, where the first term is the Crow–Siddiqui measure of kurtosis, and 2.91 corresponds to the value of this measure for a normal distribution.

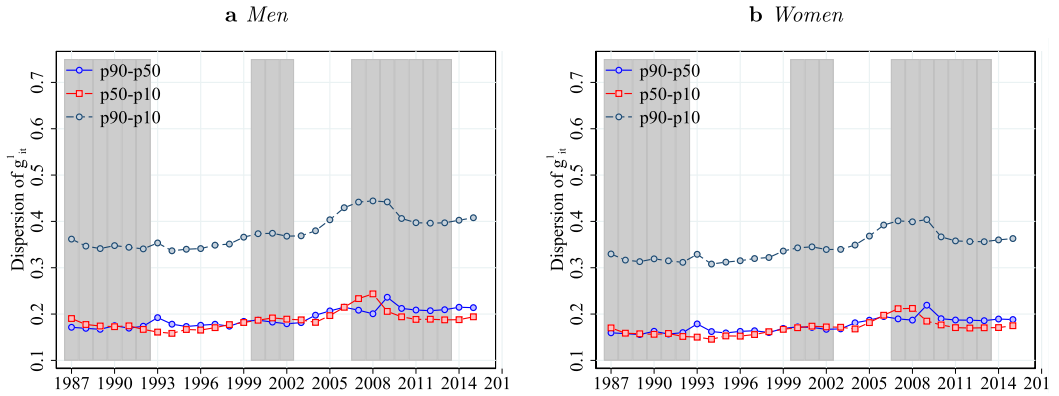APPENDIX OA-6: RESULTS BASED ON HOUSEHOLD DISPOSABLE INCOME

a *Men*   **b** *Women*

FIGURE 43. Dispersion of the distribution of 1-year residual log disposable income growth rates, households. *Notes*: Dispersion of 1-year residual household log disposable growth rates, $g_{it}^1 = \varepsilon_{it+1} - \varepsilon_{it}$. LS sample. The figure plots the following variables against time: p90–p50 (blue), p50–p10 (red), and p90–p10 (grey) for men in panel a, and for women in panel b. The shaded areas indicate recessionary periods with GDP growth below 2%.
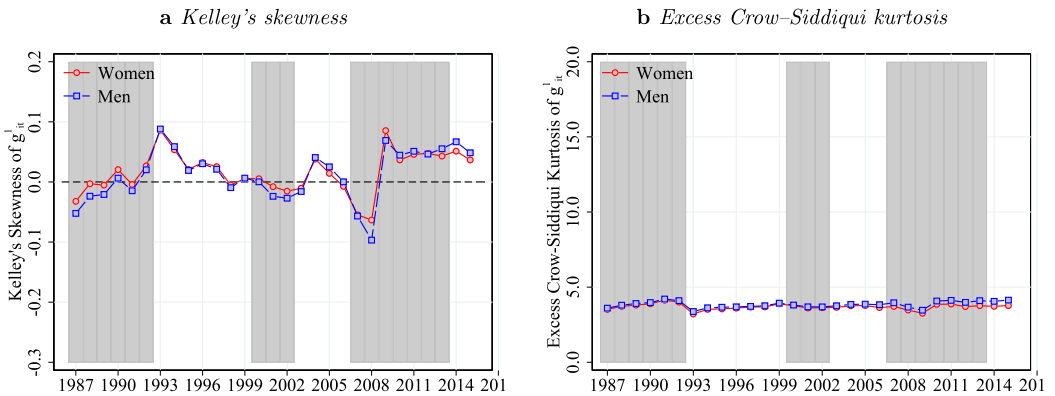
a *Kelley's skewness*   **b** *Excess Crow–Siddiqui kurtosis*

FIGURE 44. Skewness and kurtosis of the distribution of 1-year residual log disposable income growth rates, households. *Notes*: Skewness and kurtosis of distribution of 1-year residual household log disposable growth rates, $g_{it}^1 = \varepsilon_{it+1} - \varepsilon_{it}$. LS sample. The figure plots the following variables against time: (a) Men and Women: Kelley's skewness calculated as $\frac{(\text{p90–p50}) - (\text{p50–p10})}{\text{p90–p10}}$, (b) Men and Women: Excess Crow–Siddiqui kurtosis calculated as $\frac{\text{p97.5–p2.5}}{\text{p75–p25}} - 2.91$ where the first term is the Crow–Siddiqui measure of kurtosis and 2.91, corresponds to the value of this measure for a normal distribution. The shaded areas indicate recessionary periods with GDP growth below 2%.
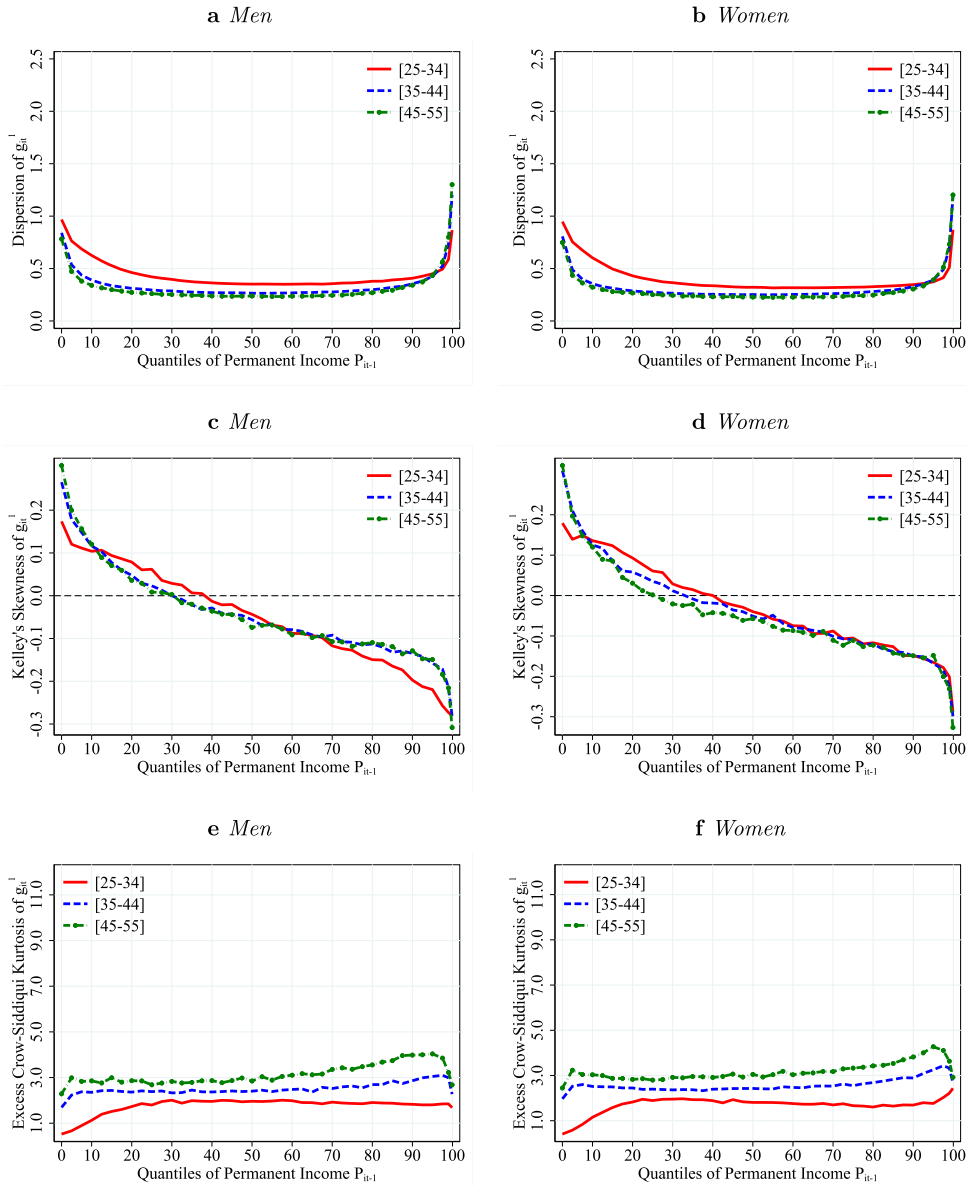
FIGURE 45. Dispersion, skewness and kurtosis of the distribution of 1-year residual log disposable income growth rates, households. *Notes*: Dispersion, skewness, and kurtosis of distribution of 1-year residual household log disposable growth rates, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. H sample in the period 1997–2016. Permanent Income is based on three years of income, $t-4$, $t-3$, $t-2$. The figure plots the following variables against permanent income quantile groups for the 3 age groups (denoted by color): (a) Men: p90–p10, (b) Women: p90–p10, (c) Men: Kelley's skewness, (d) Women: Kelley's skewness, (e) Men: Excess Crow–Siddiqui kurtosis, (f) Women: Excess Crow–Siddiqui kurtosis. Kelley's skewness is calculated as $\frac{(\text{p90–p50})-(\text{p50–p10})}{\text{p90–p10}}$. Excess Crow–Siddiqui kurtosis is calculated as $\frac{\text{p97.5–p2.5}}{\text{p75–p25}} - 2.91$, where the first term is the Crow–Siddiqui measure of kurtosis, and 2.91 corresponds to the value of this measure for a normal distribution.

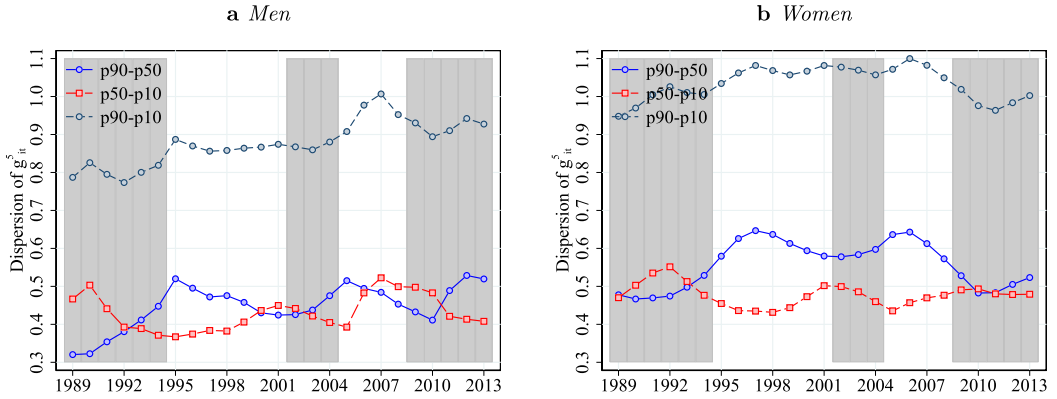APPENDIX OA-7: RESULTS BASED ON EARNINGS, 5-YEAR GROWTH RATES

a *Men*   **b** *Women*

FIGURE 46. Dispersion of the distribution of 5-year residual log earnings growth rates. *Notes*: Dispersion of distribution of 5-year residual log earnings growth rates, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. LS sample. The figure plots the following variables against time: p90–p50 (blue), p50–p10 (red), and p90–p10 (grey) for men in panel a, and for women in panel b.
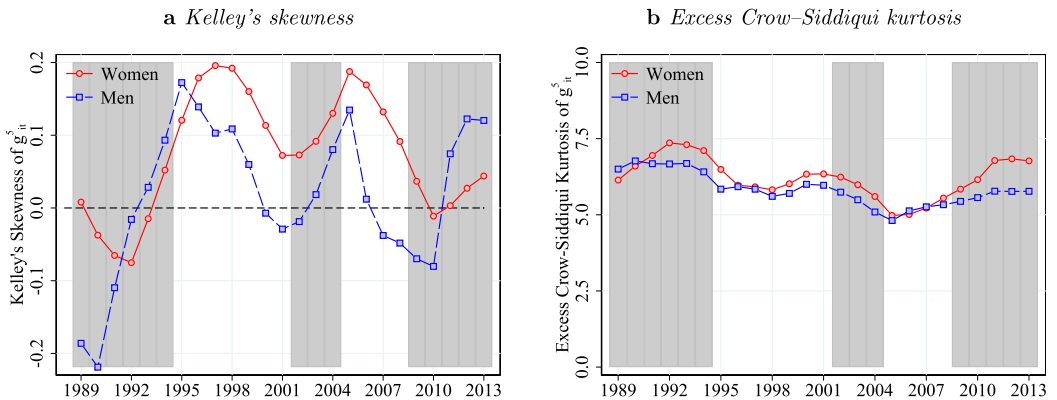
a *Kelley's skewness*   **b** *Excess Crow–Siddiqui kurtosis*

FIGURE 47. Skewness and kurtosis of the distribution of 5-year residual log earnings growth rates. *Notes*: Skewness and kurtosis of distribution of 5-year residual log earnings growth rates, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. LS sample. The figure plots the following variables against time: (a) Men and Women: Kelley's skewness calculated as $\frac{(p90–p50)-(p50–p10)}{p90–p10}$, (b) Men and Women: Excess Crow–Siddiqui kurtosis calculated as $\frac{p97.5–p2.5}{p75–p25} - 2.91$ where the first term is the Crow–Siddiqui measure of kurtosis and 2.91, corresponds to the value of this measure for a normal distribution.
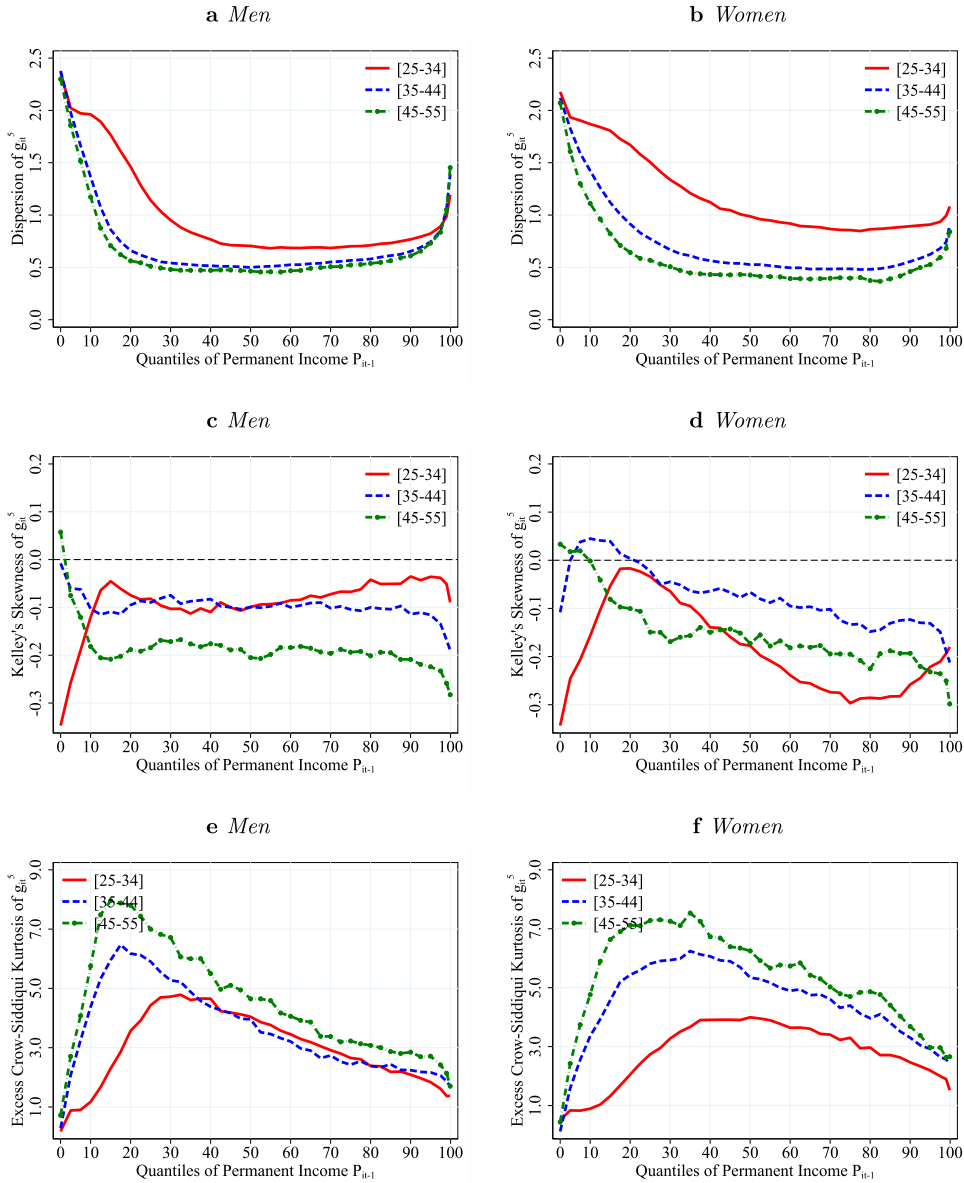
FIGURE 48. Dispersion, skewness and kurtosis of the distribution of 5-year residual log earnings growth rates. *Notes*: Dispersion, skewness, and kurtosis of distribution of 5-year residual log earnings growth rates, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. H sample in the period 1997–2016. Permanent Income is based on three years of income, $t-4, t-3, t-2$. The figure plots the following variables against permanent income quantile groups for the 3 age groups (denoted by color): (a) Men: p90–p10, (b) Women: p90–p10, (c) Men: Kelley's skewness, (d) Women: Kelley's skewness, (e) Men: Excess Crow–Siddiqui kurtosis, (f) Women: Excess Crow–Siddiqui kurtosis. Kelley's skewness is calculated as $\frac{(p90-p50)-(p50-p10)}{p90-p10}$. Excess Crow–Siddiqui kurtosis is calculated as $\frac{p97.5-p2.5}{p75-p25} - 2.91$, where the first term is the Crow–Siddiqui measure of kurtosis, and 2.91 corresponds to the value of this measure for a normal distribution.

APPENDIX OA-8: RESULTS BASED ON DISPOSABLE INCOME, 5-YEAR GROWTH RATES
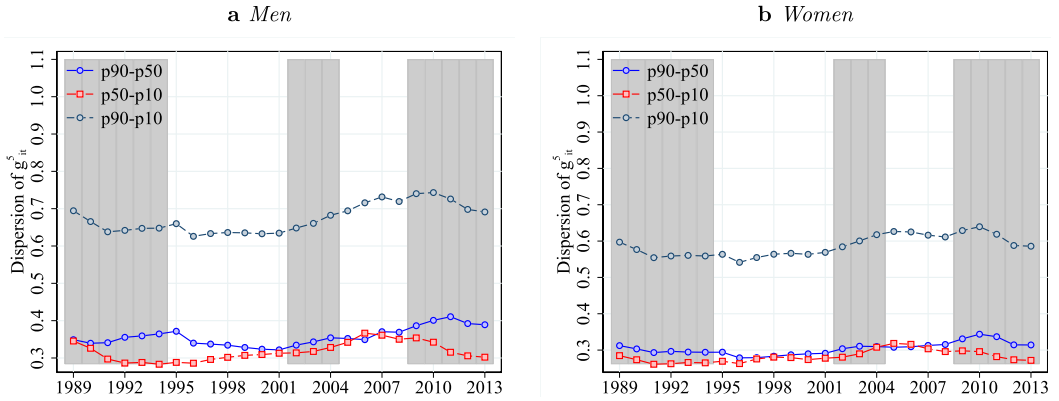

FIGURE 49. Dispersion of the distribution of 5-year residual log disposable income growth rates. *Notes*: Dispersion of distribution of 5-year residual log disposable income growth rates, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. LS sample. The figure plots the following variables against time: p90–p50 (blue), p50–p10 (red), and p90–p10 (grey) for men in panel a, and for women in panel b.
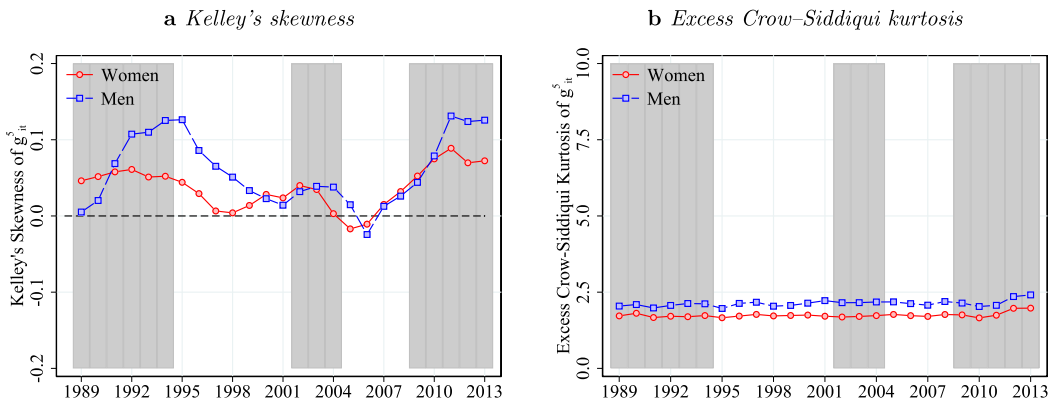

FIGURE 50. Skewness and kurtosis of the distribution of 5-year residual log disposable income growth rates. *Notes*: Skewness and kurtosis of distribution of 5-year residual log disposable income growth rates, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. LS sample. The figure plots the following variables against time: (a) Men and Women: Kelley's skewness calculated as $\frac{(p90–p50)−(p50–p10)}{p90–p10}$, (b) Men and Women: Excess Crow–Siddiqui kurtosis calculated as $\frac{p97.5–p2.5}{p75–p25} − 2.91$ where the first term is the Crow–Siddiqui measure of kurtosis and 2.91, corresponds to the value of this measure for a normal distribution.
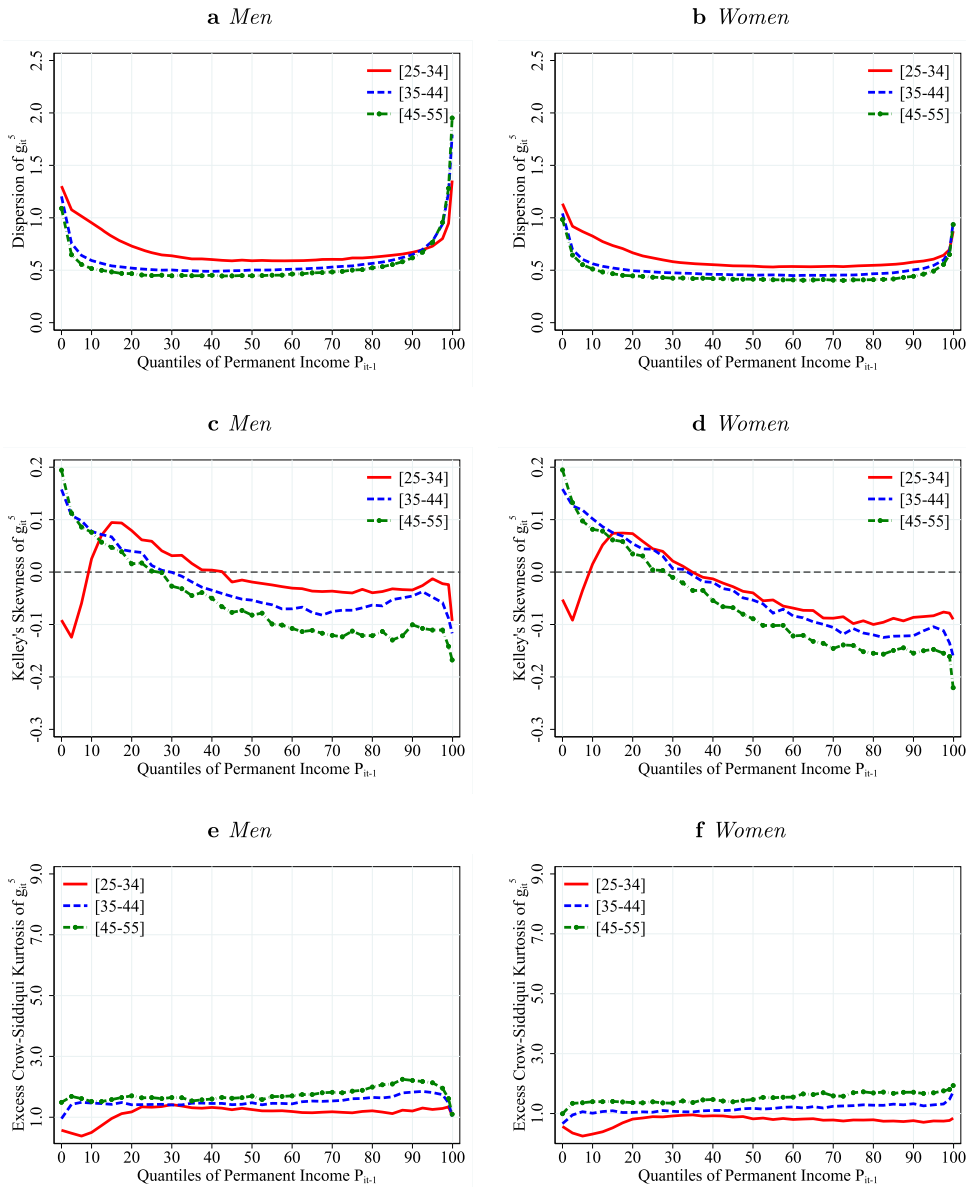
FIGURE 51. Dispersion, skewness and kurtosis of the distribution of 5-year residual log disposable income growth rates. *Notes*: Dispersion, skewness, and kurtosis of distribution of 5-year residual log disposable income growth rates, $g_{it}^5 = \varepsilon_{it+3} - \varepsilon_{it-2}$. H sample in the period 1997–2016. Permanent Income is based on three years of income, $t-4$, $t-3$, $t-2$. The figure plots the following variables against permanent income quantile groups for the 3 age groups (denoted by color): (a) Men: p90–p10, (b) Women: p90–p10, (c) Men: Kelley's skewness, (d) Women: Kelley's skewness, (e) Men: Excess Crow–Siddiqui kurtosis, (f) Women: Excess Crow–Siddiqui kurtosis. Kelley's skewness is calculated as $\frac{(p90-p50)-(p50-p10)}{p90-p10}$. Excess Crow–Siddiqui kurtosis is calculated as $\frac{p97.5-p2.5}{p75-p25} - 2.91$, where the first term is the Crow–Siddiqui measure of kurtosis, and 2.91 corresponds to the value of this measure for a normal distribution.